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Magnetoacoustic and Alfvénic black holes

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Abstract We introduce analogue black holes (BHs) based on ideal magnetohydrodynamic equations. Similar to acoustic BHs, which trap phonons and emit Hawking radiation (HR) at the sonic horizon where the flow speed changes from super- to sub-sonic, in the horizon of magnetoacoustic and Alfvénic BHs, the magnetoacoustic and Alfvén waves will be trapped and emit HR made of quantized vibrations similar to phonons which we call magnephonons and Alphonons. We proposed that magnetoacoustic and Alfvénic BHs may be created in the laboratory using a tube with variable cross section embedded in a uniform magnetic field, and a super-magnetoacoustic or a super-Alfvénic flow. We show that the Hawking temperature for both BHs is a function of the background magnetic field, number density of fluid, and radius of the tube. For a typical setup, the temperature is estimated to be about 0.0266 K.

1 Introduction

In 1916 Schwarzschild gave a metric as a solution of the Einstein field equation. Singularity of such a metric predicted a gravitational BH with event horizon at Schwarzschild radius [1]. Although, based on classical physics everything, even light, is absorbed by BHs and cannot escape, in the context of quantum field theory in curved space, Hawking showed that BHs should emit black body radiation [2,3]. HR in the universe has not been observed yet, but numerous attempts have been done to simulate the interesting phenomena in the laboratory. Unruh showed that HR is not only a characteristic of gravitational BHs, it is also a characteristic of the acoustic analogue BH [4,5]. After 1981, most of attempts are proposed based on Bose–Einstein condensates of quantum fluid, [6–10] quasi particles in superfluid [11], ultra-cold fermions [12], in plasmas and ion rings, [13–17] slow light in an atomic vapor, [18–21] in water [22,23], etc. Recently, observation of self-amplifying HR in an analogue BH laser suggested a very promising experiment method for probing the inside of a BH [24]. From a theoretical point of view, the acoustic analogue BH models are developed in geometrical acoustics and physical acoustics [25]. Using the linearized hydrodynamic equations in the presence of initial material flow, a wave equation for velocity potential was obtained. Tensorial form of the wave equation results in an acoustic metric. The acoustic metric is singular at a point where the local sound speed is equal to the flow speed [26]. This was interpreted as characteristic of a sonic BH [25,26].The effect of magnetic field on the acoustic BH and HR has not been studied, yet.

Alternatively, the idea for definition of acoustic BH can be applied to introduce new analogue magnetoacoustic and Alfvénic BHs in the magnetohydrodynamics (MHD) framework. Magnetohydrodynamics (MHD) is an useful approach to analyze characteristics (flow, wave and dissipation etc) of the laboratory and astrophysical plasma [33]. After Alfvén [34], MHD waves (Alfvén and magnetoacoustic) have been detected, using the laboratory experiments Alfvén wave (e.g. [35–37], in the earth-ionosphere and magnetic field [38,39], a variety normal modes of the solar corona [40–44].

Here, first we treat the magnetoacoustic, Alfvén, and acoustic waves based on Helmholtz theorem for a uniform and stationary medium with constant background magnetic field. Second, mimicking the definition of acoustic BH in a nozzle, we introduce magnetoacoustic and Alfvénic BHs with using a slightly variable cross section tube. We conclude that at the horizon of magnetoacoustic and Alfvénic BHs should emit radiations made of the Magnephonon and Alphonon, respectively. We define two quasi-particles Magnephonon and Alphonon correspond to quantum of the magnetoacoustic and Alfvén waves, respectively.

This paper is organized as follows: Sect. 2 gives the properties of magnetoacoustic waves (fast, slow, and Alfvén waves) using the Helmholtz decomposition and explains a derivation of magnetoacoustic metric in the basis of linearized ideal

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magnetohydrodynamic equations. Section 4 introduces the acoustic, magnetoacoustic, and Alfvénic black holes, respectively. Section 5 calculate the Hawking temperature for the acoustic, magnetoacoustic, and Alfvénic black holes. Section 6 describes the conclusions.

2 Magnetoacoustic waves and metric

Here, we give the conditions for a definition of a magnetoacoustic metric, in the non-relativistic magnetohydrodynamic (MHD) framework. In the MHD approach, the behavior of continuous plasma is governed by a non-relativistic form of Maxwell's equations, together with Ohm's law, a gas law, equation of mass continuity, motion and energy equations [28]. The ideal MHD equations for an adiabatic process and irrotational flow $\nabla\times\mathbf{v}=0$ are given by:

$$\frac{\partial\rho}{\partial t}+\nabla.(\rho\mathbf{v})=0, \tag{1}$$

$$\rho\left(\frac{\partial\mathbf{v}}{\partial t}+\frac{1}{2}\nabla \mathrm{v}^2\right)=-\nabla p+\frac{1}{\mu}(\nabla\times\mathbf{B})\times\mathbf{B}, \tag{2}$$

$$\frac{\partial\mathbf{B}}{\partial t}=\nabla\times(\mathbf{v}\times\mathbf{B}), \tag{3}$$

$$\nabla.\mathbf{B}=0, \tag{4}$$

$$p=k\rho^{\gamma}, \tag{5}$$

where, $\rho, p, \mathbf{v}, \mathbf{B}, \mu, \gamma$, and k are density, pressure, flow velocity, magnetic field, magnetic permeability, atomicity coefficient, and a constant, respectively. For the derivation of a metric for magnetoacoustic wave we need to linearised the MHD equations with choosing the velocity disturbances as a velocity potential in similar analytical process for derivation of acoustic metric from wave equation. To do this, first we treat the propagation of magnetoacoustic waves in homogenous unbounded medium choosing velocity disturbance as a velocity potential and a vector potential. Second, the magnetoacoustic and Alfvénic metrics are calculated.

