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Study of CP violations in $B^- \to K^-\pi^+\pi^-$ and $B^- \to K^- f_0(500)$ decays in the QCD factorization approach

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Abstract Within the QCD factorization approach, we study the CP violations in $B^- \to K^-\pi^+\pi^-$ and $B^- \to K^- f_0(500)$ decays. We find the experimental data of the localized CP asymmetry in $B^- \to K^-\pi^+\pi^-$ decays in the region $m^2_{K^-\pi^+} < 15$ GeV^2 and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2 can be explained by the interference of two intermediate resonances, $\rho^0(770)$ and $f_0(500)$ when the parameters in our interference model are in the allowed ranges, i.e. the relative strong phase $\delta \in [2.124, 5.976]$ and the end-point divergence parameters $\rho_S \in [5.692, 8]$ and $\phi_S \in [0, 2\pi]$. With the obtained allowed ranges for ρ_S and ϕ_S, we obtain the predictions for the CP asymmetry parameter $A_{CP} \in [-0.115, -0.151]$ and the branching fraction $\mathcal{B} \in [3.763, 20.014] \times 10^{-5}$ for $B^- \to K^- f_0(500)$ decay modes.

1 Introduction

Charge-Parity (CP) violation is essential to our understanding of both particle physics and the evolution of the early universe. It is one of the most fundamental and important properties of weak interaction, and has gained extensive attentions ever since its first discovery in 1964 [1]. In the Standard Model (SM), CP violation is related to the weak complex phase in the Cabibbo–Kobayashi–Maskawa (CKM) matrix, which describes the mixing of different generations of quarks [2,3]. Besides the weak phase, a large strong phase is also needed for a large CP asymmetry. Generally, this strong phase is provided by QCD loop corrections and some phenomenological models.

In recent years, prompted by a large number of experimental measurements, three-body hadronic B meson decays have been studied by using different theoretical frameworks [4–8]. Strong dynamics contained in three-body hadronic B meson decays is much more complicated than that in two-body cases, e.g. how to factorize B to three-body final states matrix elements. Both $BABAR$ [9] and Belle [10] Collaborations claimed evidence of partial rate asymmetries in the channels $B^\pm \to \rho^0(770)K^\pm$ in the Dalitz plot analysis of $B^- \to K^-\pi^+\pi^-$. LHCb also observed the large CP asymmetry in the localized region of the phase space [11,12], $A_{CP}(K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$, for $m^2_{K^-\pi^+} < 15$ GeV^2 and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2, which spans the $\pi^+\pi^-$ masses around the $\rho^0(770)$ resonance. Such three-body decays in this region have been studied in Refs. [13,14] using a simple model based on the framework of the factorization approach. The authors of Refs [15,16] considered the possibility of obtaining a large local CP violation in $B^- \to \pi^+\pi^-\pi^-$ decay from the interference of the resonances $\rho^0(770)$ and $f_0(500)$. In principle, in region of $m^2_{K^-\pi^+} < 15$ GeV^2 and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2 for $B^- \to K^-\pi^+\pi^-$ decay should cover both $\pi\pi$ and $K\pi$ resonances, however, because of the narrow width of K^*, there will be weaker interference between $K\pi$ and $\pi\pi$ systems compared with the interference between $\pi\pi$ systems. So in our work, we only consider the $\pi\pi$ system resonances, such as the $\rho^0(770)$ and $f_0(500)$ resonances in the above region.

In contrast to vector and tensor mesons, the identification of scalar mesons is a long-standing puzzle, because some of them have large decay widths which cause strong overlaps between resonances and backgrounds in experiments [17]. Up to now, there have been some progresses in the study of charmless hadronic B decays with scalar mesons in the final states both experimentally and theoretically. On the experi-

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mental side, measurements of B decays to the scalar mesons such as $f_0(980)$, $f_0(1370)$, $f_0(1500)$, $f_0(1710)$, $a_0(980)$, $a_0(1450)$, and $K_0^*(1430)$ have been reported by $BABAR$ and Belle Collaborations, but the decays to $f_0(500)$ have not been reported and the CP violation and the branching fractions have not been measured for such processes. So it is important to predict the values of $A_{CP}(B^- \to K^- f_0(500))$ and $\mathcal{B}(B^- \to K^- f_0(500))$. In spite of the striking success of QCD theory, the underlying structure of the light scalar mesons is still under controversy, scalar mesons have been identified as ordinary a $\bar{q}q$ states, four-quark states, or meson-meson bound states, or even those supplemented with a scalar glueball, in our work we will follow the $\bar{q}q$ quark model and consider the mixing between $f_0(500)$ and $f_0(980)$ [18]. However, this does not necessarily imply that other models for $f_0(500)$ are ruled out.

Theoretically, to calculate the hadronic matrix elements of B nonleptonic weak decays, some approaches, including the naive factorization [19,20], the QCD factorization (QCDF) [21–23], the perturbative QCD (pQCD) approach [24–26], and the soft-collinear effective theory (SCET) [27,28], have been fully developed and extensively employed in recent years. Unfortunately, in the collinear factorization approximation, the calculation of annihilation corrections always suffers from end-point divergence. In the pQCD approach, such divergence is regulated by introducing the parton transverse momentum k_T and the Sudakov factor at the expense of modeling the additional k_T dependence of meson wave functions, and large complex annihilation corrections are presented. In the SCET approach, such divergence is removed by separating the physics at different momentum scales and using zero-bin subtraction to avoid double counting the soft degrees of freedom. Within the QCDF framework, to estimate the annihilation amplitudes and regulate the endpoint divergency, the logarithmically divergent integral is usually parameterized in a model-independent manner and explicitly expressed as $\int_0^1 \frac{dx}{x} \to X_A$ which will be explained specifically in Sect. 2. In this work, within the framework of QCDF [29,30], we will study the decays of $B^- \to K^-\pi^+\pi^-$ via the interference of $\rho^0(770)$ and $f_0(500)$ and $B^- \to K^- f_0(500)$.

The remainder of this paper is organized as follows. In Sect. 2, we briefly present the formalism of the QCD factorization approach. In Sect. 3, we present the formalisms for CP violation of $B^- \to K^- f_0(500)$ and $B^- \to K^-\pi^+\pi^-$. The numerical results are given in Sect. 4 and we summarize our work in Sect 5.

