

Weak decays of B_c into two hadrons under flavor SU(3) symmetry

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Abstract A large number of B_c meson events have been recorded at the LHCb detector, especially some two-body hadronic decay modes. We analyzed the weak decays of the B_c meson into two hadron states under the flavor SU(3) symmetry. The relations among amplitudes of B_c into $D + P(V)$, $B + P(V)$, $P(V) + P(V)$, $T_8 + \bar{T}_8$ and $T_{10} + \bar{T}_{10}$ were investigated systematically, where $P(V)$ denotes a light pseudoscalar (vector) meson and $T_{8,10}$ denotes a light baryon. The $\eta - \eta'$ mixing and $\omega - \phi$ mixing effects are also considered for the phenomenological discussions. We obtained the relations among decay widths of different B_c decay channels. These results are helpful to study the two-body decay properties of the B_c meson and test the flavor SU(3) symmetry.

1 Introduction

The B_c meson family is unique because it is composed of two different heavy flavor quarks, the charm and bottom. The lifetime of the B_c meson is greatly longer than that of heavy quarkonia since it has weak decays only. Of course, the B_c meson's decays become vivid and complicated. The B_c decays have three kinds of decay modes: (i) the bottom quark decays through $b \rightarrow c, u$, which accounts for around 20% to the total decay width; (ii) the charm quark decays through $\bar{c} \rightarrow \bar{s}, \bar{d}$, which accounts for around 70% to the total decay width; (iii) the weak annihilation, which accounts for around 10% to the total decay width [1].

Due to the running of the Large Hadron Collider (LHC), new experimental data on the decays of B_c meson are collected, and several new rare decay channels have been discovered in recent years [2–4]. Therein, the $B_c^+ \rightarrow B_s^0 \pi^+$ decay channel by the charm weak transition was first observed by the LHCb Collaboration [2] and the measured product of the ratio of cross sections and branching fraction

is $[\sigma(B_c^+)/\sigma(B_s^0)] \times Br(B_c^+ \rightarrow B_s^0 \pi^+) = [2.37 \pm 0.31(stat) \pm 0.11(syst)_{-0.13}^{+0.17}(\tau_{B_c})] \times 10^{-3}$. Except $B_c^+ \rightarrow B_s^0 \pi^+$, other decay channels with the charm weak transition have not been observed currently. It is worth while to note that a baryonic decay of the B_c meson, $B_c^+ \rightarrow J/\psi p \bar{p} \pi^+$, is observed for the first time, with a significance of 7.3σ [3]. These measurements will certainly help us to understand the production and decay properties of the B_c meson.

Very recently, the LHCb Collaboration have firstly measured the $B_c^+ \rightarrow D^0 K^+$ decay mode with a statistical significance of 5.1σ using proton–proton collision data corresponding to an integrated luminosity of $3.0 fb^{-1}$ at 7 and 8 TeV. The ratio between the branching fraction and that of $B_c^+ \rightarrow J/\psi \pi^+$ decay mode is given to be $B(B_c^+ \rightarrow D^0 K^+)/B(B_c^+ \rightarrow J/\psi \pi^+) = 0.13 \pm 0.04(stat) \pm 0.01(syst) \pm 0.01(R_{J/\psi \pi})$ [4]. For $B_c^+ \rightarrow D^0 \pi^+$ channel, however, there is no clear event excess in the distribution of the invariant-mass $m_{D^0 \pi^+}$. The LHCb Collaboration only gave the upper limit as $R_{D^0 \pi^+} = (f_c/f_u)B(B_c^+ \rightarrow D^0 \pi^+) < 3.9 \times 10^{-7}$ [4].

Theoretically, the decays of the B_c meson have been investigated in different approaches. People employed the theoretical frames such as perturbative QCD (PQCD) approach [5–20], QCD sum rules (QCD SR) [21–23], Light-cone sum rules (LCSR) [24], the relativistic quark model (RQM) [25–28], the nonrelativistic constituent quark model (NCQM) [29], the light-front quark model (LFQM) [30–33], the Bethe-Salpeter equation method [34,35], the nonrelativistic QCD (NRQCD) approach [36–45], Principle of maximum conformality (PMC) [46], and Internal and external emission formulae [47]. From some of references, one can see that different theoretical frames may provide rather different predictions for the decay width of the same decay channel [43]. The testing of these theoretical predictions has to refer to future LHCb experiments.

On the other hand, in order to determine the dynamics-independent nature among different decay channels, the flavor SU(3) symmetry approach is a powerful tool to deal with

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the decays into light hadrons. Under the flavor $SU(3)$ symmetry, the decay amplitudes are parameterized in terms of $SU(3)$ -irreducible and model-independent amplitudes. Even though the size of the amplitudes can not be determined by itself in the flavor $SU(3)$ symmetry approach, the constraints on certain decay modes are clear. Thus the flavor $SU(3)$ symmetry approach has been widely employed in many works on the weak decays of heavy flavor mesons and baryons into two or three hadrons [48–67]. The flavor $SU(3)$ symmetry approach plays an important role to bridge dynamic theory and experimental data in the understanding of the decay properties of the heavy flavor mesons and baryons.

Up to now, the flavor $SU(3)$ symmetry approach has employed to analyze the B_c meson decays into charmed tetraquarks [58] and charmed or bottomed mesons [67]. In this paper, we will systematically explore B_c decay into $D + P(V)$, $B + P(V)$, $P(V) + P(V)$, $T_8 + \bar{T}_8$ and $T_{10} + \bar{T}_{10}$ under the flavor $SU(3)$ symmetry respectively, where $P(V)$ denotes the light pseudoscalar(vector) meson while $T_{8,10}$ denotes a light baryon. In particular we derive relations for decay widths and CP violations among different decay channels, which shall be tested by future precise experimental measurements.

The work is divided into four parts. In Sect. 2, we will give an overview of flavor $SU(3)$ classification of the hadronic states with different light quarks and their associated members. In Sect. 3 we will study the $SU(3)$ decay amplitudes for the weak B_c decays into two mesons or two baryons. We will discuss the relations for decay widths and CP violations in B_c decays in Sect. 4. We summarize and conclude in the end.

2 Particle multiplets

Using the standard flavor $SU(3)$ group representation, the B_c meson is a singlet, while the heavy mesons transform as $\mathbf{3}$ representation [68–70], and can be written as $B_i = (B_u(u\bar{b}), B_d(d\bar{b}), B_s(s\bar{b}))$ and $D_i = (D_u(u\bar{c}), D_d(d\bar{c}), D_s(s\bar{c}))$. The light pseudoscalar mesons P with spin-parity $J^P = 0^-$ has an octet

$$P_j^i = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -2\frac{\eta_8}{\sqrt{6}} \end{pmatrix}. \quad (1)$$

Considering that the $SU(3)$ singlet pseudoscalar state η_1 can be written as $(P_{\eta_1})_j^i = \delta_j^i \eta_1$, thus the physical eigenstates η and η' can be described by the mixing between η_1 and η_8 ¹

$$\begin{aligned} |\eta\rangle &= \cos\theta|\eta_8\rangle - \sin\theta|\eta_1\rangle, \\ |\eta'\rangle &= \sin\theta|\eta_8\rangle + \cos\theta|\eta_1\rangle. \end{aligned} \quad (2)$$

Similarly, for the light vector meson with spin-parity $J^P = 1^-$, we have the multiplet as

$$V_j^i = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -2\frac{\omega_8}{\sqrt{6}} \end{pmatrix}. \quad (3)$$

Considering that the $SU(3)$ singlet vector state ϕ_1 can be written as $(V_{\phi_1})_j^i = \delta_j^i \phi_1$, thus the physical eigenstates ω and ϕ can be described by the mixing between ϕ_1 and ω_8

$$\begin{aligned} |\omega\rangle &= \cos\theta_V|\omega_8\rangle - \sin\theta_V|\phi_1\rangle, \\ |\phi\rangle &= \sin\theta_V|\omega_8\rangle + \cos\theta_V|\phi_1\rangle, \end{aligned} \quad (4)$$

The light baryons with spin-parity $J^P = \frac{1}{2}^+$ form a $SU(3)$ octet T_8 , which can be described as

$$(T_8)^{ij} = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & \Sigma^+ & p^+ \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda^0}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -2\frac{\Lambda^0}{\sqrt{6}} \end{pmatrix}. \quad (5)$$

