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Energy conditions and conservation laws in LTB metric via Noether symmetries

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Abstract In this paper, we have investigated Noether symmetries in Lemaitre–Tolman–Bondi (LTB) metric. Using the Lagrangian associated with the LTB metric, the set of determining equations for Noether symmetries is obtained and then integrated in several cases. It is shown that the LTB metric can be classified in to eight distinct classes corresponding to Noether algebra of dimension 4, 5, 6, 7, 8, 9, 11 and 17. The obtained Noether symmetries are compared with Killing and homothetic vectors. The well known Noether's theorem is used to find the expressions for conservation laws in each case. Moreover, it is shown that most of the obtained metrics are anisotropic or perfect fluid models which satisfy certain energy conditions and the equation of state.

Keywords LTB metric · Noether symmetry · Energy conditions · Equation of state

1 Introduction

The Einstein's field equations (EFEs) form a base for the mathematical structure of general theory of relativity. These equations consist of a system of ten coupled nonlinear partial differential equations which states that $G_{ab} = kT_{ab}$, where T_{ab} is the stress-energy tensor, giving the description of density and flux of energy and momentum in the spacetime, G_{ab} is the Einstein tensor which expresses the curvature of spacetime, and k denotes the gravitational constant. The Einstein tensor contains all the basic geometric properties of the spacetimes and it is given by $G_{ab} = R_{ab} - \frac{R}{2}g_{ab}$, where R_{ab} and g_{ab} are the Ricci and metric tensors respectively and R is the scalar curvature. For a given distribution of energy and momentum in the form of T_{ab}, the metric tensor is regarded as an exact solution of the EFEs. If we could find the g_{ab}, the other terms appearing in the EFEs can be found by a nonlinear manner.

Finding the exact solutions of EFEs is an important problem in the theory of general relativity. In fact, the search for new solutions of these equations had opened new avenues to our understanding of the universe. However, finding the exact solutions of EFEs is quite challenging, the reason being their highly nonlinear nature. Since the development of the theory of general relativity, a limited number of exact solutions have been found [1].

One of the known approaches for finding the exact solutions of EFEs is to use some symmetry restrictions on the metric tensor. This symmetry is expressed in terms of Killing vectors (KVs), which requires that the Lie derivative of the metric g_{ab} vanishes along the vector field X, that is $\mathcal{L}_X g_{ab} = 0$, where $\mathcal{L}$ denotes the Lie derivative operator. A comprehensive detail about KVs and the corresponding conservation laws in spacetimes can be found in [1–3].

Some other spacetime symmetries appearing in the literature include homothetic vectors ($\mathcal{L}_X g_{ab} = 2\psi g_{ab}$), where ψ is a constant, curvature collineations ($\mathcal{L}_X R^a_{bcd} = 0$), Ricci collineations ($\mathcal{L}_X R_{ab} = 0$) and matter collineations ($\mathcal{L}_X T_{ab} = 0$). These symmetries are also widely discussed in the literature [4–9].

Noether symmetry was introduced by Emmy Noether [10]. According to Noether theorem, every continuous symmetry admitted by the Lagrangian of a system corresponds to a conservation law. Consequently, this theorem gives conservation of energy and linear and angular momenta of a physical system if it is invariant under time translation and spacial translations and rotations. Noether symmetries usually provide the additional conservation laws, not given by KVs. Moreover, the Lie algebra of KVs is always a subset of the Lie algebra of Noether symmetries. Homothetic vectors also have a close link with Noether symmetries. In fact, if X is a homothetic vector, then $X + 2\psi s \partial_s$ is a Noether symmetry associated with X. Conversely, if $X + 2\psi s \partial_s$ is a Noether symmetry and X is independent of s, then X is a homothetic vector [11].

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In recent literature, Noether symmetries are investigated for some well known spacetimes. Bokhari and Kara [12] studied Noether symmetries in conformally flat Friedmann metric and compared their results with KVs. According to their analysis, the flat Friedmann metric admits additional conservation laws not given by KVs. A similar comparison of KVs and Noether symmetries was done by Bokhari et al. [13] which led them to the conjectur that the Noether symmetries obtained by considering the Lagrangian provide additional conservation laws, not given by KVs. Hickman and Yazdan [11] presented a complete classification of Bianchi type II spacetimes via Noether symmetries where they also showed that the set of Noether symmetries contains Killing as well as homothetic vectors. Ali and his collaborators [14–16] investigated Noether symmetry of static plane, static spherical and static cylindrically symmetric spacetimes. A complete classification of Bianchi type V spacetimes according to their Noether symmetries was given in [17], where some useful physical interpretation of new cosmological solutions were also given. Recently, the Noether symmetries in non static plane symmetric spacetimes were explored by Usamah et al. [18].

Spacetime symmetries, including Noether symmetries and KVs, help in finding the new exact solutions of EFEs. However, although the vacuum EFEs are a well-defined mathematical and physical system, the EFEs with matter do not become such a system until the matter content is specified as a set of fields and the EFEs are supplemented with field equations for each of the matter fields. Thus the study of EFEs with non zero energy-momentum tensor that does not come from particular matter fields may not give physically interesting results. Depending upon the source, the energy-momentum tensor has a particular form. For example, if the source is a perfect fluid, then $T_{ab} = (p + \rho)u_a u_b + p g_{ab}$, where p, ρ and u_a represent pressure, density and four-velocity of the perfect fluid. Similarly, for an isotropic fluid we have $T_{ab} = (\rho + p_\perp)u_a u_b + (p_{||} - p_\perp)n_a n_b + p_\perp g_{ab}$, where ρ, u_a and n_a respectively represent the energy density, four-velocity and spacelike unit vector, whereas $p_\perp$ and $p_{||}$ are the pressures perpendicular and parallel to n_a respectively. Also $u_a u^a = -1$, $n_a n^a = 1$ and $u_a n^a = 0$ [19]. It can be seen that when $p_{||} = p_\perp$, then the T_{ab} of anisotropic fluid reduces to that of a perfect fluid.

