Eur. Phys. J. C (2018) 78:739
https://doi.org/10.1140/epjc/s10052-018-6231-0

THE EUROPEAN PHYSICAL JOURNAL C

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Letter

A solution to the electroweak horizon problem in the $R_h = ct$ universe

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Received: 8 August 2018 / Accepted: 8 September 2018 / Published online: 15 September 2018

Abstract Particle physics suggests that the Universe may have undergone several phase transitions, including the well-known inflationary event associated with the separation of the strong and electroweak forces in grand unified theories. The accelerated cosmic expansion during this transition, at cosmic time $t \sim 10^{-36} - 10^{-33}$ s, is often viewed as an explanation for the uniformity of the CMB temperature, T, which would otherwise have required inexplicable initial conditions. With the discovery of the Higgs particle, it is now quite likely that the Universe underwent another (electroweak) phase transition, at $T = 159.5 \pm 1.5$ GeV – roughly $\sim 10^{-11}$ s after the big bang. During this event, the fermions gained mass and the electric force separated from the weak force. There is currently no established explanation, however, for the apparent uniformity of the vacuum expectation value of the Higgs field which, like the uniformity in T, gives rise to its own horizon problem in standard ΛCDM cosmology. We show in this paper that a solution to the electroweak horizon problem may be found in the choice of cosmological model, and demonstrate that this issue does not exist in the alternative Friedmann–Robertson–Walker cosmology known as the $R_h = ct$ universe.

1 Introduction

Several phase transitions in particle physics have potentially deep implications for cosmology. A well known example is the phase transition associated with grand unified theories (GUTs), during which the strong and electroweak (EW) forces are believed to have separated. This well-studied case was originally motivated by missing magnetic monopoles, but was quickly identified as an inflationary event [1] that could solve the horizon problem in standard ΛCDM cosmology.

Today, with the discovery of the Higgs particle [2], the consequences of a second well-motivated transition – the electroweak phase transition (EWPT) – occurring at a critical temperature of 159.5 ± 1.5 GeV, are being studied with increasing interest. In ΛCDM, this temperature would have been reached at cosmic time $t \sim 10^{-11}$ s, well past the first (inflationary) transition at $t \sim 10^{-36} - 10^{-33}$ s. The standard model EWPT is now known to be a 'crossover,' i.e., one that does not depart far from equilibrium, rather than first order (with a discontinuity), that would have provided a ready explanation for the origin of baryon asymmetry, i.e., the matter left over after the annihilations between matter and anti-matter ended at very early times. But many extensions to the standard model of particle physics allow additional Higgs fields that reopen the possibility of a first-order phase transition at the EW scale [3] which would, in addition, generate gravitational waves (e.g., [4]) measurable with LISA and other next-generation detectors [5,6]. Learning about a possible first-order EWPT by measuring the Higgs self-interaction will also be a goal of the High-Luminosity Large Hadron Collider (LHC) and other future colliders (see, e.g., [7–9]). Of course, first and foremost, the crucial function of the EWPT is the generation of fermionic mass and the consequent separation of the electric and weak forces, both crucial events in the history of the Universe.

As the Universe cooled down further following the EWPT, a third phase transition is believed to have occurred at roughly 100 MeV, corresponding to a time $t \sim 10^{-6}$ s in ΛCDM. This would have arisen out of quantum chromodynamics, associated with the transformation of quarks behaving like free particles (in a quark-gluon plasma at asymptotically high temperatures) into the 'confined states' of baryons and mesons in the hadronic phase as the Universe continued to expand.

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Our focus in this paper is the EWPT because, as we shall see, the nature of the Higgs field appears to lead inevitably to yet another horizon problem, not unlike what happened with the CMB temperature, though this time having to do with the vacuum expectation value (vev) of the Higgs field, which appears to be universal – even on scales exceeding causally-connected regions. Inflation was invoked to account for the uniformity of the CMB on large scales, but the accelerated expansion it spawned would have occurred well *before* the EWPT, and would therefore have been largely irrelevant to the Higgs vev.

