

Holographic RG flows in $N = 4$ SCFTs from half-maximal gauged supergravity

Parinya Karndumri^a, Khem Upathambhakul^b

String Theory and Supergravity Group, Department of Physics, Faculty of Science, Chulalongkorn University, 254 Phayathai Road, Pathumwan, Bangkok 10330, Thailand

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Abstract We study four-dimensional $N = 4$ gauged supergravity coupled to six vector multiplets with semisimple gauge groups $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. All of these gauge groups are dyonically embedded in the global symmetry group $SO(6, 6)$ via its maximal subgroup $SO(3, 3) \times SO(3, 3)$. For $SO(4) \times SO(4)$ gauge group, there are four $N = 4$ supersymmetric AdS_4 vacua with $SO(4) \times SO(4)$, $SO(4) \times SO(3)$, $SO(3) \times SO(4)$ and $SO(3) \times SO(3)$ symmetries, respectively. These AdS_4 vacua correspond to $N = 4$ SCFTs in three dimensions with $SO(4)$ R-symmetry and different flavor symmetries. We explicitly compute the full scalar mass spectra at all these vacua. Holographic RG flows interpolating between these conformal fixed points are also given. The solutions describe supersymmetric deformations of $N = 4$ SCFTs by relevant operators of dimensions $\Delta = 1, 2$. A number of these solutions can be found analytically although some of them can only be obtained numerically. These results provide a rich and novel class of $N = 4$ fixed points in three-dimensional Chern–Simons–Matter theories and possible RG flows between them in the framework of $N = 4$ gauged supergravity in four dimensions. Similar studies are carried out for non-compact gauge groups, but the $SO(4) \times SO(4)$ gauge group exhibits a much richer structure.

1 Introduction

The study of holographic RG flows is one of the most interesting results from the celebrated AdS/CFT correspondence since its original proposal in [1]. The solutions take the form of domain walls interpolating between AdS vacua, for RG flows between two conformal fixed points, or between an AdS vacuum and a singular geometry, for RG flows from a

conformal fixed point to a non-conformal field theory. Many of these fixed points are described by superconformal field theories (SCFTs) which are also believed to give some insight into the dynamics of various branes in string/M-theory.

Rather than finding holographic RG flow solutions directly in string/M-theory, a convenient and effective way to find these solutions is to look for domain wall solutions in lower dimensional gauged supergravities. In some cases, the resulting solutions can be uplifted to interesting brane configurations within string/M-theory, see for example [2–4]. Apart from rendering the computation simpler, working in lower dimensional gauged supergravities also has an advantage of being independent of higher dimensional embedding. Results obtained within this framework are applicable in any models described by the same effective gauged supergravity regardless of their higher dimensional origins.

Many results along this direction have been found in maximally gauged supergravities, see for examples [5–13]. A number of RG flow solutions in half-maximally gauged supergravities in various dimensions have, on the other hand, been studied only recently in [14–21], see also [22, 23] for earlier results. In this paper, we will give holographic RG flow solutions within $N = 4$ gauged supergravity in four dimensions. Solutions in the case of non-semisimple gauge groups with known higher dimensional origins have already been considered in [19, 20]. This non-semisimple gauging, however, turns out to have a very restricted number of supersymmetric AdS_4 vacua. In this work, we will consider semisimple gauging of $N = 4$ supergravity similar to the study in other dimensions. Although some general properties of AdS_4 vacua and RG flows have been pointed out recently in [18], to the best of our knowledge, a detailed analysis and explicit RG flow solutions in $N = 4$ gauged supergravity have not previously appeared.

Gaugings of $N = 4$ supergravity coupled to an arbitrary number n of vector multiplets have been studied and classi-

^a e-mail: parinya.ka@hotmail.com

^b e-mail: keima.tham@gmail.com

fied for a long time [24–26], and the embedding tensor formalism which includes all possible deformations of $N = 4$ supergravity has been given in [27]. For the case of $n < 6$, the relation between the resulting $N = 4$ supergravity and ten-dimensional supergravity is not known. Therefore, we will consider only the case of $n \geq 6$ which is capable of embedding in ten dimensions. Furthermore, we are particularly interested in $N = 4$ gauged supergravity coupled to six vector multiplets to simplify the computation. In this case, possible gauge groups are embedded in the global symmetry group $SL(2, \mathbb{R}) \times SO(6, 6)$. From a general result of [28], the existence of $N = 4$ supersymmetric AdS_4 vacua requires that the gauge group is purely embedded in the $SO(6, 6)$ factor. Furthermore, the gauge group must contain an $SO(3) \times SO(3)$ subgroup with one of the $SO(3)$ factors embedded electrically and the other one embedded magnetically.

We will consider gauge groups in the form of a simple product $G_1 \times G_2$ in which one of the two factors is embedded electrically in $SO(3, 3) \subset SO(6, 6)$ while the other is embedded magnetically in the other $SO(3, 3)$ subgroup of $SO(6, 6)$. Taking the above criteria for having supersymmetric AdS_4 vacua into account, we will study the case of $G_1, G_2 = SO(4)$ and $SO(3, 1)$. There are then three different product gauge groups to be considered, $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. We will identify possible supersymmetric AdS_4 vacua and supersymmetric RG flows interpolating between these vacua. These solutions should describe RG flows in the dual $N = 4$ Chern–Simons–Matter (CSM) theories driven by relevant operators dual to the scalar fields of the $N = 4$ gauged supergravity. As shown in [29], some of the $N = 4$ CSM theories can be obtained from a non-chiral orbifold of the ABJM theory [30]. Other classes of $N = 4$ CSM theories are also known, see [31, 32] for example. These theories play an important role in describing the dynamics of M2-branes on various backgrounds. The solutions obtained in this paper should also be useful in this context via the AdS/CFT correspondence. It should also be emphasized that all gauge groups considered here are currently not known to have higher dimensional origins. Therefore, the corresponding holographic duality in this case is still not firmly established.

The paper is organized as follows. In Sect. 2, we review $N = 4$ gauged supergravity coupled to vector multiplets in the embedding tensor formalism. This sets up the framework we will use throughout the paper and collects relevant formulae and notations used in subsequent sections. In Sect. 3, $N = 4$ gauged supergravity with $SO(4) \times SO(4)$ gauge group is constructed, and the scalar potential for scalars which are singlets under $SO(4)_{\text{inv}} \subset SO(4) \times SO(4)$ is computed. We will identify possible supersymmetric AdS_4 vacua and compute the full scalar mass spectra at these vacua. The section ends with supersymmetric RG flow solutions interpo-

lating between AdS_4 vacua and RG flows to non-conformal field theories. A similar study is performed in Sects. 4 and 5 for non-compact $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$ gauge groups. Conclusions and comments on the results will be given in Sect. 6. An appendix containing the convention on 't Hooft matrices is included at the end of the paper.

2 $N = 4$ gauged supergravity coupled to vector multiplets

To set up our framework, we give a brief review of four-dimensional $N = 4$ gauged supergravity. We mainly give relevant information and necessary formulae to find supersymmetric AdS_4 vacua and domain wall solutions. More details on the construction can be found in [27].

$N = 4$ supergravity can couple to an arbitrary number n of vector multiplets. The supergravity multiplet consists of the graviton $e_{\mu}^{\hat{\mu}}$, four gravitini ψ_{μ}^i , six vectors A_{μ}^m , four spin- $\frac{1}{2}$ fields χ^i and one complex scalar τ containing the dilaton ϕ and the axion χ . The complex scalar can be parametrized by $SL(2, \mathbb{R})/SO(2)$ coset. Each vector multiplet contains a vector field A_{μ} , four gaugini λ^i and six scalars ϕ^m . Similar to the dilaton and the axion in the gravity multiplet, the $6n$ scalar fields can be parametrized by $SO(6, n)/SO(6) \times SO(n)$ coset.

Throughout the paper, space-time and tangent space indices are denoted respectively by $\mu, \nu, \dots = 0, 1, 2, 3$ and $\hat{\mu}, \hat{\nu}, \dots = 0, 1, 2, 3$. The $SO(6) \sim SU(4)$ R-symmetry indices will be described by $m, n = 1, \dots, 6$ for the $SO(6)$ vector representation and $i, j = 1, 2, 3, 4$ for the $SO(6)$ spinor or $SU(4)$ fundamental representation. The n vector multiplets will be labeled by indices $a, b = 1, \dots, n$. All fields in the vector multiplets accordingly carry an additional index in the form of $(A_{\mu}^a, \lambda^{ia}, \phi^{ma})$.

Fermionic fields and the supersymmetry parameters transforming in the fundamental representation of $SU(4)_R \sim SO(6)_R$ R-symmetry are subject to the chirality projections

$$\gamma_5 \psi_{\mu}^i = \psi_{\mu}^i, \quad \gamma_5 \chi^i = -\chi^i, \quad \gamma_5 \lambda^i = \lambda^i \quad (1)$$

while the fields transforming in the anti-fundamental representation of $SU(4)_R$ satisfy

$$\gamma_5 \psi_{\mu i} = -\psi_{\mu i}, \quad \gamma_5 \chi_i = \chi_i, \quad \gamma_5 \lambda_i = -\lambda_i. \quad (2)$$

Gaugings of the matter-coupled $N = 4$ supergravity can be described by two components of the embedding tensor $\xi^{\alpha M}$ and $f_{\alpha MNP}$ with $\alpha = (+, -)$ and $M, N = (m, a) = 1, \dots, n + 6$ denoting fundamental representations of $SL(2, \mathbb{R})$ and $SO(6, n)$, respectively. Under the full $SL(2, \mathbb{R}) \times SO(6, n)$ duality symmetry, the electric vector fields $A^{+M} = (A_{\mu}^m, A_{\mu}^a)$, appearing in the ungauged

Lagrangian, together with their magnetic dual A^{-M} form a doublet under $SL(2, \mathbb{R})$ denoted by $A^{\alpha M}$. A general gauge group is embedded in both $SL(2, \mathbb{R})$ and $SO(6, n)$, and the magnetic vector fields can also participate in the gauging. However, each magnetic vector field must be accompanied by an auxiliary two-form field in order to remove the extra degrees of freedom.

From the analysis of supersymmetric AdS_4 vacua in [28], see also [25, 26], purely electric gaugings do not admit AdS_4 vacua unless an $SL(2, \mathbb{R})$ phase is included [25]. The latter is however incorporated in the magnetic component f_{-MNP} [27]. Therefore, only gaugings involving both electric and magnetic vector fields, or dyonic gaugings, lead to AdS_4 vacua. Furthermore, the existence of maximally supersymmetric AdS_4 vacua requires $\xi^{\alpha M} = 0$. Accordingly, we will from now on restrict ourselves to the case of dyonic gaugings and $\xi^{\alpha M} = 0$.