2 QCD factorization

With the operator product expansion, the effective weak Hamiltonian for B meson decays can be written as [31]

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_{D=d,s} \lambda_p^{(D)} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + h.c., \tag{1}$$

where G_F represents the Fermi constant, $\lambda_p^{(D)} = V_{pb} V_{pD}^*$, V_{pb} and V_{pD} are the CKM matrix elements, C_i ($i = 1, 2, \ldots, 10$) are the Wilson coefficients, $Q_{1,2}^p$ are the tree level operators and Q_{3-10} are the penguin ones, and $Q_{7\gamma}$ and Q_{8g} are the electromagnetic and chromomagnetic dipole operators, respectively. The explicit forms of the operators Q_i are [32]

$$\begin{aligned}
Q_1^p &= \bar{p}\gamma_\mu(1-\gamma_5)b\bar{D}\gamma^\mu(1-\gamma_5)p, \\
Q_2^p &= \bar{p}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\bar{D}_\beta\gamma^\mu(1-\gamma_5)p_\alpha, \\
Q_3 &= \bar{D}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\
Q_4 &= \bar{D}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\
Q_5 &= \bar{D}\gamma_\mu(1-\gamma_5)b\sum_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\
Q_6 &= \bar{D}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\
Q_7 &= \frac{3}{2}\bar{D}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1+\gamma_5)q', \\
Q_8 &= \frac{3}{2}\bar{D}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1+\gamma_5)q'_\alpha, \\
Q_9 &= \frac{3}{2}\bar{D}\gamma_\mu(1-\gamma_5)b\sum_{q'}e_{q'}\bar{q}'\gamma^\mu(1-\gamma_5)q', \\
Q_{10} &= \frac{3}{2}\bar{D}_\alpha\gamma_\mu(1-\gamma_5)b_\beta\sum_{q'}e_{q'}\bar{q}'_\beta\gamma^\mu(1-\gamma_5)q'_\alpha, \\
Q_{7\gamma} &= \frac{-e}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)F^{\mu\nu}b, \\
Q_{8g} &= \frac{-g_s}{8\pi^2}m_b\bar{s}\sigma_{\mu\nu}(1+\gamma_5)G^{\mu\nu}b,
\end{aligned} \tag{2}$$

where α and β are color indices, $q' = u, d, s, c$ or b quarks.

In dealing with the charmless B decay into two mesons M_1 and M_2, the decay amplitude is usually divided into the emission part and the annihilation part in terms of the structures of the topological diagrams. In the heavy quark limit, the former part can be written as the product of the decay constant and the form factor, while for the latter part, it is always regarded as being power suppressed. With the standard procedure of the QCDF, the emission part of the decay amplitude has the following form:

$$\mathcal{M}(B^- \to M_1 M_2) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \sum_i V_{pb} V_{pd}^* \alpha_i^p(\mu) \times \langle M_1 M_2 | Q_i | B \rangle, \quad (3)$$

where $\alpha_i^p(\mu)$ are flavour parameters which can be expressed in terms of the effective parameters a_i^p, which can be calculated perturbatively, with the expressions given by [31]

$$a_i^p(M_1 M_2) = \left(C_i' + \frac{C_{i\pm 1}'}{N_c} \right) N_i(M_2) + \frac{C_{i\pm 1}'}{N_c} \frac{C_F \alpha_s}{4\pi} \left[V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1 M_2) \right] + P_i^p(M_2), \quad (4)$$

where C_i' are effective Wilson coefficients which are defined as $C_i(m_b)\langle Q_i(m_b)\rangle = C_i'\langle Q_i\rangle^{tree}$ with $\langle Q_i\rangle^{tree}$ being the matrix element at the tree level, the upper (lower) signs apply when i is odd (even), $N_i(M_2)$ are leading-order coefficients, $C_F = (N_c^2 - 1)/2N_c$ with $N_c = 3$, the quantities $V_i(M_2)$ account for one-loop vertex corrections, $H_i(M_1 M_2)$ describe hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the B meson, and $P_i^p(M_1 M_2)$ are from penguin contractions [31]. Similarly, weak annihilation contributions are described by the terms b_i and $b_{i,EW}$, which have the following expressions:

$$\begin{aligned}
b_1 &= \frac{C_F}{N_c^2} C_1' A_1^i, \quad b_2 = \frac{C_F}{N_c^2} C_2' A_1^i, \\
b_3^p &= \frac{C_F}{N_c^2} \left[C_3' A_1^i + C_5'(A_3^i + A_3^f) + N_c C_6' A_3^f \right], \\
b_4^p &= \frac{C_F}{N_c^2} \left[C_4' A_1^i + C_6' A_2^i \right], \\
b_{3,EW}^p &= \frac{C_F}{N_c^2} \left[C_9' A_1^i + C_7'(A_3^i + A_3^f) + N_c C_8' A_3^f \right], \\
b_{4,EW}^p &= \frac{C_F}{N_c^2} \left[C_{10}' A_1^i + C_8' A_2^i \right],
\end{aligned} \quad (5)$$

where the subscripts 1, 2, 3 of $A_n^{i,f}$ $(n = 1, 2, 3)$ stand for the annihilation amplitudes induced from $(V-A)(V-A)$, $(V-A)(V+A)$, and $(S-P)(S+P)$ operators, respectively, and the superscripts i and f refer to gluon emission from the initial- and final-state quarks, respectively. The explicit expressions for $A_n^{i,f}$ can be found in Ref. [31].

When dealing with the weak annihilation contributions and the hard spectator contributions, one suffers from the infrared endpoint singularity $X = \int_0^1 dx/(1-x)$. The treatment of the endpoint divergence is model dependent, and we follow Ref. [22] to parameterize the endpoint divergence in the annihilation and hard spectator diagrams as

$$X_{A,H}^{B\to O_1 O_2} = \left(1 + \rho_{A,H}^{O_1 O_2} e^{i\phi_{A,H}^{O_1 O_2}} \right) \ln \frac{m_B}{\Lambda_h}, \quad (6)$$

where O_1 and O_2 denote species of mesons in the final state including scalar (S), vector (V) and pseudoscalar (P) mesons, Λ_h is a typical scale of order 500 MeV, $\rho_{A,H}^{O_1 O_2}$ is an unknown real parameter, $\phi_{A,H}^{O_1 O_2}$ is the free strong phase in the range $[0, 2\pi]$. In our work, we will use:

$$X^{O_1 O_2} = X_{A,H}^{B\to O_1 O_2} = \left(1 + \rho^{O_1 O_2} e^{i\phi^{O_1 O_2}} \right) \ln \frac{m_B}{\Lambda_h}. \quad (7)$$

In fact, the QCDF approach itself cannot give information or constraints on the phenomenological parameters $\rho^{O_1 O_2}$ and $\phi^{O_1 O_2}$, both of them should be fixed by experimental data such as branching fractions and CP asymmetries.

3 Calculation of CP violation

3.1 CP violation formalism for $B^- \to K^-\pi^+\pi^-$

In this section, we will consider a B meson three-body decay process, $B \to M_1 M_2 M_3$, where M_i $(i = 1, 2, 3)$ are light mesons. There are two resonaces, X and Y, appearing during this process: $B \to X(Y) M_3$, then both X and Y decay to $M_1 M_2$. The amplitude for $B \to X(Y) M_3 \to M_1 M_2 M_3$ around the X and Y resonance region can be expressed as [15]

$$\mathcal{M} = \mathcal{M}_X + \mathcal{M}_Y e^{i\delta}, \quad (8)$$

where δ is a relative strong phase, $\mathcal{M}_X$ and $\mathcal{M}_Y$ are the amplitudes for $B \to X M_3 \to M_1 M_2 M_3$ and $B \to Y M_3 \to M_1 M_2 M_3$, respectively.