There is no well-established solution yet to this so-called electroweak horizon problem (EHP), which has been recognized in various forms over the past half century. At first, the inclination was to search for sub-horizon features produced in the EWPT. For example, Zeldovic et al. [10] and Kibble [11] offered an early assessment of the possibility that domain walls might have been created in the cosmos as a result of such scale transitions in the early Universe. These topological defects would have significant observational consequences, e.g., producing measurable anisotropies in the CMB temperature [12–14]. As cosmological observations have improved, however, it has become increasingly clear that the most likely resolution of the EHP is to avoid it in the first place. More recent attempted solutions have therefore included a late-time weak-scale inflation [15–20], though no particular proposal has had any impact with our interpretation of the observations thus far.

In this paper, we suggest that the EHP may be due to an incorrect choice of the cosmology, and propose that the solution may be found – not in a tweaked ΛCDM but, rather – in the alternative Friedmann-Robertson-Walker (FRW) cosmology known as the $R_\mathrm{h} = ct$ Universe [21–24]. We shall demonstrate that, just as $R_\mathrm{h} = ct$ avoids the horizon problem with the CMB temperature [25], it is equally free of any subsequent horizon problem with the EWPT. The critical difference between ΛCDM and $R_\mathrm{h} = ct$ that allows this to happen is that, while the former has an early decelerated expansion, the latter does not.

2 Background

The Higgs mechanism for generating fermionic mass is now widely accepted [26,27]. There are actually two parts to this story: the first has to do with when (and if) a non-zero vev is acquired (which particle physicists commonly refer to as 'turning on the Higgs field'); the second has to do with the size of the coupling constants with which the various elementary particles sense the Higgs field. At asymptotically high temperatures, the EW symmetry is unbroken. In simple terms, this means that all the 'messenger' particles carrying the electroweak force transfer the same amount of momentum per unit energy from one fermion to the next. In this regime, the relativistic expression for energy, $E^2 = m^2c^4 + p^2c^2$, does not differentiate among them based on the value of p/E because they all have $m = 0$, regardless of whether the particle is a photon, a $W^\pm$ or a Z.

The electric and weak forces separate, however, when p/E changes due to the emergence of a non-zero mass. This 'spontaneous symmetry breaking' happens only when the Higgs acquires a non-zero vev and the particles have unequal coupling constants, so that their masses are different. But note that even if the Higgs mechanism did not exist, symmetry would be attained at asymptotically high temperatures anyway, because in that limit $E/mc^2 \gg 1$, which would make the ratio p/E virtually identical for all the bosons. The symmetry would still have been broken as the temperature dropped, as long as their inertial masses were different. The viability of the Higgs mechanism just makes the spontaneous symmetry breaking cleaner and more precisely localized in temperature – and therefore redshift, or cosmic time. As noted in the introduction, we now know that the EWPT must have occurred at the critical temperature $T = 159.5 \pm 1.5$ GeV, when $t \sim 10^{-11}$ s in ΛCDM.

But what sets the Higgs vev? As of today, there is no known theoretical constraint on this important property of the Higgs field. This critical temperature could have been something else. One may reasonably argue, however, that whatever conditions establish the vev, it is manifested uniformly throughout a causally-connected region of spacetime. There is no reason, though, why the same vev should emerge everywhere, even at distances exceeding an observer's causal horizon.

Donoghue et al. [28] took an interesting approach to this question by estimating the likelihood function for the Higgs vev based on anthropic constraints on the existence of atoms. It is known that nuclei and atoms would not exist if the masses of light quarks and the electron were modestly different from their measured values [29–31]. And since the fermionic masses are proportional to the Higgs vev, these anthropically permitted bounds may be interpreted as constraints on the Higgs vev distribution accessible to us, given the other parameters in the standard cosmological model.

They explored how the Higgs vev distribution function is shaped by possible variations in the cosmology, always with the constraint that nuclei and atoms should appear. This assumes, of course, that there exists an a priori range of vev's, based on an unknown property of the fundamental theory. Particle physicists expect that the Higgs vev can take on any value throughout a very large domain, extending at least up to the GUT scale, many orders of magnitude above the EW scale (a disparity in energy known as the 'hierarchy problem').

They found that even within an anthropic framework, there is no reason to expect a single, observed uniform Higgs vev, commonly referred to as v_0. The distribution estimated in this fashion peaks near v_0, though it extends over several orders of magnitude. Its median value is actually $2.25v_0$, and its 2σ range extends from $0.10v_0$ to $11.7v_0$. Recalling that fermionic masses are proportional to the vev, we therefore see that nuclear and atomic properties could in principle have varied by at least one to two orders of magnitude across the Universe, from one causally-connected region to another. Yet no such variation has ever been confirmed.