An energy condition is a relation one demands the energy-momentum tensor of matter satisfy in order to try to capture the idea that energy density, given by T_{00}, should be non negative. The importance of this condition is that if positive and negative energy regions are allowed, the empty vacuum would become unstable. There are many ways of the generalization of the condition $T_{00} \geq 0$ to the whole tensor. The simplest example is the weak energy condition which stipulates that for any timelike vector v^a at any point of the spacetime manifold, T_{ab} satisfies $T_{ab}v^a v^b \geq 0$. Some other well known energy conditions are null, dominant and strong energy conditions. For an anisotropic fluid, the energy conditions take the following forms:

$$
\begin{aligned}
\text{Dominant energy condition} \; : \quad & \rho \geq 0, \quad \rho \geq |p_{||}|, \quad \rho \geq |p_\perp|, \\
\text{Strong energy condition} \; : \quad & \rho + p_{||} \geq 0, \quad \rho + p_\perp \geq 0, \\
& \rho + p_{||} + 2p_\perp \geq 0, \\
\text{Weak energy condition} \; : \quad & \rho \geq 0, \quad \rho + p_{||} \geq 0, \quad \rho + p_\perp \geq 0, \\
\text{Null energy condition} \; : \quad & \rho + p_{||} \geq 0, \quad \rho + p_\perp \geq 0. \quad (1.1)
\end{aligned}
$$

The above energy conditions reduce to those for a perfect fluid if $p_{||} = p_\perp$. With these notions, we may introduce the equation of state, which is the relation between energy density and components of anisotropic pressure, that is:

$$
p_{||} = p_{||}(\rho), \qquad p_\perp = p_\perp(\rho), \tag{1.2}
$$

which is also equivalent to the relations given by:

$$
\frac{\partial p_{||}}{\partial t}\frac{\partial \rho}{\partial r} = \frac{\partial p_{||}}{\partial r}\frac{\partial \rho}{\partial t}, \qquad \frac{\partial p_\perp}{\partial t}\frac{\partial \rho}{\partial r} = \frac{\partial p_\perp}{\partial r}\frac{\partial \rho}{\partial t}. \tag{1.3}
$$

In this paper, we present Noether symmetries and the corresponding conservation laws admitted by LTB metric. The energy conditions and equation of state are also discussed for the obtained models. In the next section, we derive the determining equations for Noether symmetries using the Lagrangian associated with LTB metric. In Sects. 3, 4, 5, 6, 7, 8, 9 and 10, we present Noether algebras of different dimensions admitted by the LTB metric. The physical implications of all the obtained metrics are also presented in these sections. We give a brief summary of the work at the end of the paper.

2 Derivation of determining equations

The LTB metric is a spherically symmetric but inhomogeneous metric which provides an exact toy model for an inhomogeneous universe. This metric is represented by [20–22]:

$$
ds^2 = -dt^2 + w^2(t, r)dr^2 + u^2(t, r)\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{2.1}
$$

where w and u are non zero functions having both temporal and spatial dependence. If w and u are dependent on t only, this metric reduces to the well known Kantowski–Sachs metric [23]. The non zero components of the energy-momentum tensor for the above metric are:

$$
\begin{aligned}
T_{00} = -\frac{1}{w^3 u^2} \Big(& u'^2 w - \dot{u}^2 w^3 - 2w^2 u \dot{w}\dot{u} \\
& -2uu'w' - w^3 + 2wuu'' \Big),
\end{aligned}
$$

$$T_{11} = -\frac{1}{u^2}\left(2uw^2\ddot{u} + w^2\dot{u}^2 - u'^2 + w^2\right),$$

$$T_{22} = -\frac{u}{w^3}\left(w^3\ddot{u} + w^2\dot{u}\dot{w} + uw^2\ddot{w} + w'u' - wu''\right),$$

$$T_{33} = \sin^2\theta\, T_{22},$$

$$T_{01} = \frac{2}{uw}\left(\dot{w}u' - w\dot{u}'\right). \tag{2.2}$$

If the source is an anisotropic fluid, then the above components of energy-momentum tensor become:

$$T_{00} = \rho, \quad T_{11} = w^2 p_{||}, \quad T_{22} = u^2 p_{\perp},$$

$$T_{33} = \sin^2\theta\, T_{22}, \quad T_{01} = 0, \tag{2.3}$$

and if the source is a perfect fluid, then $p_{||} = p_{\perp} = p$ and we have:

$$T_{00} = \rho, \quad T_{11} = w^2 p, \quad T_{22} = u^2 p,$$

$$T_{33} = \sin^2\theta\, T_{22}, \quad T_{01} = 0. \tag{2.4}$$

The usual Lagrangian corresponding to the metric given in (2.1) is:

$$L = -\dot{t}^2 + w^2(t,r)\dot{r}^2 + u^2(t,r)\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right). \tag{2.5}$$

A vector field V of the form

$$V = \eta\frac{\partial}{\partial s} + V^i\frac{\partial}{\partial x_i}, \tag{2.6}$$

represents a Noether symmetry if it leaves its Lagrangian invariant such that the following condition holds:

$$V^{(1)}L + L(D\eta) = DA, \tag{2.7}$$

where

$$V^{(1)} = V + V_s^i\frac{\partial}{\partial\dot{x}_i} \tag{2.8}$$

is the first prolongation of V such that $V_s^i = DV^i - \dot{x}_i D\eta$. Also D is the differential operator which is defined as:

$$D = \frac{\partial}{\partial s} + \dot{x}_i\frac{\partial}{\partial x_i}. \tag{2.9}$$

Moreover, η, V^i and the Gauge functions A all are functions of (s, t, r, θ, ϕ) and $x_i = (t, r, \theta, \phi)$ are depending variables of s such that $\dot{x}_i = \frac{\partial x_i}{\partial s}$. With the help of Noether theorem, the invariants (conservation laws) corresponding to each Noether symmetry can be found using the following expression:

$$I = \eta L + \left(V^i - \dot{x}_i\eta\right)\frac{\partial L}{\partial\dot{x}_i} - A. \tag{2.10}$$

Simplifying Eq. (2.7) using the Lagrangian (2.5), we obtain the following system of 19 partial differential equations,

known as determining equations:

$$A_{,s} = \eta_{,t} = \eta_{,r} = \eta_{,\theta} = \eta_{,\phi} = 0, \tag{2.11}$$

$$2V_{,t}^0 = \eta_{,s}, \tag{2.12}$$

$$2\dot{w}V^0 + 2w'V^1 + 2wV_{,r}^1 = w\eta_{,s}, \tag{2.13}$$

$$2\dot{u}V^0 + 2u'V^1 + 2uV_{,\theta}^2 = u\eta_{,s}, \tag{2.14}$$

$$2\dot{u}V^0 + 2u'V^1 + 2u\cot\theta V^2 + 2uV_{,\phi}^3 = u\eta_{,s}, \tag{2.15}$$