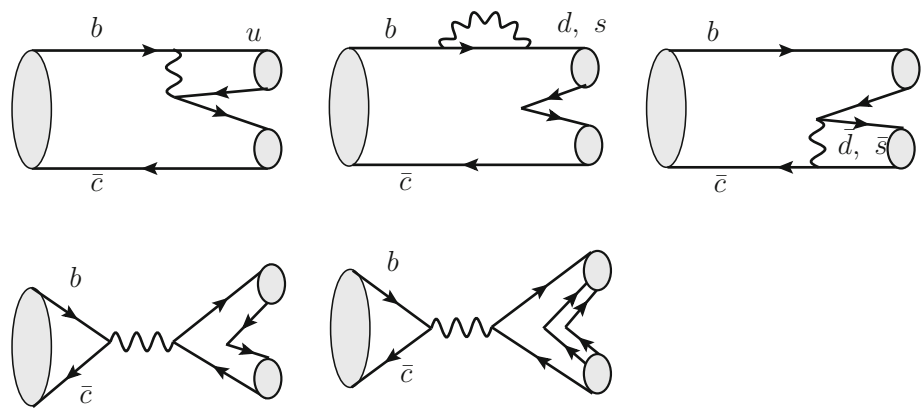
The light baryons with spin-parity $J^P = \frac{3}{2}^+$ form a $SU(3)$ decuplet T_{10} , the components of which are

$$\begin{aligned} (T_{10})^{111} &= \Delta^{++}, \quad (T_{10})^{112} = (T_{10})^{121} = (T_{10})^{211} = \frac{\Delta^+}{\sqrt{3}}, \\ (T_{10})^{222} &= \Delta^-, \quad (T_{10})^{122} = (T_{10})^{211} = (T_{10})^{221} \\ &= \frac{\Delta^0}{\sqrt{3}}, \quad (T_{10})^{333} = \Omega^-, \\ (T_{10})^{113} &= (T_{10})^{131} = (T_{10})^{311} = \frac{\Sigma'^+}{\sqrt{3}}, \\ (T_{10})^{223} &= (T_{10})^{232} = (T_{10})^{322} = \frac{\Sigma'^-}{\sqrt{3}}, \\ (T_{10})^{123} &= (T_{10})^{132} = (T_{10})^{213} = (T_{10})^{231} = (T_{10})^{312} \\ &= (T_{10})^{321} = \frac{\Sigma'^0}{\sqrt{6}}, \\ (T_{10})^{133} &= (T_{10})^{313} = (T_{10})^{331} = \frac{\Xi'^0}{\sqrt{3}}, \\ (T_{10})^{233} &= (T_{10})^{323} = (T_{10})^{332} = \frac{\Xi'^-}{\sqrt{3}}. \end{aligned} \quad (6)$$

From them, one can see the decuplet T_{10} is symmetrical when changing the order of the superscript i, j, k .

¹ Here we only treated the η and η' as quark-antiquark configuration, thus the gluonium contribution in the η' is not considered.

Fig. 1 Typical Feynman diagrams for the weak B_c decays into two hadrons



3 SU(3) decay amplitudes for weak B_c decays into two hadrons

In this section, we study the $B_c \rightarrow D_i + P(V)$, $B_c \rightarrow B_i + P(V)$, $B_c \rightarrow P(V) + P(V)$, and $B_c \rightarrow T_{10} + \bar{T}_{10}$ decays in the flavor SU(3) symmetry, respectively. The typical Feynman diagrams for the weak B_c decays into two hadrons are plotted in Fig. 1. Their decay amplitudes will be parameterized in terms of SU(3)-irreducible amplitudes. They are helpful to get the decay widths relations.

3.1 SU(3) decay amplitudes for $B_c \rightarrow D_i + P(V)$

First, we will study the bottom quark decays, i.e. $B_c \rightarrow D_i + P(V)$ channels. As already mentioned before the B_c is a singlet in the flavor SU(3) group, while the D_i transforms as $\mathbf{3}$ representation, and the light pseudo-scalar meson P and vector meson V belong to octets.

The $B_c \rightarrow D_q + P(V)$ decays are controlled by the bottom to light quark transition, thus the weak Hamiltonian \mathcal{H}_{eff} is

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}V_{uq}^* \left[C_1 O_1^{\bar{u}u} + C_2 O_2^{\bar{u}u} \right] - V_{tb}V_{tq}^* \left[\sum_{i=3}^{10} C_i O_i \right] \right\} + \text{H.c.}, \quad (7)$$

where the V_{ij} is the CKM matrix element and the O_i are the four-fermion effective operators. According to the group multiplication and decomposition, we have $\bar{\mathbf{3}} \otimes \mathbf{3} \otimes \bar{\mathbf{3}} = \bar{\mathbf{3}} \oplus \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \bar{\mathbf{15}}$. The tree operators is described as a vector $H^i(\bar{\mathbf{3}})$, an asymmetrical tensor $H_k^{[ij]}(\mathbf{6})$, and a symmetrical tensor $H_k^{(ij)}(\bar{\mathbf{15}})$. From the above formulae, the penguin operators belong to the $\bar{\mathbf{3}}$ representation.

From the weak Hamiltonian, the B_c decay amplitudes into a charmed meson D_i and a light pseudo-scalar meson P can be expressed with the tree amplitude $A_{B_c}^T$ and the penguin amplitude $A_{B_c}^P$

$$A(B_c \rightarrow D_i P) = \langle D_i + P | \mathcal{H}_{eff} | B_c \rangle = V_{ub}V_{uq}^* A_{B_c}^T + V_{tb}V_{tq}^* A_{B_c}^P, \quad (8)$$

where q denotes the light down or strange quark. $A_{B_c}^T$ and $A_{B_c}^P$ can be described as the flavor SU(3) amplitudes. After removing the CKM matrix elements, the weak Hamiltonian can be rewritten as the sum of the flavor irreducible representations.

For the strangeless decays, i.e. $\Delta S = 0(b \rightarrow d)$, the flavor irreducible representations have the nonzero components [48, 51, 56]:

$$\begin{aligned} H^2(\bar{\mathbf{3}}) &= 1, \quad H_1^{12}(\mathbf{6}) = -H_1^{21}(\mathbf{6}) \\ &= H_3^{23}(\mathbf{6}) = -H_3^{32}(\mathbf{6}) = 1, \\ 2H_1^{12}(\bar{\mathbf{15}}) &= 2H_1^{21}(\bar{\mathbf{15}}) = -3H_2^{22}(\bar{\mathbf{15}}) = -6H_3^{23}(\bar{\mathbf{15}}) \\ &= -6H_3^{32}(\bar{\mathbf{15}}) = 6. \end{aligned} \quad (9)$$

Similarly, one can get the nonzero components of the related irreducible representations for the $\Delta S = 1(b \rightarrow s)$ decays. The explicit expressions can be obtained by Eq. (12) with the exchange $2 \leftrightarrow 3$.

Thus it is easily to rewrite the penguin amplitude $A_{B_c}^P$ and the tree amplitude $A_{B_c}^T$ as

$$\begin{aligned} A_{B_c}^P &= \alpha H^i(\bar{\mathbf{3}}) D_j P_i^j, \\ A_{B_c}^T &= a_3 H^i(\bar{\mathbf{3}}) D_j P_i^j \\ &\quad + a_6 H_1^{[ij]}(\mathbf{6}) D_j P_i^l + a_{15} H_k^{(ij)}(\bar{\mathbf{15}}) D_j P_i^k, \end{aligned} \quad (10)$$

where the α represents the hadronic parameter from the strong and electro-weak interactions for the penguin topology, while a_i with $i = 3, 6, 15$ represent the hadronic parameters from the strong and electro-weak interactions for the tree topology. Under the flavor SU(3) symmetry, the B_c decay amplitudes into a charmed meson and a light meson can be obtained. We gave the results in Tables 1 and 2. In Table 1, $\alpha' = \alpha + V_{ub}V_{ud}^* a_3 / (V_{tb}V_{td}^*)$ and $\alpha'' = \alpha + V_{ub}V_{us}^* a_3 / (V_{tb}V_{ts}^*)$. In Ref. [67], the B_c decay amplitudes into a charmed meson and a light meson are also studied in the flavor SU(3) symmetry. Compared with their results,

Table 1 Decay amplitudes of $B_c \rightarrow D_i + P$ decays. Here and in the following tables, the α and a_i represent the hadronic parameters. Besides, $\alpha' = \alpha + V_{ub}V_{ud}^*a_3/(V_{tb}V_{td}^*)$ and $\alpha'' = \alpha + V_{ub}V_{us}^*a_3/(V_{tb}V_{ts}^*)$