3 The electroweak horizon problem

To understand why this is a problem with standard cosmology, let us examine the size of a causally-connected region within which one may expect to find a uniform Higgs vev (see Fig. 1). In Ref. [25], we showed that the null geodesic equation for a flat Universe may be written

$$\dot{R}_\gamma = c\left(\frac{R_\gamma}{R_{\rm h}} - 1\right), \tag{1}$$

where $R_{\rm h} \equiv c/H(t)$ is the Hubble (or gravitational) radius in terms of the Hubble parameter $H(t)$ [21,22], and R_γ is the proper radius of a photon propagating along the null geodesic reaching the observer at $R_\gamma = 0$ (point B in this figure). In the FRW framework, a proper radius may also be written as $R \equiv a(t)r$, in terms of the comoving radius r and the

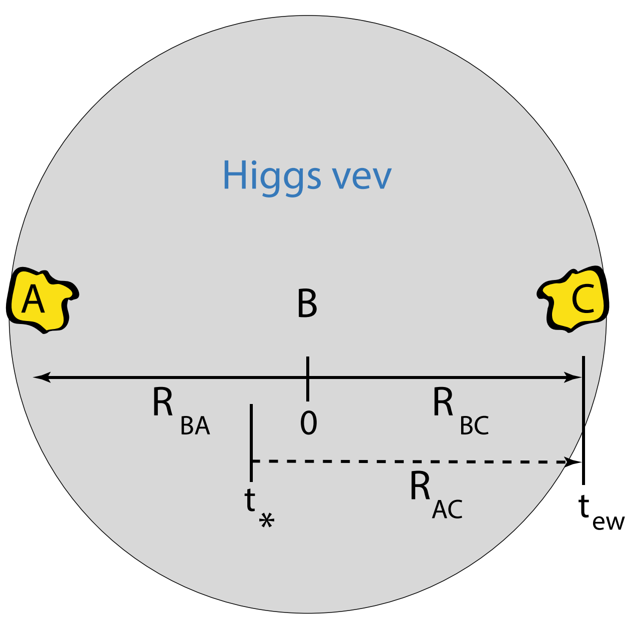

Fig. 1 Schematic diagram showing observer B connecting causally with two opposite patches (A and C) in the Higgs vev, a proper distance $R_{\rm BA}(t) = R_{\rm BC}(t)$ away. Patch A emitted a light signal at time t_* that reached C, a proper distance $R_{\rm AC}(t_{\rm ew})$ away, at the electroweak phase transition time $t_{\rm ew}$

universal expansion factor $a(t)$. When the cosmic equation of state is written in the form $p = w\rho$, for the total pressure p and energy density ρ, the gravitational radius satisfies the dynamical equation [32]

$$\dot{R}_{\rm h} = \frac{3}{2}(1+w)c. \tag{2}$$

For example, in a radiation dominated universe, with $w = 1/3$, the Hubble (or gravitational) radius expands at twice the speed of light.

Solving Eqs. (1) and (2) simultaneously yields the null geodesic $R_\gamma(t)$ linking a source emitting photons at $R_{\rm src}(t_e) = R_\gamma(t_e)$ at time t_e, with the observer who receives them at $R_\gamma(t_o) = 0$ at time t_o. Note that for a given observer at time t_o, there is a unique (radial) null geodesic arriving at his location (see Fig. 2). This function $R_\gamma(t)$ begins at $R_\gamma(0) = 0$ at time $t = 0$ (i.e., the big bang), increases while $R_\gamma > R_{\rm h}$ and reaches a maximum at $t_{\rm max}$ defined by the condition $R_\gamma(t_{\rm max}) = R_{\rm h}(t_{\rm max})$, and then decreases again towards $R_\gamma(t_o) = 0$ once $R_{\rm h}$ overtakes R_γ so that $\dot{R}_\gamma < 0$ in Eq. (1) (see Ref. [25]).

