

Symmetry constrained two Higgs doublet models

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Abstract We study two-Higgs-doublet models (2HDM) where Abelian symmetries have been introduced, leading to a drastic reduction in the number of free parameters in the 2HDM. Our analysis is inspired in BGL models, where, as the result of a symmetry of the Lagrangian, there are tree-level scalar mediated Flavour-Changing-Neutral-Currents, with the flavour structure depending only on the CKM matrix. A systematic analysis is done on the various possible schemes, which are classified in different classes, depending on the way the extra symmetries constrain the matrices of couplings defining the flavour structure of the scalar mediated neutral currents. All the resulting flavour textures of the Yukawa couplings are stable under renormalisation since they result from symmetries imposed at the Lagrangian level. We also present a brief phenomenological analysis of the most salient features of each class of symmetry constrained 2HDM.

1 Introduction

One of the simplest extensions of the Standard Model (SM) consists of the introduction of one or more additional scalar doublets to its spectrum. The first 2 Higgs doublet model (2HDM) was proposed by Lee [1] in order to generate spontaneous CP violation, at a time when only two incomplete generations were known. The general 2HDM [2, 3] has a priori two flavour problems:

- (i) it has potentially dangerous scalar mediated Flavour Changing Neutral Currents (FCNC) at tree level,
- (ii) it leads to a large increase in the number of flavour parameters in the scalar sector, parametrised by two arbitrary 3×3 complex matrices, which we denote by N_d and N_u .

The first problem was elegantly solved by Glashow and Weinberg [4] through the introduction of a \mathbb{Z}_2 discrete symmetry. However, an exact or unbroken \mathbb{Z}_2 symmetry renders it impossible to generate either spontaneous or explicit CP violation in the scalar sector, in the context of 2HDM. Both explicit [5] and spontaneous [6] CP violation in the scalar sector can be obtained if one introduces a third scalar doublet while maintaining FCNC in the scalar sector. Recently, it was pointed out [7] that an intriguing correlation exists between the possibility of a given scalar potential to generate explicit and spontaneous CP violation. Indeed in most examples studied, if a given scalar potential can generate spontaneous CP violation, it can also have explicit CP violation in the scalar sector.

In a separate development, which addresses simultaneously the above two problems of 2HDM, it was shown [8] by Branco, Grimus and Lavoura (BGL) that one may have a scenario where there are tree level FCNC, but with N_d and N_u fixed entirely by the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. In some BGL models, the suppression of FCNC couplings resulting from the smallness of CKM elements, is such that the new neutral scalars need not be too massive in order to conform with experiment. BGL models have been studied in the literature [9, 10] and their phenomenological consequences have been analysed in the context of the LHC [11–13]. A generalisation of BGL models has been recently proposed in the framework of 2HDM [14].

Regarding symmetries, Ferreira and Silva [15] classified all possible implementations of Abelian symmetries in

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2HDM with fermions which lead to non-vanishing quark masses and a CKM matrix which is not block diagonal (see also [16]).

In this paper we study in a systematic way scenarios arising from different implementations of Abelian symmetries in the context of 2HDM which can lead to a natural reduction in the number of parameters in these models. In the search for these scenarios, we were inspired by BGL and generalised BGL (gBGL) models where the coupling matrices N_d, N_u (see Eqs. (8)–(9)) can be written in terms of the quark mass matrices and projection operators. Thus we classify the different models according to the structures of N_d, N_u . We identify the symmetry leading to each of the models and the corresponding flavour textures of the Yukawa couplings. These textures are stable under renormalisation, since they result from symmetries of the Lagrangian.

The organisation of the paper is the following. The notation is set up in Sect. 2. We then present our main results in Sects. 3 and 4, obeying what we denote the *Left* and *Right conditions* introduced in Eqs. (13) and (16), respectively. We show that, besides BGL and gBGL there is a new type of model obeying *Left conditions* and that there are six classes of models obeying *Right conditions* which, as far as we can tell, are presented in full generality here for the first time. For definiteness, we concentrate on the quark sector. Some of the most salient phenomenological implications are presented in Sect. 5, and our conclusions appear in Sect. 6. We defer some technical details to Appendix A. In particular, we present in Appendix A.4 conditions for the identification of the various models which are invariant under basis transformations in the spaces of left-handed doublets and of up-type and down-type right-handed singlets.

2 Generalities and notation

The Yukawa Lagrangian, with summation over fermion generation indices implied and omitted, reads

$$\mathcal{L}_Y = -\bar{Q}_L^0[\Gamma_1\Phi_1 + \Gamma_2\Phi_2]d_R^0 - \bar{Q}_L^0[\Delta_1\tilde{\Phi}_1 + \Delta_2\tilde{\Phi}_2]u_R^0 + \text{H.c.}, \tag{1}$$

with $\tilde{\Phi}_j = i\sigma_2\Phi_j^*$. Electroweak spontaneous symmetry breaking arises from the vacuum expectation values of the scalar doublets

$$\langle\Phi_1\rangle = \begin{pmatrix} 0 \\ e^{i\xi_1}v_1/\sqrt{2} \end{pmatrix}, \quad \langle\Phi_2\rangle = \begin{pmatrix} 0 \\ e^{i\xi_2}v_2/\sqrt{2} \end{pmatrix}. \tag{2}$$

We use $v^2 \equiv v_1^2 + v_2^2$, $c_\beta \equiv \cos\beta \equiv v_1/v$, $s_\beta \equiv \sin\beta \equiv v_2/v$, $t_\beta \equiv \tan\beta$ and $\xi \equiv \xi_2 - \xi_1$. In the ‘‘Higgs basis’’ [17–19]

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ s_\beta & -c_\beta \end{pmatrix} \begin{pmatrix} e^{-i\xi_1} & \Phi_1 \\ e^{-i\xi_2} & \Phi_2 \end{pmatrix}, \tag{3}$$

only H_1 has a non-zero vacuum expectation value

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{4}$$

Expanding the scalar fields around Eq. (4), one has

$$H_1 = \begin{pmatrix} G^+ \\ (v + h^0 + iG^0)/\sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ (R^0 + iI^0)/\sqrt{2} \end{pmatrix}, \tag{5}$$

with G^0, G^\pm the would-be Goldstone bosons, h^0, R^0, I^0 neutral fields and H^\pm the charged scalar. Then, the Yukawa couplings in Eq. (1) read

$$-\frac{v}{\sqrt{2}}\mathcal{L}_Y = \bar{Q}_L^0(M_d^0H_1 + N_d^0H_2)d_R^0 + \bar{Q}_L^0(M_u^0\tilde{H}_1 + N_u^0\tilde{H}_2)u_R^0 + \text{H.c.}, \tag{6}$$

with the M_d^0, M_u^0 mass matrices, and the N_d^0, N_u^0 matrices given by

$$\begin{aligned} M_d^0 &= \frac{ve^{i\xi_1}}{\sqrt{2}}(c_\beta\Gamma_1 + e^{i\xi}s_\beta\Gamma_2), \\ N_d^0 &= \frac{ve^{i\xi_1}}{\sqrt{2}}(s_\beta\Gamma_1 - e^{i\xi}c_\beta\Gamma_2), \\ M_u^0 &= \frac{ve^{-i\xi_1}}{\sqrt{2}}(c_\beta\Delta_1 + e^{-i\xi}s_\beta\Delta_2), \\ N_u^0 &= \frac{ve^{-i\xi_1}}{\sqrt{2}}(s_\beta\Delta_1 - e^{-i\xi}c_\beta\Delta_2). \end{aligned} \tag{7}$$

$$\tag{8}$$

This Lagrangian can be written in terms of physical quantities as

$$-\frac{v}{\sqrt{2}}\mathcal{L}_Y = (\bar{u}_L V, \bar{d}_L)(M_d H_1 + N_d H_2)d_R + (\bar{u}_L, \bar{d}_L V^\dagger)(M_u \tilde{H}_1 + N_u \tilde{H}_2)u_R + \text{H.c.} \tag{9}$$

We have used the usual bidiagonalisations into the mass bases,

$$\begin{aligned} \mathcal{U}_L^{d\dagger} M_d^0 \mathcal{U}_R^d &= M_d = \text{diag}(m_d, m_s, m_b), \\ \mathcal{U}_L^{u\dagger} M_u^0 \mathcal{U}_R^u &= M_u = \text{diag}(m_u, m_c, m_t), \end{aligned} \tag{10}$$

via 3×3 unitary matrices \mathcal{U}_X^q ($q = u, d, X = L, R$). $V = \mathcal{U}_L^{u\dagger}\mathcal{U}_L^d$ in Eq. (9) is the CKM mixing matrix. While the quark masses M_d and M_u in Eq. (10) are characterised by $3 + 3 = 6$ physical parameters, in a general 2HDM the complex matrices

$$\mathcal{U}_L^{d\dagger} N_d^0 \mathcal{U}_R^d = N_d, \quad \mathcal{U}_L^{u\dagger} N_u^0 \mathcal{U}_R^u = N_u, \tag{11}$$

are free. This introduces in principle $2 \times 3 \times 3 \times 2 = 36$ new real parameters.¹ This large freedom is certainly a source of concern since, for example, FCNC can put significant constraints on N_d and N_u . Invariance under some (symmetry) transformation is the best motivated requirement which can limit this inflation of parameters. Following [15], we consider in particular Abelian symmetry transformations²

$$\begin{aligned} \Phi_1 &\mapsto \Phi_1, \quad \Phi_2 \mapsto e^{i\theta} \Phi_2, \quad Q_{Lj}^0 \mapsto e^{i\alpha_j\theta} Q_{Lj}^0, \\ d_{Rj}^0 &\mapsto e^{i\beta_j\theta} d_{Rj}^0, \quad u_{Rj}^0 \mapsto e^{i\gamma_j\theta} u_{Rj}^0, \end{aligned} \tag{12}$$

where $\alpha_j, \beta_j, \gamma_j$, are the charges of the different fermion doublets and singlets normalized to the charge of the second scalar doublet Φ_2 . As already mentioned, all possible realistic implementations of Eq. (12) were classified in [15]. In BGL models and their generalization in [14], the symmetry properties had an interesting translation into relations among the N_q^0 and M_q^0 matrices (very useful for example in the study of the renormalization group evolution of the Yukawa matrices). Having such a connection between a symmetry and matrix relations is not always possible. Inspired by the existence of that property in those two interesting classes of models, we focus on 2HDMs which obey an Abelian symmetry, Eq. (12), and which fulfill an additional requirement; either (a) or (b) below:

- (a) The Yukawa coupling matrices are required to obey *Left conditions*

$$N_d^0 = L_d^0 M_d^0, \quad N_u^0 = L_u^0 M_u^0, \tag{13}$$

with

$$L_q^0 = \ell_1^{[q]} P_1 + \ell_2^{[q]} P_2 + \ell_3^{[q]} P_3, \tag{14}$$

where $\ell_j^{[q]}$ are, a priori, arbitrary numbers. Here and henceforth we shall often use the index q to refer to matrices in the up ($q = u$) or down ($q = d$) sectors. We have used the projection operators P_i defined by $[P_i]_{jk} = \delta_{ij}\delta_{jk}$ (no sum in j). In matrix form:

$$P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{15}$$

These projection operators satisfy $P_i P_j = \delta_{ij} P_i$ (no sum in i) and $\sum_i P_i = \mathbf{1}$.