Channel $\Delta S = 0$	Amplitude
$B_c^- \rightarrow D^- \pi^0$	$\frac{1}{\sqrt{2}} (-V_{tb}V_{td}^*\alpha + V_{ub}V_{ud}^*(-a_3 + a_6 + 5a_{15}))$
$B_c^- \rightarrow \bar{D}^0 \pi^-$	$V_{tb}V_{td}^*\alpha + V_{ub}V_{ud}^*(a_3 - a_6 + 3a_{15})$
$B_c^- \rightarrow D_s^- K^0$	$V_{tb}V_{td}^*\alpha + V_{ub}V_{ud}^*(a_3 + a_6 - a_{15})$
$B_c^- \rightarrow D^- \eta$	$\frac{1}{\sqrt{6}} \cos \theta (V_{tb}V_{td}^*\alpha + V_{ub}V_{ud}^*(a_3 + 3a_6 + 3a_{15})) - \frac{1}{\sqrt{3}} \sin \theta V_{tb}V_{td}^*\alpha'$
$B_c^- \rightarrow D^- \eta'$	$\frac{1}{\sqrt{6}} \sin \theta (V_{tb}V_{td}^*\alpha + V_{ub}V_{ud}^*(a_3 + 3a_6 + 3a_{15})) + \frac{1}{\sqrt{3}} \cos \theta V_{tb}V_{td}^*\alpha''$
Channel $\Delta S = 1$	Amplitude
$B_c^- \rightarrow D^- \bar{K}^0$	$V_{tb}V_{ts}^*\alpha + V_{ub}V_{us}^*(a_3 + a_6 - a_{15})$
$B_c^- \rightarrow \bar{D}^0 K^-$	$V_{tb}V_{ts}^*\alpha + V_{ub}V_{us}^*(a_3 - a_6 + 3a_{15})$
$B_c^- \rightarrow D_s^- \pi^0$	$\sqrt{2}V_{ub}V_{us}^*(a_6 + 2a_{15})$
$B_c^- \rightarrow D_s^- \eta$	$\sqrt{\frac{2}{3}} \cos \theta (-V_{tb}V_{ts}^*\alpha + V_{ub}V_{us}^*(-a_3 + 3a_{15})) - \frac{1}{\sqrt{3}} \sin \theta V_{tb}V_{ts}^*\alpha'$
$B_c^- \rightarrow D_s^- \eta'$	$\sqrt{\frac{2}{3}} \sin \theta (-V_{tb}V_{ts}^*\alpha + V_{ub}V_{us}^*(-a_3 + 3a_{15})) + \frac{1}{\sqrt{3}} \cos \theta V_{tb}V_{ts}^*\alpha''$

Table 2 Decay amplitudes of $B_c \rightarrow D_i + V$ decays. Here and in the following, the α^V and a_i^V represent the hadronic parameters. Besides, $\alpha^{V'} = \alpha^V + V_{ub}V_{ud}^*a_3^V/(V_{tb}V_{td}^*)$ and $\alpha^{V''} = \alpha^V + V_{ub}V_{us}^*a_3^V/(V_{tb}V_{ts}^*)$

Channel $\Delta S = 0$	Amplitude
$B_c^- \rightarrow D^- \rho^0$	$\frac{1}{\sqrt{2}} (-V_{tb}V_{td}^*\alpha^V + V_{ub}V_{ud}^*(-a_3^V + a_6^V + 5a_{15}^V))$
$B_c^- \rightarrow \bar{D}^0 \rho^-$	$V_{tb}V_{td}^*\alpha^V + V_{ub}V_{ud}^*(a_3^V - a_6^V + 3a_{15}^V)$
$B_c^- \rightarrow D_s^- K^{*0}$	$V_{cb}V_{cd}^*\alpha^V + V_{ub}V_{ud}^*(a_3^V + a_6^V - a_{15}^V)$
$B_c^- \rightarrow D^- \omega$	$\frac{1}{\sqrt{6}} \cos \theta_V (V_{tb}V_{td}^*\alpha^V + V_{ub}V_{ud}^*(a_3^V + 3a_6^V + 3a_{15}^V)) - \frac{1}{\sqrt{3}} \sin \theta_V V_{tb}V_{td}^*\alpha^{V'}$
$B_c^- \rightarrow D^- \phi$	$\frac{1}{\sqrt{6}} \sin \theta_V (V_{tb}V_{td}^*\alpha^V + V_{ub}V_{ud}^*(a_3^V + 3a_6^V + 3a_{15}^V)) + \frac{1}{\sqrt{3}} \cos \theta_V V_{tb}V_{td}^*\alpha^{V''}$
Channel $\Delta S = 1$	Amplitude
$B_c^- \rightarrow D^- \bar{K}^{*0}$	$V_{tb}V_{ts}^*\alpha^V + V_{ub}V_{us}^*(a_3^V + a_6^V - a_{15}^V)$
$B_c^- \rightarrow \bar{D}^0 K^{*-}$	$V_{tb}V_{ts}^*\alpha^V + V_{ub}V_{us}^*(a_3^V - a_6^V + 3a_{15}^V)$
$B_c^- \rightarrow D_s^- \rho^0$	$\sqrt{2}V_{ub}V_{us}^*(a_6^V + 2a_{15}^V)$
$B_c^- \rightarrow D_s^- \omega$	$\sqrt{\frac{2}{3}} \cos \theta_V (-V_{tb}V_{ts}^*\alpha^V + V_{ub}V_{us}^*(-a_3^V + 3a_{15}^V)) - \frac{1}{\sqrt{3}} \sin \theta_V V_{tb}V_{ts}^*\alpha^{V'}$
$B_c^- \rightarrow D_s^- \phi$	$\sqrt{\frac{2}{3}} \sin \theta_V (-V_{tb}V_{ts}^*\alpha^V + V_{ub}V_{us}^*(-a_3^V + 3a_{15}^V)) + \frac{1}{\sqrt{3}} \cos \theta_V V_{tb}V_{ts}^*\alpha^{V''}$

we considered the $\eta - \eta'$ mixing effects and expanding the amplitude relations into the decay channels involving the vector mesons.

3.2 SU(3) decay amplitudes for $B_c \rightarrow B_i + P(V)$

The second largest decay mode is from the charm decay. According to the estimation of the decay widths, there are three kinds of decay strength: Cabibbo-allowed by $c \rightarrow s\bar{d}u$, singly Cabibbo-suppressed by $c \rightarrow u\bar{d}/\bar{s}s$, and doubly Cabibbo-suppressed by $c \rightarrow d\bar{s}u$.

We write the nonzero components of the Hamiltonian for the Cabibbo-allowed decay channels

$$H_2^{31}(\mathbf{6}) = -H_2^{13}(\mathbf{6}) = 1, \quad H_2^{31}(\mathbf{15}) = H_2^{13}(\mathbf{15}) = 1. \quad (11)$$

Combing the $c \rightarrow u\bar{d}d$ and $c \rightarrow u\bar{s}s$ decays, we write the nonzero components of the Hamiltonian for the singly

Cabibbo suppressed channels as follows

$$\begin{aligned} H_2^{12}(\mathbf{6}) &= -H_2^{21}(\mathbf{6}) = H_3^{31}(\mathbf{6}) = -H_3^{13}(\mathbf{6}) = \sin \theta_C, \\ H_3^{31}(\mathbf{15}) &= H_3^{13}(\mathbf{15}) = -H_2^{12}(\mathbf{15}) = -H_2^{21}(\mathbf{15}) = \sin \theta_C, \end{aligned} \quad (12)$$

where the relation $V_{ud}V_{cd}^* \simeq -V_{us}V_{cs}^* \simeq \sin \theta_C$ is employed.

The nonzero components of the Hamiltonian for the doubly Cabibbo suppressed $c \rightarrow d\bar{s}u$ decays are

$$\begin{aligned} H_3^{21}(\mathbf{6}) &= -H_3^{12}(\mathbf{6}) = \sin^2 \theta_C, \\ H_3^{21}(\mathbf{15}) &= H_3^{12}(\mathbf{15}) = \sin^2 \theta_C, \end{aligned} \quad (13)$$

where the relation $V_{us}V_{cd}^* \simeq |V_{ud}V_{cd}^*|^2 \simeq \sin^2 \theta_C$ is employed.