For a broad range of conditions, previous studies have shown that an observer receiving light at time t ($> t_{\rm max}$) sees a maximum photon excursion $R_{\gamma{\rm o}}(t_{\rm max}) \lesssim R_{\rm h}(t)/2$ away from his position [33–36]. As explained more extensively in Ref. [35], this behaviour of null geodesics in FRW is not difficult to understand. All models other than de Sitter had no pre-existing detectable sources away from the observer's location before the big bang. The photons we detect at time t from the most remote distances were emitted only *after* their sources had sufficient time to reach these extreme locations, which lie at roughly half of $R_{\rm h}(t)$. Therefore the proper size of our *visible* Universe at any given time t is only about half of the Hubble (or gravitational) radius $R_{\rm h}(t)$ (see also Ref. [36]). Claims that we see sources today (at time t_0) beyond $R_{\rm h}(t_0)$ (see, e.g., Ref. [37]) are simply confusing where the sources are today with where they were when they emitted the light we are receiving now. Our causally-connected region is based solely on the proper size (i.e., $R_\gamma[t_{\rm max}]$) of the volume within which light signals have been exchanged. Photons radiated by sources beyond $R_{\rm h}$ may be detectable in our future, but they have no relevance to the causally connected spacetime points today.

We shall discuss the implications of the null geodesics shown in Fig. 2 for $R_{\rm h} = ct$ shortly, but first we examine why an EWPT horizon problem emerges in ΛCDM. We shall write the Hubble parameter for flat ΛCDM cosmology in the form

$$H(a) = H_0\sqrt{\Omega_{\rm m}\, a^{-3} + \Omega_{\rm r}\, a^{-4} + \Omega_\Lambda}, \tag{3}$$

where today's Hubble constant ($H_0 = 67.8$ km s^{-1} Mpc^{-1}) and the scaled densities for matter ($\Omega_m = 0.308$), radiation ($\Omega_r = 5.37 \times 10^{-5}$) and dark energy ($\Omega_\Lambda = 1 - \Omega_m - \Omega_r$), are assumed to have their *Planck* values [38]. The redshift and age at decoupling are, respectively, $z_{cmb} = 1089.9$ and $t_{cmb} = 377,700$ years. Therefore, the expansion factor at this epoch was $a(t_{cmb}) = (1 + z_{cmb})^{-1} \approx 9.17 \times 10^{-4}$, implying a Hubble constant $H(t_{cmb}) \approx 4.78 \times 10^{-14}$ s^{-1}. The Hubble (gravitational) radius at decoupling was therefore $R_h(t_{cmb}) = c/H(t_{cmb}) \approx 0.20$ Mpc.

Integrating the equation

$$t_{cmb} - t = \int_a^{a_{cmb}} \frac{da}{a\,H(a)}, \tag{4}$$

it is straightforward to find $a(t)$ and $H(t)$ with the use of Eq. (3) at any time $t < t_{cmb}$, since we terminate the calculation at $t_{ew} = 10^{-11}$ s, well after the inflationary phase ended at $t_f \sim 10^{-33}$ s. At the EWPT, one finds an expansion factor $a(t_{ew}) \approx 1.93 \times 10^{-4}$ and a corresponding Hubble (gravitational) radius $R_h(t_{ew}) \approx 0.016$ Mpc.

Solutions to the geodesic Eq. (1) therefore suggest that the size of a causally-connected region at t_{ew} would have been $R_{ew}(t_{ew}) \lesssim R_h(t_{ew})/2 \approx 0.008$ Mpc, and shifting forward to today, this scale expands to $R_{ew}(t_0) \sim [a(t_0)/a(t_{ew})]R_{ew}(t_{ew}) \sim 41.5$ Mpc. This proper radius would represent the size of the largest region we should expect to see with a uniform Higgs vev today, yet we also know from Eq. (1) that the proper size of our visible Universe right now is $\lesssim R_h(t_0)/2 \approx 2,212$ Mpc – more than 50 times bigger. If our understanding of the EWPT is correct, we should therefore be seeing a variation of fermionic and atomic properties across the Universe, which is absolutely not the case. This is the electroweak horizon problem. And to amplify this point, recall that one of the 'hopes' for the EWPT is that it may explain the baryon asymmetry. But this mechanism would not work because an $R_{ew}(t_0)$ much smaller than $R_h(t_0)$ would mean we should see pockets of antimatter at proper distances exceeding the electroweak horizon, and we simply have no evidence of such exotic domains.