With $\xi^{\alpha M} = 0$, the gauge covariant derivative can be written as

$$D_\mu = \nabla_\mu - g A_\mu^{\alpha M} f_{\alpha M}{}^{NP} t_{NP} \tag{3}$$

where ∇_μ is the usual space-time covariant derivative including the spin connection. t_{MN} are $SO(6, n)$ generators in the fundamental representation and can be chosen as

$$(t_{MN})_P{}^Q = 2\delta_{[M}^Q \eta_{N]P}. \tag{4}$$

$\eta_{MN} = \text{diag}(-1, -1, -1, -1, -1, -1, 1, \dots, 1)$ is the $SO(6, n)$ invariant tensor, and g is the gauge coupling constant that can be absorbed in the embedding tensor $f_{\alpha MNP}$. For a product gauge group consisting of many simple subgroups, there can be as many independent coupling constants as the simple groups within the product. Note also that with the component $\xi^{\alpha M} = 0$, the gauge group is embedded solely in $SO(6, n)$.

To define a consistent gauging, the embedding tensor has to satisfy a set of quadratic constraints

$$f_{\alpha R[MN} f_{\beta PQ]}{}^R = 0, \quad \epsilon^{\alpha\beta} f_{\alpha MNR} f_{\beta PQ}{}^R = 0. \tag{5}$$

as well as the linear or representation constraint $f_{\alpha MNP} = f_{\alpha[MNP]}$.

The scalar coset $SL(2, \mathbb{R})/SO(2)$ can be described by the coset representative \mathcal{V}_α . We will choose the explicit form of \mathcal{V}_α as follow

$$\mathcal{V}_\alpha = e^{\frac{\phi}{2}} \begin{pmatrix} \chi - ie^{-\phi} \\ 1 \end{pmatrix}. \tag{6}$$

For the $SO(6, n)/SO(6) \times SO(n)$ coset, we introduce a coset representative \mathcal{V}_M^A transforming under global

$SO(6, n)$ and local $SO(6) \times SO(n)$ by left and right multiplications, respectively. By splitting the index $A = (m, a)$, we can write the coset representative as

$$\mathcal{V}_M^A = (\mathcal{V}_M^m, \mathcal{V}_M^a). \tag{7}$$

Being an element of $SO(6, n)$, the matrix \mathcal{V}_M^A satisfies the relation

$$\eta_{MN} = -\mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a. \tag{8}$$

In addition, we can parametrize the $SO(6, n)/SO(6) \times SO(n)$ coset in term of a symmetric matrix

$$M_{MN} = \mathcal{V}_M^m \mathcal{V}_N^m + \mathcal{V}_M^a \mathcal{V}_N^a \tag{9}$$

which is manifestly $SO(6) \times SO(n)$ invariant.

In this paper, we are mainly interested in supersymmetric solutions with only the metric and scalars non-vanishing. The bosonic Lagrangian with vector and auxiliary two-form fields vanishing can be written as

$$e^{-1} \mathcal{L} = \frac{1}{2} R + \frac{1}{16} \partial_\mu M_{MN} \partial^\mu M^{MN} - \frac{1}{4(\text{Im}\tau)^2} \partial_\mu \tau \partial^\mu \tau^* - V \tag{10}$$

where e is the vielbein determinant. The scalar potential is given in terms of the scalar coset representative and the embedding tensor by

$$V = \frac{g^2}{16} \left[f_{\alpha MNP} f_{\beta QRS} M^{\alpha\beta} \left[\frac{1}{3} M^{MQ} M^{NR} M^{PS} + \left(\frac{2}{3} \eta^{MQ} - M^{MQ} \right) \eta^{NR} \eta^{PS} \right] - \frac{4}{9} f_{\alpha MNP} f_{\beta QRS} \epsilon^{\alpha\beta} M^{MNPQRS} \right] \tag{11}$$

where M^{MN} is the inverse of M_{MN} , and M^{MNPQRS} is obtained from

$$M^{MNPQRS} = \epsilon_{mnpqrs} \mathcal{V}_M^m \mathcal{V}_N^n \mathcal{V}_P^p \mathcal{V}_Q^q \mathcal{V}_R^r \mathcal{V}_S^s \tag{12}$$

with indices raised by η^{MN} . Similar to M^{MN} , $M^{\alpha\beta}$ is the inverse of a symmetric 2×2 matrix $M_{\alpha\beta}$ defined by

$$M_{\alpha\beta} = \text{Re}(\mathcal{V}_\alpha \mathcal{V}_\beta^*). \tag{13}$$

Fermionic supersymmetry transformations are given by

$$\delta \psi_\mu^i = 2D_\mu \epsilon^i - \frac{2}{3} g A_1^{ij} \gamma_\mu \epsilon_j, \tag{14}$$

$$\delta \chi^i = i \epsilon^{\alpha\beta} \mathcal{V}_\alpha D_\mu \mathcal{V}_\beta \gamma^\mu \epsilon^i - \frac{4}{3} i g A_2^{ij} \epsilon_j, \tag{15}$$

$$\delta \lambda_a^i = 2i \mathcal{V}_a^M D_\mu \mathcal{V}_M^{ij} \gamma^\mu \epsilon_j + 2i g A_{2aj}^i \epsilon^j. \tag{16}$$

The fermion shift matrices, also appearing in fermionic mass-like terms of the gauged Lagrangian, are given by

$$\begin{aligned} A_1^{ij} &= \epsilon^{\alpha\beta} (\mathcal{V}_\alpha)^* \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_2^{ij} &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_{kl}^M \mathcal{V}_N^{ik} \mathcal{V}_P^{jl} f_{\beta M}^{NP}, \\ A_{2ai}^j &= \epsilon^{\alpha\beta} \mathcal{V}_\alpha \mathcal{V}_a^M \mathcal{V}_{ik}^N \mathcal{V}_P^{jk} f_{\beta MN}^P \end{aligned} \quad (17)$$

where \mathcal{V}_M^{ij} is defined in terms of the 't Hooft symbols G_m^{ij} and \mathcal{V}_M^m as

$$\mathcal{V}_M^{ij} = \frac{1}{2} \mathcal{V}_M^m G_m^{ij}. \quad (18)$$

Similarly, the inverse elements \mathcal{V}_{ij}^M can be written as

$$\mathcal{V}_{ij}^M = -\frac{1}{2} \mathcal{V}_m^M (G_m^{ij})^*. \quad (19)$$

G_m^{ij} convert an index m in the vector representation of $SO(6)$ into an anti-symmetric pair of indices $[ij]$ in the $SU(4)$ fundamental representation. They satisfy the relations

$$G_{mij} = -(G_m^{ij})^* = -\frac{1}{2} \epsilon_{ijkl} G_m^{kl}. \quad (20)$$

The explicit form of these matrices can be found in the appendix.

The scalar potential can also be written in terms of the fermion shift matrices A_1 and A_2 as

$$V = -\frac{1}{3} A_1^{ij} A_{1ij} + \frac{1}{9} A_2^{ij} A_{2ij} + \frac{1}{2} A_{2ai}^j A_{2a}^i. \quad (21)$$

Together with the fermionic supersymmetry transformations, it then follows that unbroken supersymmetry corresponds to an eigenvalue of A_1^{ij} , α , satisfying $V_0 = -\frac{\alpha^2}{3}$ where V_0 is the value of the scalar potential at the vacuum or the cosmological constant.

3 Supersymmetric AdS_4 vacua and holographic RG flows in $SO(4) \times SO(4)$ gauged supergravity

We are interested in gauge groups that can be embedded in $SO(3, 3) \times SO(3, 3) \subset SO(6, 6)$. These gauge groups take the form of a product $G_1 \times G_2$ with $G_1, G_2 \subset SO(3, 3)$ being six-dimensional. Semisimple groups of dimension six that can be embedded in $SO(3, 3)$ are $SO(4)$, $SO(3, 1)$ and $SO(2, 2)$. The embedding tensors for these gauge groups are given in [33]. Since gauge groups involving $SO(2, 2)$ factors do not give rise to AdS_4 vacua, we will not consider these gauge groups in this paper. In this section, we will study $N = 4$ gauged supergravity with compact $SO(4) \times SO(4) \sim SO(3) \times SO(3) \times SO(3) \times SO(3)$ gauge group.

3.1 AdS_4 vacua

Non-vanishing components of the embedding tensor for $SO(4) \times SO(4)$ gauge group are given by

$$\begin{aligned} f_{+\hat{m}\hat{n}\hat{p}} &= \sqrt{2}(g_1 - \tilde{g}_1) \epsilon_{\hat{m}\hat{n}\hat{p}}, & f_{+\hat{a}\hat{b}\hat{c}} &= \sqrt{2}(g_1 + \tilde{g}_1) \epsilon_{\hat{a}\hat{b}\hat{c}}, \\ f_{-\tilde{m}\tilde{n}\tilde{p}} &= \sqrt{2}(g_2 - \tilde{g}_2) \epsilon_{\tilde{m}\tilde{n}\tilde{p}}, & f_{-\tilde{a}\tilde{b}\tilde{c}} &= \sqrt{2}(g_2 + \tilde{g}_2) \epsilon_{\tilde{a}\tilde{b}\tilde{c}}, \end{aligned} \quad (22)$$

where we have used the indices $M = (m, a) = (\hat{m}, \tilde{m}, \hat{a}, \tilde{a})$ with $\hat{m} = 1, 2, 3$, $\tilde{m} = 4, 5, 6$, $\hat{a} = 7, 8, 9$ and $\tilde{a} = 10, 11, 12$. As mentioned before, the first and second $SO(4)$ factors are embedded electrically and magnetically, respectively.

Non-compact generators of $SO(6, 6)$ are given by

$$Y_{ma} = e_{m,a+6} + e_{a+6,m} \quad (23)$$

in which the 12×12 matrices e_{MN} are defined by

$$(e_{MN})_{PQ} = \delta_{MP} \delta_{NQ}. \quad (24)$$

The 36 scalars in $SO(6, 6)/SO(6) \times SO(6)$ transform as $(\mathbf{6}, \mathbf{6})$ under the compact group $SO(6) \times SO(6)$. Under the gauge group $SO(4)_+ \times SO(4)_- \sim SO(3)_+^1 \times SO(3)_+^2 \times SO(3)_-^1 \times SO(3)_-^2$, these scalars transform as

$$(\mathbf{6}, \mathbf{6}) \rightarrow (\mathbf{3}, \mathbf{3}, \mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}, \mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{3}, \mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{1}, \mathbf{3}, \mathbf{3}). \quad (25)$$

We will consider scalars which are singlets under the diagonal subgroup $SO(4)_{\text{inv}} \sim [SO(3)_+^1 \times SO(3)_+^2]_D \times [SO(3)_-^1 \times SO(3)_-^2]_D$. Under $SO(4)_{\text{inv}}$, the scalars transform as

$$2(\mathbf{1}, \mathbf{1}) + (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{3}) + (\mathbf{1}, \mathbf{5}) + (\mathbf{5}, \mathbf{1}) + 2(\mathbf{3}, \mathbf{3}). \quad (26)$$