For the specific process $B^- \to K^-\pi^+\pi^-$ in the region $m_{K^\pm\pi^\pm}^2 < 15$ GeV^2 and $0.08 < m_{\pi^+\pi^-}^2 < 0.66$ GeV^2, $\rho^0(770)$ and $f_0(500)$ are the dominate resonances. We will adopt the naive Breit-Wigner form for $\rho^0(770)$ with the pole mass $m_{\rho^0(770)} = 0.775$GeV and the width $\Gamma_{\rho^0(770)} = 0.149$GeV [17], but for the $f_0(500)$ meson we will use the Bugg model [33–35] with the pole mass $m_{f_0(500)} = 0.5$GeV and the width $\Gamma_{f_0(500)} = 0.5$GeV, $\mathcal{M}_{\rho^0(770)}$ and $\mathcal{M}_{f_0(500)}$ take the following form:

$$\mathcal{M}_{\rho^0(770)} = \frac{\langle \rho^0(770) K^- | \mathcal{H}_{eff} | B^- \rangle \langle \pi^+\pi^- | \mathcal{H}_{\rho^0(770)\pi^+\pi^-} | \rho^0(770) \rangle}{S_{\rho^0(770)}}, \quad (9)$$

$$\mathcal{M}_{f_0(500)} = \langle f_0(500) K^- | \mathcal{H}_{eff} | B^- \rangle \langle \pi^+\pi^- | \mathcal{H}_{f_0\pi\pi} | f_0(500) \rangle R_{f_0(500)}(s). \quad (10)$$

In Eq. (9), $S_{\rho^0(770)}$ is the reciprocal of the propagator of $\rho^0(770)$ which takes the form $s - m_{\rho^0(770)}^2 + i m_{\rho^0(770)} \Gamma_{\rho^0(770)}$, where s is the invariant mass squared of mesons π^+ and π^- and $\Gamma_{\rho^0} \approx \Gamma_{\rho^0 \to \pi^+\pi^-}$.

In Eq. (10), the parameterization of $f_0(500)$ in the Bugg model [33–35] is given by

$$R_{f_0(500)}(s) = M\Gamma_1(s)/[M^2 - s - g_1^2(s)\frac{s-s_A}{M^2-s_A}z(s) - iM\Gamma_{tot}(s)], \tag{11}$$

where $z(s) = j_1(s) - j_1(M^2)$ with $j_1(s) = \frac{1}{\pi}[2 + \rho_1 \ln(\frac{1-\rho_1}{1+\rho_1})]$, $\Gamma_{tot}(s) = \sum_{i=1}^{4}\Gamma_i(s)$ and other relevant functions are giving in the following:

$$\begin{aligned}
M\Gamma_1(s) &= g_1^2(s)\frac{s-s_A}{M^2-s_A}\rho_1(s),\\
M\Gamma_2(s) &= 0.6g_1^2(s)(s/M^2)\exp(-\alpha|s-4m_K^2|)\rho_2(s),\\
M\Gamma_3(s) &= 0.2g_1^2(s)(s/M^2)\exp(-\alpha|s-4m_\eta^2|)\rho_3(s),\\
M\Gamma_4(s) &= Mg_{4\pi}\rho_{4\pi}(s)/\rho_{4\pi}(M^2),\\
g_1^2(s) &= M(b_1+b_2s)\exp[-(s-M^2)/A],\\
\rho_{4\pi}(s) &= 1.0/[1+\exp(7.082-2.845s)].
\end{aligned} \tag{12}$$

For the parameters in Eqs. (11,12), they are fixed to $M = 0.953\text{GeV}$, $s_A = 0.14m_\pi^2$, $b_1 = 1.302\,\text{GeV}^2$, $b^2 = 0.340$, $A = 2.426\,\text{GeV}^2$ and $g_{4\pi} = 0.011$ GeV which are given in the fourth column of Table I in Ref. [33]. The parameters $\rho_{1,2,3}$ are the phase-space factors of the decay channels $\pi\pi$, KK and $\eta\eta$, respectively, and are defined as [33]

$$\rho_i(s) = \sqrt{1 - 4\frac{m_i^2}{s}}, \tag{13}$$

with $m_1 = m_\pi$, $m_2 = m_K$ and $m_3 = m_\eta$.

The effective Hamiltonians for strong processes $\rho^0(770) \to \pi^+\pi^-$ and $f_0(500) \to \pi^+\pi^-$ in Eqs. (9,10) can be formally expressed as

$$\begin{aligned}
\mathcal{H}_{\rho^0\pi\pi} &= -ig_{\rho^0\pi\pi}\rho^0_\mu\pi^+\overleftrightarrow{\partial}^\mu\pi^-,\\
\mathcal{H}_{f_0\pi\pi} &= g_{f_0\pi\pi}f_0(2\pi^+\pi^- + \pi^0\pi^0),
\end{aligned} \tag{14}$$

where ρ^0_μ, f_0 and $\pi^\pm$ are the field operators for $\rho^0(770)$, $f_0(500)$ and π mesons, respectively, $g_{\rho^0\pi\pi}$ and $g_{f_0\pi\pi}$ are the effective coupling constants which can be expressed in terms of the decay widths of $\rho^0 \to \pi^+\pi^-$ and $f_0 \to \pi^+\pi^-$, respectively,

$$\begin{aligned}
g^2_{\rho^0\pi\pi} &= \frac{48\pi}{\left(1-\frac{4m_\pi^2}{m_\rho^2}\right)^{3/2}} \times \frac{\Gamma_{\rho^0\to\pi^+\pi^-}}{m_\rho},\\
g^2_{f_0\pi\pi} &= \frac{4\pi m_{f_0}\Gamma_{f_0\to\pi^+\pi^-}}{\left(1-\frac{4m_\pi^2}{m_{f_0}^2}\right)^{1/2}}.
\end{aligned} \tag{15}$$

Both $\rho^0(770)$ and $f_0(500)$ decay into one pion pair dominantly through the strong interaction, and the isospin symmetry of the strong interaction tells us that $\Gamma_{\rho^0} \approx \Gamma_{\rho^0\to\pi^+\pi^-}$ and $\Gamma_{f_0} \approx \frac{3}{2}\Gamma_{f_0\to\pi^+\pi^-}$.

The differential CP asymmetry parameter can be defined as

$$A_{CP} = \frac{|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2}. \tag{16}$$

By integrating the denominator and numerator of A_{CP} in the region R, we get the localized integrated CP asymmetry, which can be measured in experiments and takes the following form:

$$A^R_{CP} = \frac{\int_R ds_{12}ds_{13}(|\mathcal{M}|^2 - |\bar{\mathcal{M}}|^2)}{\int_R ds_{12}ds_{13}(|\mathcal{M}|^2 + |\bar{\mathcal{M}}|^2)}, \tag{17}$$

where R represents certain region of the phase space, in our work R includes $m^2_{K^-\pi^+} < 15\,\text{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66\,\text{GeV}^2$ in $B^- \to K^-\pi^+\pi^-$ decay.