As far as we can tell observationally, the Higgs vev is the same throughout our visible Universe, even on opposite sides of us, with a separation more than 50 times larger than the electroweak horizon. As noted earlier, there is currently no established explanation for how this could happen. This is the principal reason we are proposing in this paper that the emergence of such a major hurdle with the standard ΛCDM model is yet more evidence of a fundamental problem with the cosmology, not the particle physics. In the next section, we shall see that the alternative FRW cosmology known as the $R_h = ct$ universe has neither a CMB horizon problem, nor a horizon problem with the EWPT.

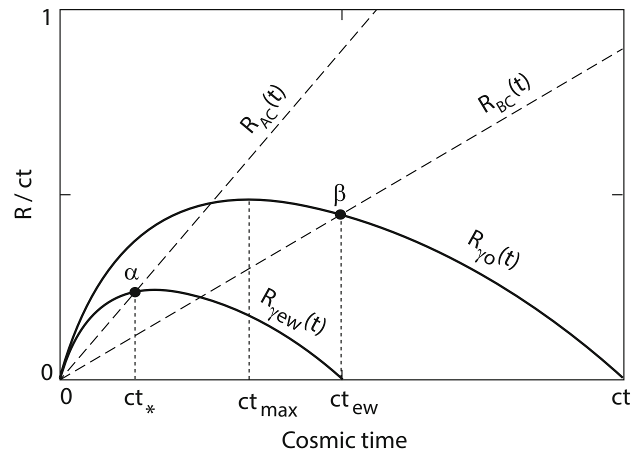

Fig. 2 Photon trajectories as seen by an observer in patch C (see Fig. 1) for the $R_h = ct$ universe. The curve $R_{\gamma ew}(t)$ represents the null geodesic arriving at C at the electroweak phase transition time t_{ew} from any source emitting light at proper distance $R_{src}(t) = R_{\gamma ew}(t)$ at time $t < t_{ew}$. The other solid curve $R_{\gamma o}(t)$ is the null geodesic for light arriving at C at time $t > t_{ew}$. The dashed lines $R_{AC}(t)$ and $R_{BC}(t)$ show the proper distance from A to C and from B to C, respectively. The other labels and symbols are defined in the text

4 A solution of the EHP in $R_h = ct$

Our goal here is to understand how and why observer B sees a uniform Higgs vev throughout his visible universe at time t, which necessitates a causal connection between patches A and C by the time (t_{ew}) the electroweak phase transition occurred. We shall find it easier to describe the events from the perspective of an observer in C, however. This won't affect how the proper distances are calculated because t is the same everywhere and, given the evident symmetry, C receives a signal from B at the same time that B receives a signal from C.

In Fig. 2 we show two relevant null geodesics and the worldlines of patch A and observer B as seen by C (see Fig. 1). The vertical axis gives the proper distance as a fraction of ct for C receiving light signals at cosmic time t_{ew} and t. The label 't_{max}' refers to the time at which the null geodesic $R_{\gamma 0}(t)$ attains its maximum proper distance. A light signal is emitted by A at time t_* (labeled α in Fig. 2), traveling towards C along the trajectory $R_{\gamma ew}(t)$. As long as these photons reach C before a subsequent signal is emitted by B at t_{ew} (labeled β), A and C will have been causally connected at the EWPT.

For a given cosmology, in this case $R_h = ct$, the null geodesic $R_{\gamma 0}(t)$ is unique and, for a given time t_{ew}, there exists a single point β satisfying the necessary conditions for C to receive the signal from B at t. Turning this around, from B's perspective this happens identically from two opposite sides in the sky, and therefore $R_{AC}(t_{ew}) = 2R_{BC}(t_{ew})$. The key question is thus whether there exists a time t_* such that A and C were causally connected at $t_{\rm ew}$, with a proper distance $R_{\rm AC}(t_{\rm ew})$ that grew to fill the entire visible Universe by time t_0 today.