The two singlets correspond to the following non-compact generators

$$\hat{Y}_1 = Y_{11} + Y_{22} + Y_{33}, \quad \hat{Y}_2 = Y_{44} + Y_{55} + Y_{66} \quad (27)$$

in terms of which the coset representative is given by

$$L = e^{\phi_1 \hat{Y}_1} e^{\phi_2 \hat{Y}_2}. \quad (28)$$

Together with the $SL(2, \mathbb{R})/SO(2)$ scalars which are $SO(4) \times SO(4)$ singlets, there are four $SO(4)_{\text{inv}}$ singlet scalars. The scalar potential for these singlets can be computed to

$$\begin{aligned}
 V = & \frac{1}{8} e^{-\phi-6\phi_1-6\phi_2} [e^{\phi+3\phi_2} [e^{\phi+3\phi_2} g_1^2 + e^{\phi+12\phi_1+3\phi_2} \tilde{g}_1^2 \\
 & - 3e^{\phi+4\phi_1+3\phi_2} (2g_1^2 + \tilde{g}_1^2) - 3e^{\phi+8\phi_1+3\phi_2} (g_1^2 + 2\tilde{g}_1^2)] \\
 & + 2e^{3\phi_1} g_1 [g_2(1 + 3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3 + e^{4\phi_2})] \\
 & + 6e^{7\phi_1} g_1 [g_2(1 + 3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3 + e^{4\phi_2})] \\
 & - 6e^{5\phi_1} \tilde{g}_1 [g_2(1 + 3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3 + e^{4\phi_2})] \\
 & - 2e^{9\phi_1} \tilde{g}_1 [g_2(1 + 3e^{4\phi_2}) - e^{2\phi_2} \tilde{g}_2(3 + e^{4\phi_2})] \\
 & + e^{6\phi_1} [[1 - 3e^{4\phi_2} (2 + e^{4\phi_2})] (1 + e^{2\phi} \chi^2) g_2^2 \\
 & + 16g_2 \tilde{g}_2 e^{6\phi_2} (1 + e^{2\phi} \chi^2) + e^{4\phi_2} [e^{2\phi} (16e^{2\phi_2} g_1 \tilde{g}_1 \\
 & - 3\tilde{g}_2^2 \chi^2 - 6\tilde{g}_2^2 e^{4\phi_2} \chi^2 + e^{8\phi_2} \chi^2 \tilde{g}_2^2) \\
 & + (e^{8\phi_2} - 6e^{4\phi_2} - 3)\tilde{g}_2^2]]. \tag{29}
 \end{aligned}$$

This potential admits a maximally supersymmetric AdS_4 critical point with $SO(4) \times SO(4)$ symmetry at $\chi = \phi_1 = \phi_2 = 0$ and

$$\phi = \ln \left| \frac{g_2 - \tilde{g}_2}{g_1 - \tilde{g}_1} \right|. \tag{30}$$

For convenience, we will denote this AdS_4 vacuum with $SO(4) \times SO(4)$ symmetry by critical point I.

Without loss of generality, we can shift the dilaton such that this critical point occurs at $\phi = 0$. For definiteness, we will choose

$$\tilde{g}_2 = g_1 + g_2 - \tilde{g}_1. \tag{31}$$

At this critical point, we find the value of the cosmological constant and the AdS_4 radius

$$V_0 = -6(g_1 - \tilde{g}_1)^2 \quad \text{and} \quad L = \frac{1}{\sqrt{2}(g_1 - \tilde{g}_1)} \tag{32}$$

where we have assumed that $\tilde{g}_1 > g_1$. All scalars have masses $m^2 L^2 = -2$. In general, using the relation $m^2 L^2 = \Delta(\Delta - 3)$, we find that these scalars can be dual to operators of dimensions $\Delta = 1$ or $\Delta = 2$. These correspond to mass terms of scalars ($\Delta = 1$) or fermions ($\Delta = 2$) in the dual three-dimensional SCFTs. Usually, the correct choice is fixed by supersymmetry as in the case of ABJM theory. However, in the present case, the identification is not so clear.

Furthermore, the scalar potential in (29) also admits additional three supersymmetric AdS_4 vacua:

- II. This critical point has $SO(3)_+ \times SO(4)_-$ symmetry with

$$\phi = \ln \left[\frac{2\sqrt{g_1 \tilde{g}_1}}{g_1 + \tilde{g}_1} \right], \quad \phi_1 = \frac{1}{2} \ln \left[\frac{g_1}{\tilde{g}_1} \right], \quad \phi_2 = 0,$$

$$\begin{aligned}
 V_0 = & -\frac{3(g_1 + \tilde{g}_1)(g_1 - \tilde{g}_1)^2}{\sqrt{g_1 \tilde{g}_1}}, \\
 L = & \frac{(g_1 \tilde{g}_1)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{g_1 + \tilde{g}_1}}. \tag{33}
 \end{aligned}$$

- III. This critical point has $SO(4)_+ \times SO(3)_-$ symmetry with

$$\begin{aligned}
 \phi = & -\ln \left[\frac{2\sqrt{g_2 \tilde{g}_2}}{g_2 + \tilde{g}_2} \right], \quad \phi_2 = \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \quad \phi_1 = 0, \\
 V_0 = & -\frac{3(g_2 + \tilde{g}_2)(g_1 - \tilde{g}_1)^2}{\sqrt{g_2 \tilde{g}_2}}, \\
 L = & \frac{(g_2 \tilde{g}_2)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{g_2 + \tilde{g}_2}}. \tag{34}
 \end{aligned}$$

- IV. This critical point is invariant under a smaller symmetry $SO(4)_{\text{inv}}$ with

$$\begin{aligned}
 \phi = & \ln \left[\sqrt{\frac{g_1 \tilde{g}_1 g_2 + \tilde{g}_2}{g_2 \tilde{g}_2 g_1 + \tilde{g}_1}} \right], \quad \phi_1 = \frac{1}{2} \ln \left[\frac{g_1}{\tilde{g}_1} \right], \\
 \phi_2 = & \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \\
 V_0 = & -\frac{3(g_2 + \tilde{g}_2)^2 (g_1 - \tilde{g}_1)^2}{2\sqrt{g_1 \tilde{g}_1 g_2 \tilde{g}_2}}, \\
 L = & \frac{\sqrt{2}(g_1 \tilde{g}_1 g_2 \tilde{g}_2)^{\frac{1}{4}}}{(\tilde{g}_1 - g_1)\sqrt{(g_1 + \tilde{g}_1)(g_2 + \tilde{g}_2)}}. \tag{35}
 \end{aligned}$$

We have written the above equations in term of \tilde{g}_2 for brevity. All of these critical points preserve the full $N = 4$ supersymmetry and correspond to $N = 4$ SCFTs in three dimensions. Scalar masses at these critical points are given in Tables 1, 2 and 3.

It should also be noted that three massless scalars at critical points II and III are Goldstone bosons corresponding to the symmetry breaking $SO(4)_+ \times SO(4)_- \rightarrow SO(3)_+ \times SO(4)_-$ and $SO(4)_+ \times SO(4)_- \rightarrow SO(4)_+ \times SO(3)_-$.

Table 1 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3)_+ \times SO(4)_-$ symmetry and the corresponding dimensions of the dual operators

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1, 1) | 4 | 4 |
| (3, 1, 1) | $0_{\times 3}$ | 3 |
| (1, 3, 3) | $0_{\times 9}$ | 3 |
| (5, 1, 1) | $-2_{\times 5}$ | 1, 2 |
| (3, 1, 3) + (3, 3, 1) | $-2_{\times 18}$ | 1, 2 |

Table 2 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(4)_+ \times SO(3)_-$ symmetry and the corresponding dimensions of the dual operators

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1, 1) | 4 | 4 |
| (1, 1, 3) | $0_{\times 3}$ | 3 |
| (3, 3, 1) | $0_{\times 9}$ | 3 |
| (1, 1, 5) | $-2_{\times 5}$ | 1, 2 |
| (1, 3, 3) + (3, 1, 3) | $-2_{\times 18}$ | 1, 2 |

Table 3 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3) \sim SO(4)_{inv}$ symmetry and the corresponding dimensions of the dual operators

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1) | $4_{\times 2}$ | 4 |
| (1, 5) + (5, 1) | $-2_{\times 10}$ | 1, 2 |
| (1, 3) + (3, 1) | $0_{\times 6}$ | 3 |
| (3, 3) | $0_{\times 18}$ | 3 |

These scalars live in representations (3, 1, 1) and (1, 1, 3), respectively. Similarly, for critical point IV, six of the massless scalars in the representation (1, 3) + (3, 1) are Goldstone bosons of the symmetry breaking $SO(4)_+ \times SO(4)_- \rightarrow SO(4)_{inv}$. The remaining massless scalars correspond to marginal deformations in the SCFTs dual to these AdS_4 vacua. These deformations necessarily break some amount of supersymmetry since the $N = 4$ AdS_4 vacua have no moduli preserving $N = 4$ supersymmetry [28]. It should also be noted that the vacuum structure of this gauged supergravity is very similar to two copies of $SO(3) \times SO(3) \sim SO(4)$ $N = 3$ gauged supergravity considered in [34].

3.2 Holographic RG flows between $N = 4$ SCFTs

We now consider holographic RG flow solutions interpolating between supersymmetric AdS_4 vacua previously identified. To find supersymmetric flow solutions, we begin with the metric ansatz

$$ds^2 = e^{2A(r)} dx_{1,2}^2 + dr^2 \tag{36}$$

where $dx_{1,2}^2$ is the flat Minkowski metric in three dimensions.

For spinor conventions, we will use the Majorana representation with all γ^μ real and γ_5 purely imaginary. This choice implies that ϵ_i is a complex conjugate of ϵ^i . All scalar fields will be functions of only the radial coordinate r in order to preserve Poincaré symmetry in three dimensions. The BPS

conditions coming from setting $\delta\chi^i = 0$ and $\delta\lambda_a^i = 0$ require the following projection

$$\gamma_r \epsilon^i = e^{i\Lambda} \epsilon_i. \tag{37}$$

It follows from the $\delta\psi_{\mu i} = 0$ conditions for $\mu = 0, 1, 2$, that

$$A' = \pm W, \quad e^{i\Lambda} = \pm \frac{\mathcal{W}}{W} \tag{38}$$

where $W = |\mathcal{W}|$, and $'$ denotes the r -derivative. The superpotential \mathcal{W} is defined by

$$\mathcal{W} = \frac{2}{3} \alpha \tag{39}$$

where α is the eigenvalue of A_1^{ij} corresponding to the unbroken supersymmetry. The detailed analysis leading to Eq. (38) can be found, for example, in [34].