$$V_{,r}^0 - w^2 V_{,t}^1 = 0, \tag{2.16}$$

$$V_{,\theta}^0 - u^2 V_{,t}^2 = 0, \tag{2.17}$$

$$V_{,\phi}^0 - u^2\sin^2\theta V_{,t}^3 = 0, \tag{2.18}$$

$$w^2 V_{,\theta}^1 + u^2 V_{,r}^2 = 0, \tag{2.19}$$

$$w^2 V_{,\phi}^1 + u^2\sin^2\theta V_{,r}^3 = 0, \tag{2.20}$$

$$V_{,\phi}^2 + \sin^2\theta V_{,\theta}^3 = 0, \tag{2.21}$$

$$2V_{,s}^0 = -A_{,t}, \tag{2.22}$$

$$2w^2 V_{,s}^1 = A_{,r}, \tag{2.23}$$

$$2u^2 V_{,s}^2 = A_{,\theta}, \tag{2.24}$$

$$2u^2\sin^2\theta V_{,s}^3 = A_{,\phi}, \tag{2.25}$$

where dot and prime over the metric functions represent derivatives with respect to t and r respectively. The solution of the above determining equations would give the exact form of LTB metrics along with their Noether symmetries. During the process of integrating these equations, many cases arise which restrict the metric functions to satisfy certain conditions, giving the exact form of LTB metric admitting Noether algebra of dimension 4, 5, 6, 7, 8, 9, 11 and 17. We skip to write the basic algebraic manipulation and present only the exact form of metrics along with their Noether symmetries, invariants and some physical implications including energy conditions and equation of state in the forthcoming sections.

3 Four Noether symmetries

The minimal set of Noether symmetries admitted by LTB metric is:

$$V_0 = \partial_s, \quad V_1 = \sin\phi\partial_\theta + \cot\theta\cos\phi\partial_\phi,$$

$$V_2 = -\cos\phi\partial_\theta + \cot\theta\sin\phi\partial_\phi, \quad V_3 = \partial_\phi. \tag{3.1}$$

Out of these four Noether symmetries, V_1, V_2 and V_3 are the minimum three KVs of LTB metric and V_0 is the symmetry corresponding to the Lagrangian. The above set of minimal Noether symmetries is admitted by the LTB metric for following forms of metric functions.

Using (2.10), the corresponding invariants for the above set of minimal Noether symmetries are obtained as:

$$I_0 = \dot{t}^2 - w^2(t)\dot{r}^2 - u^2(r)\dot{\theta}^2 - u^2(r)\sin^2\theta\dot{\phi}^2,$$

$$I_1 = -2u^2(r)\left(\sin\phi\dot{\theta} + \cos\theta\cos\phi\sin\theta\dot{\phi}\right),$$

Table 1 Metrics admitting four Noether symmetries

No.	$w(t)$	$u(r)$
4a	$(w\dot{w})_{,tt} \neq 0$	$\left(\frac{u}{u'}\right)_{,rr} \neq 0,\ u' \neq 0$
4b	$w = \sqrt{c_1 t^2 + 2c_2 t + c_3};\ c_1 \neq 0$	$\left(\frac{u}{u'}\right)_{,rr} \neq 0,\ u' \neq 0$
4c	$w = \sqrt{c_1 t^2 + 2c_2 t + c_3};\ c_1 \neq 0$	$u = (\alpha r + \beta)^{\frac{1}{\alpha}};\ \alpha \neq 0$
4d	$w = \sqrt{2c_2 t + c_3}$	$\left(\frac{u}{u'}\right)_{,rr} \neq 0,\ u' \neq 0$
4e	$w = \sqrt{2c_2 t + c_3}$	$u = (\alpha r + \beta)^{\frac{1}{\alpha}};\ \alpha \neq 0$

$$I_2 = 2u^2(r)\left(\cos\phi\dot{\theta} - \cos\theta\sin\phi\sin\theta\dot{\phi}\right),$$
$$I_3 = -2u^2(r)\sin^2\theta\dot{\phi}. \tag{3.2}$$

For the models given in **4d** and **4e**, the condition $T_{01} = 0$ implies $w = \sqrt{c_3}$. With this value of w and u satisfying the constraints given in Table 1, the metrics **4d** and **4e** represent anisotropic fluids with the following values of energy density and parallel and perpendicular pressures:

$$\rho = \frac{1}{c_3 u^2}\left(c_3 - u'^2 - 2uu''\right), \quad p_{\parallel} = \frac{u'^2 - c_3}{c_3 u^2},$$
$$p_{\perp} = \frac{u''}{c_3 u}. \tag{3.3}$$

As the above expressions are dependent on one variable, the equation of state $p_{\parallel} = p_{\parallel}(\rho)$, $p_{\perp} = p_{\perp}(\rho)$ is clearly satisfied. As far the energy conditions are concerned, they may impose some extra conditions on the metric function u. For example, the positive energy condition $T_{00} = \rho \geq 0$ will be satisfied if u is chosen such that $u'^2 + 2uu'' \leq c_3$.

The metrics given in **4a, 4b** and **4c** do not satisfy the condition of anisotropic fluid because here the condition $T_{01} = \dot{w}u' = 0$ requires either $\dot{w} = 0$ or $u' = 0$ but none of these two conditions holds true because of the constraints satisfied by w and u given in Table 1.

4 Five Noether symmetries

When the metric functions are dependent on t only, then along with the four basic Noether symmetries given in (3.1), we obtain an extra Noether symmetry (KV) $V_4 = \partial_r$ with the corresponding invariant $I_4 = -2w^2(t)\dot{r}^2$. Table 2 contains the exact forms of the metric functions admitting the set of these five Noether symmetries:

The models **5a** and **5b** represent anisotropic fluids with

$$\rho = \frac{1 + \dot{u}^2}{u^2} + \frac{2\dot{w}\dot{u}}{uw}, \quad p_{\parallel} = -\frac{1}{u^2}\left(1 + \dot{u}^2 + 2u\ddot{u}\right),$$
$$p_{\perp} = -\frac{1}{uw}\left(u\ddot{w} + w\ddot{u} + \dot{u}\dot{w}\right), \tag{4.1}$$

which immediately satisfy the equation of state while the energy conditions may require some extra conditions. For

Table 2 Metrics admitting five Noether symmetries

No.	$w(t)$	$u(t)$
5a	$\dot{w} \neq 0$	$\ddot{u} \neq 0$ and $u \neq \cosh t$
5b	$w \neq \sinh t$	$u = \cosh t$
5c	$(w\dot{w})_{,tt} \neq 0$	$u = \text{const.} = \xi$
5d	$w = \sqrt{c_1 t^2 + 2c_2 t + c_3};\ c_1 \neq 0$	$u = \text{const.} = \xi$
5e	$w = \sqrt{2c_2 t + c_3}$	$u = \text{const.} = \xi$

model **5c** the physical terms are given by:

$$\rho = \frac{1}{\xi^2}, \quad p_{\parallel} = -\frac{1}{\xi^2}, \quad p_{\perp} = -\frac{\ddot{w}}{w}. \tag{4.2}$$

Here the energy density is always positive and the dominant and weak energy conditions respectively require $\xi^2 \mid \frac{\ddot{w}}{w} \mid \leq 1$ and $\xi^2 \frac{\ddot{w}}{w} \leq 1$. Moreover, the strong energy condition will be satisfied if $\xi^2 \frac{\ddot{w}}{w} \leq 1$ and $\frac{\ddot{w}}{w} \leq 0$. The equation of state for these models is given by $p_{\parallel} = -\rho$, $p_{\perp} = -\frac{\xi^2 \ddot{w}\rho}{w}$.