¹ Notice however that the bidiagonalisation of the mass matrices still leaves the freedom to rephase individual quark fields. Together with the CKM matrix, the N_d, N_u matrices should enter physical observables in rephasing invariant combinations [19].

² The global continuous symmetry transformation in Eq. (12) would give massless Nambu-Goldstone bosons upon electroweak symmetry breaking: they can be avoided by adding soft breaking terms in the scalar potential.

- (b) The Yukawa coupling matrices are instead required to obey *Right conditions*

$$N_d^0 = M_d^0 R_d^0, \quad N_u^0 = M_u^0 R_u^0, \tag{16}$$

with

$$R_q^0 = r_1^{[q]} P_1 + r_2^{[q]} P_2 + r_3^{[q]} P_3, \tag{17}$$

where $r_j^{[q]}$ are, again a priori, arbitrary numbers and, as in Eq. (14), P_i are the projection operators in Eq. (15).

Upper (and lower) case L's and R's are used in correspondence with the matrices (and parameters) acting on the left or the right of M_q^0 in Eqs. (13) and (16). Although it is not required a priori, the matrices L_q^0 and R_q^0 are non-singular. All the resulting models, that is all 2HDMs obeying Eq. (12) and either *Left* or *Right conditions* are analysed in Sects. 3 and 4, respectively.

We emphasize that our aim is to reduce the number of parameters. As shown in Ref. [15], imposing Abelian symmetries leaves only a reduced set of possible models, each with a significantly reduced number of independent parameters. Here, we consider only those Abelian models which can in some sense be seen as generalizations of the BGL models, by imposing, in addition, the *Left conditions* in Eq. (13), or the *Right conditions* in Eq. (16). As anticipated, the number of independent parameters of the models is significantly reduced with respect to the most general 2HDM. It is to be noticed that, as shown in Sects. 3.2 and 4.2, $\ell_j^{[q]}$ or $r_j^{[q]}$, which are a priori arbitrary, turn out to be unavoidably fixed in terms of t_β . Quite significantly, as analysed in Appendix A.1, Eqs. (13) and (16) have an elegant interpretation. In the popular 2HDMs of types I, II, X and Y [18, 20–22], a \mathbb{Z}_2 symmetry is incorporated and it eliminates the FCNC at tree level. But, in those cases, the \mathbb{Z}_2 assignment is universal for the different fermion families; all fermions of a given charge couple to the *same* scalar doublet. Here, Eqs. (13) and (16) have a different non-universal interpretation which leads to controlled FCNC:

- in the models of Sect. 3, obtained by imposing the *Left conditions* in Eq. (13), each left-handed doublet Q_{Li}^0 couples exclusively, i.e. to one and only one, of the scalar doublets Φ_k ,
- in the models of Sect. 4, obtained by imposing the *Right conditions* in Eq. (16), each right-handed singlet d_{Ri}^0, u_{Rj}^0 , couples exclusively to one scalar doublet Φ_k .

In particular, we stress that here, and in contrast to type I, II, X, and Y models, fermions of a given electric charge but different families need not couple all to the same scalar

doublet. In this sense, conditions (13) and (16) – applied in the context of models with Abelian symmetries – can also be seen as a generalization of the Glashow, Weinberg conditions [4] for Natural Flavour Conservation (NFC). In the present approach, having L_d^0 and L_u^0 proportional to the identity (or R_d^0 and R_u^0 proportional to the identity) enforces the NFC type I and type II 2HDM.

3 Symmetry controlled models with ‘Left’ Conditions

We present in this section the different models arising from an Abelian symmetry and for which there are matrices L_d^0 and L_u^0 such that Eq. (13) is verified. To this end, we have constructed a program which produces *all* models satisfying the Abelian symmetries in Eq. (12), and which lead to non-vanishing quark masses and a CKM matrix which is not block diagonal, thus verifying the results in ref. [15].³ For each Abelian model, the program then checks if it satisfies in addition Eq. (13). Thus, our final list will be complete. Before addressing the models themselves, it is convenient to make some observations on the effect of rotating into mass bases of the up and down quarks.

3.1 Conditions in the mass basis

In the mass bases, given by the unitary transformations in Eqs. (10), (13) reads

$$N_d = L_d M_d, \quad N_u = L_u M_u, \quad (18)$$

with the transformed matrices

$$L_d = \mathcal{U}_L^{d\dagger} L_d^0 \mathcal{U}_L^d, \quad L_u = \mathcal{U}_L^{u\dagger} L_u^0 \mathcal{U}_L^u. \quad (19)$$

Introducing transformed projection operators

$$P_j^{[dL]} \equiv \mathcal{U}_L^{d\dagger} P_j \mathcal{U}_L^d, \quad P_j^{[uL]} \equiv \mathcal{U}_L^{u\dagger} P_j \mathcal{U}_L^u, \quad (20)$$

one simply has

$$\begin{aligned} L_d &= \ell_1^{[d]} P_1^{[dL]} + \ell_2^{[d]} P_2^{[dL]} + \ell_3^{[d]} P_3^{[dL]}, \\ L_u &= \ell_1^{[u]} P_1^{[uL]} + \ell_2^{[u]} P_2^{[uL]} + \ell_3^{[u]} P_3^{[uL]}. \end{aligned} \quad (21)$$

Furthermore, since the CKM matrix is $V = \mathcal{U}_L^{u\dagger} \mathcal{U}_L^d$, one has the straightforward relation

$$P_k^{[uL]} = V P_k^{[dL]} V^\dagger, \quad (22)$$

which is relevant for the parametrisation of the FCNC couplings in the discussion to follow.

³ In fact, there is a misprint in Eq. (89) of the published version of [15], which however is correct in the arxiv version.

3.2 How to determine ℓ_i

Here we show how one determines the coefficients ℓ_i ($i = 1, 2, 3$) just by examining the form of the Yukawa matrices Γ_1 and Γ_2 . For definiteness, we concentrate on the down sector. The reasoning for the up sector follows similar lines and yields the same conclusions.

As a first step, we notice that, under the assumption of an Abelian symmetry [15],

$$(\Gamma_1)_{ia} \neq 0 \Rightarrow (\Gamma_2)_{ia} = 0, \quad (23)$$

(and the converse $1 \leftrightarrow 2$ also holds); notice that this implication involves the *same* matrix element of Γ_1 and Γ_2 .

As a second step, consider $(\Gamma_1)_{ia} \neq 0$. We already know that this implies $(\Gamma_2)_{ia} = 0$. But then, the (ia) entries in Eqs. (7) yield

$$(M_d^0)_{ia} = \frac{ve^{i\xi_1}}{\sqrt{2}} c_\beta (\Gamma_1)_{ia}, \quad (N_d^0)_{ia} = \frac{ve^{i\xi_1}}{\sqrt{2}} s_\beta (\Gamma_1)_{ia}, \quad (24)$$

and we obtain

$$(N_d^0)_{ia} = t_\beta (M_d^0)_{ia}. \quad (25)$$

Now we use the *Left conditions* in Eqs. (13)–(15):

$$(N_d^0)_{ia} = \ell_i^{[d]} (M_d^0)_{ia}. \quad (26)$$

Combining Eqs. (25) and (26), we find that

$$(\Gamma_1)_{ia} \neq 0 \Rightarrow \ell_i^{[d]} = t_\beta. \quad (27)$$

As a third step, we consider the possibility that $(\Gamma_2)_{ib} \neq 0$. A similar argument entails

$$(\Gamma_2)_{ib} \neq 0 \Rightarrow \ell_i^{[d]} = -t_\beta^{-1}. \quad (28)$$

Comparing Eqs. (27) and (28), we conclude that the combination of an Abelian symmetry, *c.f.* Eq. (12), with the *Left conditions* of Eq. (13) implies that one cannot have simultaneously $(\Gamma_1)_{ia} \neq 0$ and $(\Gamma_2)_{ib} \neq 0$, for *any* choices of a and b . So, for the *Left condition*, Γ_1 and Γ_2 cannot both have nonzero matrix elements in the same row. This has the physical consequence that each doublet Q_{Li}^0 couples to one and only one doublet Φ_k .

Moreover, we find the rule book for the assignment of $\ell_i^{[d]}$ in our models with *Left conditions*:

$$\begin{aligned} \text{if } (\Gamma_1)_{ia} \text{ exists, then } \ell_i^{[d]} &= t_\beta; \\ \text{if } (\Gamma_2)_{ia} \text{ exists, then } \ell_i^{[d]} &= -t_\beta^{-1}. \end{aligned} \quad (29)$$

One can easily see that the up sector matrices Δ_1 and Δ_2 , and the corresponding $\ell_i^{[u]}$ follow exactly the same rule.