For $SO(4)_{inv}$ singlet scalars, the tensor A_1^{ij} takes the form of a diagonal matrix

$$A_1^{ij} = \frac{3}{2} \mathcal{W} \delta^{ij} \tag{40}$$

with the superpotential given by

$$\begin{aligned} \mathcal{W} = \frac{1}{4\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1 - 3\phi_2} & \left[3i \tilde{g}_1 e^{\phi + 2\phi_1 + 3\phi_2} + i \tilde{g}_1 e^{\phi + 6\phi_1 + 3\phi_2} \right. \\ & + e^{3\phi_1} (i + e^\phi \chi) \left[g_2 (1 + 3e^{4\phi}) - \tilde{g}_2 e^{2\phi_2} (3 + e^{4\phi_2}) \right] \\ & \left. - i g_1 e^{\phi + 3\phi_2} - 3i g_1 e^{\phi + 4\phi_1 + 3\phi_2} \right]. \end{aligned} \tag{41}$$

The variation of λ_a^i leads to the following BPS equations

$$\phi'_1 = -\frac{i}{2\sqrt{2}} e^{i\Lambda} e^{\frac{\phi}{2} - 3\phi_1} (e^{4\phi_1} - 1) (e^{2\phi_1} \tilde{g}_1 - g_1), \tag{42}$$

$$\phi'_2 = -\frac{1}{2\sqrt{2}} e^{i\Lambda} e^{-\frac{\phi}{2} - 3\phi_2} (e^{4\phi_2} - 1) (e^{2\phi_2} \tilde{g}_2 - g_2) (e^\phi \chi - i). \tag{43}$$

Consistency of the first condition implies that the phase $e^{i\Lambda}$ is purely imaginary, $e^{i\Lambda} = \pm i$. With this choice, the second condition requires that $\chi = 0$. This is also consistent with the variation of the dilatini χ^i . Furthermore, with $\chi = 0$, the superpotential (41) is purely imaginary in agreement with the spinor phase $e^{i\Lambda}$ in Eq. (38).

We will choose a definite sign in order to identify the $SO(4) \times SO(4)$ critical point with the limit $r \rightarrow \infty$. The BPS equations for ϕ , ϕ_1 and ϕ_2 can then be written as

$$\phi' = -4 \frac{\partial W}{\partial \phi}, \quad \phi'_1 = -\frac{2}{3} \frac{\partial W}{\partial \phi_1}, \quad \phi'_2 = -\frac{2}{3} \frac{\partial W}{\partial \phi_2} \tag{44}$$

together with the A' equation

$$A' = W. \tag{45}$$

Explicitly, these equations read

$$\phi'_1 = -\frac{1}{2\sqrt{2}}e^{\frac{\phi}{2}-3\phi_1}(e^{4\phi_1}-1)(e^{2\phi_1}\tilde{g}_1-g_1), \tag{46}$$

$$\phi'_2 = \frac{1}{2\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_2}(e^{4\phi_2}-1)(e^{2\phi_2}\tilde{g}_2-g_2), \tag{47}$$

$$\begin{aligned} \phi' = & -\frac{1}{2\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_1-3\phi_2} \\ & \times \left[3\tilde{g}_1e^{\phi+2\phi_1+3\phi_2} - g_1e^{\phi+3\phi_2} - 3g_1e^{\phi+4\phi_1+3\phi_2} \right. \\ & + \tilde{g}_1e^{\phi+6\phi_1+3\phi_2} + e^{3\phi_1} \\ & \left. \times \left[\tilde{g}_2e^{2\phi_2}(3+e^{4\phi_2}) - g_2(1+3e^{4\phi_2}) \right] \right], \tag{48} \end{aligned}$$

$$\begin{aligned} A' = & \frac{1}{4\sqrt{2}}e^{-\frac{\phi}{2}-3\phi_1-3\phi_2} \\ & \times \left[3\tilde{g}_1e^{\phi+2\phi_1+3\phi_2} - g_1e^{\phi+3\phi_2} - 3g_1e^{\phi+4\phi_1+3\phi_2} \right. \\ & + \tilde{g}_1e^{\phi+6\phi_1+3\phi_2} + e^{3\phi_1} \\ & \left. \times \left[g_2(1+3e^{4\phi_2}) - \tilde{g}_2e^{2\phi_2}(3+e^{4\phi_2}) \right] \right] \tag{49} \end{aligned}$$

The scalar potential can be written in term of the real superpotential W as

$$V = 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_2} \right)^2 - 3W^2. \tag{50}$$

It can be verified that the above BPS equations are compatible with the second-order field equations. It should be noted that the consistency between BPS equations and field equations also requires $\chi = 0$.

We now consider various possible RG flows interpolating between the $N = 4$ supersymmetric fixed points. Some of these flows can be obtained analytically, but the others require some sort of numerical analysis. Near the $SO(4) \times SO(4)$ critical point as $r \rightarrow \infty$, the BPS equations give

$$\phi, \phi_1, \phi_2 \sim e^{-\frac{r}{L_I}} \tag{51}$$

in agreement with the fact that all these scalars are dual to operators of dimensions $\Delta = 1, 2$. L_I is the AdS_4 radius at critical point I.

We begin with the flow between critical points I and II. In this case, we can consistently set $\phi_2 = 0$. By considering ϕ and A as functions of ϕ_1 , we can combine the above BPS equations into

$$\frac{d\phi}{d\phi_1} = -\frac{g_1(1+3e^{4\phi_1}) + e^{2\phi_1}[4(g_2-\tilde{g}_2)e^{\phi_1-\phi} - \tilde{g}_1(e^{4\phi_1}+3)]}{(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1)} \tag{52}$$

$$\frac{dA}{d\phi_1} = \frac{g_1(1+3e^{4\phi_1}) - e^{2\phi_1}[4(g_2-\tilde{g}_2)e^{\phi_1-\phi} + \tilde{g}_1(3+e^{4\phi_1})]}{2(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1)}. \tag{53}$$

The first equation can be solved by

$$\phi = \ln \left[\frac{g_2 - \tilde{g}_2 + C_1(e^{4\phi_1} - 1)}{\tilde{g}_1e^{3\phi_1} - g_1e^{\phi_1}} \right]. \tag{54}$$

The integration constant C_1 will be chosen such that the solution interpolates between $\phi = 0$ at the $SO(4) \times SO(4)$ critical point and $\phi = \ln \left[\frac{\sqrt{g_1\tilde{g}_1}}{g_1+\tilde{g}_1} \right]$ at the $SO(3)_+ \times SO(4)_-$ critical point. This is achieved by choosing $C_1 = \frac{\tilde{g}_1^2(g_2-\tilde{g}_2)}{\tilde{g}_1^2-g_1^2}$, and the solution for ϕ is given by

$$\phi = \ln \left[\frac{(g_2 - \tilde{g}_2)(g_1 + \tilde{g}_1e^{2\phi_1})e^{-\phi_1}}{\tilde{g}_1^2 - g_1^2} \right]. \tag{55}$$

With this solution, Eq. (53) can be solved by

$$A = \frac{\phi_1}{2} - \ln(1 - e^{4\phi_1}) + \ln(g_1 - \tilde{g}_1e^{2\phi_1}) + \frac{1}{2} \ln(g_1 + \tilde{g}_1e^{2\phi_1}) \tag{56}$$

where an irrelevant additive integration constant has been removed.

By changing to a new radial coordinate \tilde{r} defined by $\frac{d\tilde{r}}{dr} = e^{\frac{\phi}{2}}$, Eq. (46) becomes

$$\frac{d\phi_1}{d\tilde{r}} = -\frac{1}{2\sqrt{2}}e^{-3\phi_1}(e^{4\phi_1}-1)(\tilde{g}_1e^{2\phi_1}-g_1) \tag{57}$$

whose solution is given by

$$\begin{aligned} \frac{(g_1^2 - \tilde{g}_1^2)\tilde{r}}{\sqrt{2}} = & (g_1 - \tilde{g}_1) \tan^{-1} e^{\phi_1} - (g_1 + \tilde{g}_1) \tanh^{-1} e^{\phi_1} \\ & + 2\sqrt{g_1\tilde{g}_1} \tanh^{-1} \left[\sqrt{\frac{\tilde{g}_1}{g_1}} e^{\phi_1} \right]. \tag{58} \end{aligned}$$

Near critical point II, the operator dual to ϕ_1 becomes irrelevant with dimension $\Delta = 4$, but the operator dual to the dilaton ϕ is still relevant with dimensions $\Delta = 1, 2$. This can be seen by looking at the behavior of scalars near critical point II

$$\phi \sim e^{-\frac{r}{L_{II}}} \text{ and } \phi_1 \sim e^{\frac{r}{L_{II}}} \tag{59}$$

as $r \rightarrow -\infty$.

We can then consider a flow from critical point II to critical point IV. Along this flow, we have $\phi_1 = \frac{1}{2} \ln \frac{g_1}{\tilde{g}_1}$, and a similar analysis gives the solution

$$\phi = \ln \left[\frac{2\sqrt{g_1\tilde{g}_1}(g_2 + \tilde{g}_2)e^{2\phi_2}}{(g_1 + \tilde{g}_1)(g_2 + \tilde{g}_2e^{2\phi_2})} \right], \tag{60}$$

$$A = \frac{\phi_2}{2} - \ln(1 - e^{4\phi_2}) + \ln(\tilde{g}_2e^{2\phi_2} - g_2) + \frac{1}{2} \ln(g_2 + \tilde{g}_2e^{2\phi_2}). \tag{61}$$

Along this flow, the running of ϕ_2 is described by

$$\begin{aligned} & \frac{(g_1 - \tilde{g}_1)(g_2 + \tilde{g}_2)}{\sqrt{2}} \bar{r} \\ &= (\tilde{g}_1 - g_1) \tan^{-1} e^{\phi_2} - (g_2 + \tilde{g}_2) \tanh^{-1} e^{\phi_2} \\ &+ 2\sqrt{g_2\tilde{g}_2} \tanh^{-1} \left[e^{\phi_2} \sqrt{\frac{\tilde{g}_2}{g_2}} \right] \end{aligned} \tag{62}$$

with \bar{r} defined by $\frac{d\bar{r}}{d\tau} = e^{-\frac{\phi}{2}}$.

Similarly, the flows between critical points I and III and between critical points III and IV are given respectively by

$$\phi_1 = 0, \tag{63}$$

$$\phi = \ln \left[\frac{e^{\phi_2}(g_2 + \tilde{g}_2)}{g_2 + e^{2\phi_2}\tilde{g}_2} \right], \tag{64}$$

$$A = \frac{\phi_2}{2} - \ln(1 - e^{4\phi_2}) + \ln(e^{2\phi_2}\tilde{g}_2 - g_2) + \frac{1}{2} \ln(g_2 + \tilde{g}_2e^{2\phi_2}) \tag{65}$$

and

$$\phi_2 = \frac{1}{2} \ln \left[\frac{g_2}{\tilde{g}_2} \right], \tag{66}$$

$$\phi = \ln \left[\frac{e^{-\phi_1}(g_2 + \tilde{g}_2)(g_1 + \tilde{g}_1e^{2\phi_1})}{2(g_1 + \tilde{g}_1)\sqrt{g_2\tilde{g}_2}} \right], \tag{67}$$

$$A = \frac{\phi_1}{2} - \ln(1 - e^{4\phi_1}) + \ln(g_1 - e^{2\phi_1}\tilde{g}_1) + \frac{1}{2} \ln(g_1 + \tilde{g}_1e^{2\phi_1}) \tag{68}$$

In these cases, the r -dependent of ϕ_1 and ϕ_2 can be obtained in the same way as Eqs. (58) and (62).

For a direct flow from critical point I to critical point IV, a numerical solution is needed. This solution is given in Fig. 1. More generally, a flow from critical point I to critical point II and finally to critical point IV can also be found. This solution is given in Fig. 2 and describes a cascade of RG flows with smaller flavor symmetry along the flow.