The decay amplitudes $A(B_c \rightarrow B_i + P) = \langle B_i + P | \mathcal{H}_{eff} | B_c \rangle$ can be written as $A_{B_c}^T(B_c \rightarrow B_i + P)$, where the representation $H^i(\mathbf{3})$ will vanish in the flavor SU(3) sym-

Table 3 Decay amplitudes of $B_c \rightarrow B_i + P$ decays

Cabibbo allowed channel	Amplitude
$B_c^- \rightarrow \bar{B}_s^0 \pi^-$	$V_{cs} V_{ud}^* (-a_6 + a_{15})$
$B_c^- \rightarrow B^- K^0$	$V_{cs} V_{ud}^* (a_6 + a_{15})$
Cabibbo suppressed channel	Amplitude
$B_c^- \rightarrow \bar{B}_s^- K^-$	$V_{cs} V_{us}^* (-a_6 + a_{15})$
$B_c^- \rightarrow B^- \eta$	$-\sqrt{\frac{3}{2}} \cos \theta V_{cs} V_{us}^* (a_6 + a_{15})$
$B_c^- \rightarrow B^- \eta'$	$-\sqrt{\frac{3}{2}} \sin \theta V_{cs} V_{us}^* (a_6 + a_{15})$
$B_c^- \rightarrow \bar{B}^0 \pi^-$	$V_{cd} V_{ud}^* (a_6 - a_{15})$
$B_c^- \rightarrow B^- \pi^0$	$\frac{1}{\sqrt{2}} V_{cd} V_{ud}^* (a_6 + a_{15})$
Doubly Cabibbo suppressed channel	Amplitude
$B_c^- \rightarrow \bar{B}^0 K^-$	$V_{cd} V_{us}^* (-a_6 + a_{15})$
$B_c^- \rightarrow B^- \bar{K}^0$	$V_{cd} V_{us}^* (a_6 + a_{15})$

Table 4 Decay amplitudes of $B_c \rightarrow B_i + V$ decays

Cabibbo allowed channel	Amplitude
$B_c^- \rightarrow \bar{B}_s^0 \rho^-$	$V_{cs} V_{ud}^* (-a_6^V + a_{15}^V)$
$B_c^- \rightarrow B^- K^{*0}$	$V_{cs} V_{ud}^* (a_6^V + a_{15}^V)$
Cabibbo suppressed channel	Amplitude
$B_c^- \rightarrow \bar{B}_s^- K^{*-}$	$V_{cs} V_{us}^* (-a_6^V + a_{15}^V)$
$B_c^- \rightarrow B^- \omega$	$-\sqrt{\frac{3}{2}} \cos \theta_V V_{cs} V_{us}^* (a_6^V + a_{15}^V)$
$B_c^- \rightarrow B^- \phi$	$-\sqrt{\frac{3}{2}} \sin \theta_V V_{cs} V_{us}^* (a_6^V + a_{15}^V)$
$B_c^- \rightarrow \bar{B}^0 \rho^-$	$V_{cd} V_{ud}^* (a_6^V - a_{15}^V)$
$B_c^- \rightarrow B^- \rho^0$	$\frac{1}{\sqrt{2}} V_{cd} V_{ud}^* (a_6^V + a_{15}^V)$
Doubly Cabibbo suppressed channel	Amplitude
$B_c^- \rightarrow \bar{B}^0 K^{*-}$	$V_{cd} V_{us}^* (-a_6^V + a_{15}^V)$
$B_c^- \rightarrow B^- \bar{K}^{*0}$	$V_{cd} V_{us}^* (a_6^V + a_{15}^V)$

metry [64]. The effective Hamiltonian can be written as

$$A_{B_c \rightarrow B_i + P}^T = a_6 H_i^{[ij]}(\mathbf{6}) B_j P_i^l + a_{15} H_k^{[ij]}(\mathbf{15}) B_j P_i^k. \quad (14)$$

The B_c decay amplitudes into a bottom meson and a light meson can be obtained, which are listed in Tables 3 and 4.

3.3 SU(3) decay amplitudes for $B_c \rightarrow P(V) + P(V)$

In this subsection and the following subsection, we will study the weak annihilation accounting for around 10% of the B_c total decay width. Let us begin with the decays into two light mesons, i.e. $B_c \rightarrow P(V) + P(V)$ decay channels. As already mentioned before the light pseudo-scalar mesons P and light vector mesons V belong to flavor octets.

The $B_c \rightarrow P(V) + P(V)$ decays are induced by the bottom transition into charm quark. The effective weak Hamiltonian is written as

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \left\{ V_{cb} V_{uq}^* [C_1 O_1^{\bar{c}u} + C_2 O_2^{\bar{c}u}] + \text{H.c.} \right\}, \quad (15)$$

Since the light quarks in this transition form an octet, we have the nonzero component for the bottom transition into $c\bar{u}d$

$$H_1^2(\mathbf{8}) = V_{cb} V_{ud}^*. \quad (16)$$

For the bottom transition into $b \rightarrow c\bar{u}s$, the non-zero component becomes

$$H_1^3(\mathbf{8}) = V_{cb} V_{us}^*. \quad (17)$$

The decay amplitudes $A(B_c \rightarrow P + P) = \langle P + P | \mathcal{H}_{eff} | B_c \rangle$ can be expressed as

$$A(B_c \rightarrow P + P) = a_8 H_i^j(\mathbf{8}) P_j^i P_k^i. \quad (18)$$

The results of the decay amplitudes of $B_c \rightarrow P(V) + P(V)$ can be found in Table 5.

3.4 SU(3) decay amplitudes for $B_c \rightarrow T_8 + \bar{T}_8$ and $B_c \rightarrow T_{10} + \bar{T}_{10}$

In this subsection, we will study the two-body baryonic decays of the B_c meson. The light baryons with spin-parity $J^P = \frac{1}{2}^+$ form a SU(3) octet T_8 , while the light baryons with spin-parity $J^P = \frac{3}{2}^+$ form a SU(3) decuplet T_{10} .

The decay amplitudes $A(B_c \rightarrow T_8 + \bar{T}_8) = \langle T_8 + \bar{T}_8 | \mathcal{H}_{eff} | B_c \rangle$ can be expressed as

$$A(B_c \rightarrow T_8 + \bar{T}_8) = a'_8 H_i^j(\mathbf{8}) T^{ki}(\mathbf{8}) \bar{T}_{jk}(\mathbf{8}). \quad (19)$$

The corresponding amplitudes results are listed in Table 6.

The decay amplitudes $A(B_c \rightarrow T_{10} + \bar{T}_{10}) = \langle T_{10} + \bar{T}_{10} | \mathcal{H}_{eff} | B_c \rangle$ can be expressed as

$$A(B_c \rightarrow T_{10} + \bar{T}_{10}) = a'_8 H_i^j(\mathbf{8}) T_{10}^{kil}(\mathbf{10}) \bar{T}_{jkl}(\mathbf{10}). \quad (20)$$

The corresponding amplitudes results are listed in Table 7.

4 Decay widths relations for weak B_c decays into two hadrons

Doing a square of the decay amplitudes and integrating the phase space, their decay widths can be obtained. Under flavor SU(3) symmetry, there is no difference in the phase space. Thus the relation between decay widths shall be obtained accordingly. Let us define a ratio between two different channels as

$$R = \Gamma_i / \Gamma_j. \quad (21)$$

Table 5 Decay amplitudes of $B_c \rightarrow P(V) + P(V)$ decays

Channel $\Delta S = 0$	Amplitude	Channel $\Delta S = 0$	Amplitude
$B_c^- \rightarrow \bar{K}^0 K^-$	$V_{cb} V_{ud}^* a_8$	$B_c^- \rightarrow \bar{K}^{*0} K^{*-}$	$V_{cb} V_{ud}^* a_8^V$
$B_c^- \rightarrow \pi^- \eta$	$\sqrt{\frac{2}{3}} V_{cb} V_{ud}^* \cos \theta a_8$	$B_c^- \rightarrow \rho^- \omega$	$\sqrt{\frac{2}{3}} V_{cb} V_{ud}^* \cos \theta_V a_8^V$
$B_c^- \rightarrow \pi^- \eta'$	$\sqrt{\frac{2}{3}} V_{cb} V_{ud}^* \sin \theta a_8$	$B_c^- \rightarrow \rho^- \phi$	$\sqrt{\frac{2}{3}} V_{cb} V_{ud}^* \sin \theta_V a_8^V$
Channel $\Delta S = 1$	Amplitude	Channel $\Delta S = 1$	Amplitude
$B_c^- \rightarrow K^- \eta$	$-\sqrt{\frac{1}{6}} V_{cb} V_{us}^* \cos \theta a_8$	$B_c^- \rightarrow K^{*-} \omega$	$-\sqrt{\frac{1}{6}} V_{cb} V_{us}^* \cos \theta_V a_8^V$
$B_c^- \rightarrow K^- \eta'$	$-\sqrt{\frac{1}{6}} V_{cb} V_{us}^* \sin \theta a_8$	$B_c^- \rightarrow K^{*-} \phi$	$-\sqrt{\frac{1}{6}} V_{cb} V_{us}^* \sin \theta_V a_8^V$
$B_c^- \rightarrow \pi^0 K^-$	$\frac{1}{\sqrt{2}} V_{cb} V_{us}^* a_8$	$B_c^- \rightarrow \rho^0 K^{*-}$	$\frac{1}{\sqrt{2}} V_{cb} V_{us}^* a_8^V$
$B_c^- \rightarrow \pi^- K^0$	$V_{cb} V_{us}^* a_8$	$B_c^- \rightarrow \rho^- K^{*0}$	$V_{cb} V_{us}^* a_8^V$