3.3 Left models

Omitting the trivial cases of type I or type II 2HDMs, for which the transformation properties in Eq. (12) have no flavour dependence (both L_d and L_u are in that case proportional to the identity matrix $\mathbf{1}$), we now address the different possible models which obey *Left conditions*.

3.3.1 BGL models

We start with the well known case of BGL models [8]. The symmetry transformation is

$$\begin{aligned} \Phi_2 &\mapsto e^{i\theta} \Phi_2, & Q_{L3}^0 &\mapsto e^{-i\theta} Q_{L3}^0, \\ d_{R3}^0 &\mapsto e^{-i2\theta} d_{R3}^0, & \theta &\neq 0, \pi. \end{aligned} \tag{30}$$

The corresponding Yukawa coupling matrices are

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \\ \Delta_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \end{aligned} \tag{31}$$

where \times denote arbitrary, independent, and (in general) non-vanishing matrix entries. Following the rule book in Eq. (29) for the *Left conditions*, we find immediately

$$\begin{aligned} N_d^0 &= (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3) M_d^0, \\ N_u^0 &= (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3) M_u^0. \end{aligned} \tag{32}$$

Here, the right-handed singlet transforming non-trivially in Eq. (30) is a down quark. Such models are sometimes known as down-type BGL models, “dBGL”. In the particular implementation shown in Eq. (30), it is the third generation down quark which is involved; this is known as a “*b* model”. We could equally well have substituted the $d_{R3}^0 \mapsto e^{-i2\theta} d_{R3}^0$ transformation in Eq. (30) by $d_{R1}^0 \mapsto e^{-i2\theta} d_{R1}^0$, or by $d_{R2}^0 \mapsto e^{-i2\theta} d_{R2}^0$. These are known as “*d* model” and “*s* model”, respectively.

Parametrisation

Following Eqs. (32) and (20), one can write

$$\begin{aligned} N_d &= (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[dL]}) M_d, \\ N_u &= (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[uL]}) M_u. \end{aligned} \tag{33}$$

Since Γ_1 and Γ_2 are block diagonal, M_d^0 is block-diagonal too and then

$$P_3^{[dL]} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{34}$$

Using Eq. (22),

$$P_3^{[uL]} = V P_3^{[dL]} V^\dagger, \quad \text{i.e. } (P_3^{[uL]})_{ij} = V_{i3} V_{j3}^* = V_{ib} V_{jb}^*, \tag{35}$$

and one obtains the final parametrisation for the physical couplings

$$\begin{aligned} (N_d)_{ij} &= \delta_{ij} (t_\beta - (t_\beta + t_\beta^{-1}) \delta_{j3}) m_{dj}, \\ (N_u)_{ij} &= (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ib} V_{jb}^*) m_{uj}. \end{aligned} \tag{36}$$

Equation (36) involves quarks masses, CKM mixings and t_β , but *no new parameters*. BGL models implement in a renormalizable 2HDM the ideas of Minimal Flavour Violation. Besides this important property, BGL models are special in some respects that deserve comment: tree level FCNC are present either in the up or in the down quark sector, not in both (in this example, the *b*-dBGL model, they only appear in the up sector). The transformation properties in Eq. (30) give a block diagonal form for the down Yukawa coupling matrices: this corresponds to the fact that some matrix conditions of the *Right* type are also fulfilled for BGL models (this is not the case for the models in the next sections). Finally, when the lepton sector is included in the picture, it was shown in [10] that the appropriate symmetry transformation group is \mathbb{Z}_4 , that is $\theta \rightarrow \pi/2$ in Eq. (30).

3.3.2 Generalised BGL: gBGL

This second class of models is a generalisation of BGL models, introduced in [14] (see also [23]); the defining transformation properties are

$$\Phi_2 \mapsto -\Phi_2, \quad Q_{L3}^0 \mapsto -Q_{L3}^0, \tag{37}$$

and the symmetry group is just \mathbb{Z}_2 . The corresponding Yukawa matrices are

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \\ \Delta_2 &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}. \end{aligned} \tag{38}$$

Following the rule book in Eq. (29) for the *Left conditions*, we find immediately

$$\begin{aligned} N_d^0 &= (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3) M_d^0, \\ N_u^0 &= (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3) M_u^0. \end{aligned} \tag{39}$$

Parametrisation

While in the BGL model (of Sect. 3.3.1) Γ_1 and Γ_2 are block-diagonal, this is not the case here. However, Eqs. (32) and (39) are identical,⁴ giving again

$$\begin{aligned} N_d &= (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[dL]}) M_d, \\ N_u &= (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[uL]}) M_u. \end{aligned} \tag{40}$$

Recalling Eq. (20), one can introduce complex unitary vectors $\hat{n}_{[d]}$ and $\hat{n}_{[u]}$ by

$$\hat{n}_{[d]j} \equiv (P_3 \mathcal{U}_L^d)_{3j}, \quad \hat{n}_{[u]j} \equiv (P_3 \mathcal{U}_L^u)_{3j}, \tag{41}$$

in terms of which

$$(P_3^{[dL]})_{ij} = \hat{n}_{[d]i}^* \hat{n}_{[d]j}, \quad (P_3^{[uL]})_{ij} = \hat{n}_{[u]i}^* \hat{n}_{[u]j}. \tag{42}$$

The N_d and N_u matrices are then given by

$$\begin{aligned} (N_d)_{ij} &= (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[d]i}^* \hat{n}_{[d]j}) m_{dj}, \\ (N_u)_{ij} &= (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[u]i}^* \hat{n}_{[u]j}) m_{uj}. \end{aligned} \tag{43}$$

It is important to stress that $\hat{n}_{[d]}$ and $\hat{n}_{[u]}$ are not independent. From Eq. (22),

$$\hat{n}_{[u]i} V_{ij} = \hat{n}_{[d]j}, \tag{44}$$

and thus only four new independent parameters (besides quark masses, CKM mixings and t_β) appear in Eq. (43): two moduli, the third being fixed by normalization, and two relative phases, since the products $\hat{n}_{[q]i}^* \hat{n}_{[q]j}$ are insensitive to an overall phase.

3.3.3 jBGL

The last case in this section is a new model presented here for the first time (see also [24]). It is a sort of ‘‘Flipped’’ generalised BGL, which follows from

$$\begin{aligned} \Phi_2 &\mapsto e^{i\theta} \Phi_2, \quad Q_{L3}^0 \mapsto e^{-i\theta} Q_{L3}^0, \\ d_{Rj}^0 &\mapsto e^{-i\theta} d_{Rj}^0, \quad j = 1, 2, 3. \end{aligned} \tag{45}$$

The corresponding Yukawa coupling matrices are

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ 0 & 0 & 0 \end{pmatrix},$$

⁴ This is consistent with the fact that BGL can be recovered as a particular limit of generalised BGL models.

$$\Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \times & \times & \times \end{pmatrix}. \tag{46}$$

The *Left conditions* read in this case

$$\begin{aligned} N_d^0 &= (-t_\beta^{-1} P_1 - t_\beta^{-1} P_2 + t_\beta P_3) M_d^0, \\ N_u^0 &= (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3) M_u^0. \end{aligned} \tag{47}$$

Notice how, with respect to Eq. (38), the structures of the down Yukawa matrices Γ_1 and Γ_2 are interchanged (while the Δ matrices remain the same).

Parametrisation

Benefiting from the details given in the parametrisation of the gBGL models of Sect. 3.3.2, it is now straightforward to obtain

$$\begin{aligned} (N_d)_{ij} &= (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{n}_{[d]i}^* \hat{n}_{[d]j}) m_{dj}, \\ (N_u)_{ij} &= (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[u]i}^* \hat{n}_{[u]j}) m_{uj}, \end{aligned} \tag{48}$$

where, again, $\hat{n}_{[u]i} V_{ij} = \hat{n}_{[d]j}$. Notice the difference in the t_β dependence of N_d in Eq. (48), with respect to the gBGL case in Eq. (43).

One can see that BGL is *not* a particular case of jBGL. Also, BGL is a particular limit of gBGL, and jBGL is a sort of ‘‘Flipped’’ gBGL. One might wonder whether there is some sort of ‘‘Flipped’’ BGL, obtainable from an Abelian symmetry, which arises as a suitable limit of jBGL. It is possible to see by inspection of the symmetry transformations in Eq. (12) that such a case is not allowed.

3.4 Summary of models with Left conditions

We summarize in Table 2 the main properties of the different models discussed in the previous sections, which obey *Left conditions*. For the BGL models of Sect. 3.3.1 we display separately up and down type models (uBGL and dBGL respectively). Since we have started from *all* Abelian models consistent with non-zero masses and a CKM matrix not block diagonal [15], we are certain that Table 2 contains all models satisfying the *Left condition*. We recovered the BGL [8] and gBGL [14] models already present in the literature, and proved that there exists only one such new class of models, which we dubbed ‘‘jBGL’’.