3.3 RG flows to $N = 4$ non-conformal theory

A consistent truncation of the above $N = 4$ $SO(4) \times SO(4)$ gauged supergravity is obtained by setting $\phi_1 = \phi_2 = 0$.

In this case, only scalars in the gravity multiplet are present. As previously mentioned, the axion χ cannot be turned on simultaneously with ϕ_1 and ϕ_2 .

For $\phi_1 = \phi_2 = 0$, the superpotential is complex and given by

$$\mathcal{W} = \frac{1}{\sqrt{2}} e^{-\frac{\phi}{2}} [(g_2 - \tilde{g}_2)\chi e^{\phi} - i(\tilde{g}_2 - g_2 + e^{\phi}(g_1 - \tilde{g}_1))]. \tag{69}$$

With $\tilde{g}_2 = g_1 + g_2 - \tilde{g}_1$, the scalar potential takes a simpler form

$$V = 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + 4e^{-2\phi} \left(\frac{\partial W}{\partial \chi} \right)^2 - 3W^2 = -(g_1 - \tilde{g}_1)^2 e^{-\phi} [1 + 4e^{\phi} + e^{2\phi}(1 + \chi^2)] \tag{70}$$

which has only one AdS_4 critical point at $\phi = \chi = 0$. This is critical point I of the previous subsection.

The BPS equations in this truncation are given by

$$\phi' = -4 \frac{\partial W}{\partial \phi} = -\frac{\sqrt{2}(\tilde{g}_1 - g_1)[e^{2\phi}(1 + \chi^2) - 1]}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}}, \tag{71}$$

$$\chi' = -4e^{-2\phi} \frac{\partial W}{\partial \chi} = -\frac{2\sqrt{2}(\tilde{g}_1 - g_1)e^{-\frac{\phi}{2}}\chi}{\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}}, \tag{72}$$

$$A' = W = \frac{1}{\sqrt{2}}(\tilde{g}_1 - g_1)e^{-\frac{\phi}{2}}\sqrt{(1 + e^{\phi})^2 + e^{2\phi}\chi^2}. \tag{73}$$

Near the AdS_4 critical point, we find

$$\phi \sim \chi \sim e^{-\frac{r}{L_4}} \tag{74}$$

implying that ϕ and χ correspond to relevant operators of dimensions $\Delta = 1, 2$.

By considering ϕ and A as functions of χ , we can combine the BPS equations into

$$\frac{d\phi}{d\chi} = \frac{e^{2\phi}(1 + \chi^2) - 1}{2\chi}, \tag{75}$$

$$\frac{dA}{d\chi} = -\frac{1 + 2e^{\phi} + e^{2\phi}(1 + \chi^2)}{4\chi} \tag{76}$$

which can be solved by

$$\phi = -\frac{1}{2} \ln(1 - 2C\chi - \chi^2), \tag{77}$$

$$A = -\ln \chi + \frac{1}{2} \ln[1 - C\chi + \sqrt{1 - 2C\chi - \chi^2}] + \frac{1}{4} \ln(1 - 2C\chi - \chi^2) \tag{78}$$

in which an additive integration constant for A has been neglected. It should also be noted that we must keep the constant $C \neq 0$ in order to obtain the correct behavior near the AdS_4 critical point as given in Eq. (74).

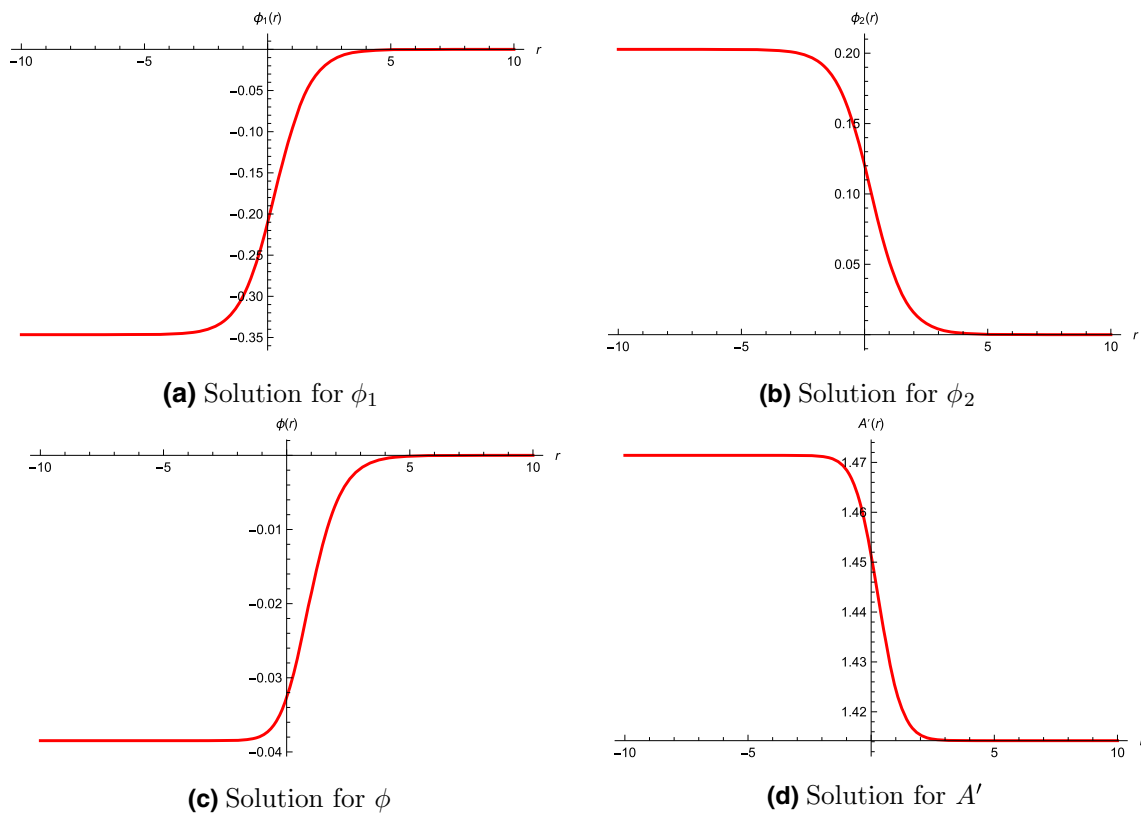


Fig. 1 An RG flow between critical points I and IV with $g_1 = 1, \tilde{g}_1 = \tilde{g}_2 = 2$ and $g_2 = 3$

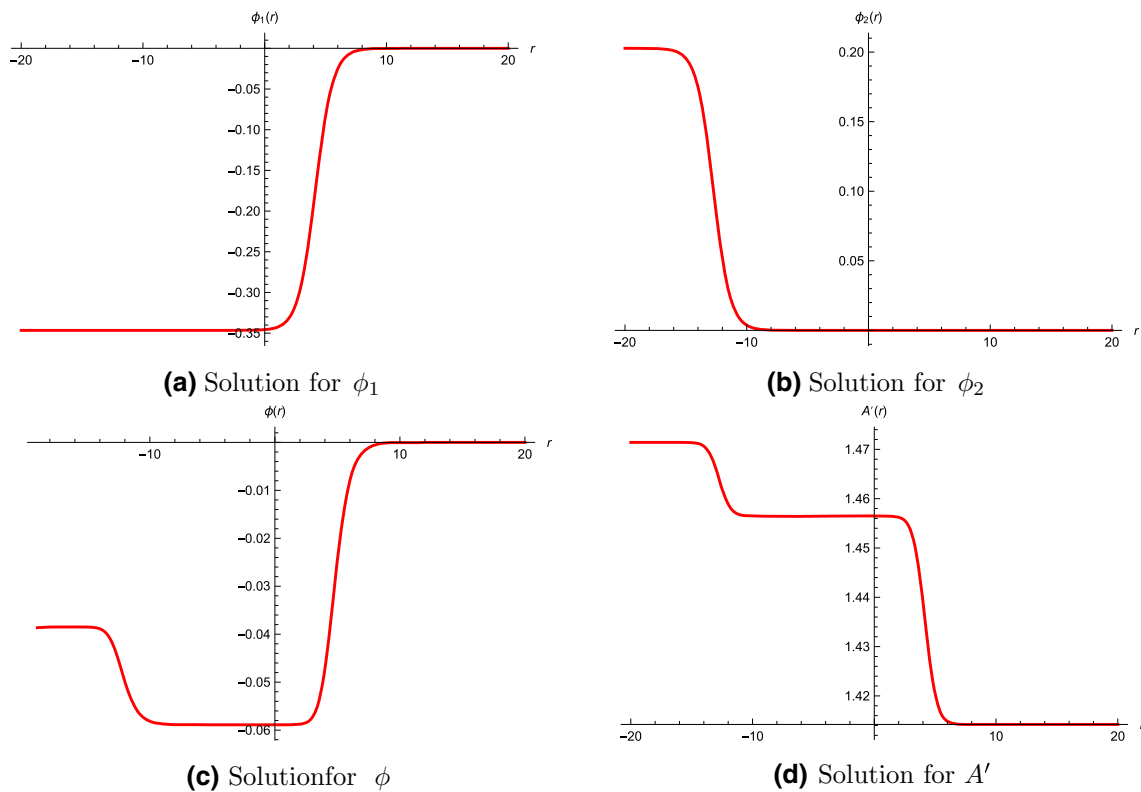


Fig. 2 An RG flow from critical point I to critical point II and continue to critical point IV with $g_1 = 1, \tilde{g}_1 = \tilde{g}_2 = 2$ and $g_2 = 3$

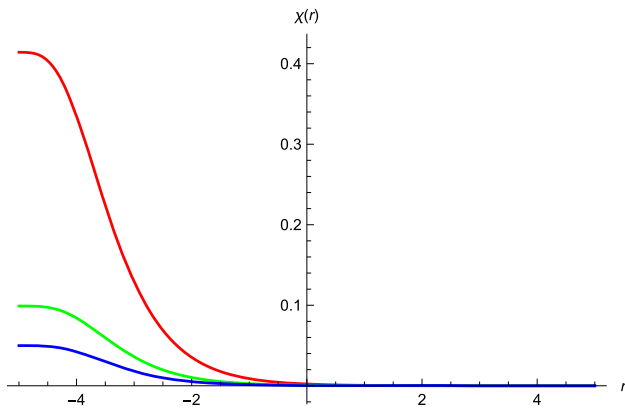


Fig. 3 Solutions for χ with $g_1 = 1, \tilde{g}_1 = 2$ and $\chi_0 = \sqrt{1 + C^2} - C$ for $C = 1$ (red), $C = 5$ (green) and $C = 10$ (blue)

Finally, we can substitute the ϕ solution in (77) in Eq. (72) and in principle solve for χ as a function of r . However, we are not able to solve for χ analytically. We then look for numerical solutions. From Eq. (77), we see that $\phi \rightarrow 0$ as $\chi \rightarrow 0$. This limit, as usual, corresponds to the AdS_4 critical point. We can also see that ϕ is singular at χ_0 for which $1 - 2C\chi_0 - \chi_0^2 = 0$ or

$$\chi_0 = -C \pm \sqrt{1 + C^2}. \tag{79}$$

This implies that ϕ flows from the value $\phi = 0$ at the critical point to a singular value $\phi \rightarrow \infty$ while χ flows between the values $\chi = 0$ and $\chi = \chi_0$. Examples of solutions for χ is shown in Fig. 3.