Table 6 Decay amplitudes of $B_c \rightarrow T_8 + \bar{T}_8$ decays

Channel $\Delta S = 0$	Amplitude	Channel $\Delta S = 0$	Amplitude
$B_c^- \rightarrow \Lambda^0 \bar{\Sigma}^-$	$\sqrt{\frac{1}{6}} V_{cb} V_{ud}^* a'_8$	$B_c^- \rightarrow \bar{\Lambda}^0 \Sigma^-$	$\sqrt{\frac{1}{6}} V_{cb} V_{ud}^* a'_8$
$B_c^- \rightarrow \Sigma^0 \bar{\Sigma}^-$	$-\sqrt{\frac{1}{2}} V_{cb} V_{ud}^* a'_8$	$B_c^- \rightarrow \bar{\Sigma}^0 \Sigma^-$	$\sqrt{\frac{1}{2}} V_{cb} V_{ud}^* a'_8$
$B_c^- \rightarrow n \bar{p}^-$	$V_{cb} V_{ud}^* a'_8$	-	-
Channel $\Delta S = 1$	Amplitude	Channel $\Delta S = 1$	Amplitude
$B_c^- \rightarrow \bar{\Lambda}^0 \Xi^-$	$\sqrt{\frac{1}{6}} V_{cb} V_{us}^* a'_8$	$B_c^- \rightarrow \bar{\Sigma}^- \Xi^0$	$V_{cb} V_{us}^* a'_8$
$B_c^- \rightarrow \bar{\Sigma}^0 \Xi^-$	$\sqrt{\frac{1}{2}} V_{cb} V_{us}^* a'_8$	$B_c^- \rightarrow \bar{p}^- \Lambda^0$	$-\sqrt{\frac{2}{3}} V_{cb} V_{us}^* a'_8$

Table 7 Decay amplitudes of $B_c \rightarrow T_{10} + \bar{T}_{10}$ decays

Channel $\Delta S = 0$	Amplitude	Channel $\Delta S = 0$	Amplitude
$B_c^- \rightarrow \Delta^0 \bar{\Delta}^-$	$\frac{2}{3} V_{cb} V_{ud}^* a'_8$	$B_c^- \rightarrow \Delta^- \bar{\Delta}^0$	$\sqrt{\frac{1}{3}} V_{cb} V_{ud}^* a'_8$
$B_c^- \rightarrow \Delta^+ \bar{\Delta}^-$	$\sqrt{\frac{1}{3}} V_{cb} V_{ud}^* a'_8$	$B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0$	$\frac{1}{3} V_{cb} V_{ud}^* a'_8$
$B_c^- \rightarrow \Sigma'^0 \bar{\Sigma}'^-$	$\frac{\sqrt{2}}{3} V_{cb} V_{ud}^* a'_8$	$B_c^- \rightarrow \Sigma'^- \bar{\Sigma}'^0$	$\frac{\sqrt{2}}{3} V_{cb} V_{ud}^* a'_8$
Channel $\Delta S = 1$	Amplitude	Channel $\Delta S = 1$	Amplitude
$B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0$	$\frac{1}{3} V_{cb} V_{us}^* a'_8$	$B_c^- \rightarrow \Sigma'^0 \bar{\Delta}^-$	$\frac{\sqrt{2}}{3} V_{cb} V_{us}^* a'_8$
$B_c^- \rightarrow \Sigma'^+ \bar{\Delta}^-$	$\sqrt{\frac{1}{3}} V_{cb} V_{us}^* a'_8$	$B_c^- \rightarrow \Xi'^0 \bar{\Sigma}'^-$	$\frac{2}{3} V_{cb} V_{us}^* a'_8$
$B_c^- \rightarrow \Omega^- \bar{\Xi}'^0$	$\sqrt{\frac{1}{3}} V_{cb} V_{us}^* a'_8$	$B_c^- \rightarrow \Xi'^- \bar{\Sigma}'^0$	$\frac{\sqrt{2}}{3} V_{cb} V_{us}^* a'_8$

The results for the weak two-body decays of B_c into hadrons are given in Tables 8, 9 and 10. These ratios could be tested by the future LHCb experiments.

At last, let us study the CP asymmetry effects in the B_c meson decays, which may bring about the effect at $\mathcal{O}(10^{-3})$. From the above amplitudes in tables, the CP conjugated amplitudes can be obtained accordingly. For example, the CP conjugated amplitudes of the B_c meson decays into a charmed meson and a light meson are of the form of

$$\frac{\bar{A}(B_c^+ \rightarrow \bar{D}_i + \bar{P})}{A(B_c^- \rightarrow D_i + P)} = \frac{V_{ub}^* V_{uq} A_{B_c}^T + V_{tb}^* V_{tq} A_{B_c}^P}{V_{ub} V_{uq}^* A_{B_c}^T + V_{tb} V_{tq}^* A_{B_c}^P}. \quad (22)$$

The direct CP asymmetry can then be defined as

$$\begin{aligned} A_{CP}^{dir} &= \frac{\Gamma(B_c^+ \rightarrow \bar{f}_1 \bar{f}_2) - \Gamma(B_c^- \rightarrow f_1 f_2)}{\Gamma(B_c^+ \rightarrow \bar{f}_1 \bar{f}_2) + \Gamma(B_c^- \rightarrow f_1 f_2)} \\ &= \frac{|\bar{A}|^2 - |A|^2}{|\bar{A}|^2 + |A|^2}. \end{aligned} \quad (23)$$

From this equation, the direct CP asymmetry can be obtained when inputting the corresponding decay amplitudes. We do not list these repeated results. The decay channels of the B_c meson into a bottom meson and a light meson, two light mesons, or two baryons have no direct CP asymme-

Table 8 The decay width ratios for $B_c \rightarrow B_i + P(V)$

$B_c \rightarrow B_i + P$		$B_c \rightarrow B_i + V$	
$\Gamma_i / \Gamma(B_c^- \rightarrow B^- K^0)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow B^- K^{*0})$	R
$\frac{\Gamma(B_c^- \rightarrow B^- \pi^0)}{\Gamma(B_c^- \rightarrow B^- K^0)}$	$\frac{ V_{cd} ^2}{2 V_{cs} ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- \rho^0)}{\Gamma(B_c^- \rightarrow B^- K^{*0})}$	$\frac{ V_{cd} ^2}{2 V_{cs} ^2}$
$\frac{\Gamma(B_c^- \rightarrow B^- \eta')}{\Gamma(B_c^- \rightarrow B^- K^0)}$	$\frac{3 V_{us}^* ^2 \sin^2 \theta}{2 V_{ud} ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- \phi)}{\Gamma(B_c^- \rightarrow B^- K^{*0})}$	$\frac{3 V_{us}^* ^2 \sin^2 \theta}{2 V_{ud} ^2}$
$\frac{\Gamma(B_c^- \rightarrow B^- \eta)}{\Gamma(B_c^- \rightarrow B^- K^0)}$	$\frac{3 V_{us}^* ^2 \cos^2 \theta}{2 V_{ud} ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- \omega)}{\Gamma(B_c^- \rightarrow B^- K^{*0})}$	$\frac{3 V_{us}^* ^2 \cos^2 \theta}{2 V_{ud} ^2}$
$\frac{\Gamma(B_c^- \rightarrow B^- \bar{K}^0)}{\Gamma(B_c^- \rightarrow B^- K^0)}$	$\frac{ V_{cd} V_{us}^* ^2}{ V_{cs} V_{ud} ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- \bar{K}^{*0})}{\Gamma(B_c^- \rightarrow B^- K^{*0})}$	$\frac{ V_{cd} V_{us}^* ^2}{ V_{cs} V_{ud} ^2}$
$\Gamma_i / \Gamma(B_c^- \rightarrow \bar{B}^0 K^-)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow \bar{B}^0 K^{*-})$	R
$\frac{\Gamma(B_c^- \rightarrow \bar{B}^0 \pi^-)}{\Gamma(B_c^- \rightarrow \bar{B}^0 K^-)}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$	$\frac{\Gamma(B_c^- \rightarrow \bar{B}^0 \rho^-)}{\Gamma(B_c^- \rightarrow \bar{B}^0 K^{*-})}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$
$\frac{\Gamma(B_c^- \rightarrow \bar{B}_s^- K^-)}{\Gamma(B_c^- \rightarrow \bar{B}^0 K^-)}$	$\frac{ V_{cs} ^2}{ V_{cd} ^2}$	$\frac{\Gamma(B_c^- \rightarrow \bar{B}_s^- K^{*-})}{\Gamma(B_c^- \rightarrow \bar{B}^0 K^{*-})}$	$\frac{ V_{cs} ^2}{ V_{cd} ^2}$