Moreover, for model **5d**, we have $\rho = \frac{1}{\xi^2}$, $p_{\parallel} = -\frac{1}{\xi^2}$, and $p_{\perp} = \frac{c_2^2 - c_1 c_3}{(c_1 t^2 + 2c_2 t + c_3)^2}$. Here the equation of state holds and the strong and weak energy conditions are satisfied if $c_2^2 \geq c_1 c_3$. Finally, the model **5e** is a special case of the metric **5d** when $c_1 = 0$ and this model immediately satisfies the strong and weak energy conditions.

5 Six Noether symmetries

Table 3 shows different metrics admitting six Noether symmetries along with their Noether generators and invariants. Among these six, four are the basic Noether symmetries which are same as given in (3.1) and the extra two Noether symmetries are presented for each metric.

For all the above cases, V_5 is a KV. One can see that the Noether symmetry V_4 for the metric **6c** corresponds to a homothetic vector $\frac{t}{2}\partial_t + \frac{(\alpha r + \beta)}{2}\partial_r$ with the homothety constant $\psi = \frac{1}{2}$. For the remaining three metric, V_4 is a proper Noether symmetry.

Table 3 Metrics admitting six Noether symmetries

No.	$w(t,r)$	$u(t,r)$	Noether symmetries	Invariants
6a	const. $\gamma \neq 0$	$u' = 0, \ddot{u} \neq 0,$	$V_4 = \frac{s}{\gamma^2}\partial_r; \quad A = -2r$	$I_4 = -2(s\dot{r} - r)$
		$u \neq \cosh t$	$V_5 = \partial_r;$	$I_5 = -2\gamma^2 \dot{r}^2$
6b	$w(r)$	$u' = 0, \ddot{u} \neq 0,$	$V_4 = \frac{s\partial_r}{2w}; \quad A = \int w\,dr$	$I_4 = -(sw\dot{r} + \int w\,dr)$
		$u \neq \cosh t$	$V_5 = \frac{\partial_r}{w}$	$I_5 = -2w\dot{r}$
6c	$(c_1 t + 2c_3)^{(1-\alpha)}$	$(\alpha r + \beta)^{\frac{1}{\alpha}}$	$V_4 = s\partial_s + \frac{t\partial_t}{2} + \frac{(\alpha r + \beta)\partial_r}{2}$	$I_4 = sL + t\dot{i} - (c_1 t + 2c_3)^{2(1-\alpha)}(\alpha r + \beta)\dot{r}$
			$V_5 = \partial_t$	$I_5 = 2\dot{i}$
6d	$w(r);$	$u(r);$	$V_4 = -\frac{s\partial_t}{2}; \quad A = t$	$I_4 = -s\dot{i} - t$
	$w \neq u_r$	$u_r \neq 0$	$V_5 = \partial_t$	$I_5 = 2\dot{i}$

The metrics **6a** and **6b** are anisotropic models whose energy-momentum tensor give:

$$\rho = \frac{1 + \dot{u}^2}{u^2}, \quad p_{||} = -\frac{1}{u^2}\left(1 + \dot{u}^2 + 2u\ddot{u}\right), \quad p_\perp = -\frac{\ddot{u}}{u}, \tag{5.1}$$

which satisfy equation of state and the energy density remains positive for any value of u. The energy conditions for this model are satisfied conditionally and the simplified form of such conditions can be easily obtained by using (5.1) in (1.1).

For the metric **6c**, the condition $T_{01} = 0$ implies either $c_1 = 0$ or $\alpha = 1$. If $c_1 = 0$, then w becomes a constant and u is a function of r only. Consequently, the quantities ρ, $p_{||}$ and $p_\perp$ are all functions of single variable and hence satisfy equation of state. The second case, that is when $\alpha = 1$, leads to the vacuum spacetime with $\rho = p_{||} = p_\perp = 0$.

Finally, the metric **6d** represents an anisotropic model whose energy-momentum tensor gives:

$$\rho = -\frac{2u''}{uw^2} + \frac{2u'w'}{uw^3} + \frac{1}{u^2} - \frac{u'^2}{u^2 w^2},$$
$$p_{||} = \frac{u'^2 - w^2}{w^2 u^2}, \quad p_\perp = \frac{u''}{uw^2} - \frac{u'w'}{uw^3}. \tag{5.2}$$

Since all the above quantities are functions of single variable, so they clearly satisfy the equation of state.

6 Seven Noether symmetries

Different metrics admitting seven Noether symmetries are presented in Table 4. Out of these seven Noether symmetries, four are same as given in by (3.1). The fifth Noether symmetry (KV), given by $V_4 = \partial_r$, is same for all these metrics while the remaining two Noether symmetries are given along with each metric.

In all the above cases except **7d** and **7f**, V_5 and V_6 are KVs. In case **7d**, V_6 corresponds to a homothetic vector $\frac{\alpha t + \beta}{2\alpha}\partial_t$,

while V_5 is a proper Noether symmetry. Finally in case **7f**, V_6 is a KV while V_5 corresponds to a homothetic vector $\frac{\alpha t + \beta}{2\alpha}\partial_t$.

The models **7a** and **7c** are anisotropic fluids with $\rho = \frac{1}{\xi^2}$, $p_{||} = -\frac{1}{\xi^2}$, and $p_\perp = k^2$. These quantities clearly satisfy the equation of state, $p_{||} = -\rho$, $p_\perp = \xi^2 k^2 \rho$. Moreover, the strong, weak and null energy conditions are satisfied, while the dominant energy condition requires $k^2 \xi^2 \leq 1$.

Similarly, the models **7b**, **7e** and **7g** are also anisotropic fluids whose energy-momentum tensor gives $\rho = -p_{||} = \frac{1}{\xi^2}$ and $p_\perp = -k^2$. Here the strong energy condition is failed, while the dominant, weak and null energy conditions are satisfied if $k^2 \xi^2 \leq 1$.