Near the singularity $\phi \rightarrow \infty$ and $\chi \rightarrow \chi_0$, we find that

$$\chi - \chi_0 \sim r^4, \quad \phi \sim -\ln r^2, \quad A \sim \ln r. \tag{80}$$

This gives the metric

$$ds^2 = r^2 dx_{1,2}^2 + dr^2. \tag{81}$$

From the scalar potential (70), we find $V \rightarrow -\infty$ for any value of χ_0 . Therefore, the singularity is physical according to the criterion of [35]. We then conclude that the solution describes an RG flow from the $N = 4$ SCFT in the UV to a non-conformal field theory in the IR corresponding to the above singularity. The deformations break conformal symmetry but preserve the $SO(4)$ flavor symmetry and $N = 4$ Poincaré supersymmetry in three dimensions.

4 $N = 4$ $SO(3, 1) \times SO(3, 1)$ gauged supergravity

In this section, we consider non-compact gauge group $SO(3, 1) \times SO(3, 1)$ with the embedding tensor

$$\begin{aligned} f_{+123} &= f_{+189} = f_{+729} = -f_{+783} = \frac{1}{\sqrt{2}}(g_1 - \tilde{g}_1), \\ f_{+789} &= f_{+183} = f_{+723} = -f_{+129} = \frac{1}{\sqrt{2}}(g_1 + \tilde{g}_1), \\ f_{-456} &= f_{-4,11,12} = f_{-10,5,12} = -f_{-10,11,6} \\ &= \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2), \\ f_{-10,11,12} &= f_{-4,11,6} = f_{-10,5,6} = -f_{-45,12} \\ &= \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2). \end{aligned} \tag{82}$$

We now repeat the analysis performed in the previous section.

4.1 Supersymmetric AdS_4 vacuum

We will parametrize the $SO(6, 6)/SO(6) \times SO(6)$ coset by using scalars that are $SO(3) \times SO(3) \subset SO(3, 1) \times SO(3, 1)$ invariant. From the embedding of $SO(3, 1)$ in $SO(3, 3)$, there are two $SO(3) \times SO(3)$ singlets corresponding to the non-compact generators

$$\tilde{Y}_1 = Y_{11} + Y_{22} - Y_{33}, \quad \tilde{Y}_2 = Y_{44} + Y_{55} - Y_{66}. \tag{83}$$

The coset representative can be parametrized as

$$L = e^{\phi_1 \tilde{Y}_1} e^{\phi_2 \tilde{Y}_2}. \tag{84}$$

In this case, the scalar potential is given by

$$\begin{aligned} V &= \frac{1}{8} e^{-\phi - 6\phi_1 - 6\phi_2} \left[2g_2 e^{\phi + 3\phi_1 + 9\phi_2} \right. \\ &\quad \times (e^{6\phi_1} g_1 - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad - 6g_2 e^{\phi + 3\phi_1 + 5\phi_2} (g_1 e^{6\phi_1} - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad + 6\tilde{g}_2 e^{\phi + 3\phi_1 + 7\phi_2} (3\tilde{g}_1 e^{4\phi_1} - 3g_1 e^{2\phi_1} + g_1 e^{6\phi_1} - \tilde{g}_1) \\ &\quad - 2\tilde{g}_2 e^{\phi + 3\phi_1 + 3\phi_2} (g_1 e^{6\phi_1} - 3g_1 e^{2\phi_1} - \tilde{g}_1 + 3\tilde{g}_1 e^{4\phi_1}) \\ &\quad + 3e^{6\phi_1 + 4\phi_2} [e^{4\phi_2} (2g_2^2 - \tilde{g}_2^2)(1 + \chi^2 e^{2\phi}) \\ &\quad - 3(g_2^2 - 2\tilde{g}_2^2)(1 + \chi^2 e^{2\phi})] \\ &\quad + g_2^2 e^{6\phi_1 + 6\phi_2} (1 + \chi^2 e^{2\phi}) + \tilde{g}_2^2 e^{6\phi_1} (1 + \chi^2 e^{2\phi}) \\ &\quad + e^{6\phi_2} \left[3(2g_1^2 - \tilde{g}_1^2) e^{2\phi + 8\phi_1} \right. \\ &\quad + 16e^{6\phi_1} [g_2 \tilde{g}_2 + e^{2\phi} (g_1 \tilde{g}_1 + g_2 \tilde{g}_2 \chi^2)] \\ &\quad \left. + g_1^2 e^{2\phi + 12\phi_1} \tilde{g}_1^2 e^{2\phi} - 3(g_1^2 - 2\tilde{g}_1^2) e^{\phi + 4\phi_1} \right] \left. \right]. \end{aligned} \tag{85}$$

This potential admits only one supersymmetric AdS_4 critical point at

$$\begin{aligned} \phi &= \frac{1}{2} \ln \left[\frac{g_1 \tilde{g}_1 (g_2^2 + \tilde{g}_2^2) (g_1^2 + \tilde{g}_1^2)^2}{g_2 \tilde{g}_2} \right], \quad \chi = 0, \\ \phi_1 &= \frac{1}{2} \ln \left[-\frac{\tilde{g}_1}{g_1} \right], \quad \phi_2 = \frac{1}{2} \ln \left[-\frac{\tilde{g}_2}{g_2} \right]. \end{aligned} \tag{86}$$

Table 4 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(3, 1) \times SO(3, 1)$ gauge group

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1) | $4_{\times 2}$ | 4 |
| (1, 5) + (5, 1) | $-2_{\times 10}$ | 1, 2 |
| (1, 3) + (3, 1) | $0_{\times 6}$ | 3 |
| (3, 3) | $0_{\times 18}$ | 3 |

This critical point preserves $N = 4$ supersymmetry and $SO(3) \times SO(3)$ symmetry. The latter is the maximal compact subgroup of $SO(3, 1) \times SO(3, 1)$ gauge group. Without loss of generality, we can shift the scalars such that the critical point occurs at $\phi = \phi_1 = \phi_2 = 0$. This can be achieved by setting

$$\tilde{g}_1 = -g_1, \quad \tilde{g}_2 = -g_2, \quad g_2 = -g_1. \tag{87}$$

With these values, the cosmological constant and AdS_4 radius are given by

$$V_0 = -6g_1^2 \quad \text{and} \quad L^2 = \frac{1}{2g_1^2}. \tag{88}$$

It should be noted that the choice $\tilde{g}_1 = -g_1, \tilde{g}_2 = -g_2$ and $g_2 = g_1$ makes the critical point at $\phi = \phi_1 = \phi_2 = \chi = 0$ a dS_4 with $V_0 = 2g_1^2$.

At the $N = 4$ AdS_4 critical point, the gauge group $SO(3, 1) \times SO(3, 1)$ is broken down to its maximal compact subgroup $SO(3) \times SO(3)$. All scalar masses at this critical point are given in Table 4. The two singlet representations (1, 1) corresponding to ϕ_1 and ϕ_2 are dual to irrelevant operators of dimensions $\Delta = 4$, and six massless scalars in representation (1, 3) + (3, 1) are Goldstone bosons.

4.2 RG flows without vector multiplet scalars

Since there is only one supersymmetric AdS_4 critical point, there is no supersymmetric RG flow between the dual SCFTs. In this case, we instead consider RG flows from the SCFT dual to the $N = 4$ AdS_4 vacuum with $SO(3) \times SO(3)$ symmetry. We begin with a simple truncation to scalar fields in the supergravity multiplet obtained by setting $\phi_1 = \phi_2 = 0$. Within this truncation, the superpotential is given by

$$\mathcal{W} = \frac{3i}{2\sqrt{2}} g_1 e^{-\frac{\phi}{2}} [1 + e^\phi (1 - i\chi)] \tag{89}$$

in term of which the scalar potential can be written as

$$V = \frac{16}{9} \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{16}{9} e^{-2\phi} \left(\frac{\partial W}{\partial \chi} \right)^2 - \frac{4}{3} W^2 = -g_1^2 e^{-\phi} [1 + 4e^\phi + e^{2\phi} (1 + \chi^2)]. \tag{90}$$

The flow equations obtained from $\delta\chi^i = 0$ conditions are given by

$$\phi' = -\frac{8}{3} \frac{\partial W}{\partial \phi} = -\frac{\sqrt{2} g_1 e^{-\frac{\phi}{2}} [e^{2\phi} (1 + \chi^2) - 1]}{\sqrt{(1 + e^\phi)^2 + e^{2\phi} \chi^2}}, \tag{91}$$

$$\chi' = -\frac{8}{3} e^{-2\phi} \frac{\partial W}{\partial \chi} = -\frac{2\sqrt{2} g_1 e^{-\frac{\phi}{2}} \chi}{\sqrt{(1 + e^\phi)^2 + e^{2\phi} \chi^2}}. \tag{92}$$

The BPS conditions from $\delta\lambda_a^i = 0$ are, of course, identically satisfied by setting $\phi_1 = \phi_2 = 0$.

The flow equation for the metric function is simply given by

$$A' = W = \frac{3}{2\sqrt{2}} g_1 e^{-\frac{\phi}{2}} \sqrt{(1 + e^\phi)^2 + e^{2\phi} \chi^2}. \tag{93}$$

Near the AdS_4 critical point, we find

$$\phi \sim \chi \sim e^{-\sqrt{2} g_1 r} \sim e^{-\frac{r}{L}} \tag{94}$$

as expected for the dual operators of dimensions $\Delta = 1, 2$.

Apart from some numerical factors involving gauge coupling constants, the structure of the resulting BPS equations are very similar to the $SO(4) \times SO(4)$ case. We therefore only give the solution without going into any details here

$$\phi = -\frac{1}{2} \ln(1 - \chi^2 - 2C\chi), \tag{95}$$

$$A = -\frac{3}{2} \ln \chi + \frac{3}{8} \ln(1 - 2C\chi - \chi^2) + \frac{3}{4} \ln(1 - C\chi + \sqrt{1 - 2C\chi - \chi^2}). \tag{96}$$

As in the $SO(4) \times SO(4)$ case, we are able to solve for χ only numerically. An example of solutions for χ is shown in Fig. 4. From the figure, it can be readily seen that, along the flow, χ interpolates between $\chi = 0$ and $\chi_0 = -C \pm \sqrt{1 + C^2}$. The ϕ solution, on the other hand, interpolates between $\phi = 0$ and $\phi \rightarrow \infty$ as can be seen from the solution (95). The singularity $\phi \rightarrow \infty$ also gives rise to $V \rightarrow -\infty$ for any value of χ_0 . Therefore, the singularity is physical, and the solution describes an RG flow from the $N = 4$ SCFT in the UV with $SO(3) \times SO(3)$ symmetry to a non-conformal field theory in the IR corresponding to this singularity.