Table 9 The decay width ratios for $B_c \rightarrow P(V) + P(V)$

$B_c \rightarrow P + P$		$B_c \rightarrow V + V$	
$\Gamma_i / \Gamma(B_c^- \rightarrow \bar{K}^0 K^-)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow K^{*0} K^{*-})$	R
$\frac{\Gamma(B_c^- \rightarrow \pi^- \eta)}{\Gamma(B_c^- \rightarrow \bar{K}^0 K^-)}$	$\frac{2 \cos^2 \theta}{3}$	$\frac{\Gamma(B_c^- \rightarrow \rho^- \omega)}{\Gamma(B_c^- \rightarrow K^{*0} K^{*-})}$	$\frac{2 \cos^2 \theta_V}{3}$
$\frac{\Gamma(B_c^- \rightarrow \pi^- \eta')}{\Gamma(B_c^- \rightarrow \bar{K}^0 K^-)}$	$\frac{2 \sin^2 \theta}{3}$	$\frac{\Gamma(B_c^- \rightarrow \rho^- \phi)}{\Gamma(B_c^- \rightarrow K^{*0} K^{*-})}$	$\frac{2 \sin^2 \theta_V}{3}$
$\Gamma_i / \Gamma(B_c^- \rightarrow \pi^- K^0)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow \rho^- K^{*0})$	R
$\frac{\Gamma(B_c^- \rightarrow \pi^0 K^-)}{\Gamma(B_c^- \rightarrow \pi^- K^0)}$	$\frac{1}{2}$	$\frac{\Gamma(B_c^- \rightarrow \rho^0 K^{*-})}{\Gamma(B_c^- \rightarrow \rho^- K^{*0})}$	$\frac{1}{2}$
$\frac{\Gamma(B_c^- \rightarrow K^- \eta')}{\Gamma(B_c^- \rightarrow \pi^- K^0)}$	$\frac{\sin^2 \theta}{6}$	$\frac{\Gamma(B_c^- \rightarrow K^{*-} \phi)}{\Gamma(B_c^- \rightarrow \rho^- K^{*0})}$	$\frac{\sin^2 \theta_V}{6}$
$\frac{\Gamma(B_c^- \rightarrow K^- \eta)}{\Gamma(B_c^- \rightarrow \pi^- K^0)}$	$\frac{\cos^2 \theta}{6}$	$\frac{\Gamma(B_c^- \rightarrow K^{*-} \omega)}{\Gamma(B_c^- \rightarrow \rho^- K^{*0})}$	$\frac{\cos^2 \theta_V}{6}$
$\frac{\Gamma(B_c^- \rightarrow \bar{K}^0 K^-)}{\Gamma(B_c^- \rightarrow \pi^- K^0)}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- K^{*0})}{\Gamma(B_c^- \rightarrow \rho^- K^{*0})}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$

try, because only tree topology diagrams contribute. From the interference of tree diagrams and penguin diagrams, these decay channels of the B_c meson decays into a charmed meson and a light meson have the direct CP asymmetry.

Employing the unitarity of the CKM matrix with $(VV^\dagger)_{ij} = (V^\dagger V)_{ij} = \delta_{ij}$, we get

$$Im [V_{tb}^* V_{td} V_{ub} V_{ud}^*] = -Im [V_{tb}^* V_{ts} V_{ub} V_{us}^*]. \tag{24}$$

The relations for the direct CP asymmetry of the B_c meson decays into a charmed meson and a light meson are

$$\frac{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow \bar{D}^0 \pi^-)}{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow \bar{D}^0 K^-)} = -\frac{\Gamma(B_c^- \rightarrow \bar{D}^0 K^-)}{\Gamma(B_c^- \rightarrow \bar{D}^0 \pi^-)}, \tag{25}$$

$$\frac{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow D_s^- \bar{K}^0)}{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow D^- \bar{K}^0)} = -\frac{\Gamma(B_c^- \rightarrow D^- \bar{K}^0)}{\Gamma(B_c^- \rightarrow D_s^- \bar{K}^0)}, \tag{26}$$

$$\frac{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow \bar{D}^0 \rho^-)}{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow \bar{D}^0 K^{*-})} = -\frac{\Gamma(B_c^- \rightarrow \bar{D}^0 K^{*-})}{\Gamma(B_c^- \rightarrow \bar{D}^0 \rho^-)}, \tag{27}$$

$$\frac{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow D_s^- \bar{K}^{*0})}{\mathcal{A}_{CP}^{dir}(B_c^- \rightarrow D^- \bar{K}^{*0})} = -\frac{\Gamma(B_c^- \rightarrow D^- \bar{K}^{*0})}{\Gamma(B_c^- \rightarrow D_s^- \bar{K}^{*0})}. \tag{28}$$

These relations actually are very general and similar to the relations in B meson decays [71,72], which shall be tested by future LHCb experiments.

5 conclusions

In this paper, we investigated the decay width relations for the B_c weak decays into two hadrons under the flavor SU(3) symmetry. The corresponding decay amplitudes are described by the summation of the flavor SU(3) irreducible amplitudes. The decay channels of the B_c meson into a charmed meson and a light meson, a bottom meson and a light meson, two light mesons, or two baryons were studied systematically. The direct CP asymmetry effects only exist in the decay channels of the B_c meson into a charmed meson and a light meson. And we obtained some direct CP asymmetry relations in the flavor SU(3) symmetry. Hadron-hadron colliders provide a solid platform and the precision tests for the various decay modes of the double heavy B_c meson. The theoretical predictions for the decay width ratios R and the direct

Table 10 The decay width ratios for $B_c \rightarrow T_8 + \bar{T}_8$ and $B_c \rightarrow T_{10} + \bar{T}_{10}$

$B_c \rightarrow T_8 + \bar{T}_8$		$B_c \rightarrow T_{10} + \bar{T}_{10}$	
$\Gamma_i / \Gamma(B_c^- \rightarrow n \bar{p}^-)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)$	R
$\frac{\Gamma(B_c^- \rightarrow \Lambda^0 \bar{\Sigma}^-)}{\Gamma(B_c^- \rightarrow n \bar{p}^-)}$	$\frac{1}{6}$	$\frac{\Gamma(B_c^- \rightarrow \Delta^0 \bar{\Delta}^-)}{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)}$	4
$\frac{\Gamma(B_c^- \rightarrow \bar{\Lambda}^0 \bar{\Sigma}^-)}{\Gamma(B_c^- \rightarrow n \bar{p}^-)}$	$\frac{1}{6}$	$\frac{\Gamma(B_c^- \rightarrow \Delta^- \bar{\Delta}^0)}{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)}$	3
$\frac{\Gamma(B_c^- \rightarrow \Sigma^0 \bar{\Sigma}^-)}{\Gamma(B_c^- \rightarrow n \bar{p}^-)}$	$\frac{1}{2}$	$\frac{\Gamma(B_c^- \rightarrow \Delta^+ \bar{\Delta}^-)}{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)}$	3
$\frac{\Gamma(B_c^- \rightarrow \bar{\Sigma}^0 \bar{\Sigma}^-)}{\Gamma(B_c^- \rightarrow n \bar{p}^-)}$	$\frac{1}{2}$	$\frac{\Gamma(B_c^- \rightarrow \Sigma'^0 \bar{\Sigma}'^-)}{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)}$	2
–	–	$\frac{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Xi}'^0)}{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Xi}'^0)}$	2
$\Gamma_i / \Gamma(B_c^- \rightarrow \bar{\Sigma}^- \Xi^0)$	R	$\Gamma_i / \Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)$	R
$\frac{\Gamma(B_c^- \rightarrow \bar{\Lambda}^0 \Xi^-)}{\Gamma(B_c^- \rightarrow \bar{\Sigma}^- \Xi^0)}$	$\frac{1}{6}$	$\frac{\Gamma(B_c^- \rightarrow \Sigma'^0 \bar{\Delta}^-)}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	2
$\frac{\Gamma(B_c^- \rightarrow \bar{\Sigma}^0 \Xi^-)}{\Gamma(B_c^- \rightarrow \bar{\Sigma}^- \Xi^0)}$	$\frac{1}{2}$	$\frac{\Gamma(B_c^- \rightarrow \Sigma'^\rho \bar{\Delta}^-)}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	3
$\frac{\Gamma(B_c^- \rightarrow \bar{p}^- \Lambda^0)}{\Gamma(B_c^- \rightarrow \bar{\Sigma}^- \Xi^0)}$	$\frac{2}{3}$	$\frac{\Gamma(B_c^- \rightarrow \Xi^0 \bar{\Xi}'^-)}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	4
–	–	$\frac{\Gamma(B_c^- \rightarrow \Omega^- \bar{\Xi}'^0)}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	3
–	–	$\frac{\Gamma(B_c^- \rightarrow \Xi'^- \bar{\Sigma}'^0)}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	2
$\frac{\Gamma(B_c^- \rightarrow n \bar{p}^-)}{\Gamma(B_c^- \rightarrow \bar{\Sigma}^- \Xi^0)}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$	$\frac{\Gamma(B_c^- \rightarrow B^- K^{*0})}{\Gamma(B_c^- \rightarrow \Sigma'^- \bar{\Delta}^0)}$	$\frac{ V_{ud}^* ^2}{ V_{us}^* ^2}$