For the metric **7f**, we have:

$$\rho = \frac{c_1 + 3c_1\alpha^2 - 4c_3\alpha^2}{c_1(\alpha t + \beta)^2}, \quad p_{||} = -\frac{1 + \alpha^2}{(\alpha t + \beta)^2},$$
$$p_\perp = -\left(\frac{\alpha c_1 - 2c_3\alpha}{c_1(\alpha t + \beta)}\right)^2, \tag{6.1}$$

where the equation of state clearly holds and the strong energy condition is satisfied if either $c_1 \leq 2c_3 \leq 0$ or $c_1 \geq 2c_3 \geq 0$. The model given in case **7d** is a special case of the metric of case **7f** and for this model, the strong energy condition is immediately satisfies.

7 Eight Noether symmetries

There are two metrics admitting eight Noether symmetries:

8a. $\quad ds^2 = -dt^2 + \gamma^2 dr^2 + (\alpha t + \beta)^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right),$ (7.1)

where γ is a non zero constant. Among these eight symmetries, four are given in (3.1), while the remaining four are:

$$V_4 = \partial_r,$$
$$V_5 = s\partial_s + \frac{\alpha t + \beta}{2\alpha}\partial_t + \frac{r}{2}\partial_r,$$

Table 4 Metrics admitting seven Noether symmetries

No.	$w(t,r)$	$u(t,r)$	Noether symmetries	Invariants
7a	$\sin(kt); k \neq 0$	const. $\xi \neq 0$	$V_5 = \cosh(kr)\partial_t - \cot(kt)\sinh(kr)\partial_r$ $V_6 = \sinh(kr)\partial_t - \cot(kt)\cosh(kr)\partial_r$	$I_5 = 2\cosh(kr)\dot{t} + 2\cos(kt)\sin(kr)\sinh(kr)\dot{r}$ $I_6 = 2\sinh(kr)\dot{t} + 2\cos(kt)\sin(kr)\cosh(kr)\dot{r}$
7b	e^{kt}	const. $\xi \neq 0$	$V_5 = \partial_t - kr\partial_r$ $V_6 = r\partial_t - \dfrac{e^{-2kt}}{2k}\partial_r - \dfrac{kr^2}{2}\partial_r$	$I_5 = 2\dot{t} + 2kre^{2kt}\dot{r}$ $I_6 = 2r\dot{t} + \dfrac{\dot{r}}{k} + kr^2 e^{2kt}\dot{r}$
7c	$\cos(kt)$	const. $\xi \neq 0$	$V_5 = \cosh(kr)\partial_t + \tan(kt)\sinh(kr)\partial_r$ $V_6 = \sinh(kr)\partial_t + \tan(kt)\cosh(kr)\partial_r$	$I_5 = 2\cosh(kr)\dot{t} - 2\cos(kt)\sin(kr)\sinh(kr)\dot{r}$ $I_6 = 2\sinh(kr)\dot{t} - 2\cos(kt)\sin(kr)\cosh(kr)\dot{r}$
7d	$\alpha t + \beta$	$\alpha t + \beta$	$V_5 = \dfrac{s^2}{2}\partial_s + \dfrac{s(\alpha t+\beta)}{2\alpha}\partial_t; \quad A = \dfrac{t(\alpha t+2\beta)}{2\alpha}$ $V_6 = s\partial_s + \dfrac{\alpha t+\beta}{2\alpha}\partial_t$	$I_5 = \dfrac{s^2}{2}L + \dfrac{s(\alpha t+\beta)\dot{t}}{\alpha} - \dfrac{t^2}{2} - \dfrac{\beta t}{\alpha}$ $I_6 = sL + \dfrac{(\alpha t+\beta)\dot{t}}{\alpha}$
7e	$\sinh(kt); k \neq 0$	const. $\xi \neq 0$	$V_5 = \cosh(kr)\partial_t - \coth(kt)\sinh(kr)\partial_r$ $V_6 = \sinh(kr)\partial_t - \coth(kt)\cosh(kr)\partial_r$	$I_5 = 2\cosh(kr)\dot{t} + 2\cosh(kt)\sinh(kt)\sinh(kr)\dot{r}$ $I_6 = 2\sinh(kr)\dot{t} + 2\cosh(kt)\sinh(kt)\cosh(kr)\dot{r}$
7f	$(\alpha t + \beta)^{\frac{c_1-2c_3}{c_1}};$ $c_1 \neq 0$	$\alpha t + \beta$	$V_5 = s\partial_s + \dfrac{\alpha t+\beta}{2\alpha}\partial_t$ $V_6 = r\partial_r$	$I_5 = sL + \dfrac{(\alpha t+\beta)\dot{t}}{\alpha}$ $I_6 = -2r(\alpha t+\beta)\dot{t} + \dfrac{2(c_1-2c_3)}{c_1}r$
7g	$\cosh(kt)$	const. $\xi \neq 0$	$V_5 = \cos(kr)\partial_t - \tanh(kt)\sin(kr)\partial_r$ $V_6 = \sin(kr)\partial_t + \tanh(kt)\cos(kr)\partial_r$	$I_5 = 2\cos(kr)\dot{t} + 2\cosh(kt)\sinh(kt)\sin(kr)\dot{r}$ $I_6 = 2\sin(kr)\dot{t} - 2\cosh(kt)\sinh(kt)\cos(kr)\dot{r}$

$$V_6 = \frac{s^2}{2}\partial_s + \frac{s(\alpha t+\beta)}{2\alpha}\partial_t + \frac{sr}{2}\partial_r; \quad A = \frac{t^2}{2} + \frac{\beta t}{\alpha} - \frac{r^2\gamma^2}{2},$$

$$V_7 = \frac{s}{\gamma^2}\partial_r; \quad A = -2r. \tag{7.2}$$

Out of these four Noether symmetries, V_4 is a KV, V_5 corresponds to a homothetic vector $\frac{\alpha t+\beta}{2\alpha}\partial_t + \frac{r}{2}\partial_r$ and the remaining two are proper Noether symmetries. The conservation laws for these four symmetries are given by:

$$I_4 = -2w^2(t)\dot{r}^2,$$

$$I_5 = sL + \frac{(\alpha t+\beta)\dot{t}}{\alpha} - r\gamma^2\dot{r},$$

$$I_6 = \frac{s^2}{2}L + \frac{s(\alpha t+\beta)\dot{t}}{\alpha} - sr\gamma^2\dot{r} - \frac{t^2}{2} - \frac{\beta t}{\alpha} + \frac{r^2\gamma^2}{2},$$

$$I_7 = -2s\dot{r} + 2r. \tag{7.3}$$

The metric (7.1) represents anisotropic fluid whose energy-momentum tensor give:

$$\rho = \frac{1+\alpha^2}{(\alpha t+\beta)^2}, \quad p_\parallel = -\frac{1+\alpha^2}{(\alpha t+\beta)^2}, \quad p_\perp = 0, \tag{7.4}$$

which shows that there is no pressure in the perpendicular direction and all the energy conditions are satisfied. The equation of state for the above model is given by $p_\parallel = -\rho$.