3.2 Calculation of amplitudes of $B^- \to \rho^0(770)(f_0(500))K^- \to \pi^+\pi^-K^-$

In the QCDF, including the emission and annihilation contributions, the decay amplitudes of $B^- \to \rho^0(770)K^-$ and $B^- \to f_0(500)K^-$ can be finally given as

$$\begin{aligned}
&\mathcal{M}(B^- \to \rho^0(770)K^-)\\
&= \langle\rho^0(770)K^-|\mathcal{H}_{eff}|B^-\rangle\\
&= \sum_{p=u,c}\lambda_p^{(s)}\frac{-iG_F}{2}\Big\{\Big[\alpha_1(\rho K)\delta_{pu} + \alpha_4^p(\rho K) + \alpha^p_{4,EW}(\rho K)\Big]\\
&\times m_B^2A_0^{B\to\rho}(0)f_K + \Big[\alpha_2(K\rho)\delta_{pu} + \frac{3}{2}\alpha^p_{3,EW}(K\rho)\Big]\\
&\times m_B^2F_+^{B\to f}(0)f_\rho\\
&+ \Big[b_2(\rho K)\delta_{pu} + b_3^p(\rho K) + b^p_{3,EW}(\rho K)\Big] \times f_Bf_\rho f_K\Big\},
\end{aligned} \tag{18}$$

for $B^- \to \rho^0(770)K^-$, and

$$\begin{aligned}
&\mathcal{M}(B^- \to f_0(500)K^-)\\
&= \langle f_0(500)K^-|\mathcal{H}_{eff}|B^-\rangle\\
&= \sum_{p=u,c}\lambda_p^{(s)}\frac{G_F}{2}\Big\{\Big[\alpha_1(fK)\delta_{pu} + \alpha_4^p(fK) + \alpha^p_{4,EW}(fK)\Big]\\
&\times (m_f^2 - m_B^2)F_0^{B\to f}(m_K^2)f_K\\
&+ \Big[\alpha_2(Kf)\delta_{pu} + 2\alpha_3(Kf) + \frac{1}{2}\alpha^p_{3,EW}(Kf))\Big]\\
&\times (m_B^2 - m_K^2)F_0^{B\to K}(m_f^2)\bar{f}^u_{f_0(500)}\\
&+ \Big[\sqrt{2}\alpha_3^p(Kf) + \sqrt{2}\alpha_4^p(Kf) - \frac{1}{\sqrt{2}}\alpha^p_{3,EW}(Kf)\\
&- \frac{1}{\sqrt{2}}\alpha^p_{4,EW}(Kf)\Big]\\
&\times (m_B^2 - m_K^2)F_0^{B\to K}(m_f^2)\bar{f}^s_{f_0(500)} - \Big[b_2(fK)\delta_{pu}\\
&+ b_3^p(fK) + b^p_{3,EW}(fK)\Big]\\
&\times f_Bf_K\bar{f}^u_{f_0(500)} - \sqrt{2}\Big[b_2(Kf)\delta_{pu} + b_3^p(Kf) + b^p_{3,EW}(Kf)\Big]\\
&\times f_Bf_K\bar{f}^s_{f_0(500)}\Big\},
\end{aligned} \tag{19}$$

for $B^- \to f_0(500)K^-$, where ρ and f are the abbreviations for $\rho^0(770)$ and $f_0(500)$, respectively, $A_0^{B\to M_1}(0)$ and $F_{+,0}^{B\to M_2}(q^2)$ are form factors for B to M_1 and M_2 transitions, f_K, f_ρ and f_B are decay constants of K, ρ and B mesons, respectively, $\bar{f}^u_{f_0(500)}$ and $\bar{f}^s_{f_0(500)}$ are decay constants of $f_0(500)$ coming from the up and strange quark components, respectively.

From Eq. (14), we can obtain the amplitudes for $\rho^0(770) \to \pi^+\pi^-$ and $f_0(500) \to \pi\pi$ as

$$\begin{aligned}\mathcal{M}(\rho^0(770) \to \pi^+\pi^-) &= \langle \pi^+\pi^- | \mathcal{H}_{\rho^0\pi\pi} | \rho^0(770)\rangle \\ &= g_{\rho\pi^+\pi^-}\varepsilon_{\rho^0}\cdot(p_{\pi^-}-p_{\pi^+}), \qquad (20)\\ \mathcal{M}(f_0(500) \to \pi^+\pi^-) &= \langle \pi^+\pi^- | \mathcal{H}_{f_0\pi\pi} | f_0(500)\rangle = 2g_{f_0\pi^+\pi^-},\end{aligned}$$

where ε_{ρ^0} is the polarization vector of $\rho^0(770)$, p_{π^+} and p_{π^-} are the momenta of π^+ and π^-, respectively.

Considering the total processes, one can get

$$\begin{aligned}&\mathcal{M}(B^- \to \rho^0(770)K^- \to \pi^+\pi^-K^-)\\ &= \sum_{p=u,c}\lambda_p^{(s)}\frac{-iG_F g_{\rho\pi^+\pi^-}}{S_{\rho^0(770)}}(\hat{s}_{K\pi}-s_{K\pi})\\ &\times\Big\{A_0^{B\to\rho}(0)f_K\left[\alpha_1(\rho K)\delta_{pu}+\alpha_4^p(\rho K)+\alpha^p_{4,EW}(\rho K)\right] \qquad (21)\\ &+F_+^{B\to K}(0)f_\rho\left[\alpha_2(K\rho)\delta_{pu}+\frac{3}{2}\alpha^p_{3,EW}(K\rho)\right]\\ &+\frac{f_Bf_\rho f_K}{m_B^2}\left[b_2(\rho K)\delta_{pu}+b_3^p(\rho K)+b^p_{3,EW}(\rho K)\right]\Big\},\end{aligned}$$

for the $B^- \to \rho^0(770)K^- \to \pi^+\pi^-K^-$ decay mode, where $\hat{s}_{K\pi}$ is the midpoint of the allowed range of $s_{K^-\pi^+}$, i.e. $\hat{s}_{K\pi} = (s_{K^-\pi^+,\max}+s_{K^-\pi^+,\min})/2$, with $s_{K^-\pi^+,\max}$ and $s_{K^-\pi^+,\min}$ being the maximum and minimum values of $s_{K^-\pi^+}$ for fixed $s_{\pi^+\pi^-}$.