CP asymmetries for the considered B_c weak decays into two hadrons, as listed in the Tables of this paper, could be tested in the future LHCb experiments.

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References

- S. Kar, P.C. Dash, M. Priyadarsini, S. Naimuddin, N. Barik, Phys. Rev. D **88**(9), 094014 (2013). <https://doi.org/10.1103/PhysRevD.88.094014>
- R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **111**(18), 181801 (2013). <https://doi.org/10.1103/PhysRevLett.111.181801>. arXiv:1308.4544 [hep-ex]
- R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **113**(15), 152003 (2014). <https://doi.org/10.1103/PhysRevLett.113.152003>. arXiv:1408.0971 [hep-ex]
- R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **118**(11), 111803 (2017). <https://doi.org/10.1103/PhysRevLett.118.111803>. arXiv:1701.01856 [hep-ex]
- Ds Du, Z. Wang, Phys. Rev. D **39**, 1342 (1989). <https://doi.org/10.1103/PhysRevD.39.1342>
- J.F. Sun, D.S. Du, Y.L. Yang, Eur. Phys. J. C **60**, 107 (2009). <https://doi.org/10.1140/epjc/s10052-009-0872-y>. arXiv:0808.3619 [hep-ph]
- W.F. Wang, Y.Y. Fan, Z.J. Xiao, Chin. Phys. C **37**, 093102 (2013). <https://doi.org/10.1088/1674-1137/37/9/093102>. arXiv:1212.5903 [hep-ph]
- X. Liu, Z.J. Xiao, C.D. Lu, Phys. Rev. D **81**, 014022 (2010). <https://doi.org/10.1103/PhysRevD.81.014022>. arXiv:0912.1163 [hep-ph]
- X. Liu, Z.J. Xiao, Phys. Rev. D **82**, 054029 (2010). <https://doi.org/10.1103/PhysRevD.82.054029>. arXiv:1008.5201 [hep-ph]
- X. Liu, Z.J. Xiao, J. Phys. G **38**, 035009 (2011). <https://doi.org/10.1088/0954-3889/38/3/035009>. arXiv:1003.3929 [hep-ph]
- Z.J. Xiao, X. Liu, Chin. Sci. Bull. **59**, 3748 (2014). <https://doi.org/10.1007/s11434-014-0418-z>. arXiv:1401.0151 [hep-ph]
- Z.T. Zou, Y. Li, X. Liu, Phys. Rev. D **97**(5), 053005 (2018). <https://doi.org/10.1103/PhysRevD.97.053005>. arXiv:1712.02239 [hep-ph]
- Z. Rui, Z.T. Zou, Phys. Rev. D **90**(11), 114030 (2014). <https://doi.org/10.1103/PhysRevD.90.114030>. arXiv:1407.5550 [hep-ph]
- Z. Rui, Phys. Rev. D **97**(3), 033001 (2018). <https://doi.org/10.1103/PhysRevD.97.033001>. arXiv:1712.08928 [hep-ph]
- J. Zhang, X.Q. Yu, Eur. Phys. J. C **63**, 435 (2009). <https://doi.org/10.1140/epjc/s10052-009-1112-1>. arXiv:0905.0945 [hep-ph]
- Q. Chang, N. Wang, J. Sun, L. Han, J. Phys. G **44**(8), 085005 (2017). <https://doi.org/10.1088/1361-6471/aa7bca>. arXiv:1707.03691 [hep-ph]
- C.H. Chen, Y.H. Lin, arXiv:1710.05531 [hep-ph]
- S. Dubnicka, A.Z. Dubnickova, A. Issadykov, M.A. Ivanov, A. Liptaj, Phys. Rev. D **96**(7), 076017 (2017). <https://doi.org/10.1103/PhysRevD.96.076017>. arXiv:1708.09607 [hep-ph]