4 Symmetry controlled models with right conditions

In the previous section we have explored 2HDM whose symmetry under the Abelian transformations in Eq. (12) is supplemented by the requirement that the M_q^0 and N_q^0 obey the relations in Eq. (13), where L_q^0 in Eq. (14) acts *on the left*. In this section we analyse symmetry based models where we impose the conditions of Eq. (16), $N_q^0 = M_q^0 R_q^0$, where R_q^0

Table 1 Models obeying *Right conditions*, Eq. (49). As in Table 2, the first row shows Glashow–Weinberg models without tree level FCNC for comparison. In the “Parameters” column, (+4) or (+8) is the number of new parameters associated to $\hat{r}_{[q]}$

Model	Properties		Parameters	Transformation
	Sym.	Tree FCNC		
G–W	\mathbb{Z}_2	$(N_u)_{ij} \propto m_{u_i} \delta_{ij}$ $(N_d)_{ij} \propto m_{d_i} \delta_{ij}$	t_β, m_{q_k}	$\Phi_2 \mapsto -\Phi_2$
Type A	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i} t_\beta \delta_{ij}$	t_β, m_{q_k} $\hat{r}_{[u]}(+4)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $u_{R3}^0 \mapsto e^{i\theta} u_{R3}^0$
Type B	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i} t_\beta \delta_{ij}$	t_β, m_{q_k} $\hat{r}_{[u]}(+4)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $u_{R1}^0 \mapsto e^{i\theta} u_{R1}^0, u_{R2}^0 \mapsto e^{i\theta} u_{R2}^0$
Type C	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} t_\beta \delta_{ij}$ $(N_d)_{ij} = m_{d_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[d]}(+4)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0$
Type D	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} t_\beta \delta_{ij}$ $(N_d)_{ij} = m_{d_i} (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[d]}(+4)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $d_{R1}^0 \mapsto e^{-i\theta} d_{R1}^0, d_{R2}^0 \mapsto e^{-i\theta} d_{R2}^0$
Type E	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[u]}, \hat{r}_{[d]}(+8)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0, u_{R3}^0 \mapsto e^{i\theta} u_{R3}^0$
Type F	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = m_{u_i} (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j})$ $(N_d)_{ij} = m_{d_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j})$	t_β, m_{q_k} $\hat{r}_{[u]}, \hat{r}_{[d]}(+8)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2, d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0$ $u_{R1}^0 \mapsto e^{i\theta} u_{R1}^0, u_{R2}^0 \mapsto e^{i\theta} u_{R2}^0$

Table 2 Models obeying the *Left conditions* of Eq. (18). For the uBGL and dBGL models we only show the FCNC corresponding to one case, the *top* and *bottom* models, respectively. The first row shows

Glashow–Weinberg models without tree level FCNC for comparison. In the “Parameters” column, (+4) is the number of new real parameters corresponding to $\hat{n}_{[q]}$

Model	Properties		Parameters	Transformation
	Sym.	Tree FCNC		
G–W	\mathbb{Z}_2	$(N_u)_{ij} \propto \delta_{ij} m_{u_j}$ $(N_d)_{ij} \propto \delta_{ij} m_{d_j}$	t_β, m_{q_k}	$\Phi_2 \mapsto -\Phi_2$
uBGL (<i>t</i>)	$\mathbb{Z}_{n \geq 4}$	$(N_u)_{ij} = \delta_{ij} (t_\beta - (t_\beta + t_\beta^{-1}) \delta_{j3}) m_{u_j}$ $(N_d)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ii}^* V_{ij}) m_{d_j}$	V, t_β, m_{q_k}	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $Q_{L3}^0 \mapsto e^{i\theta} Q_{L3}^0, u_{R3}^0 \mapsto e^{i2\theta} u_{R3}^0$
dBGL (<i>b</i>)	$\mathbb{Z}_{n \geq 4}$	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) V_{ib} V_{jb}^*) m_{u_j}$ $(N_d)_{ij} = \delta_{ij} (t_\beta - (t_\beta + t_\beta^{-1}) \delta_{j3}) m_{d_j}$	V, t_β, m_{q_k}	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $Q_{L3}^0 \mapsto e^{-i\theta} Q_{L3}^0, d_{R3}^0 \mapsto e^{-i2\theta} d_{R3}^0$
gBGL	\mathbb{Z}_2	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[u]i}^* \hat{n}_{[u]j}) m_{u_j}$ $(N_d)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[d]i}^* \hat{n}_{[d]j}) m_{d_j}$	V, t_β, m_{q_k} $\hat{n}_{[q]}(+4)$	$\Phi_2 \mapsto -\Phi_2$ $Q_{L3}^0 \mapsto -Q_{L3}^0$
jBGL	$\mathbb{Z}_{n \geq 2}$	$(N_u)_{ij} = (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{n}_{[u]i}^* \hat{n}_{[u]j}) m_{u_j}$ $(N_d)_{ij} = (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{n}_{[d]i}^* \hat{n}_{[d]j}) m_{d_j}$	V, t_β, m_{q_k} $\hat{n}_{[q]}(+4)$	$\Phi_2 \mapsto e^{i\theta} \Phi_2$ $Q_{L3}^0 \mapsto e^{-i\theta} Q_{L3}^0, d_{Rj}^0 \mapsto e^{-i\theta} d_{Rj}^0$

in Eq. (17) acts on the right, that is, models which obey *Right conditions*.

4.1 Conditions in the mass basis

In the mass basis, Eq. (16) reads

$$N_d = M_d R_d, \quad N_u = M_u R_u, \tag{49}$$

with the transformed matrices

$$R_d = \mathcal{U}_R^{d\dagger} R_d^0 \mathcal{U}_R^d, \quad R_u = \mathcal{U}_R^{u\dagger} R_u^0 \mathcal{U}_R^u, \tag{50}$$

and

$$R_d = r_1^{[d]} P_1^{[dR]} + r_2^{[d]} P_2^{[dR]} + r_3^{[d]} P_3^{[dR]},$$

$$R_u = r_1^{[u]} P_1^{[uR]} + r_2^{[u]} P_2^{[uR]} + r_3^{[u]} P_3^{[uR]}. \tag{51}$$

The transformed projection operators are now

$$P_j^{[dR]} \equiv \mathcal{U}_R^{d\dagger} P_j \mathcal{U}_R^d, \quad P_j^{[uR]} \equiv \mathcal{U}_R^{u\dagger} P_j \mathcal{U}_R^u. \tag{52}$$

$P_j^{[dR]}$ and $P_j^{[uR]}$ are related via $\mathcal{U}_R^{u\dagger} \mathcal{U}_R^d$, but, contrary to Sect. 3.1, this right-handed analog of the CKM matrix is completely arbitrary. This straightforward yet crucial difference

among models with *Left* and *Right conditions* will ultimately be responsible for the wider parametric freedom of the latter.

4.2 How to determine r_i

Repeating the steps in Sect. 3.2, one can easily establish here the following rule book for the assignment of r_i in our models with *Right conditions*:

$$\begin{aligned} \text{if } (\Gamma_1)_{ai} \text{ exists, then } r_i^{[d]} &= t_\beta; \\ \text{if } (\Gamma_2)_{ai} \text{ exists, then } r_i^{[d]} &= -t_\beta^{-1}. \end{aligned} \tag{53}$$

One can also see that the up sector matrices Δ_1 and Δ_2 , and the corresponding $r_i^{[u]}$ follow exactly the same rule.

4.3 Right models

It is obvious that cases in which both R_d and R_u are proportional to the identity matrix have been discarded automatically by the discussion of models with *Left conditions*. But, for *Right conditions* it is still possible to have either $R_d \propto \mathbf{1}$ or $R_u \propto \mathbf{1}$ (but not both). Among the six different types of models which obey *Right conditions*, the first four have that property.

4.3.1 Type A

The first model follows from symmetry under

$$\Phi_2 \mapsto e^{i\theta} \Phi_2, \quad u_{R3}^0 \mapsto e^{i\theta} u_{R3}^0. \tag{54}$$

The Yukawa coupling matrices in this case are

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \\ \Delta_2 &= \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}. \end{aligned} \tag{55}$$

We should mention that, as explained in Appendix A.3, it is immaterial whether Δ_1 contains the first two columns and Δ_2 the third, or some other permutation is chosen.

Following the rule book in Eq. (53) for the *Right conditions*, we find immediately

$$N_d^0 = M_d^0 t_\beta \mathbf{1}, \quad N_u^0 = M_u^0 (t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3). \tag{56}$$

Parametrisation

Since only Γ_1 is non-zero, the down sector is trivial: $N_d = t_\beta M_d$. For the up sector, however,

$$N_u = M_u (t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[u_R]}). \tag{57}$$

Similarly to the models in Sect. 3, one can introduce a complex unitary vector $\hat{r}_{[u]}$

$$\hat{r}_{[u]j} \equiv (P_3 U_R^u)_{3j}, \tag{58}$$

in terms of which

$$(P_3^{[u_R]})_{ij} = \hat{r}_{[u]i}^* \hat{r}_{[u]j}, \tag{59}$$

and thus

$$\begin{aligned} (N_d)_{ij} &= m_{d_i} t_\beta \delta_{ij}, \\ (N_u)_{ij} &= m_{u_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j}). \end{aligned} \tag{60}$$

Therefore, besides quark masses and t_β , only four new independent parameters appear in Eq. (60).

4.3.2 Type B

The second model follows from the symmetry

$$\Phi_2 \mapsto e^{i\theta} \Phi_2, \quad u_{R1}^0 \mapsto e^{i\theta} u_{R1}^0, \quad u_{R2}^0 \mapsto e^{i\theta} u_{R2}^0. \tag{61}$$

The corresponding Yukawa coupling matrices are

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \\ \Delta_2 &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}. \end{aligned} \tag{62}$$

Notice how, with respect to the previous model in Eq. (55), the forms of Δ_1 and Δ_2 are interchanged in Eq. (62). Thus, our Type B model is a sort of Flipped Type A model. The *Right conditions* become

$$N_d^0 = M_d^0 t_\beta \mathbf{1}, \quad N_u^0 = M_u^0 (-t_\beta^{-1} P_1 - t_\beta^{-1} P_2 + t_\beta P_3). \tag{63}$$

Parametrisation

Given the parametrisation of the previous case, it follows immediately that in this case:

$$\begin{aligned} (N_d)_{ij} &= m_{d_i} t_\beta \delta_{ij}, \\ (N_u)_{ij} &= m_{u_i} (-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{r}_{[u]i}^* \hat{r}_{[u]j}). \end{aligned} \tag{64}$$

Notice the different t_β dependence in Eq. (64) with respect to Eq. (60).

4.3.3 Type C

The transformation properties for this model are

$$\Phi_2 \mapsto e^{i\theta} \Phi_2, \quad d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0, \tag{65}$$

and the *Right conditions* read

$$N_d^0 = M_d^0(t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3), \quad N_u^0 = M_u^0 t_\beta \mathbf{1}. \quad (66)$$

The Yukawa coupling matrices are in this case

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix},$$

$$\Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (67)$$

Parametrisation

Since

$$N_d = M_d(t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[dR]}), \quad N_u = t_\beta M_u, \quad (68)$$

defining

$$\hat{r}_{[d]j} \equiv (P_3 \mathcal{U}_R^d)_{3j}, \quad (69)$$

and

$$(P_3^{[dR]})_{ij} = \hat{r}_{[d]i}^* \hat{r}_{[d]j}, \quad (70)$$

we find

$$(N_d)_{ij} = m_{d_i} (t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j}), \quad (N_u)_{ij} = t_\beta m_{u_i} \delta_{ij}, \quad (71)$$

implying that, besides quark masses and t_β , only four new independent parameters appear in Eq. (71). A particular case of these models appears in Ref. [25], with all coefficients taken as real in order to have an exclusive spontaneous origin for CP violation (no CKM CP violation). As such, there are in ref. [25] only two instead of four parameters arising from $\hat{r}_{[d]j}$.