For the $B^- \to f_0(500)K^- \to \pi^+\pi^-K^-$ decay modes.

$$\begin{aligned}&\mathcal{M}(B^- \to f_0(500)K^- \to \pi^+\pi^-K^-)\\ &= \sum_{p=u,c}\lambda_p^{(s)}\frac{G_F g_{f\pi^+\pi^-}}{S_{f_0(500)}}\Big\{(m_f^2-m_B^2)F_0^{B\to f}(m_K^2)\\ &\times f_K\left[\delta_{pu}\alpha_1(fK)+\alpha_4^p(fK)+\alpha^p_{4,EW}(fK)\right]\\ &-f_Bf_K\bar{f}^u_{f_0(500)}\left[\delta_{pu}b_2(fK)+b_3^p(fK)+b^p_{3,EW}(fK)\right]\\ &+(m_B^2-m_K^2)F_0^{B\to K}(m_f^2)\bar{f}^u_{f_0(500)} \qquad (22)\\ &\times\left[\delta_{pu}\alpha_2(Kf)+2\alpha_3^p(Kf)+\frac{1}{2}\alpha^p_{3,EW}(Kf)\right]\\ &+(m_B^2-m_K^2)F_0^{B\to K}(m_f^2)\bar{f}^s_{f_0(500)}\\ &\times\left[\sqrt{2}\alpha_3^p(Kf)+\sqrt{2}\alpha_4^p(Kf)-\frac{1}{\sqrt{2}}\alpha^p_{3,EW}(Kf)-\frac{1}{\sqrt{2}}\alpha^p_{4,EW}(Kf)\right]\\ &-f_Bf_K\bar{f}^s_{f_0(500)}\left[\sqrt{2}\delta_{pu}b_2(Kf)+\sqrt{2}b_3^p(Kf)+\sqrt{2}b^p_{3,EW}(Kf)\right]\Big\}.\end{aligned}$$

The amplitude for $B^- \to K^-\pi^+\pi^-$ around the $f_0(500)$ and $\rho^0(770)$ resonance region can be expressed as

$$\begin{aligned}\mathcal{M} &= \mathcal{M}(B^- \to f_0(500)K^- \to \pi^+\pi^-K^-)\\ &+\mathcal{M}(B^- \to \rho^0(770)K^- \to \pi^+\pi^-K^-)e^{i\delta}\\ &= \sum_{p=u,c}\lambda_p^{(s)}\frac{G_F g_{f\pi^+\pi^-}}{S_{f_0(500)}}\Big\{(m_f^2-m_B^2)F_0^{B\to f}(0)f_K\big[\delta_{pu}\alpha_1(fK)\\ &+\alpha_4^p(fK)+\alpha^p_{4,EW}(fK)\big]\\ &-f_Bf_K\bar{f}^u_{f_0(500)}\left[\delta_{pu}b_2(fK)+b_3^p(fK)+b^p_{3,EW}(fK)\right]\\ &-f_Bf_K\bar{f}^s_{f_0(500)}\big[\sqrt{2}\delta_{pu}b_2(Kf)+\sqrt{2}b_3^p(Kf)\\ &+\sqrt{2}b^p_{3,EW}(Kf)\big]\\ &+(m_B^2-m_K^2)F_0^{B\to K}(0)\bar{f}^u_{f_0(500)}\big[\delta_{pu}\alpha_2(Kf)+2\alpha_3^p(Kf)\\ &+\frac{1}{2}\alpha^p_{3,EW}(Kf)\big] \qquad (23)\\ &+(m_B^2-m_K^2)F_0^{B\to K}(0)\bar{f}^s_{f_0(500)}\big[\sqrt{2}\alpha_3^p(Kf)+\sqrt{2}\alpha_4^p(Kf)\\ &-\frac{1}{\sqrt{2}}\alpha^p_{3,EW}(Kf)-\frac{1}{\sqrt{2}}\alpha^p_{4,EW}(Kf)\big]\Big\}\\ &+\sum_{p=u,c}\lambda_p^{(s)}\frac{-iG_F g_{\rho\pi^+\pi^-}}{S_{\rho^0(770)}}(\hat{s}_{K\pi}-s_{K\pi})\Big\{A_0^{B\to\rho}(0)f_K\\ &\times\left[\alpha_1(\rho K)\delta_{pu}+\alpha_4^p(\rho K)+\alpha^p_{4,EW}(\rho K)\right]\\ &+F_0^{B\to K}(0)f_\rho\left[\alpha_2(K\rho)\delta_{pu}+\frac{3}{2}\alpha^p_{3,EW}(K\rho)\right]\\ &+\frac{f_Bf_\rho f_K}{m_B^2}\left[b_2(\rho K)\delta_{pu}+b_3^p(\rho K)+b^p_{3,EW}(\rho K)\right]\Big\}e^{i\delta},\end{aligned}$$

where $\delta \in [0, 2\pi]$. Substituting Eq. (23) into Eq. (17) and taking the integral region R as $m^2_{K^-\pi^+} < 15$ GeV^2 and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2, we can get the expression of the localized $A_{CP}(B \to K^-\pi^+\pi^-)$, which is a function of $X(\rho_S, \phi_S)$ and δ.

3.3 Calculation of differential CP violation and branching fraction of $B^- \to K^-f_0(500)$

Using Eq. (16), the differential CP asymmetry parameter of $B \to M_1M_2$ can be expressed as

$$A_{CP}(B \to M_1M_2) = \frac{|\mathcal{M}(B \to M_1M_2)|^2-|\bar{\mathcal{M}}(B \to M_1M_2)|^2}{|\mathcal{M}(B \to M_1M_2)|^2+|\bar{\mathcal{M}}(B \to M_1M_2)|^2}. \qquad (24)$$

The branching fraction of $B \to M_1M_2$ decay has the following form:

$$\mathcal{B}(B \to M_1M_2) = \tau_B\frac{|p_c|}{8\pi m_B^2}|\mathcal{M}(B \to M_1M_2)|^2, \qquad (25)$$

where τ_B is the lifetime of B meson, m_B is the mass of B meson, $|p_c|$ is the norm of a hadron's three momentum in the final state which can be expressed as

$$|p_c| = \frac{1}{2m_B}\sqrt{[m_B^2 - (m_{M_1} + m_{M_2})^2][m_B^2 - (m_{M_1} - m_{M_2})^2]}, \tag{26}$$

where m_{M_1} and m_{M_2} are the two final state mesons' masses, respectively.