8b. $\quad ds^2 = -dt^2 + w^2(r)dr^2 + (\alpha t+\beta)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$ (7.5)

Out of the eight Noether symmetries admitted by the above metric, four are same as given in (3.1) and the extra four Noether symmetries are listed below:

$$V_4 = \frac{\partial_r}{w},$$

$$V_5 = s\partial_s + \frac{\alpha t+\beta}{2\alpha}\partial_t + \frac{\int w\,dr}{2w}\partial_r,$$

$$V_6 = \frac{s\partial_r}{2w}; \quad A = \int w\,dr,$$

$$V_7 = \frac{s^2}{2}\partial_s + \frac{s(\alpha t+\beta)}{2\alpha}\partial_t + \frac{s\int w\,dr}{2w}\partial_r;$$

$$A = -\frac{t^2}{2} - \frac{\beta t}{\alpha} + \frac{(\int w\,dr)^2}{2}. \tag{7.6}$$

Here V_4 is a KV, V_5 corresponds to a homothetic vector $\frac{\alpha t+\beta}{2\alpha}\partial_t + \frac{\int w\,dr}{2w}\partial_r$ and V_6 and V_7 are proper Noether symmetries. The conservation laws for these four symmetries are:

$$I_4 = -2w\dot{r}, \quad I_5 = sL + \frac{(\alpha t+\beta)\dot{t}}{\alpha} - w\int w\,dr\,\dot{r},$$

$$I_6 = -\left(sw\dot{r} + \int w\,dr\right),$$

$$I_7 = \frac{s^2}{2}L + \frac{s(\alpha t+\beta)\dot{t}}{\alpha} - sw\int w\,dr\,\dot{r}$$

Table 5 Metrics admitting nine Noether symmetries

No.	$w(t,r)$	$u(t,r)$	Noether symmetries	Invariants
9a	$\beta \neq 0$	$\cosh t$	$V_4 = s\partial_r;\quad A = -2r$	$I_4 = -2(s\beta^2\dot{r} - r)$
			$V_5 = \partial_r$	$I_5 = -2\beta^2\dot{r}^2$
			$V_6 = \sin\theta\sin\phi\partial_t + \tanh t\cos\theta\sin\phi\partial_\theta$	$I_6 = -2\sinh t\cosh t(\cos\theta\sin\phi\dot\theta +$
				$\sin\theta\cos\phi\dot\phi)$
			$+\csc\theta\tanh t\cos\phi\partial_\phi$	$+2\sin\theta\sin\phi\dot{t}$
			$V_7 = -\sin\theta\cos\phi\partial_t - \tanh t\cos\theta\cos\phi\partial_\theta$	$I_7 =$
				$2\sinh t\cosh t(\cos\theta\cos\phi\dot\theta - \sin\theta\sin\phi\dot\phi)$
			$+\csc\theta\tanh t\sin\phi\partial_\phi$	$-2\sin\theta\cos\phi\dot{t}$
			$V_8 = -\cos\theta\partial_t + \tanh t\sin\theta\partial_\theta$	$I_8 = -2\cos\theta\dot{t} - 2\sinh t\cosh t\sin\theta\dot\theta$
9b	$w(r)$	$\cosh t$	$V_4 = \frac{s\partial_r}{2w};\quad A = \int w\,dr$	$I_4 = -(sw\dot{r} + \int w\,dr)$
			$V_5 = \frac{\partial_r}{w}$	$I_5 = -2w\dot{r}$
			V_6, V_7 and V_8 are same as given in **9a**	I_6, I_7 and I_8 are same as in **9a**
9c	$\alpha \neq 0$	$\xi \neq 0$	$V_4 = \frac{s}{\alpha^2}\partial_r;\quad A = -2r$	$I_4 = -2s\dot{r} + 2r$
			$V_5 = -s\partial_t;\quad A = -2t$	$I_5 = -2s\dot{t} + 2t$
			$V_6 = \partial_t,\quad V_7 = r\partial_t + \frac{t}{\alpha^2}\partial_r$	$I_6 = 2\dot{t},\quad I_7 = 2r\dot{t} - 2t\dot{r}$
			$V_8 = \partial_r$	$I_8 = -2\alpha^2\dot{r}^2$
9d	$w(r)$	$\xi \neq 0$	$V_4 = \frac{s\partial_r}{2w};\quad A = \int w\,dr$	$I_4 = -(sw\dot{r} + \int w\,dr)$
			$V_5 = \frac{-s}{2}\partial_t;\quad A = t$	$I_5 = -(s\dot{t} + t)$
			$V_6 = \frac{\partial_r}{w}$	$I_6 = -2w\dot{r}$
			$V_7 = \int w\,dr\,\partial_t + \frac{t}{w}\partial_r,\quad V_8 = \partial_t$	$I_7 = 2(\int w\,dr\,\partial_t - tw\dot{r}),\quad I_8 = 2\dot{r}$
9e	$\alpha t + \beta$	$\xi \neq 0$	$V_4 = -s\sinh(\alpha r)\partial_t + \frac{s}{\alpha t+\beta}\cosh(\alpha r)\partial_r;$	$I_4 =$
			$A = -\frac{2\beta}{\alpha}\sinh(\alpha r)$	$-2s\left(\sinh(\alpha r)\dot{t} + (\alpha t + \beta)\cosh(\alpha r)\dot{r}\right) +$
				$\frac{2\beta}{\alpha}\sinh(\alpha r)$
			$V_5 = -s\cosh(\alpha r)\partial_t + \frac{s}{\alpha t+\beta}\sinh(\alpha r)\partial_r;$	$I_5 =$
			$A = -\frac{2\beta}{\alpha}\cosh(\alpha r)$	$-2s\left(\cosh(\alpha r)\dot{t} + (\alpha t + \beta)\sinh(\alpha r)\dot{r}\right) +$
				$\frac{2\beta}{\alpha}\cosh(\alpha r)$
			$V_6 = \partial_r$	$I_6 = -2(\alpha t + \beta)^2\dot{r}^2$
			$V_7 = \cosh(\alpha r)\partial_t - \frac{\sinh(\alpha r)}{\alpha t+\beta}\partial_r$	$I_7 = 2\left(\cosh(\alpha r)\dot{t} + (\alpha t + \beta)\sinh(\alpha r)\dot{r}\right)$
			$V_8 = \sinh(\alpha r)\partial_t - \frac{\cosh(\alpha r)}{\alpha t+\beta}\partial_r$	$I_8 = 2\left(\sinh(\alpha r)\dot{t} + (\alpha t + \beta)\cosh(\alpha r)\dot{r}\right)$

$$+\frac{t^2}{2} + \frac{\beta t}{\alpha} - \frac{(\int w\,dr)^2}{2}. \tag{7.7}$$