4.3.4 Type D

The transformation properties for this model are

$$\Phi_2 \mapsto e^{i\theta} \Phi_2, \quad d_{R1}^0 \mapsto e^{-i\theta} d_{R1}^0, \quad d_{R2}^0 \mapsto e^{-i\theta} d_{R2}^0, \quad (72)$$

and the *Right conditions* read

$$N_d^0 = M_d^0(-t_\beta^{-1} P_1 - t_\beta^{-1} P_2 + t_\beta P_3), \quad N_u^0 = M_u^0 t_\beta \mathbf{1}. \quad (73)$$

The Yukawa coupling matrices are in this case

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Gamma_2 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix},$$

$$\Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (74)$$

Parametrisation

Here

$$N_d = M_d(-t_\beta^{-1} \mathbf{1} + (t_\beta + t_\beta^{-1}) P_3^{[dR]}), \quad N_u = t_\beta M_u, \quad (75)$$

from which

$$(N_d)_{ij} = m_{d_i} (-t_\beta \delta_{ij} + (t_\beta + t_\beta^{-1}) \hat{r}_{[d]i}^* \hat{r}_{[d]j}),$$

$$(N_u)_{ij} = t_\beta m_{u_i} \delta_{ij}. \quad (76)$$

Therefore, besides quark masses and t_β , only four new independent parameters appear in Eq. (76).

4.3.5 Type E

The transformation properties for this model are

$$\Phi_2 \mapsto e^{i\theta} \Phi_2, \quad d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0, \quad u_{R3}^0 \mapsto e^{i\theta} u_{R3}^0, \quad (77)$$

and the corresponding Yukawa coupling matrices are

$$\Gamma_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix},$$

$$\Delta_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \quad (78)$$

leading to the *Right conditions*

$$N_d^0 = M_d^0(t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3),$$

$$N_u^0 = M_u^0(t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3). \quad (79)$$

Parametrisation

While in the previous models one quark sector had a trivial structure (since $\Gamma_2 = 0$ in types A and B, while $\Delta_2 = 0$ in types C and D), that is not the case in Eq. (78), and one naturally expects an increase in the number of parameters. An appropriate parametrisation is obtained along the same lines as before. With

$$N_d = M_d(t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[dR]}),$$

$$N_u = M_u(t_\beta \mathbf{1} - (t_\beta + t_\beta^{-1}) P_3^{[uR]}), \quad (80)$$

but *two* complex unitary vectors are now necessary, $\hat{r}_{[d]}$ and $\hat{r}_{[u]}$, defined by

$$\hat{r}_{[d]j} \equiv (P_3 \mathcal{U}_R^d)_{3j}, \quad \hat{r}_{[u]j} \equiv (P_3 \mathcal{U}_R^u)_{3j}, \quad (81)$$

and in terms of which

$$(P_3^{[dR]})_{ij} = \hat{r}_{[d]i}^* \hat{r}_{[d]j}, \quad (P_3^{[uR]})_{ij} = \hat{r}_{[u]i}^* \hat{r}_{[u]j}. \quad (82)$$

The parametrisation of this model is then

$$\begin{aligned} (N_d)_{ij} &= m_{d_i}(t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}_{[d]i}^* \hat{r}_{[d]j}), \\ (N_u)_{ij} &= m_{u_i}(t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}_{[u]i}^* \hat{r}_{[u]j}). \end{aligned} \tag{83}$$

It is important to notice that now, besides the quark masses and t_β , four new independent real parameters enter Eq. (83) via $\hat{r}_{[d]j}$ and another four via $\hat{r}_{[u]j}$. Contrary to the situation in models with *Left conditions* in Sect. 3, where the CKM matrix ties $\hat{h}_{[u]}$ and $\hat{h}_{[d]}$, and it is fixed or given by another sector of the complete model (the couplings of quarks to the W gauge boson), in models with *Right conditions* there is no analog of the CKM matrix to connect $\hat{r}_{[u]}$ and $\hat{r}_{[d]}$ in a fixed manner.⁵

4.3.6 Type F

The transformation properties of this last model are

$$\begin{aligned} \Phi_2 &\mapsto e^{i\theta} \Phi_2, \quad d_{R3}^0 \mapsto e^{-i\theta} d_{R3}^0, \\ u_{R1}^0 &\mapsto e^{i\theta} u_{R1}^0, \quad u_{R2}^0 \mapsto e^{i\theta} u_{R2}^0, \end{aligned} \tag{84}$$

and the corresponding Yukawa coupling matrices have the following form:

$$\begin{aligned} \Gamma_1 &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}, \Gamma_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \Delta_1 = \begin{pmatrix} 0 & 0 & \times \\ 0 & 0 & \times \\ 0 & 0 & \times \end{pmatrix}, \\ \Delta_2 &= \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ \times & \times & 0 \end{pmatrix}. \end{aligned} \tag{85}$$

Notice how, with respect to the previous model in Eq. (78), the forms of Δ_1 and Δ_2 are interchanged in Eq. (85). Thus, our Type F model is a sort of Flipped Type E model. The *Right conditions* become

$$\begin{aligned} N_d^0 &= M_d^0(t_\beta P_1 + t_\beta P_2 - t_\beta^{-1} P_3), \\ N_u^0 &= M_u^0(-t_\beta^{-1} P_1 - t_\beta^{-1} P_2 + t_\beta P_3). \end{aligned} \tag{86}$$

Parametrisation

Parametrising this last model follows trivially from the previous one:

$$\begin{aligned} (N_d)_{ij} &= m_{d_i}(t_\beta \delta_{ij} - (t_\beta + t_\beta^{-1})\hat{r}_{[d]i}^* \hat{r}_{[d]j}), \\ (N_u)_{ij} &= m_{u_i}(-t_\beta^{-1} \delta_{ij} + (t_\beta + t_\beta^{-1})\hat{r}_{[u]i}^* \hat{r}_{[u]j}). \end{aligned} \tag{87}$$

The same comments made in Type E apply to the parameter count in Type F models: besides the quark masses and t_β , as in Eq. (83), four new independent real parameters enter Eq. (87) via $\hat{r}_{[d]j}$ and another four via $\hat{r}_{[u]j}$.

⁵ Interpreting the situation the other way around, Eq. (83) would provide a window of sensitivity to the right-handed analog of CKM (for example, in extensions to models with a gauged $SU(2)_L \otimes SU(2)_R$ symmetry).

4.4 Summary of models with *Right conditions*

We summarize in Table 1 the main properties of the different models discussed in the previous sections, which obey *Right conditions*.

5 Phenomenology

In the previous sections we have presented different classes of models which include controlled tree-level FCNC; different cases within the same class share the same number of parameters, and this number varies among different classes. This section is devoted to a discussion of aspects related to the phenomenology of the different models.

Equation (9) shows the relevant Lagrangian. We can read from it directly the couplings of the charged scalars, involving $V N_d$, $V^\dagger N_u$, and the Hermitian conjugates $N_d^\dagger V^\dagger$ and $N_u^\dagger V$:

$$-\frac{v}{\sqrt{2}} \mathcal{L}_Y \supset H^+ \bar{u}_\alpha [(V N_d)_{\alpha k} \gamma_R - (N_u^\dagger V)_{\alpha k} \gamma_L] d_k + \text{H.c.}, \tag{88}$$

where $\gamma_{R,L} = (1 \pm \gamma_5)/2$, and sums over the up quark (down quark) index α (k) are implicit. To find the couplings with the neutral scalars, one must specify the scalar potential. In models with a \mathbb{Z}_2 symmetry softly broken, one can have CP violation in the scalar sector, spontaneous [26] or explicit – for recent reviews, see for example [27–29]. Conversely, if CP is conserved, then I^0 in Eq. (5) is a CP-odd mass eigenstate, usually denoted by A . Still, the scalars h^0 and R^0 written in the Higgs basis of Eq. (5) mix into the mass eigenstate basis of CP-even neutral scalars h and H via an angle $\beta - \alpha$. As a result, the couplings of these scalars become of the type

$$\begin{aligned} s_{\beta-\alpha} M_q + c_{\beta-\alpha} N_q, \\ -c_{\beta-\alpha} M_q + s_{\beta-\alpha} N_q, \end{aligned} \tag{89}$$

for the lighter h and heavier H scalars, respectively ($\cos x \equiv c_x$, $\sin x \equiv s_x$). We know from the decays of the 125 GeV scalar [30] that $s_{\beta-\alpha}$ should lie close to 1. Besides the $\beta - \alpha$ mixing effect present in the usual type I and type II (and X and Y), we see that there are now FCNC controlled by N_q , even for the 125 GeV scalar (which we take to be the lighter state h). These effects are $c_{\beta-\alpha}$ suppressed in h , but not in H (or the charged scalars H^\pm). The effects of N_q can appear in both flavour changing and in flavour conserving couplings. The former require a non-diagonal N_q , while the latter exist even if N_q turned out to be diagonal. An important result of our paper is that for the models discussed here, non diagonal couplings, when they exist, are in every case proportional to

$$\pm(t_\beta + t_\beta^{-1})\hat{h}_{[q]i}^* \hat{h}_{[q]j}^* m_{q_j}, \quad \pm m_{q_i}(t_\beta + t_\beta^{-1})\hat{r}_{[q]i}^* \hat{r}_{[q]j}^*, \tag{90}$$

for *Left* and *Right* models, respectively. Since $(t_\beta + t_\beta^{-1}) = 2/s_{2\beta}$ is equal or larger than 2, and could in principle be arbitrarily large, this could overcome the $c_{\beta-\alpha}$ suppression of FCNC for the 125 GeV scalar mentioned above.