Substituting the amplitude of $B^- \to K^- f_0(500)$,

$$\begin{aligned}
&\mathcal{M}(B^- \to f_0(500)K^-)\\
&= \langle f_0(500)K^-|\mathcal{H}_{eff}|B^-\rangle\\
&= \sum_{p=u,c} \lambda_p^{(s)} \frac{G_F}{2}\Big\{\Big[\alpha_1(fK)\delta_{pu} + \alpha_4^p(fK) + \alpha_{4,EW}^p(fK)\Big]\\
&\times (m_f^2 - m_B^2)F_0^{B\to f}(m_K^2) f_K\\
&+ \Big[\alpha_2(Kf)\delta_{pu} + 2\alpha_3(Kf) + \frac{1}{2}\alpha_{3,EW}^p(Kf))\Big]\\
&\times (m_B^2 - m_K^2)F_0^{B\to K}(m_f^2)\bar{f}_{f_0(500)}^u\\
&+ \Big[\sqrt{2}\alpha_3^p(Kf) + \sqrt{2}\alpha_4^p(Kf) - \frac{1}{\sqrt{2}}\alpha_{3,EW}^p(Kf) - \frac{1}{\sqrt{2}}\alpha_{4,EW}^p(Kf)\Big]\\
&\times (m_B^2 - m_K^2)F_0^{B\to K}(m_f^2)\bar{f}_{f_0(500)}^s\\
&- \Big[b_2(fK)\delta_{pu} + b_3^p(fK) + b_{3,EW}^p(fK)\Big]\\
&\times f_B f_K \bar{f}_{f_0(500)}^u - \sqrt{2}\Big[b_2(Kf)\delta_{pu} + b_3^p(Kf) + b_{3,EW}^p(Kf)\Big]\\
&\times f_B f_K \bar{f}_{f_0(500)}^s\Big\},
\end{aligned}$$

into Eq. (24) we can get the expression of $A_{CP}(B^- \to K^- f_0(500))$. Substituting Eqs. (19) and (26) into Eq. (25), one can obtain the branching fraction of $B^- \to K^- f_0(500)$. Both of them are functions of $X(\rho_S, \phi_S)$.

4 Numerical results

The expressions for $A_{CP}(B^- \to K^-\pi^+\pi^-)$, $A_{CP}(B^- \to K^- f_0(500))$ and $\mathcal{B}(B^- \to K^- f_0(500))$ obtained in the QCD factorization approach depend on many input parameters including CKM matrix elements, effective Wilson coefficients, light-cone distribution amplitudes of mesons, form factors and decay constants. CKM matrix elements can be expressed in the terms of Wolfenstein parameters A, λ, ρ and η. In our work, we take values $A = 0.811^{+0.023}_{-0.024}$, $\lambda = 0.225 \pm 0.00061$, $\bar{\rho} = 0.117 \pm 0.021$, and $\bar{\eta} = 0.353 \pm 0.013$ with $\bar{\rho} = \rho(1 - \frac{\lambda^2}{2})$, $\bar{\eta} = \eta(1 - \frac{\lambda^2}{2})$ [17]. The effective Wilson coefficients used in our calculations are taken from Ref. [36]:

$$\begin{aligned}
&C_1' = -0.3125, \quad C_2' = -1.1502,\\
&C_3' = 2.120 \times 10^{-2} + 5.174 \times 10^{-3}i,\\
&C_4' = -4.869 \times 10^{-2} - 1.552 \times 10^{-2}i,\\
&C_5' = 1.420 \times 10^{-2} + 5.174 \times 10^{-3}i,\\
&C_6' = -5.792 \times 10^{-2} - 1.552 \times 10^{-2}i,\\
&C_7' = -8.340 \times 10^{-5} - 9.938 \times 10^{-5}i,\\
&C_8' = 3.839 \times 10^{-4},\\
&C_9' = -1.017 \times 10^{-2} - 9.938 \times 10^{-5}i,\\
&C_{10}' = 1.959 \times 10^{-3}.
\end{aligned} \tag{27}$$

The twist-2 light-cone distribution amplitudes (LCDA) for the pseudoscalar (P) and vector (V) mesons are

$$\Phi_{P,V}(x,\mu) = 6x(x-1)\left[1 + \sum_{m=1}^{\infty} \alpha_m^{P,V}(\mu) C_m^{3/2}(2x-1)\right], \tag{28}$$

and twist-3 ones are

$$\Phi_p(x) = 1, \quad \Phi_\sigma(x) = 6x(x-1), \tag{29}$$

$$\Phi_\upsilon(x) = 3\left[2x - 1 + \sum_{m=1}^{\infty} \alpha_{m,\perp}^V(\mu) P_{m+1}(2x-1)\right], \tag{30}$$

where $C_m^{3/2}$ and P_m are the Gegenbauer and Legendre polynomials, respectively, $\alpha_m^{P,V}(\mu)$ and $\alpha_{m,\perp}^V(\mu)$ are Gegenbauer moments which depend on the scale μ. The Gegenbauer moments of K and ρ are $\alpha_1^K = 0.06 \pm 0.03$, $\alpha_2^K = 0.25 \pm 0.15$, and $\alpha_1^\rho = 0$, $\alpha_2^\rho = 0.14 \pm 0.06$, $\alpha_{1,\perp}^\rho = 0$, and $\alpha_{2,\perp}^\rho = 0.15 \pm 0.07$ [37], respectively, at the scale $\mu = 1$ GeV.

In general, the twist-2 LCDA of a scalar meson, Φ_S, has the following form [18]:

$$\Phi_S(x,\mu) = \bar{f}_S 6x(x-1) \sum_{m=1}^{\infty} B_m(\mu) C_m^{3/2}(2x-1), \tag{31}$$

where $\bar{f}_S$ are the decay constants of the scalar meson S, B_m are Gegenbauer moments. Based on the QCD sum rule methods [38,39] and the effect of the width of $\Gamma_{f_0(500)}$, one can set $m_{f_0(500)} - \frac{\Gamma_{f_0(500)}}{2} \leqslant m_S \leqslant m_{f_0(500)} + \frac{\Gamma_{f_0(500)}}{2}$ roughly. Then we derive the decay constants $\bar{f}_{f_0(500)}^q$ $(q = u, s)$ with the $f_0(980)$-$f_0(500)$ mixing angle $\theta = 17^0$ [29]: $\bar{f}_{f_0(500)}^u = (0.4829 \pm 0.14)$ GeV and $\bar{f}_{f_0(500)}^s = (-0.21 \pm 0.10)$ GeV, $B_1^u = -0.42 \pm 0.074$, $B_3^u = -0.58 \pm 0.23$, $B_1^s = -0.35 \pm 0.061$, and $B_3^s = -0.43 \pm 0.18$ at the scale $\mu = 1$ GeV.

As for the twist-3 distribution amplitudes, we use [18]

$$\Phi_S^s(x) = \bar{f}_S, \quad \Phi_S^\sigma(x) = \bar{f}_S 6x(x-1). \tag{32}$$

For the form factors of mesons, we neglect corrections quadratic in the light meson masses and we adopt the values at $q^2 = 0$ in Ref. [31] (At this kinematic point, the form factors F_+ and F_0 coincide.), $A_0^{B\to\rho}(0) = 0.303 \pm 0.029$, $F_+^{B\to K}(0) = F_0^{B\to K}(0) = 0.35 \pm 0.04$ [37] and $F_0^{B\to f}(m_K^2) \approx 0.45 \pm 0.15$ [40]. The decay constants used in our calculations are $f_K = 0.156 \pm 0.7$GeV [17], $f_\rho = 0.216 \pm 0.003$ GeV, and $f_B = 0.21 \pm 0.02$GeV [37].