19. Z. Rui, W.F. Wang, G.x. Wang, L.h. Song, C.D.L., Eur. Phys. J. C **75**(6), 293 (2015). <https://doi.org/10.1140/epjcs/10052-015-3528-0>. arXiv:1505.02498 [hep-ph]
20. Z. Rui, H. Li, G x Wang, Y. Xiao, Eur. Phys. J. C **76**(10), 564 (2016). <https://doi.org/10.1140/epjcs/10052-016-4424-y>. arXiv:1602.08918 [hep-ph]
21. P. Colangelo, G. Nardulli, N. Paver, Z. Phys. C **57**, 43 (1993). <https://doi.org/10.1007/BF01555737>
22. V.V. Kiselev, A.V. Tkabladze, Phys. Rev. D **48**, 5208 (1993). <https://doi.org/10.1103/PhysRevD.48.5208>
23. K. Azizi, H. Sundu, M. Bayar, Phys. Rev. D **79**, 116001 (2009). <https://doi.org/10.1103/PhysRevD.79.116001>. arXiv:0902.1467 [hep-ph]
24. T. Huang, F. Zuo, Eur. Phys. J. C **51**, 833 (2007). <https://doi.org/10.1140/epjcs/10052-007-0333-4>. arXiv:hep-ph/0702147 [HEP-PH]
25. M.A. Nobes, R.M. Woloshyn, J. Phys. G **26**, 2000 (1079). <https://doi.org/10.1088/0954-3899/26/7/308>. arXiv:hep-ph/0005056
26. D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **68**, 094020 (2003). <https://doi.org/10.1103/PhysRevD.68.094020>. arXiv:hep-ph/0306306
27. M.A. Ivanov, J.G. Korner, P. Santorelli, Phys. Rev. D **71**, 094006 (2005). Erratum: [Phys. Rev. D **75**, 019901 (2007)]. <https://doi.org/10.1103/PhysRevD.75.019901>, <https://doi.org/10.1103/PhysRevD.71.094006>. arXiv:hep-ph/0501051
28. D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Rev. D **82**, 034019 (2010). <https://doi.org/10.1103/PhysRevD.82.034019>. arXiv:1007.1369 [hep-ph]
29. E. Hernandez, J. Nieves, J.M. Verde-Velasco, Phys. Rev. D **74**, 074008 (2006). <https://doi.org/10.1103/PhysRevD.74.074008>. arXiv:hep-ph/0607150
30. W. Wang, Y.L. Shen, C.D. Lu, Phys. Rev. D **79**, 054012 (2009). <https://doi.org/10.1103/PhysRevD.79.054012>. arXiv:0811.3748 [hep-ph]
31. X .X. Wang, W. Wang, C .D. Lu, Phys. Rev. D **79**, 114018 (2009). <https://doi.org/10.1103/PhysRevD.79.114018>. arXiv:0901.1934 [hep-ph]
32. H .W. Ke, T. Liu, X.Q. Li, Phys. Rev. D **89**(1), 017501 (2014). <https://doi.org/10.1103/PhysRevD.89.017501>. arXiv:1307.5925 [hep-ph]
33. Y.J. Shi, W. Wang, Z.X. Zhao, Eur. Phys. J. C **76**(10), 555 (2016). <https://doi.org/10.1140/epjcs/10052-016-4405-1>. arXiv:1607.00622 [hep-ph]
34. H.F. Fu, Y. Jiang, C.S. Kim, G.L. Wang, JHEP **1106**, 015 (2011). [https://doi.org/10.1007/JHEP06\(2011\)015](https://doi.org/10.1007/JHEP06(2011)015). arXiv:1102.5399 [hep-ph]
35. R. Dhir, R.C. Verma, Phys. Rev. D **79**, 034004 (2009). <https://doi.org/10.1103/PhysRevD.79.034004>. arXiv:0810.4284 [hep-ph]
36. C.H. Chang, Y.Q. Chen, Phys. Rev. D **49**, 3399 (1994). <https://doi.org/10.1103/PhysRevD.49.3399>
37. G. Bell, T. Feldmann, Nucl. Phys. Proc. Suppl. **164**, 189 (2007). <https://doi.org/10.1016/j.nuclphysbps.2006.11.053>. arXiv:hep-ph/0509347
38. C.F. Qiao, P. Sun, JHEP **1208**, 087 (2012). [https://doi.org/10.1007/JHEP08\(2012\)087](https://doi.org/10.1007/JHEP08(2012)087). arXiv:1103.2025 [hep-ph]
39. G. Chiladze, A.F. Falk, A.A. Petrov, Phys. Rev. D **60**, 034011 (1999). <https://doi.org/10.1103/PhysRevD.60.034011>. arXiv:hep-ph/9811405
40. C.F. Qiao, R.L. Zhu, Phys. Rev. D **87**(1), 014009 (2013). <https://doi.org/10.1103/PhysRevD.87.014009>. arXiv:1208.5916 [hep-ph]
41. C .F. Qiao, P. Sun, D. Yang, R.L. Zhu, Phys. Rev. D **89**(3), 034008 (2014). <https://doi.org/10.1103/PhysRevD.89.034008>. arXiv:1209.5859 [hep-ph]
42. R. Zhu, Y. Ma, X.L. Han, Z.J. Xiao, Phys. Rev. D **95**(9), 094012 (2017). <https://doi.org/10.1103/PhysRevD.95.094012>. arXiv:1703.03875 [hep-ph]
43. R. Zhu, Nucl. Phys. B **931**, 359 (2018). <https://doi.org/10.1016/j.nuclphysb.2018.04.018>. arXiv:1710.07011 [hep-ph]
44. R. Zhu, Y. Ma, X.L. Han, Z.J. Xiao, arXiv:1805.06588 [hep-ph]
45. W. Wang, R.L. Zhu, Eur. Phys. J. C **75**(8), 360 (2015). <https://doi.org/10.1140/epjcs/10052-015-3583-6>. arXiv:1501.04493 [hep-ph]
46. J.M. Shen, X.G. Wu, H.H. Ma, S.Q. Wang, Phys. Rev. D **90**(3), 034025 (2014). <https://doi.org/10.1103/PhysRevD.90.034025>. arXiv:1407.7309 [hep-ph]
47. W.H. Liang, E. Oset, Eur. Phys. J. C **78**(6), 528 (2018). <https://doi.org/10.1140/epjcs/10052-018-5997-4>. arXiv:1804.00938 [hep-ph]
48. M.J. Savage, M.B. Wise, Phys. Rev. D **39**, 3346 (1989). Erratum: [Phys. Rev. D **40**, 3127 (1989)]. <https://doi.org/10.1103/PhysRevD.39.3346>. <https://doi.org/10.1103/PhysRevD.40.3127>
49. M. Gronau, O.F. Hernandez, D. London, J.L. Rosner, Phys. Rev. D **52**, 6356 (1995). <https://doi.org/10.1103/PhysRevD.52.6356>. arXiv:hep-ph/9504326
50. X.G. He, Eur. Phys. J. C **9**, 443 (1999). <https://doi.org/10.1007/s100529900064>. arXiv:hep-ph/9810397
51. X.G. He, Y.K. Hsiao, J.Q. Shi, Y.L. Wu, Y.F. Zhou, Phys. Rev. D **64**, 034002 (2001). <https://doi.org/10.1103/PhysRevD.64.034002>. arXiv:hep-ph/0011337
52. C.W. Chiang, M. Gronau, J.L. Rosner, D.A. Suprun, Phys. Rev. D **70**, 034020 (2004). <https://doi.org/10.1103/PhysRevD.70.034020>. arXiv:hep-ph/0404073
53. Y. Li, C.D. Lu, W. Wang, Phys. Rev. D **77**, 054001 (2008). <https://doi.org/10.1103/PhysRevD.77.054001>. arXiv:0711.0497 [hep-ph]
54. W. Wang, C.D. Lu, Phys. Rev. D **82**, 034016 (2010). <https://doi.org/10.1103/PhysRevD.82.034016>. arXiv:0910.0613 [hep-ph]
55. H.Y. Cheng, S. Oh, JHEP **1109**, 024 (2011). [https://doi.org/10.1007/JHEP09\(2011\)024](https://doi.org/10.1007/JHEP09(2011)024). arXiv:1104.4144 [hep-ph]
56. Y.K. Hsiao, C.F. Chang, X.G. He, Phys. Rev. D **93**(11), 114002 (2016). <https://doi.org/10.1103/PhysRevD.93.114002>. arXiv:1512.09223 [hep-ph]
57. C.D.L, W. Wang, F.S. Yu, Phys. Rev. D **93**(5), 056008 (2016). <https://doi.org/10.1103/PhysRevD.93.056008>. arXiv:1601.04241 [hep-ph]
58. X.G. He, W. Wang, R.L. Zhu, J. Phys. G **44**(1), 014003 (2017). <https://doi.org/10.1088/0954-3899/44/1/014003>. <https://doi.org/10.1088/0022-3727/44/27/274003>. arXiv:1606.00097 [hep-ph]
59. W. Wang, F.S. Yu, Z.X. Zhao, Eur. Phys. J. C **77**(11), 781 (2017). <https://doi.org/10.1140/epjcs/10052-017-5360-1>. arXiv:1707.02834 [hep-ph]
60. W. Wang, Z.P. Xing, J. Xu, Eur. Phys. J. C **77**(11), 800 (2017). <https://doi.org/10.1140/epjcs/10052-017-5363-y>. arXiv:1707.06570 [hep-ph]
61. X.H. Hu, Y.L. Shen, W. Wang, Z.X. Zhao, arXiv:1711.10289 [hep-ph]
62. Y.J. Shi, W. Wang, Y. Xing, J. Xu, Eur. Phys. J. C **78**(1), 56 (2018). <https://doi.org/10.1140/epjcs/10052-018-5532-7>. arXiv:1712.03830 [hep-ph]
63. W. Wang, J. Xu, Phys. Rev. D **97**, 093007 (2018). <https://doi.org/10.1103/PhysRevD.97.093007>. arXiv:1803.01476 [hep-ph]
64. X.G. He, W. Wang, arXiv:1803.04227 [hep-ph]
65. W. Wang, R.L. Zhu, Phys. Rev. D **96**(1), 014024 (2017). <https://doi.org/10.1103/PhysRevD.96.014024>. arXiv:1704.00179 [hep-ph]
66. X. Yan, B. Zhong, R. Zhu, Int. J. Mod. Phys. A **33**, 1850096 (2018). <https://doi.org/10.1142/S0217751X18500963>. arXiv:1804.06761 [hep-ph]
67. B. Bhattacharya, A.A. Petrov, Phys. Lett. B **774**, 430 (2017). <https://doi.org/10.1016/j.physletb.2017.10.004>. arXiv:1708.07504 [hep-ph]
68. D. Zeppenfeld, Z. Phys. C **8**, 77 (1981). <https://doi.org/10.1007/BF01429835>

69. L.L. Chau, H.Y. Cheng, W.K. Sze, H. Yao, B. Tseng, Phys. Rev. D **43**, 2176 (1991). Erratum: [Phys. Rev. D **58**, 019902 (1998)]. <https://doi.org/10.1103/PhysRevD.43.2176>. <https://doi.org/10.1103/PhysRevD.58.019902>
70. M. Gronau, O.F. Hernandez, D. London, J.L. Rosner, Phys. Rev. D **50**, 4529 (1994). <https://doi.org/10.1103/PhysRevD.50.4529>. [arXiv:hep-ph/9404283](https://arxiv.org/abs/hep-ph/9404283)
71. R. Fleischer, Eur. Phys. J. C **10**, 299 (1999). <https://doi.org/10.1007/s100529900099>. [arXiv:hep-ph/9903455](https://arxiv.org/abs/hep-ph/9903455)
72. M. Gronau, Phys. Lett. B **492**, 297 (2000). [https://doi.org/10.1016/S0370-2693\(00\)01119-9](https://doi.org/10.1016/S0370-2693(00)01119-9). [arXiv:hep-ph/0008292](https://arxiv.org/abs/hep-ph/0008292)