In short, there are two obvious ingredients of these models: there are new scalar particles, charged and neutral; and there are FCNC at the tree level. Starting from them, the possible New Physics clues motivating interest in these 2HDMs can be classified in

- deviations from SM expectations in the flavour conserving processes involving the 125 GeV Higgs-like scalar,
- possible sizable FCNC processes involving the 125 GeV Higgs-like scalar,
- proposed searches for new fundamental scalars.

The division is to some extent arbitrary since all three aspects are related: through mixing in the scalar sector, the 125 GeV Higgs-like scalar inherits tree-level FCNC and modified flavour conserving couplings. With those eventual clues, one can then ask two different questions:

1. how can one fix or extract parameters of a given model?
2. how can one tell apart different models?

The BGL models of Sect. 3.3.1 have already been extensively studied [9, 10], including phenomenological aspects [11–13], while the gBGL models of Sect. 3.3.2 were introduced in [14], including some insight into their phenomenology. All other models can be implemented via a \mathbb{Z}_2 symmetry, which we consider softly broken. As a result, there is a decoupling limit and all SM predictions can be recovered by taking the extra scalars very massive. Conversely, as one makes the scalars lighter, the matrices N_d and N_u (and their effect on both flavour changing and flavour conserving couplings) become more important. The crucial result in Eq. (90) means that the phenomenological analysis is very similar in all cases, and follows the same steps discussed for the gBGL models in Ref. [14].

From Eqs. (89) and (90) it turns out that the 125 GeV Higgs has flavour changing Yukawa coupling typically of the form

$$-\mathcal{L}_{h\bar{q}_i q_j} = -h\bar{q}_i Y_{ij} q_j, \tag{91}$$

where

$$Y_{ij} = c_{\beta-\alpha}(t_\beta + t_\beta^{-1})\hat{n}_{[q]i}^* \hat{n}_{[q]j} \frac{m_{qj}}{v} \quad \text{or} \tag{92}$$

$$Y_{ij} = c_{\beta-\alpha}(t_\beta + t_\beta^{-1})\frac{m_{qi}}{v}\hat{r}_{[q]i}^* \hat{r}_{[q]j}.$$

These couplings, appearing in all the models, contribute to the $\Delta F = 2$ neutral mesons mixing amplitude. In all the new models present in this paper we have an arbitrary complex unitary vector $\hat{v} = \hat{n}_{[q]}, \hat{r}_{[q]}$, and, therefore, the maximum

intensity of these flavour changing coupling can be reached when $|\hat{v}_i \hat{v}_j|$ takes its maximum value of 1/2. It is under this assumption that, by imposing the constraints from $K^0-\bar{K}^0$, $B_d^0-\bar{B}_d^0$, $B_s^0-\bar{B}_s^0$, and $D^0-\bar{D}^0$, we can get a global bound for $|c_{\beta-\alpha}(t_\beta + t_\beta^{-1})|$ except for the restrictions explained below. Therefore, following the analysis of Ref. [13] and the constraints in Ref. [31], one can conclude that all the models presented here are typically safe over the entire parameter space, provided we take

$$|c_{\beta-\alpha}(t_\beta + t_\beta^{-1})| \leq 0.02. \tag{93}$$

The bound in Eq. (93) comes from taking into account just the contribution of the SM-like Higgs h . Inclusion of the effects of H and A could modify this bound in both directions: if the masses are similar, $m_H \sim m_A$, or if A is light and H is heavy, nothing changes or the bound can be relaxed. Only when A is heavy and H is light the bound should be more stringent. Nevertheless, it has to be stressed that in a large region of the \hat{v} parameter space $|c_{\beta-\alpha}(t_\beta + t_\beta^{-1})|$ can span almost all the theoretically allowed parameter region, even reaching values of order one. In this paper we will not consider semileptonic $\Delta F = 1$ processes because any constraint will introduce extra model dependences coming from the specific choice one might make for the leptonic sector.

In general, Y_{ij} presents an extremely important m_{q_i}/v suppression, except in the case where $q_i = t$ corresponds to the top quark. In those models with FCNC in the up sector one must also check the constraints arising from rare top decays, such as $t \rightarrow hc$ and $t \rightarrow hu$. One finds [14]

$$\text{Br}(t \rightarrow hq) = 0.13 \left| \frac{\hat{v}_q \hat{v}_t}{V_{tq}} \right|^2 |c_{\beta-\alpha}(t_\beta + t_\beta^{-1})|^2. \tag{94}$$

Taking into account the experimental bounds from ATLAS [32,33] and CMS [34,35], we get

$$|c_{\beta-\alpha}(t_\beta + t_\beta^{-1})| \leq 0.4, \tag{95}$$

again for the maximum theoretical value $|\hat{v}_i \hat{v}_j| = 1/2$.

A full parameter scan lies beyond the scope of this work, but we concentrate here on an important aspect. Let us imagine that the new particles and FCNC effects discussed in this article had been detected. In that case, what properties could be relevant to tell apart different models?⁶

- uBGL, dBGL and types A,B,C and D only have tree level FCNC in one quark sector, up or down, not both. Further-

⁶ It goes without saying that the more experimental signals are available, the better the identification can be established. On that respect, available information on Higgs production \times decay signal strengths can provide some sensitivity to the diagonal couplings in Eq. (89). However, that avenue is not as distinctive as tree level FCNC.

more, uBGL and dBGL are fixed in terms of the CKM matrix, while the couplings $\hat{r}_{[q]j}$ in A, B, C and D are free parameters.

- gBGL, jBGL and types E and F have tree level FCNC in both sectors. However, in gBGL and jBGL, the parameters controlling them, $\hat{n}_{[u]j}$ and $\hat{n}_{[d]j}$, are not independent, they are related through CKM, Eq. (44), while that is not the case in models E and F where $\hat{r}_{[u]j}$ and $\hat{r}_{[d]j}$ are independent.

With these considerations, it is clear that FCNC allow for some discrimination among models, but it is not complete. Consider for example gBGL and jBGL models. N_u has the same structure in both cases, and so do the off diagonal couplings of neutral scalars with down quarks.⁷ A similar comment applies to models A vs. B, C vs. D and E vs. F. The relevant question is then: can one tell apart gBGL from jBGL? (And similarly A from B, C from D and E from F) It is interesting to consider the decays of the charged Higgs to quarks. We find

$$\Gamma(H^+ \rightarrow \bar{u}_\alpha d_k) = \frac{N_c m_{H^\pm}}{8\pi} \sqrt{\beta_+ \beta_-} \left[|a_{\alpha k}|^2 \beta_+ + |b_{\alpha k}|^2 \beta_- \right], \quad (96)$$

where

$$\beta_\pm = 1 - \left(\frac{m_{u_\alpha} \pm m_{d_k}}{m_{H^\pm}} \right)^2, \quad (97)$$

and

$$\begin{aligned} a_{\alpha k} &= \frac{1}{\sqrt{2}v} [(VN_d)_{\alpha k} - (N_u^\dagger V)_{\alpha k}], \\ b_{\alpha k} &= \frac{-i}{\sqrt{2}v} [(VN_d)_{\alpha k} + (N_u^\dagger V)_{\alpha k}], \end{aligned} \quad (98)$$

are the scalar and pseudoscalar couplings one finds when rewriting the Lagrangian in Eq. (88) as

$$-\mathcal{L}_Y \supset H^+ \bar{u}_\alpha [a_{\alpha k} + i b_{\alpha k} \gamma_5] d_k + \text{H.c.} \quad (99)$$

The i in the second equation (98) is crucial. In a reasoning similar to that used in Ref. [36] for flavour changing neutral couplings, one can prove that

$$\text{Re}(a_{\alpha k} b_{\alpha k}^*) \neq 0 \Rightarrow \text{CP violation.} \quad (100)$$

In our case

$$\text{Re}(a_{\alpha k} b_{\alpha k}^*) \propto \text{Re}\{i[(VN_d)_{\alpha k}^2 - (N_d^\dagger V)_{\alpha k}^2]\}, \quad (101)$$

⁷ The sign is irrelevant when considering exclusively off diagonal couplings with neutral scalars.

showing that CP is conserved if N_d , N_u , and V are real, as expected.⁸ We can now see that the decays in Eqs. (96)–(98) provides access to the beating of N_d , N_u against the CKM matrix V , thus permitting a distinction between gBGL and jBGL models.

6 Conclusions

The recent discovery of a scalar particle prompted the search for more scalars and re-spurring the study of models with two Higgs doublets. A general two Higgs doublet model has double the number of Yukawa couplings already present in the SM. It would seem that this would lead us even farther away from an understanding of the flavour sector. Moreover, 2HDM typically lead to FCNC, which are tightly constrained by experiment. In this article we entertain the possibility that these two issues are solved in a natural way by the presence of Abelian symmetries. We are inspired by the BGL [8] models, where FCNC are entirely determined by the CKM matrix elements, and by gBGL [14] models, which have a larger parametric freedom.

We show that such models can be obtained by enhancing an Abelian symmetry with the *Left condition* in Eq. (13). Since Ferreira and Silva [15] had listed all 2HDM models constrained by an Abelian symmetry and consistent with nonzero quark masses and a non-diagonal CKM, we could perform an exhaustive search for all such models, and show there is one, and only one, further class of models obeying the *Left condition*, which we dubbed jBGL.

We have developed a similar *Right condition* (16) and again performed an exhaustive search over the set of models with an Abelian symmetry. We identified six new classes of models, named Types A through F. For all cases, the FCNC matrices N_d and N_u have been written in terms of masses, $\tan \beta$, CKM entries, and vectors containing all the remaining parametric freedom. All FCNC couplings have the generic form in Eq. (90). Finally, we discussed how one could in principle tell these models apart, by concentrating on the use of charged Higgs decays to disentangle gBGL from jBGL models

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⁸ Naturally, a tree level calculation would yield $\Gamma(H^+ \rightarrow \bar{u}_\alpha d_k) = \Gamma(H^- \rightarrow u_\alpha \bar{d}_k)$ even for complex parameters, consistent with the absence of direct CP violation in the presence of a single diagram. Including some loop diagram(s), with different strong and weak phases, would yield a CP violating asymmetry.