A general fit of ρ and ϕ to the $B \to VP$ and $B \to PV$ data indicates $X^{PV} \neq X^{VP}$, i.e. $\rho^{PV} \approx 0.87$, $\rho^{VP} \approx 1.07$, $\phi^{VP} \approx -30^0$ and $\phi^{PV} \approx -70^0$, we shall assign an error of ± 0.1 to $\rho^{PV(VP)}$ and $\pm 20^0$ to $\phi^{PV(VP)}$ for the estimation of theoretical uncertainties [37]. On the other hand, for $B \to PS$ and $B \to SP$ decays, there is little experimental data so the values of ρ_S and ϕ_S are not determined very well, to make an estimation about $A_{CP}(B^- \to K^- f_0(500))$ and $\mathcal{B}(B^- \to K^- f_0(500))$, we adopt $X^{PS} = X^{SP} = (1 + \rho_S e^{i\phi_S}) \ln \frac{m_B}{\Lambda_h}$.

With all the above considerations, we only have three free parameters, which are the relative strong phase δ, and the divergence parameters ρ_S and ϕ_S for $A_{CP}(B^- \to K^- \rho^0(770)(f_0(500)) \to K^-\pi^+\pi^-)$. By fitting the theoretical result to the experimental data $A_{CP}(B^- \to K^-\pi^+\pi^-) = 0.678 \pm 0.078 \pm 0.0323 \pm 0.007$ in the region $m^2_{K^-\pi^+} < 15$ GeV^2 and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2, in the range $\delta \in [0, 2\pi]$, $\phi_S \in [0, 2\pi]$, $\rho_S \in [0, 8]$ [41,42] and varying each of these three parameters by 0.01 each time, i.e. $\Delta\delta = 0.01$, $\Delta\rho_S = 0.01$ and $\Delta\phi_S = 0.01$, it is found that there exist ranges of parameters δ, ρ_S and ϕ_S which satisfy the above experimental data. The allowed ranges are $\delta \in [2.124, 5.976]$, $\rho_S \in [5.692, 8]$ and $\phi_S \in [0, 2\pi]$. Therefore, the interference of $\rho^0(770)$ and $f_0(500)$ can indeed induce the data for the localized CP asymmetry in the $B^- \to K^-\pi^+\pi^-$ decays. It is noted that the values of $\rho_S \in [5.692, 8]$ are relative larger compared with the previously conservative choice of $\rho \leq 1$ [31,32]. In fact, both the hard spectator-scattering and weak annihilation contributions involve end-point divergences. For example, the hard spectator-scattering diagram at the twist-3 order posses soft and collinear divergences arising from the soft spectator quark and the annihilation amplitude has endpoint divergences even at the twist-2 level. Note that, we use the $X_A^{O_1O_2} = X_H^{O_1O_2}$ for the $B \to SP(PS)$ decay when dealing with the end-point divergences. When we get the large values of ρ_S, it means that not only the weak annihilation contributions but also the hard spectator-scattering ones become large, so it is still meaningful to do power expansions in $1/m_b$ and QCDF would not break down. Besides, although the weak annihilation amplitudes are formally $1/m_b$ power-suppressed, they are generally nontrivial, especially for the flavor-changing neutral-current dominated and pure annihilation decays. Furthermore, because of the possible strong phase provided by the weak annihilation amplitudes, the weak annihilation contributions also play an indispensable role for evaluating the charge-parity asymmetry. So far, the values of ρ and ϕ are utterly unknown from the first principles of QCD dynamics. Originally, a conservative choice of $\rho \leq 1$ with an arbitrary strong interaction phase ϕ was used. However, QCDF itself cannot give information and constraint on parameters ρ and it can only be obtained through the experimental data, hence there is no reason to restrict ρ to the range $\rho \leq 1$ [37,43–45], thus larger values of ρ_S might be possible when we deal with the divergence problems for $B \to SP(PS)$ decays. In this region of ρ_S, one can see that both the weak annihilation and the hard spectator scattering processes can make large contributions to $B^- \to K^- f_0(500)$ decays.

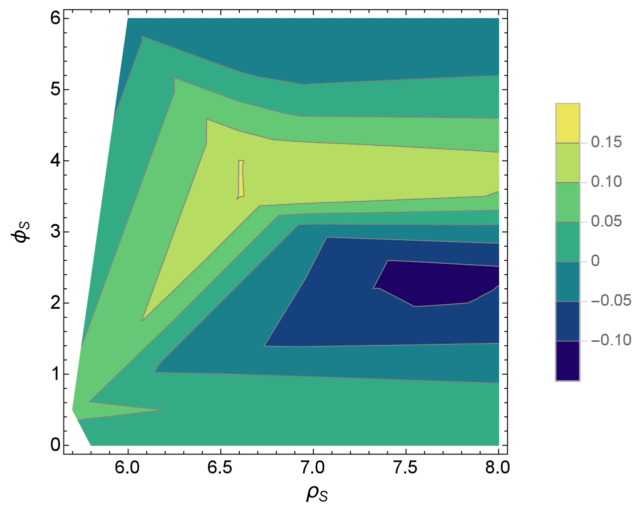

Fig. 1 Numerical results of $A_{CP}(B^- \to f_0(500)K^-)$ as functions of ρ_S and ϕ_S

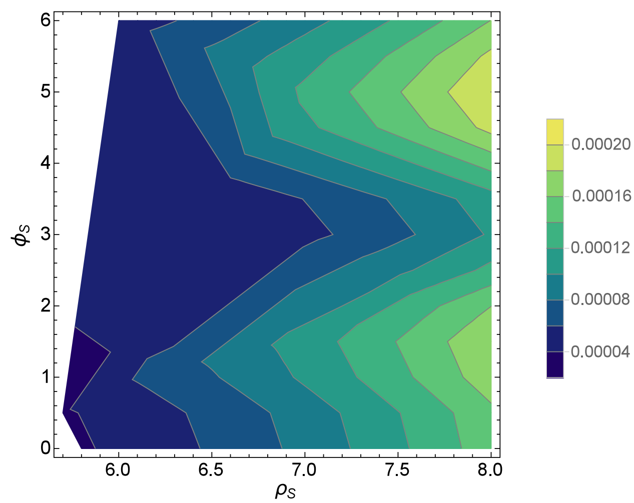

Fig. 2 Numerical results of $\mathcal{B}(B^- \to f_0(500)K^-)$ as functions of ρ_S and ϕ_S

In the obtained allowed ranges for ρ_S and ϕ_S, i.e. $\rho_S \in [5.692, 8]$ and $\phi_S \in [0, 2\pi]$, we calculate the CP asymmetry parameter and the branching fraction for $B^- \to K^- f_0(500)$ decay modes using Eqs. (24), (19), (25) and (26). The results are plotted in Figs. 1 and 2 as functions of ρ_S and ϕ_S. From these two figures and our calculated data, we obtain the predictions $A_{CP}(B^- \to K^- f_0(500)) \in [-0.115, -0.151]$ and $\mathcal{B}(B^- \to K^- f_0(500)) \in [3.763, 20.014] \times 10^{-5}$ when ρ_S and ϕ_S vary in their allowed ranges.