The metric (7.5) is anisotropic fluid with ρ, $p_{||}$ and $p_\perp$ same as given in (7.4) and hence we have the same remarks regarding the energy conditions and equation of state as for the previous metric.

8 Nine Noether symmetries

Table 5 presents different LTB metrics admitting nine Noether symmetries, four of which are same as given in (3.1), while the remaining five symmetries along with the corresponding invariants are presented below.

For the metrics **9a** and **9b**, V_4 is a proper Noether symmetry while $V_5, \ldots, V_8$ are KVs. For the metrics **9c**, **9d** and **9e**, we have two proper Noether symmetries V_4 and V_5 and three KVs, given by V_6, V_7 and V_8.

All the metrics presented in the above table are anisotropic fluids. For the metrics **9a** and **9b**, we have $\rho = 1$, $p_{||} = -3$ and $p_\perp = -1$. Hence none of the energy conditions holds except the positive energy conditions, that is $\rho \geq 0$. The equation of state is given by $p_{||} = -3\rho$, $p_\perp = -\rho$. For the remaining three models, we have $\rho = \frac{1}{\xi^2}$, $p_{||} = -\frac{1}{\xi^2}$ and $p_\perp = 0$ such that equation of state is $p_{||} = -\rho$ and all the energy conditions are satisfied.

9 Eleven Noether symmetries

Following is the only one LTB metric admitting eleven Noether symmetries:

$$ds^2 = -dt^2 + \sinh^2 t\,dr^2 + \cosh^2 t\,(d\theta^2 + \sin^2\theta\,d\phi^2). \tag{9.1}$$

Four Noether symmetries of the above metric are same as given in (3.1) and the extra seven symmetries are listed below:

$$V_4 = \partial_r,$$

$$V_5 = \sin\theta\sin\phi\sinh r\partial_t - \coth t\sin\theta\sin\phi\cosh r\partial_r$$
$$+\tanh t\sinh r\cos\theta\sin\phi\partial_\theta$$
$$+\tanh t\sinh r\csc\theta\cos\phi\partial_\phi,$$

$$V_6 = \sin\theta\sin\phi\cosh r\partial_t - \coth t\sin\theta\sin\phi\sinh r\partial_r$$
$$+\tanh t\cosh r\cos\theta\sin\phi\partial_\theta$$
$$+\tanh t\cosh r\csc\theta\cos\phi\partial_\phi),$$

$$V_7 = -\sin\theta\cos\phi\sinh r\partial_t + \coth t\sin\theta\cos\phi\cosh r\partial_r$$
$$-\tanh t\sinh r\cos\theta\cos\phi\partial_\theta$$
$$+\tanh t\sinh r\csc\theta\sin\phi\partial_\phi),$$

$$V_8 = -\sin\theta\cos\phi\cosh r\partial_t + \coth t\sin\theta\cos\phi\sinh r\partial_r$$
$$-\tanh t\cosh r\cos\theta\cos\phi\partial_\theta$$
$$+\tanh t\cosh r\csc\theta\sin\phi\partial_\phi),$$

$$V_9 = -\cos\theta\sinh r\partial_t + \coth t\cos\theta\cosh r\partial_r$$
$$+\tanh t\sinh r\sin\theta\partial_\theta,$$

$$V_{10} = -\cos\theta\cosh r\partial_t + \coth t\cos\theta\sinh r\partial_r$$
$$+\tanh t\cosh r\sin\theta\partial_\theta. \tag{9.2}$$

All the above seven symmetries are KVs. Thus the metric (9.1) admits ten KVs along with a proper Noether symmetry ∂_s. Consequently, the spacetime becomes flat. The corresponding invariant quantities are given by:

$$I_4 = -2w^2(t)\dot r^2,$$

$$I_5 = 2\sin\theta\sin\phi\left(\sinh r\dot t + \cosh t\sinh t\cosh r\dot r\right)$$
$$-2\cosh t\sinh t\sinh r\cos\theta\sin\phi\dot\theta$$
$$-2\cosh t\sinh t\sinh r\sin\theta\cos\phi\dot\phi,$$

$$I_6 = 2\sin\theta\sin\phi\left(\cosh r\dot t + \cosh t\sinh t\sinh r\dot r\right)$$
$$-2\cosh t\sinh t\cosh r\cos\theta\sin\phi\dot\theta$$
$$-2\cosh t\sinh t\cosh r\sin\theta\cos\phi\dot\phi,$$

$$I_7 = -2\sin\theta\cos\phi\left(\sinh r\dot t + \cosh t\sinh t\cosh r\dot r\right)$$
$$+2\cosh t\sinh t\sinh r\cos\theta\cos\phi\dot\theta$$
$$+2\cosh t\sinh t\sinh r\sin\theta\sin\phi\dot\phi,$$

$$I_8 = -2\sin\theta\cos\phi\left(\cosh r\dot t + \cosh t\sinh t\sinh r\dot r\right)$$
$$+2\cosh t\sinh t\cosh r\cos\theta\cos\phi\dot\theta$$
$$+2\cosh t\sinh t\cosh r\sin\theta\sin\phi\dot\phi,$$

$$I_9 = -2\cos\theta\sinh r\dot t - 2\cosh t\sinh t\cos\theta\cosh r\dot r$$
$$-2\cosh t\sinh t\sinh r\sin\theta\dot\theta,$$

$$I_{10} = -2\cos\theta\cosh r\dot t - 2\cosh t\sinh t\cos\theta\sinh r\dot r$$
$$-2\cosh t\sinh t\cosh r\sin\theta\dot\theta. \tag{9.3}$$

The metric (9.1) represents a perfect fluid model with $\rho = 3$ and $p_{||} = p_\perp = -3$. These quantities violate the strong energy condition, while the remaining energy conditions are

immediately satisfied. In this case, the equation of state is given by $p_{||} = p_\perp = -\rho$.