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Appendix A: Details on model identification

A.1 Rows and columns

Recall the arguments in Sects. 3.2 and 4.2. Consider the *Left condition*

$$N_q^0 = (\ell_1 P_1 + \ell_2 P_2 + \ell_3 P_3) M_q^0 = \begin{pmatrix} \ell_1 & 0 & 0 \\ 0 & \ell_2 & 0 \\ 0 & 0 & \ell_3 \end{pmatrix} M_q^0, \quad (102)$$

and Eqs. (7)–(8) for M_q^0 and N_q^0 ($q = d, u$) expressed in terms of the Yukawa matrices Γ_1, Γ_2 and Δ_1, Δ_2 , respectively for $q = d$ and $q = u$. If there were non-zero elements $(\Gamma_1)_{ia} \neq 0$ and $(\Gamma_2)_{ib} \neq 0$ (or $(\Delta_1)_{ia} \neq 0$ and $(\Delta_2)_{ib} \neq 0$) in the same row i of both Yukawa matrices, it would follow that

$$\begin{aligned} (N_q^0)_{ia} &= \ell_i (M_q^0)_{ia} \Rightarrow \ell_i = t_\beta \quad \text{and} \\ (N_q^0)_{ib} &= \ell_i (M_q^0)_{ib} \Rightarrow \ell_i = -t_\beta^{-1}, \end{aligned} \quad (103)$$

which is not possible. That is, the rows of the M_d^0 and N_d^0 matrices (M_u^0 and N_u^0 matrices) come either from Γ_1 or from Γ_2 (from Δ_1 or from Δ_2), never from both. In other words, each doublet Q_{Li}^0 couples to one and only one doublet Φ_j . For the *Right condition*

$$N_q^0 = M_q^0 (r_1 P_1 + r_2 P_2 + r_3 P_3) = M_q^0 \begin{pmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{pmatrix}, \quad (104)$$

it follows similarly that each singlet d_{Ri}^0, u_{Rj}^0 , couples to one and only one doublet Φ_k . However, contrary to *Left conditions*, this holds separately for the up and down sectors. Notice, finally, that the only values that the parameters ℓ_j and r_j can take, following Eq. (103) are either t_β or $-t_\beta^{-1}$.

A.2 Models

The models discussed in Sects. 3 and 4 are representative examples within each class. In the following we briefly comment on some other details on these classes of models.

Starting with the BGL models of Sect. 3.3.1, it is to be noticed that in Eq. (30) one singles out both the third generation and the down quarks. This leads to a model, the “bottom” BGL model, where tree level FCNC are absent in the down sector and are controlled by products of CKM elements $V_{ib} V_{jb}^*$ in the up sector. By choosing for example the second generation, the “strange” BGL model, one obtains again tree level FCNC in the up sector but controlled by $V_{is} V_{js}^*$ instead. Furthermore, if instead of the down sector one chooses the up sector, that is

$$\begin{aligned} \Phi_2 &\mapsto e^{i\theta} \Phi_2, \quad Q_{L3}^0 \mapsto e^{i\theta} Q_{L3}^0, \\ u_{R3}^0 &\mapsto e^{i2\theta} u_{R3}^0, \quad \theta \neq 0, \pi. \end{aligned} \quad (105)$$

instead of Eq. (30), one obtains the “top” BGL model, with no tree level FCNC in the up sector and FCNC controlled by $V_{ii} V_{ij}^*$ in the down sector. Overall, considering the quark sector alone, there are 6 BGL models, one per quark type.

For all the remaining models, shaped by either *Left* or *Right* conditions, the situation is different. Consider for example the generalised BGL model given by Eq. (37). The transformation singles out the third generation with $Q_{L3}^0 \mapsto -Q_{L3}^0$. The only trace of that election in Eq. (43) is the fact that the unitary vector $\hat{n}_{[q]}$ is given by the third row of the unitary matrix \mathcal{U}_L^q . However, if we start with $Q_{L2}^0 \mapsto -Q_{L2}^0$ instead, the form of N_d and N_u remains exactly the same, but with a different interpretation of $\hat{n}_{[q]}$ (the second row of \mathcal{U}_L^q in that case). With $\hat{n}_{[q]}$ free to vary – either $q = d$ or $q = u$, the other fixed via CKM, Eq. (44) –, it is clear that the generic parametrization in terms of $\hat{n}_{[q]}$ covers simultaneously all three initial possibilities $Q_{Lj}^0 \mapsto -Q_{Lj}^0, j = 1, 2, 3$. This consideration concerning the generalised BGL model is applicable to the remaining cases: the parametrisation of the N_d and N_u matrices involving unitary vectors $\hat{n}_{[q]}$ or $\hat{r}_{[q]}$ encompasses all initial symmetry assignments. It is to be noticed of course, that despite this fact, the models discussed in different classes are distinct: for example the jBGL model in Eq. (48) *cannot* be obtained from the gBGL model in Eq. (43) with some election of $\hat{n}_{[q]}$; they have a different dependence on t_β . The same kind of distinction applies to Eqs. (60) versus (64) and to Eqs. (83) versus (87).

A.3 Identifying Φ_1 and model discrimination

In the most general 2HDM there is nothing to disentangle Φ_1 from Φ_2 . Indeed, one can mix them through a unitary transformation without any physical consequence. The situation changes once one introduces a symmetry through some specific form. We start by noticing that the form of the Abelian symmetry chosen in Eq. (12) already singles out Φ_1 ; it is the field which remains invariant under the symmetry. Given any generic Abelian symmetry, this choice can always be made by an appropriate basis transformation in the space of

scalar doublets. Before that choice is made, the sub-indices $k = 1, 2$ in Φ_k (and, thus, in Γ_k, Δ_k , and the vevs v_k) are just unphysical labels. One should notice that models are not yet unequivocally defined, even after the basis choice is made such that the Abelian symmetry is expressed as $\Phi_1 \mapsto \Phi_1$. This is most easily seen in the simple context of the \mathbb{Z}_2 Natural Flavour Conservation models of Glashow–Weinberg [4]. In that context, after a scalar basis choice is made such that the scalars transform as $\Phi_1 \mapsto \Phi_1$ and $\Phi_2 \mapsto -\Phi_2$, one can still choose for the right handed quarks the transformations (the same for all quarks of a given charge)

$$d_R \mapsto d_R, \quad u_R \mapsto u_R; \tag{106}$$

$$d_R \mapsto -d_R, \quad u_R \mapsto -u_R; \tag{107}$$

$$d_R \mapsto d_R, \quad u_R \mapsto -u_R; \tag{108}$$

$$d_R \mapsto -d_R, \quad u_R \mapsto u_R. \tag{109}$$

In the first two equations, the up and down quarks couple to the same field (be it Φ_1 or Φ_2 ; it does not matter). This is known as Type I. In the last two equations, the up and down quarks couple to the different fields; which is known as Type II. Denoting a field by Φ_1 or Φ_2 has no physical meaning. The most direct counting can be obtained by choosing (say) Φ_2 as the field which couples to the up quarks. This is what attributes physical meaning to the labels 1 and 2. With this choice, the sub-indices of the Yukawa matrices (Γ_k and Δ_k) acquire physical meaning. The same happens with the vevs v_k [37]. Subsequent changes in the basis for fermions will alter the form of the Yukawa matrices, but not their rank.

A similar analysis can be made for the models discussed in this paper, except that here the right handed up quarks do not couple all to the same doublet. However, as can be seen from the form of the matrices shown, $\text{rank}(\Gamma_1) + \text{rank}(\Gamma_2) = 3$ and $\text{rank}(\Delta_1) + \text{rank}(\Delta_2) = 3$. As a result, one can define physically the label in Φ_1 as the scalar which couples to most of the up quarks. All subsequent choices are physically meaningful. Alternatively, one can define Φ_1 as the field which obeys $\Phi_1 \rightarrow \Phi_1$ under the Abelian symmetry, at the price of an apparent but illusory doubling of the number of model types. This is shown explicitly for the *Right* models in Table 3.⁹

In this analysis, we have used the fact that, if for example Δ_1 has two columns and Δ_2 the third, it is immaterial their placement and, moreover, their placement with respect to the placement of the columns which appear in Γ_1 and Γ_2 . To be specific, let us consider the Type A matrices of Eq. (55), where we have chosen Δ_1 to have the first two columns and Δ_2 the last, while Γ_1 has all columns. We could have chosen Δ_1 to have the first and last column, with Δ_2 having

Table 3 Identification of the *Right* models (and the usual Types I and II), in terms of the ranks of the Yukawa matrices, in the order Δ_1, Δ_2 (in columns), and Γ_1, Γ_2 (in rows)

Rank Γ_k	Rank Δ_k			
	3, 0	0, 3	2, 1	1, 2
3, 0	Type I	Type II	Type A	Type B
0, 3	Type II	Type I	Type B	Type A
2, 1	Type C	Type D	Type E	Type F
1, 2	Type D	Type C	Type F	Type E

Table 4 Identification of the *Left* models in terms of the ranks of the Yukawa matrices, in the order Δ_1, Δ_2 (in columns), and Γ_1, Γ_2 (in rows)

Rank Γ_k	Rank Δ_k	
	2, 1	1, 2
2, 1	gBGL	jBGL
1, 2	jBGL	gBGL

the second column. The different permutations refer only to the labels in the space of right handed up quarks (which is completely detached from the space of right handed down quarks). Such choices are indistinguishable.

The situation is easier for *Left* models, because left up quark and left down quark fields belong to the same doublet, leading to the restriction in Eq. (44). Hence, as seen in Sect. 3, the possible ranks of (Γ_1, Γ_2) are only (1, 2) and (2, 1). Thus, instead of Table 3 one obtains the much simpler Table 4. We are now ready to develop basis invariant conditions for the determination of the various models.