4.3 RG flows with vector multiplet scalars

We now consider solutions with non-vanishing vector multiplet scalars. In this case, we need to set $\chi = 0$ in order to

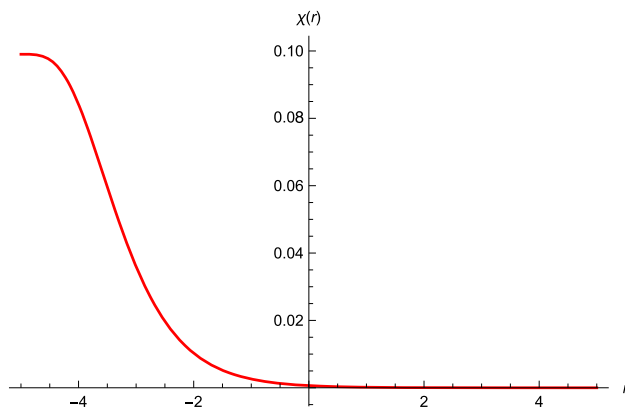


Fig. 4 Solution for χ with $\chi_0 = \sqrt{1 + C^2} - C$ for $C = 5$ and $g_1 = 1$ in $SO(3, 1) \times SO(3, 1)$ gauging

make the solutions of the BPS equations solve the second-order field equations as in the case of $SO(4) \times SO(4)$ gauging. The corresponding BPS equations are given by

$$\phi'_1 = \sqrt{2}g_1 e^{\frac{\phi}{2}} \cosh(2\phi_1) \sinh \phi_1, \tag{97}$$

$$\phi'_2 = \sqrt{2}g_1 e^{-\frac{\phi}{2}} \cosh(2\phi_2) \sinh \phi_2, \tag{98}$$

$$\phi' = \sqrt{2}g_1 e^{-\frac{\phi}{2}} [e^\phi \cosh \phi_1 (\cosh(2\phi_1) - 2) - \cosh \phi_2 (\cosh(2\phi_2) - 2)], \tag{99}$$

$$A' = \frac{1}{\sqrt{2}}g_1 e^{-\frac{\phi}{2}} [e^\phi \cosh \phi_1 (\cosh(2\phi_1) - 2) + \cosh \phi_2 (\cosh(2\phi_2) - 2)]. \tag{100}$$

With suitable boundary conditions, these equations can be solved numerically as in the previous cases. We will, however, look at particular truncations for which analytic solutions can be found. These solutions should be more interesting and more useful than the numerical ones in many aspects.

The first truncation is obtained by setting $\phi_2 = \phi_1$ and $\phi = 0$. It can be easily verified that this is a consistent truncation. The relevant BPS equations read

$$\phi'_1 = \sqrt{2}g_1 \cosh(2\phi_1) \sinh \phi_1, \tag{101}$$

$$A' = \sqrt{2}g_1 \cosh \phi_1 [\cosh(2\phi_1) - 2] \tag{102}$$

which have a solution

$$2g_1 r = \ln \left[\frac{1 - \sqrt{2} \cosh \phi_1}{1 + \sqrt{2} \cosh \phi_1} \right] - 2\sqrt{2} \tanh^{-1} e^{\phi_1}, \tag{103}$$

$$A = \ln(1 + e^{4\phi_1}) - \ln(1 - e^{2\phi_1}) - \phi_1. \tag{104}$$

The solution for ϕ_1 is clearly seen to be singular at a finite value of r .

Recall that ϕ_1 and ϕ_2 are dual to irrelevant operators, we expect that in this case, the $N = 4$ SCFT should appear in the IR. Near the singularity, we find

$$\phi_1 \sim \pm \frac{1}{3} \ln \left[C - \frac{3g_1 r}{2\sqrt{2}} \right] \quad \text{and} \quad A \sim -\frac{1}{3} \ln \left[C - \frac{3g_1 r}{2\sqrt{2}} \right] \tag{105}$$

for a constant C . It can be verified that, in this limit, the scalar potential blows up as $V \rightarrow \infty$. Therefore, the singularity is unphysical.

Another truncation is obtained by setting $\phi_2 = 0$ which gives rise to the BPS equations

$$\phi'_1 = \sqrt{2}g_1 e^{\frac{\phi}{2}} \cosh(2\phi_1) \sinh \phi_1, \tag{106}$$

$$\phi' = \sqrt{2}g_1 e^{-\frac{\phi}{2}} (1 + e^\phi \cosh \phi_1) [\cosh(2\phi_1) - 2], \tag{107}$$

$$A' = \frac{1}{\sqrt{2}}g_1 e^{-\frac{\phi}{2}} [e^\phi \cosh \phi_1 (\cosh(2\phi_1) - 2) - 1]. \tag{108}$$

An analytic solution to these equations is given by

$$\phi = \ln \left[\cosh \phi_1 - \frac{1}{2} C \cosh(2\phi_1) \operatorname{csch} \phi_1 \right], \tag{109}$$

$$\sqrt{2}g_1 \tilde{r} = \ln[C - \tanh(2\phi_1)], \tag{110}$$

$$A = \ln[\cosh(2\phi_1)] - \frac{1}{2} \ln(\sinh \phi_1) - \frac{1}{2} \ln[C \cosh(2\phi_1) - \sinh(2\phi_1)] \tag{111}$$

where the coordinate \tilde{r} is defined via $\frac{d\tilde{r}}{dr} = e^{-\frac{\phi}{2}}$. It should be noted that to give the correct behavior for ϕ and ϕ_1 near the AdS_4 critical point, we need $C \neq 0$.

The solution is singular at a finite value of \tilde{r} . Near this singularity, we find

$$\phi_1 \sim \pm \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right| \tag{112}$$

where \tilde{C} is a constant. The behavior of ϕ and A depends on the value of C .

We begin with the case $\phi_1 \rightarrow \infty$. For $C = 2$, we find from the explicit solution that

$$\begin{aligned} \phi &\sim -\phi_1 \sim \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|, \\ A &\sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|. \end{aligned} \tag{113}$$

For $C \neq 2$, we find

$$\begin{aligned} \phi &\sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|, \\ A &\sim \phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|. \end{aligned} \tag{114}$$

Both of these singularities lead to $V \rightarrow \infty$ and hence are unphysical.

We now move to another possibility with $\phi_1 \rightarrow -\infty$. In this case, we find

$$\phi \sim \phi_1 \sim \frac{1}{4} \ln \left| \sqrt{2}g_1 \tilde{r} - \tilde{C} \right|,$$

$$A \sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1\tilde{r} - \tilde{C} \right| \tag{115}$$

for $C = -2$ and

$$\begin{aligned} \phi &\sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1\tilde{r} - \tilde{C} \right|, \\ A &\sim -\phi_1 \sim -\frac{1}{4} \ln \left| \sqrt{2}g_1\tilde{r} - \tilde{C} \right| \end{aligned} \tag{116}$$

for $C \neq -2$. These behaviors also give $V \rightarrow \infty$. Therefore, we conclude that the solutions in this particular truncation do not holographically describe RG flows from $N = 4$ SCFT.

A similar analysis shows that the truncation with $\phi_1 = 0$ also leads to unphysical singularities. It would be interesting to uplift these solutions to ten or eleven dimensions and determine whether these singularities are resolved.

5 $N = 4$ $SO(4) \times SO(3, 1)$ gauged supergravity

In this section, we consider a gauge group with one compact and one non-compact factors of the form $SO(4) \times SO(3, 1)$. All the procedures are essentially the same, so we will not present much detail here. The $SO(4)$ and $SO(3, 1)$ are electrically and magnetically embedded in $SO(3, 3) \times SO(3, 3)$, respectively. The corresponding embedding tensor is given by

$$\begin{aligned} f_{+123} &= \sqrt{2}(g_1 - \tilde{g}_1), & f_{+789} &= \sqrt{2}(g_1 + \tilde{g}_1), \\ f_{-456} &= f_{-4,11,12} = f_{-10,5,12} = -f_{-10,11,6} \\ &= \frac{1}{\sqrt{2}}(g_2 - \tilde{g}_2), \\ f_{-10,11,12} &= f_{-4,11,6} = f_{-10,5,6} = -f_{-45,12} \\ &= \frac{1}{\sqrt{2}}(g_2 + \tilde{g}_2). \end{aligned} \tag{117}$$

5.1 Supersymmetric AdS_4 vacua

We consider scalar fields invariant under $SO(4)_{\text{inv}} \subset SO(4) \times SO(3) \subset SO(4) \times SO(3, 1)$. The corresponding coset representative for the $SO(6, 6)/SO(6) \times SO(6)$ coset is now given by

$$L = e^{\phi_1 \hat{Y}} e^{\phi_2 \tilde{Y}_2} \tag{118}$$

where \hat{Y}_1 and \tilde{Y}_2 are defined in (27) and (83), respectively.

The scalar potential turns out to be

$$\begin{aligned} V &= \frac{1}{8} e^{-\phi - 6\phi_1 - 6\phi_2} [(g_1 + g_2)^2 e^{2\phi + 12\phi_1 + 6\phi_2} \\ &\quad - 3(3g_1^2 + 2g_1g_2 + g_2^2) e^{2\phi + 4\phi_1 + 6\phi_2} \\ &\quad + e^{6\phi_1} [g_2^2 (1 + e^{2\phi} \chi^2) (1 + e^{4\phi_2})^3 \\ &\quad + 16e^{6\phi_2} [e^{2\phi} (g_1^2 + g_1g_2 - g_2^2 \chi^2) - g_2^2]] \\ &\quad + 8g_2 e^{\phi + 3\phi_1 + 6\phi_2} [g_1 (e^{2\phi_1} - 1)]^3 \end{aligned}$$

Table 5 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(4) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(4) \times SO(3, 1)$ gauge group

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1, 1) | 4 | 4 |
| (1, 1, 3) | $0_{\times 3}$ | 3 |
| (3, 3, 1) | $0_{\times 9}$ | 3 |
| (1, 1, 5) | $-2_{\times 5}$ | 1, 2 |
| (1, 3, 3) + (3, 1, 3) | $-2_{\times 18}$ | 1, 2 |

$$\begin{aligned} &+ g_2 e^{2\phi_1} (3 + e^{4\phi_1})] \cosh \phi_2 \\ &\times [\cosh(2\phi_2) - 2] - 3(3g_1^2 + 4g_1g_2 + 2g_2^2) \\ &\times e^{2\phi + 8\phi_1 + 6\phi_2} + g_1^2 e^{2\phi + 6\phi_2} \end{aligned} \tag{119}$$

where we have imposed the following relations

$$\tilde{g}_1 = g_1 + g_2 \quad \text{and} \quad \tilde{g}_2 = -g_2 \tag{120}$$

in order to have an $N = 4$ supersymmetric AdS_4 critical point with $SO(4) \times SO(3)$ symmetry at $\phi_1 = \phi_2 = \phi = \chi = 0$.

There are two supersymmetric AdS_4 vacua with $N = 4$ supersymmetry:

- The first critical point is a trivial one with $SO(4) \times SO(3)$ symmetry at
- A non-trivial supersymmetric critical point is given by

$$\phi = \chi = \phi_1 = \phi_2 = 0, \quad V_0 = -6g_2^2. \tag{121}$$

$$\begin{aligned} \phi_2 = \chi = 0, \quad \phi_1 &= \frac{1}{2} \ln \left[\frac{g_1}{g_1 + g_2} \right], \\ \phi &= \frac{1}{2} \ln \left[\frac{4g_1(g_1 + g_2)}{(2g_1 + g_2)^2} \right], \quad V_0 = -\frac{3g_2^2(2g_1 + g_2)}{\sqrt{g_1(g_1 + g_2)}}. \end{aligned} \tag{122}$$

This critical point is invariant under a smaller symmetry $SO(3) \times SO(3)$.