10 Seventeen Noether symmetries

In [24], it was shown that the flat Minkowski metric admits the maximum number of Noether symmetries, which is 17. Here we present another such metric admitting 17 Noether symmetric. Such a metric is given by:

$$ds^2 = -dt^2 + u_r^2(r)dr^2 + u^2(r)[d\theta^2 + \sin^2\theta d\phi^2]. \tag{10.1}$$

The thirteen Noether symmetries other than the minimal set of Noether symmetries for the above metric are given below:

$$V_4 = s\partial_s + \frac{t}{2}\partial_t + \frac{u}{2u_r}\partial_r$$

$$V_5 = \frac{s^2}{2}\partial_s + \frac{st}{2}\partial_t + \frac{su}{2u_r}\partial_r, \quad A = \frac{u^2 - t^2}{2},$$

$$V_6 = -\frac{s}{2}\partial_t, \quad A = t,$$

$$V_7 = \frac{s}{u_r}\sin\theta\sin\phi\partial_r + \frac{s}{u}(\cos\theta\sin\phi\partial_\theta + \csc\theta\cos\phi\partial_\phi);$$
$$A = 2u\sin\theta\sin\phi,$$

$$V_8 = -\frac{s}{u_r}\sin\theta\cos\phi\partial_r - \frac{s}{u}(\cos\theta\cos\phi\partial_\theta - \csc\theta\sin\phi\partial_\phi);$$
$$A = -2u\sin\theta\cos\phi,$$

$$V_9 = -\frac{s}{u_r}\cos\theta\partial_r + \frac{s}{u}\sin\theta\partial_\theta; \quad A = -2u\cos\theta,$$

$$V_{10} = \partial_t,$$

$$V_{11} = \sin\theta\sin\phi\left(u\partial_t + \frac{t}{u_r}\partial_r\right)$$
$$+\frac{t}{u}\left(\cos\theta\sin\phi\partial_\theta + \csc\theta\cos\phi\partial_\phi\right),$$

$$V_{12} = -\sin\theta\cos\phi\left(u\partial_t + \frac{t}{u_r}\partial_r\right)$$
$$-\frac{t}{u}\left(\cos\theta\cos\phi\partial_\theta - \csc\theta\sin\phi\partial_\phi\right),$$

$$V_{13} = -u\cos\theta\partial_t - \frac{t}{u_r}\cos\theta\partial_r + \frac{t}{u}\sin\theta\partial_\theta,$$

$$V_{14} = \frac{\sin\theta\sin\phi}{u_r}\partial_r + \frac{1}{u}(\cos\theta\sin\phi\partial_\theta + \csc\theta\cos\phi\partial_\phi),$$

$$V_{15} = -\frac{\sin\theta\cos\phi}{u_r}\partial_r - \frac{1}{u}(\cos\theta\cos\phi\partial_\theta - \csc\theta\sin\phi\partial_\phi),$$

$$V_{16} = -\frac{\cos\theta}{u_r}\partial_r + \frac{\sin\theta}{u}\partial_\theta. \tag{10.2}$$

In the above list, V_4 is a Noether symmetry corresponding to a homothetic vector $\frac{t}{2}\partial_t + \frac{u}{2u_r}\partial_r$. Moreover, $V_5, \ldots, V_9$ are proper Noether symmetries, while $V_{10}, \ldots, V_{16}$ are KVs. The corresponding conservation laws for these Noether symmetries are:

$$I_4 = sL + t\dot{t} - uu_r\dot{r},$$

$$I_5 = \frac{s^2}{2}L + st\dot{t} - suu_r\dot{r} + \frac{t^2 - u^2}{2},$$

$$I_6 = -(s\dot{t} + t),$$

$$I_7 = -2su_r \sin\theta \sin\phi\dot{r} - 2su \cos\theta \sin\phi\dot{\theta}$$
$$- 2su \sin\theta \cos\phi\dot{\phi} - 2u \sin\theta \sin\phi,$$

$$I_8 = 2su_r \sin\theta \cos\phi\dot{r} + 2su \cos\theta \cos\phi\dot{\theta}$$
$$- 2su \sin\theta \sin\phi\dot{\phi} + 2u \sin\theta \cos\phi,$$

$$I_9 = 2su_r \cos\theta\dot{r} - 2su \sin\theta\dot{\theta} + 2u \cos\theta,$$

$$I_{10} = t\dot{t},$$

$$I_{11} = 2 \sin\theta \sin\phi(u\dot{t} - tu_r\dot{r}) - 2tu(\cos\theta \sin\phi\dot{\theta}$$
$$+ \sin\theta \cos\phi\dot{\phi}),$$

$$I_{12} = -2 \sin\theta \cos\phi(u\dot{t} - tu_r\dot{r}) + 2tu(\cos\theta \cos\phi\dot{\theta}$$
$$- \sin\theta \sin\phi\dot{\phi}),$$

$$I_{13} = -2 \cos\theta(u\dot{t} - tu_r\dot{r}) - 2tu \sin\theta\dot{\theta},$$

$$I_{14} = -2u_r \sin\theta \sin\phi\dot{r} - 2u \cos\theta \sin\phi\dot{\theta}$$
$$- 2u \sin\theta \cos\phi\dot{\phi},$$

$$I_{15} = 2u_r \sin\theta \cos\phi\dot{r} + 2u \cos\theta \cos\phi\dot{\theta}$$
$$- 2u \sin\theta \sin\phi\dot{\phi},$$

$$I_{16} = 2u_r \cos\theta\dot{r} - 2u \sin\theta\dot{\theta}. \tag{10.3}$$

For this metric, all the energy-momentum tensor components vanish and hence it gives a vacuum solution.

11 Summary

In this paper, we have presented a classification of LTB spacetimes metric according to their Noether symmetries. The determining equations for Noether symmetries are integrated in several cases and it is concluded that the LTB metric may admit 4, 5, 6, 7, 8, 9, 11 and 17 Noether symmetries. The minimal set of Noether symmetries contains three Killing vectors and one proper Noether symmetry, while the maximum dimension of Noether symmetries turned out to be 17 which is admitted by the model given in (10.1). The obtained Noether symmetries are compared with Killing and homothetic vectors in each case and it is observed that the number of KVs for LTB metric is 3, 4, 5, 6, 7 or 10. For all the obtained metrics, the Noether theorem is used to obtain the expressions for conservations laws for each Noether symmetry. Moreover, it is shown that most of the metrics obtained are anisotropic or perfect fluid models which satisfy certain energy conditions and the equation of state.

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