A.4 Invariant conditions

Here, we present conditions for the identification of the various types of models discussed in this article, which are invariant under basis transformations in the spaces of left-handed doublets and of up-type and down-type right-handed singlets. For BGL and generalised BGL models, the following matrix conditions hold [14]:

$$\begin{aligned}
 &\text{BGL models: } \Gamma_1^\dagger \Gamma_2 = 0, \quad \Delta_1^\dagger \Delta_2 = 0, \\
 &\Gamma_1^\dagger \Delta_2 = 0, \quad \Gamma_2^\dagger \Delta_1 = 0, \\
 &\text{and } \Gamma_1 \Gamma_2^\dagger = 0 \text{ (dBGL) or } \Delta_1 \Delta_2^\dagger = 0 \text{ (uBGL),} \\
 &\text{gBGL models: } \Gamma_1^\dagger \Gamma_2 = 0, \quad \Delta_1^\dagger \Delta_2 = 0, \\
 &\Gamma_1^\dagger \Delta_2 = 0, \Gamma_2^\dagger \Delta_1 = 0. \tag{110}
 \end{aligned}$$

Their importance resides in the fact that, under a weak basis transformation (WBT) of the fermion fields

$$\begin{aligned}
 Q_L \mapsto W_L Q_L, \quad d_R \mapsto W_{dR} d_R, \\
 u_R \mapsto W_{uR} u_R, \quad W_L, W_{dR}, W_{uR} \in U(3), \tag{111}
 \end{aligned}$$

⁹ Due to Eq. (103), the change $1 \leftrightarrow 2$ implies a change $t_\beta \leftrightarrow -t_\beta^{-1}$ in the parametrization of the N_q matrices. Thus, models which could seem to differ by such a change, do in fact correspond to the same model.

the Yukawa coupling matrices are transformed as

$$\Gamma_i \mapsto W_L^\dagger \Gamma_i W_{dR}, \quad \Delta_i \mapsto W_L^\dagger \Delta_i W_{uR}, \tag{112}$$

and, although the WBT in Eqs.(111)–(112) may hide the symmetry under the Abelian transformations in Eq. (12), the conditions in Eq. (110) are in any case invariant. In general, the different combinations of Γ_i, Δ_j , which are invariant under some of the WBT are the following.

– Invariant under W_L WBT,

$$\begin{aligned} \Gamma_i^\dagger \Gamma_j &\mapsto W_{dR}^\dagger \Gamma_i^\dagger \Gamma_j W_{dR}, \\ \Delta_i^\dagger \Delta_j &\mapsto W_{uR}^\dagger \Delta_i^\dagger \Delta_j W_{uR}, \\ \Gamma_i^\dagger \Delta_j &\mapsto W_{dR}^\dagger \Gamma_i^\dagger \Delta_j W_{uR}, \end{aligned} \tag{113}$$

(and, of course, $\Delta_i^\dagger \Gamma_j = (\Gamma_j^\dagger \Delta_i)^\dagger$)

– Invariant under W_{dR} and W_{uR} WBT,

$$\begin{aligned} \Gamma_i \Gamma_j^\dagger &\mapsto W_L^\dagger \Gamma_i \Gamma_j^\dagger W_L, \\ \Delta_i \Delta_j^\dagger &\mapsto W_L^\dagger \Delta_i \Delta_j^\dagger W_L. \end{aligned} \tag{114}$$

Considering in addition the *Left* and *Right conditions* of Eqs. (13) and (16), respectively, we can straightforwardly obtain invariant conditions. This is what we turn to next.

6.0.1 Left conditions

In terms of the Yukawa matrices, the *Left conditions* are

$$\begin{aligned} \Gamma_1 &= e^{-i\xi_1} \frac{\sqrt{2}}{v} s_\beta (t_\beta^{-1} \mathbf{1} + L_d^0) M_d^0, \\ \Gamma_2 &= -e^{-i\xi_2} \frac{\sqrt{2}}{v} c_\beta (-t_\beta \mathbf{1} + L_d^0) M_d^0, \\ \Delta_1 &= e^{i\xi_1} \frac{\sqrt{2}}{v} s_\beta (t_\beta^{-1} \mathbf{1} + L_u^0) M_u^0, \\ \Delta_2 &= -e^{i\xi_2} \frac{\sqrt{2}}{v} c_\beta (-t_\beta \mathbf{1} + L_u^0) M_u^0, \end{aligned} \tag{115}$$

$$\tag{116}$$

where we have used Eqs. (7) and (8). Then,

$$\Gamma_1^\dagger \Gamma_2 = -s_\beta c_\beta e^{i\xi} \frac{2}{v^2} M_d^{0\dagger} (t_\beta^{-1} \mathbf{1} + L_d^0) (-t_\beta \mathbf{1} + L_d^0) M_d^0, \tag{117}$$

where

$$(t_\beta^{-1} \mathbf{1} + L_d^0) (-t_\beta \mathbf{1} + L_d^0) = \sum_{j=1}^3 (\ell_j^{[d]} + t_\beta^{-1}) (\ell_j^{[d]} - t_\beta) P_j = 0 \tag{118}$$

since $\ell_j^{[d]}$ is equal either to $-t_\beta^{-1}$ or to t_β , thus giving $\Gamma_1^\dagger \Gamma_2 = 0$. For the up Yukawa matrices, the conclusion is identical, and thus the conditions in Eqs. (113) involving separately the up and down quark sectors are trivially

$$\Gamma_1^\dagger \Gamma_2 = 0, \quad \Delta_1^\dagger \Delta_2 = 0. \tag{119}$$

For the conditions involving Yukawa matrices from both sectors, proceeding along similar lines, one finds

$$\begin{aligned} &e^{-i2\xi_1} \frac{v^2}{2} \Gamma_1^\dagger \Delta_1 \\ &= s_\beta^2 M_d^{0\dagger} \left[\sum_{j=1}^3 (\ell_j^{[d]} + t_\beta^{-1}) (\ell_j^{[u]} + t_\beta^{-1}) P_j \right] M_u^0, \\ &- e^{-i(\xi_1 + \xi_2)} \frac{v^2}{2} \Gamma_1^\dagger \Delta_2 \\ &= s_\beta c_\beta M_d^{0\dagger} \left[\sum_{j=1}^3 (\ell_j^{[d]} + t_\beta^{-1}) (\ell_j^{[u]} - t_\beta) P_j \right] M_u^0, \\ &- e^{-i(\xi_1 + \xi_2)} \frac{v^2}{2} \Gamma_2^\dagger \Delta_1 \\ &= s_\beta c_\beta M_d^{0\dagger} \left[\sum_{j=1}^3 (\ell_j^{[d]} - t_\beta) (\ell_j^{[u]} + t_\beta^{-1}) P_j \right] M_u^0, \\ &e^{-i2\xi_2} \frac{v^2}{2} \Gamma_2^\dagger \Delta_2 \\ &= c_\beta^2 M_d^{0\dagger} \left[\sum_{j=1}^3 (\ell_j^{[d]} - t_\beta) (\ell_j^{[u]} - t_\beta) P_j \right] M_u^0, \end{aligned} \tag{120}$$

and one can readily obtain the additional conditions

$$\begin{aligned} \text{BGL and gBGL models: } &\Gamma_1^\dagger \Delta_2 = 0, \quad \Gamma_2^\dagger \Delta_1 = 0, \\ \text{jBGL models: } &\Gamma_1^\dagger \Delta_1 = 0, \quad \Gamma_2^\dagger \Delta_2 = 0. \end{aligned} \tag{121}$$

The remaining matrix products, including $\Gamma_1 \Gamma_2^\dagger, \Delta_1 \Delta_2^\dagger$, are different from 0 and do not give invariant conditions like Eqs. (119) and (121).

6.0.1 Right conditions

For models with *Right conditions*, the analog of Eq. (119) is simply

$$\Gamma_1 \Gamma_2^\dagger = 0, \quad \Delta_1 \Delta_2^\dagger = 0. \tag{122}$$

One could naively think that conditions such as $\Gamma_2 \Delta_1^\dagger$ could be used to distinguish among different models. But, such conditions cannot be used, for they are not covariant under WBT. Fortunately, the different *Right* models can be distinguished in a basis invariant way by the rank of the Γ_1 and Δ_1 matrices.

Table 5 Summary of invariant conditions

Model	Invariant conditions
dBGL	$\Gamma_1^\dagger \Gamma_2 = 0, \Delta_1^\dagger \Delta_2 = 0, \Gamma_1^\dagger \Delta_2 = 0, \Gamma_2^\dagger \Delta_1 = 0, \Gamma_1 \Gamma_2^\dagger = 0$
uBGL	$\Gamma_1^\dagger \Gamma_2 = 0, \Delta_1^\dagger \Delta_2 = 0, \Gamma_1^\dagger \Delta_2 = 0, \Gamma_2^\dagger \Delta_1 = 0, \Delta_1 \Delta_2^\dagger = 0$
gBGL	$\Gamma_1^\dagger \Gamma_2 = 0, \Delta_1^\dagger \Delta_2 = 0, \Gamma_1^\dagger \Delta_2 = 0, \Gamma_2^\dagger \Delta_1 = 0$
jBGL	$\Gamma_1^\dagger \Gamma_2 = 0, \Delta_1^\dagger \Delta_2 = 0, \Gamma_1^\dagger \Delta_1 = 0, \Gamma_2^\dagger \Delta_2 = 0$
Type A	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 3, \text{rank}(\Delta_1) = 2$
Type B	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 3, \text{rank}(\Delta_1) = 1$
Type C	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 2, \text{rank}(\Delta_1) = 3$
Type D	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 2, \text{rank}(\Delta_1) = 0$
Type E	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 2, \text{rank}(\Delta_1) = 2$
Type F	$\Gamma_1 \Gamma_2^\dagger = 0, \Delta_1 \Delta_2^\dagger = 0, \text{rank}(\Gamma_1) = 2, \text{rank}(\Delta_1) = 1$

A.4.3 Summary

We summarize in Table 5 the invariant conditions associated with all models discussed in this article.

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