In the $R_{\rm h} = ct$ universe, the expansion factor is $a(t) \propto t$ for all cosmic time. Therefore,

$$R_{\rm AC}(t_{\rm ew}) = a(t_{\rm ew}) \int_{t_*}^{t_{\rm ew}} c\frac{dt'}{a(t')} = ct_{\rm ew}\ \ln(t_{\rm ew}/t_*). \tag{5}$$

Similarly,

$$R_{\rm BC}(t_{\rm ew}) = a(t_{\rm ew}) \int_{t_{\rm ew}}^{t_0} c\frac{dt'}{a(t')} = ct_{\rm ew}\ \ln(t_0/t_{\rm ew}). \tag{6}$$

It is trivial to verify that both of these proper distances satisfy the null geodesic Eq. (1). The constraint $R_{\rm AC}(t_{\rm ew}) = 2R_{\rm BC}(t_{\rm ew})$ therefore yields the condition

$$t_* = t_{\rm ew}\left(\frac{t_{\rm ew}}{t_0}\right)^2. \tag{7}$$

Thus, no matter when the EWPT occurred relative to t_0, there is always a time $t_* > 0$ – no matter how small – at which an exchange of signals between patches A and C could have been initiated to ensure that they were causally connected by the time the Higgs vev was manifested. Put another way, regardless of how large $R_{\rm AC}$ is today (t_0), and regardless of when the EWPT took place ($t_{\rm ew}$), there always exists a physically meaningful value of t_* that permitted A and C to be causally connected before the Higgs vev was imprinted on the cosmic structure observed by observer B at time t_0.

In this cosmology, the size of a causally-connected region at $t_{\rm ew}$ would have been $R_{\rm ew}(t_{\rm ew}) \lesssim ct_{\rm ew}/2 \approx 0.3$ cm, much smaller than the corresponding size in ΛCDM. But the critical difference between these two models is that, whereas ΛCDM underwent significant deceleration prior to $z \sim 0.7$, $R_{\rm h} = ct$ did not. In the latter cosmology, $a(t) \propto R_{\rm h}(t)$, and therefore $R_{\rm ew}(t_{\rm ew})/R_{\rm h}(t_{\rm ew}) = R_{\rm ew}(t_0)/R_{\rm h}(t_0)$. In other words, the size of the region with a uniform Higgs vev today is the same fraction of $R_{\rm h}$ as it was at the EWPT, so the entire visible Universe has a structure based on just a single Higgs vev manifested at the EWPT.

5 Conclusion

Wherever atomic and nuclear matter has been studied on cosmic scales, no reliable evidence has ever been found of a breakdown in their physical properties measured locally (see, e.g., Ref. [38]). For example, as far out as we can see in the Universe, all structures appear to be made out of matter, not antimatter. So if the baryon asymmetry is indeed due to the EWPT, its uniformity amplifies the argument of a uniform Higgs vev throughout the visible Universe.

In this paper, we have shown that the EWPT may be avoided altogether with an alternative choice of cosmology, specifically, the $R_{\rm h} = ct$ universe. We have demonstrated that, regardless of when an event took place in the early Universe, the causally-connected region at that time would have filled the entire visible Universe today. Therefore, neither the phase transition associated with GUT (producing inflation), nor the electroweak phase transition due to the Higgs field being turned on, would have created observable sub-horizon features in the $R_{\rm h} = ct$ cosmology.

We estimated the Hubble (gravitational) radius in this model assuming the same time $t_{\rm ew} \sim 10^{-11}$ s for the EWPT as in the standard model, and found that $R_{\rm h}(t_{\rm ew}) \sim 0.3$ cm, much smaller than the corresponding horizon size in ΛCDM. Even so, the fact that $R_{\rm h} = ct$ had no early deceleration means that the causally-connected volume at that time would still have expanded sufficiently to fill our entire visible Universe today. Of course, $t_{\rm ew}$ is likely to be different in $R_{\rm h} = ct$ but, as we have noted, the actual time at which the EWPT took place has no impact on the outcome.

The EHP does not receive as much discussion today as does its GUT partner, perhaps because the latter is viewed as a more critical ingredient of the standard model. But the reality is that the foundational theory behind the Higgs mechanism for creating fermionic mass and separating the electric and weak forces is now quite well established. If conflict between the EWPT and cosmology remains unresolved, it is reasonable to question the cosmological framework, as we have done in this paper. The $R_{\rm h} = ct$ model has been shown to fit many kinds of data better than ΛCDM (see, e.g., Table 1 in Ref. [39]). Indeed, there is some evidence that the largely empirical, parametric formulation of ΛCDM essentially produces optimized fits that mimic the predictions in $R_{\rm h} = ct$ [40]. We believe that this alternative model's ability to elegantly and simply avoid conceptual difficulties, such as the CMB and EW horizon problems, is yet another strong argument in its favour.