5 Summary

In this work, within the QCD factorization approach, we study the localized CP violation in $B^- \to K^-\pi^+\pi^-$ decays in the region $m^2_{K^-\pi^+} < 15\,\mathrm{GeV}^2$ and $0.08 < m^2_{\pi^+\pi^-} < 0.66$ GeV^2 by including the interference of $\rho^0(770)$ and $f_0(500)$. By fitting the experimental data of $A_{CP}(B^- \to K^-\pi^+\pi^-)$ in this region, we find that such localized CP asymmetry can be indeed induced by the interference of $\rho^0(770)$ and $f_0(500)$ when $\delta \in [2.124, 5.976]$, $\phi_S \in [0, 2\pi]$ and $\rho_S \in [5.692, 8]$. Both the hard spectator-scattering and the weak annihilation amplitudes have endpoint divergences. When we get the large values of ρ, it means that both the weak annihilation and hard spectator-scattering contributions become large, thus $1/m_b$ power expansions are still meaningful and QCDF would not break down. Furthermore, although the weak annihilation amplitudes are formally $1/m_b$ power-suppressed, they are generally nontrivial, especially for the flavor-changing neutral-current dominated and pure annihilation decays. In fact, recent experimental data require the large value of ρ to be consistent with theoretical results. This implies that ρ may be much larger than 1, thus larger values of ρ_S are acceptable while dealing with the divergence problems for $B \to SP(PS)$ decays. The large values of ρ_S indicate that the weak annihilation and the hard spectator scattering processes can make large contributions to $B^- \to K^- f_0(500)$ decays and we should take more efforts to investigate these contributions in B non-leptonic weak decays. With the obtained allowed ranges for ρ_S and ϕ_S, we predict the CP asymmetry parameter and the branching fraction for $B^- \to K^- f_0(500)$ decay modes. We find $A_{CP}(B^- \to K^- f_0(500)) \in [-0.115, -0.151]$ and $\mathcal{B}(B^- \to K^- f_0(500)) \in [3.763, 20.014] \times 10^{-5}$ in the allowed ranges of ϕ_S and ρ_S. These predictions can hopefully be tested in future experiments. In our analysis, the uncertainties coming from the CKM matrix elements, form factors, decay constants, s quark masses and Gegenbauer moments are all considered.

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References

1. J.H. Christenson, J.W. Cronin, V.L. Fitch, R. Turlay, Phys. Rev. Lett. **13**, 138 (1964)
2. N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963)
3. M. Kobayashi, T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973)
4. M. Gronau, J.L. Rosner, Phys. Lett. B **564**, 90 (2003)
5. S. Kränkl, T. Mannel, J. Virto, Nucl. Phys. B **899**, 247 (2015)
6. G. Calderón, C.H. García-Duque, Nucl. Part. Phys. Proc. **267–269**, 242 (2015)
7. W.F. Wang, Hn Li, Phys. Lett. B **763**, 29 (2016)
8. J.H.A. Nogueira, T. Frederico, O. Lourenço, Few Body Syst. **58**, 98 (2017)
9. B. Aubert et al. [BaBar Collaboration], Phys. Rev. D **78**, 012004 (2008)
10. A. Garmash et al. [Belle Collaboration], Phys. Rev. Lett. **96**, 251803 (2006)
11. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **111**, 101801 (2013)
12. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **112**, 011801 (2014)
13. H.Y. Cheng, C.K. Chua, Phys. Rev. D **88**, 114014 (2013)
14. H.Y. Cheng, Nucl. Part. Phys. Proc. **273–275**, 1290 (2016)
15. Z.H. Zhang, X.H. Guo, Y.D. Yang, Phys. Rev. D **87**, 076007 (2013)
16. B. Bhattacharya, M. Gronau, J.L. Rosner, Phys. Lett. B **726**, 337 (2013)
17. K.A. Olive et al. [Particle Data Group], Chin. Phys. C **38**, 090001 (2014)
18. H.Y. Cheng, C.K. Chua, K.C. Yang, Phys. Rev. D **73**, 014017 (2006)
19. M. Wirbel, B. Stech, M. Bauer, Z. Phys. C **29**, 637 (1985)
20. M. Bauer, B. Stech, M. Wirbel, Z. Phys. C **34**, 103 (1987)
21. M. Beneke, J. Rohrer, D. Yang, Nucl. Phys. B **774**, 64 (2007)
22. M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999)
23. H.Y. Cheng, K.C. Yang, Phys. Rev. D **78**, 094001 (2008)
24. Y.Y. Keum, Hn Li, A.I. Sanda, Phys. Lett. B **504**, 6 (2001)
25. Y.Y. Keum, H.N. Li, A.I. Sanda, Phys. Rev. D **63**, 054008 (2001)
26. C.D. Lu, K. Ukai, M.Z. Yang, Phys. Rev. D **63**, 074009 (2001)
27. C.W. Bauer, D. Pirjol, I.W. Stewart, Phys. Rev. Lett. **87**, 201806 (2001)
28. C.W. Bauer, D. Pirjol, I.W. Stewart, Phys. Rev. D **65**, 054022 (2002)
29. H.Y. Cheng, C.K. Chua, K.C. Yang, Z.Q. Zhang, Phys. Rev. D **87**, 114001 (2013)
30. Y. Li, H.Y. Zhang, Y. Xing, Z.H. Li, C.D. Lu, Phys. Rev. D **91**, 074022 (2015)
31. M. Beneke, M. Neubert, Nucl. Phys. B **675**, 333 (2003)
32. M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B **606**, 245 (2001)
33. D.V. Bugg, J. Phys. G **34**, 151 (2007)
34. R. Aaij et al. [LHCb Collaboration], Phys. Rev. D **92**(3), 032002 (2015)
35. Y. Li, A.J. Ma, W.F. Wang, Z.J. Xiao, Eur. Phys. J. C **76**(12), 675 (2016)
36. C. Wang, X.H. Guo, Y. Liu, R.C. Li, Eur. Phys. J. C **74**, 3140 (2014)
37. H.Y. Cheng, C.K. Chua, Phys. Rev. D **80**, 114008 (2009)
38. J. Govaerts, L.J. Reinders, F. de Viron, J. Weyers, Nucl. Phys. B **283**, 706 (1987)
39. H.Y. Cheng, K.C. Yang, Phys. Rev. D **71**, 054020 (2005)
40. A. Deandrea, A.D. Polosa, Phys. Rev. Lett. **86**, 216 (2001)
41. M. Ciuchini, E. Franco, A. Masiero, L. Silvestrini, Phys. Rev. D **67**, 075016 (2003)
42. C. Bobeth, M. Gorbahn, S. Vickers, Eur. Phys. J. C **75**(7), 340 (2015)
43. Q. Chang, J. Sun, Y. Yang, X. Li, Phys. Rev. D **90**, 054019 (2014)
44. J. Sun, Q. Chang, X. Hu, Y. Yang, Phys. Lett. B **743**, 444 (2015)
45. G. Zhu, Phys. Lett. B **702**, 408 (2011)