Scalar masses at these two critical points are given in Tables 5 and 6. It can be seen that the mass spectra are very similar to critical points III and IV in the case of $SO(4) \times SO(4)$ gauge group.

5.2 Holographic RG flow

In this section, we will give a supersymmetric RG flow solution interpolating between the two AdS_4 vacua identified

Table 6 Scalar masses at the $N = 4$ supersymmetric AdS_4 critical point with $SO(3) \times SO(3)$ symmetry and the corresponding dimensions of the dual operators for $SO(4) \times SO(3, 1)$ gauge group

| Scalar field representations | $m^2 L^2$ | Δ |
|------------------------------|------------------|----------|
| (1, 1) | $-2_{\times 2}$ | 1, 2 |
| (1, 1) | $4_{\times 2}$ | 4 |
| (1, 5) + (5, 1) | $-2_{\times 10}$ | 1, 2 |
| (1, 3) + (3, 1) | $0_{\times 6}$ | 3 |
| (3, 3) | $0_{\times 18}$ | 3 |

above. As in the previous cases, turning on vector multiplet scalars requires the vanishing of the axion χ . Since we are only interested in the solution interpolating between two AdS_4 vacua, we will accordingly set $\chi = 0$ from now on.

With $\chi = 0$, the superpotential is given by

$$\begin{aligned} \mathcal{W} = & \frac{i}{4\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1 - 3\phi_2} \left[g_1 e^{\phi + 3\phi_2} + 3g_1 e^{\phi + 4\phi_1 + 3\phi_2} \right. \\ & - 3(g_1 + g_2) e^{\phi + 2\phi_1 + 3\phi_2} \\ & + g_2 e^{3\phi_1} (1 + e^{2\phi_2}) (1 - 4e^{2\phi_2} + e^{4\phi_2}) \\ & \left. - (g_1 + g_2) e^{\phi + 6\phi_1 + 3\phi_2} \right] \end{aligned} \quad (123)$$

in term of which the scalar potential can be written as

$$V = 4 \left(\frac{\partial W}{\partial \phi} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_1} \right)^2 + \frac{2}{3} \left(\frac{\partial W}{\partial \phi_2} \right)^2 - 3W^2. \quad (124)$$

The BPS equations read

$$\begin{aligned} \phi_1' &= -\frac{2}{3} \frac{\partial W}{\partial \phi_1} \\ &= -\frac{1}{2\sqrt{2}} e^{\frac{\phi}{2} - 3\phi_1} (e^{4\phi_1} - 1) (e^{2\phi_1} (g_1 + g_2) - g_1), \end{aligned} \quad (125)$$

$$\begin{aligned} \phi_2' &= -\frac{2}{3} \frac{\partial W}{\partial \phi_2} \\ &= \frac{1}{2\sqrt{2}} g_2 e^{-\frac{\phi}{2} - 3\phi_1} (e^{2\phi_2} - 1) (e^{4\phi_1} + 1), \end{aligned} \quad (126)$$

$$\begin{aligned} \phi' &= -4 \frac{\partial W}{\partial \phi} \\ &= -\frac{1}{2\sqrt{2}} e^{-\frac{\phi}{2} - 3\phi_1} \left[4g_2 e^{3\phi_1} \cosh \phi_2 [\cosh(2\phi_2) - 2] \right. \\ & \quad \left. + e^{\phi} \left[(e^{2\phi_1} - 1)^3 g_1 + e^{2\phi_1} (3 + e^{4\phi_1}) g_2 \right] \right], \end{aligned} \quad (127)$$

$$\begin{aligned} A' &= \frac{1}{4} \sqrt{2} e^{-\frac{\phi}{2} - 3\phi_1} \left[e^{\phi} [(e^{2\phi_1} - 1)^3 g_1 + e^{2\phi_1} (3 + e^{4\phi_1}) g_2] \right. \\ & \quad \left. - 4g_2 e^{3\phi_1} \cosh \phi_2 [\cosh(2\phi_2) - 2] \right]. \end{aligned} \quad (128)$$

Since $\phi_2 = 0$ at both critical points, we can consistently truncate ϕ_2 out. Note also that ϕ_2 is dual to an irrelevant operator of dimension $\Delta = 4$ as can be seen from the linearized BPS equations which give

$$\phi \sim \phi_1 \sim e^{-\frac{t}{L}}, \quad \phi_2 \sim e^{\frac{t}{L}}. \quad (129)$$

With $\phi_2 = 0$, we find an RG flow solution driven by ϕ and ϕ_1 as follow

$$\begin{aligned} g_2(2g_1 + g_2)\bar{r} &= \sqrt{2}g_2 \tan^{-1} e^{\phi_1} + \sqrt{2}(2g_1 + g_2) \tanh^{-1} e^{\phi_1} \\ & - 2\sqrt{2g_1(g_1 + g_2)} \tanh^{-1} \left[e^{\phi_1} \sqrt{\frac{g_1 + g_2}{g_1}} \right], \end{aligned} \quad (130)$$

$$\phi = \ln \left[\frac{e^{-\phi_1} g_1 + e^{\phi_1} (g_1 + g_2)}{2g_1 + g_2} \right], \quad (131)$$

$$\begin{aligned} A &= \frac{1}{2} \phi_1 - \ln(1 - e^{4\phi_1}) + \ln[(e^{2\phi_1} - 1)g_1 \\ & + e^{2\phi_1} g_2] + \frac{1}{2} \ln[g_1 + (g_1 + g_2)e^{2\phi_1}] \end{aligned} \quad (132)$$

where the coordinate \bar{r} is related to r by the relation $\frac{d\bar{r}}{dr} = e^{\frac{\phi}{2}}$.

This solution preserves $N = 4$ supersymmetry in three dimensions and describes an RG flow from $N = 4$ SCFT in the UV with $SO(4) \times SO(3)$ symmetry to another $N = 4$ SCFT in the IR with $SO(3) \times SO(3)$ symmetry at which the operator dual to ϕ_1 is irrelevant. Although the number of supersymmetry is unchanged, the flavor symmetry $SO(3)$ in the UV is broken by the operator dual to ϕ_1 . We can also truncate out the vector multiplet scalars and study supersymmetric RG flows to non-conformal field theories as in the previous cases. However, we will not consider this truncation since it leads to similar structure as in the previous two gauge groups.

6 Conclusions and discussions

We have studied dyonic gaugings of $N = 4$ supergravity coupled to six vector multiplets with compact and non-compact gauge groups $SO(4) \times SO(4)$, $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$. We have identified a number of supersymmetric $N = 4$ AdS_4 vacua within these gauged supergravities and studied several RG flows interpolating between these vacua. The solutions describe supersymmetric deformations of the dual $N = 4$ SCFTs with different flavor symmetries in three dimensions. These deformations are driven by relevant operators of dimensions $\Delta = 1, 2$ which deform the UV SCFTs to other SCFTs or to non-conformal field theories in the IR.

For $SO(4) \times SO(4)$ gauge group, there are four supersymmetric AdS_4 vacua with $SO(4) \times SO(4)$, $SO(4) \times SO(3)$, $SO(3) \times SO(4)$ and $SO(4)$ symmetries. These vacua should correspond to $N = 4$ conformal fixed points of $N = 4$ CSM theories with $SO(4)$, $SO(3)$ and no flavor symmetries, respectively. We have found various RG flows interpolating between these critical points including RG flows connecting three critical points or a cascade of RG flows. These should be useful in holographic studies of $N = 4$ CSM theories.

In the case of non-compact $SO(3, 1) \times SO(3, 1)$ gauge group, we have found only one supersymmetric AdS_4 vacuum with $SO(3) \times SO(3)$ symmetry. We have given a number of RG flow solutions describing supersymmetric deformations of the dual $N = 4$ SCFT to $N = 4$ non-conformal field theories. The solutions with only scalar fields from the gravity multiplet non-vanishing give rise to physical singularities. Flows with vector multiplet scalars turned on, however, lead to physically unacceptable singularities. The mixed gauge group $SO(4) \times SO(3, 1)$ also exhibits similar structure of vacua and RG flows with two supersymmetric AdS_4 critical points.

Given our solutions, it is interesting to identify their higher dimensional origins in ten or eleven dimensions. Along this line, the result of [36,37] on $S^3 \times S^3$ compactifications could be a useful starting point for the $SO(4) \times SO(4)$ gauge group. The uplifted solutions would be desirable for a full holographic study of $N = 4$ CSM theories. This should provide an analogue of the recent uplifts of the GPPZ flow describing a massive deformation of $N = 4$ SYM [38,39]. The embedding of the non-compact gauge groups $SO(3, 1) \times SO(3, 1)$ and $SO(4) \times SO(3, 1)$ would also be worth considering.

Another direction is to find interpretations of the solutions given here in the dual $N = 4$ CSM theories with different flavor symmetries similar to the recent study in [18] for AdS_5/CFT_4 correspondence. The results found here is also in line with [18]. In particular, scalars in the gravity multiplet are dual to relevant operators at all critical points. These operators are in the same multiplet as the energy-momentum tensor. Another result is the exclusion between the operators dual to the axion and vector multiplet scalars which cannot be turned on simultaneously as required by supersymmetry in the gravity solutions. It would be interesting to find an analogous result on the field theory side.

A generalization of the present results to include more active scalars with smaller residual symmetries could provide more general holographic RG flow solutions in particular flows that break some amount of supersymmetry. We have indeed performed a partial analysis for $SO(3)_{inv}$ scalars. In this case, there are six singlets. It seems to be possible to have solutions that break supersymmetry from $N = 4$ to $N = 1$, but the scalar potential takes a highly complicated form. Therefore, we refrain from presenting it here. Solutions from other gauge groups more general than those considered here also deserve investigations. Finally, finding other types of solutions such as supersymmetric Janus and flows across dimensions to $AdS_2 \times \Sigma_2$, with Σ_2 being a Riemann surface, would also be useful in the holographic study of defect SCFTs and black hole physics. Recent works along this line include [20,40–45].

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Appendix A: Useful formulae

To convert an $SO(6)$ vector index m to a pair of anti-symmetric $SU(4)$ fundamental indices $[ij]$, we use the following 't Hooft symbols

$$\begin{aligned}
 G_1^{ij} &= \begin{bmatrix} 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{bmatrix}, & G_2^{ij} &= \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}, \\
 G_3^{ij} &= \begin{bmatrix} 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{bmatrix}, & G_4^{ij} &= \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \\
 G_5^{ij} &= \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix}, & G_6^{ij} &= \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}. \tag{133}
 \end{aligned}$$

These matrices satisfy the relation

$$G_{mij} = -\frac{1}{2} \epsilon_{ijkl} G_m^{kl} = -(G_m^{ij})^*. \tag{134}$$

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