Acknowledgements I acknowledge Amherst College for its support through a John Woodruff Simpson Fellowship.

Funded by SCOAP[3].

References

1. A.H. Guth, PRD **23**, 347 (1981)
2. G. Aad et al, (ATLAS collaboration), PLB **716**, 1 (2012)
3. P.P. Fileviez, H.H. Patel, PRD **79**, 055024 (2009)
4. D.J. Weir, Philos. Trans. R. Soc. A **376**, 20170126 (2018)

5. C. Caprini et al., JCAP **04**, 001 (2016)
6. P. Amaro-Seoane, H. Audley et al, e-print 1702.00786 (2017)
7. A. Noble, M. Perelstein, PRD **78**, 063518 (2008)
8. M.J. Dolan, C. Englert, M. Spannowsky, HEP **1210**, 112 (2012)
9. A.J. Barr, M.J. Dolan, C. Englert, D.E. Ferreira, D.E. Lima, M. Spannowsky, HEP **1502**, 016 (2015)
10. Y.B. Zeldovic, I.Y. Kobzarev, L.B. Okun, Sov. Phys. JETP **40**, 1 (1975)
11. T.W.B. Kibble, J. Math. Phys. **9**, 1387 (1976)
12. A. Vilenkin, E.P.S. Shellard, *Cosmic Strings and other Topological Defects* (Cambridge University Press, Cambridge, 1994)
13. A. Lazanu, C.J.A.P. Martins, E.P.S. Shellard, PLB **747**, 426 (2015)
14. L. Sousa, P.P. Avelino, PRD **92**, 083520 (2015)
15. L. Randall, S. Thomas, Nucl. Phys. B **449**, 229 (1995)
16. D.H. Lyth, E.D. Stewart, PRD **53**, 1784 (1996)
17. L. Randall, M. Soljacic, A.H. Guth, Nucl. Phys. B **472**, 377 (1996)
18. G. Germán, G. Ross, S. Sarkar, Nucl. Phys. B **608**, 423 (2001)
19. T. Boeckel, J. Schaffner-Bielich, PRL **105**, 041301 (2010)
20. H. Davoudiasl, D. Hooper, S.D. McDermott, PRL **116**, 031303 (2016)
21. F. Melia, MNRAS **382**, 1917 (2007)
22. F. Melia, A.S.H. Shevchuk, MNRAS **419**, 2579 (2012)
23. F. Melia, Front. Phys. **11**, 119801 (2016)
24. F. Melia, Front. Phys. **12**, 129802 (2017)
25. F. Melia, A&A **553**, A76 (2013)
26. F. Englert, R. Brout, PRL **13**, 321 (1964)
27. P. Higgs, PRL **13**, 508 (1964)
28. J.F. Donoghue, K. Dutta, A. Ross, M. Tegmark, PRD **81**, 073003 (2010)
29. V. Agrawal, S.M. Barr, J.F. Donoghue, D. Seckel, PRD **57**, 5480 (1998)
30. C.J. Hogan, Rev. Mod. Phys. **72**, 1149 (2000)
31. T. Damour, J.F. Donoghue, PRD **78**, 014014 (2008)
32. F. Melia, M. Abdelqader, IJMP D **18**, 1889 (2009)
33. O. Bikwa, F. Melia, A.S.H. Shevchuk, MNRAS **421**, 3356 (2012)
34. F. Melia, JCAP **09**, 029 (2012)
35. F. Melia, CQG **30**, 155007 (2013)
36. F. Melia, Am. J. Phys. **86**, 585 (2018)
37. T.M. Davis, C.H. Lineweaver, AIP Conference Proceedings **555**, 348 (2001)
38. P.A.R. Ade et al, (Planck Collaboration), A&A **594**, A13 (2015)
39. F. Melia, MNRAS **464**, 1966 (2017)
40. F. Melia, Astrophys. Space Sci. **356**, 393 (2015)