3 Helmholtz theorem and magnetoacoustic waves

Properties of magnetoacoustic waves (fast, slow, and Alfvén waves) in an unbounded homogenous and stationary medium with constant density ($\rho_0=cte$), constant pressure ($p_0=cte$), and uniform background magnetic field ($\mathbf{B}_0$), were investigated in the literature [28]. The linearised ideal MHD equations can be reduced to a single equation for disturbed velocity as [28],

$$\frac{\partial^2\mathbf{v}}{\partial t^2}=c_0^2\nabla(\nabla.\mathbf{v})+\mathrm{v}_A^2(\nabla\times(\nabla\times(\mathbf{v}\times\hat{B}_0)))\times\hat{B}_0. \tag{6}$$

where $c_0=\sqrt{\frac{\gamma p_0}{\rho_0}}$ is the sound speed, $\mathrm{v}_A=\frac{B_0}{\sqrt{\mu\rho_0}}$ is speed of the Alfvén wave, and $\hat{B}_0=\mathbf{B}_0/B_0$ is a unit vector. Here, we focus on studying of the characteristics of the above mentioned waves using a fundamental theorem of calculus (Helmholtz's theorem). Based on the Helmholtz decomposition, a vector field with sufficient smoothness and decay conditions [29], can be decomposed to an irrotational part ($\nabla\phi$ where ϕ is a scaler) and a solenoidal part ($\nabla\times\mathbf{A}$ where $\mathbf{A}$ is the vector potential and satisfy $\nabla.\mathbf{A}=0$). The irrotational (gradient) and divergence-free solutions can be used for treating the longitudinal and transversal waves, respectively.

Suppose an irrotational solution, $\mathbf{v}=\nabla\varphi$ with $\varphi=\tilde{\varphi}\exp{(i\mathbf{k}.\mathbf{r}-i\omega t)}$ for Eq. (6), immediately we find $\mathbf{v}=\hat{v}\exp{(i\mathbf{k}.\mathbf{r}-i\omega t)}$ where $\mathbf{k}, \omega, \tilde{\varphi}$, and $\hat{v}=i\mathbf{k}\tilde{\varphi}$, are wave vector, angular frequency of oscillations, a constant, and wave amplitude, respectively. Furthermore Eq. (6) yields,

$$\omega^2\hat{v}=c_0^2\mathbf{k}(\mathbf{k}.\hat{v})+\mathrm{v}_A^2(\mathbf{k}\times(\mathbf{k}\times(\hat{v}\times\hat{B}_0)))\times\hat{B}_0. \tag{7}$$

Equation (7) simplifies as

$$\omega^2\hat{v}=c_0^2k^2\hat{v}+\mathrm{v}_A^2(\hat{v}-\hat{B}_0(\hat{v}.\hat{B}_0))k^2. \tag{8}$$

We note that the wave vector ($\mathbf{k}$) and wave amplitude ($\hat{v}$) are parallel ($\mathbf{k}\|\hat{v}$). Considering the wave propagation parallel to the background magnetic field ($\mathbf{k}\|\mathbf{B}_0$), Eq. (8) reduces to the dispersion relation $\omega^2=c_0^2k^2$. This indicates that, in the case of an irrotational solution parallel to the magnetic field, only the acoustic wave can propagate. In the case of propagation perpendicular to the background magnetic field ($\mathbf{k}\perp\mathbf{B}_0$), Eq. (8) gives the dispersion relation $\omega^2=(\mathrm{v}_A^2+c_0^2)k^2$. This is the well-known characteristic of the fast magnetoacoustic wave with the phase speed $c_f^2=\omega^2/k^2=c_0^2+\mathrm{v}_A^2$. For the oblique propagation $\mathbf{k}.\mathbf{B}_0=kB_0\cos\theta$ (θ is the angle between $\mathbf{B}_0$ and $\mathbf{k}$), the phase speed of the slow magnetoacoustic wave is given by

$$v_{ph}^2=\frac{\omega^2}{k^2}=c_0^2-\mathrm{v}_A^2(\cos\theta-1). \tag{9}$$

From the above analysis we see that, choosing the disturbed velocity as an irrotational velocity field $\mathbf{v}=\nabla\varphi$, the Alfvén waves cannot propagate along the background magnetic field ($\mathbf{B}_0$).

Suppose a divergence-free solution, $\mathbf{v}=\nabla\times\mathbf{A}$, with a planar wave solution $\mathbf{A}=\hat{A}\exp{(i\mathbf{k}.\mathbf{r}-i\omega t)}$, $\hat{v}=i\mathbf{k}\times\hat{A}$ and $\hat{A}$ is a constant vector, Eq. (6) gives,

$$\omega^2\hat{v}=\mathrm{v}_A^2(\mathbf{k}\times(\mathbf{k}\times(\hat{v}\times\hat{B}_0)))\times\hat{B}_0. \tag{10}$$

We see that the velocity amplitude is perpendicular to the wave vector ($\mathbf{k}.\hat{v}=0$), and only the transversal Alfvén wave with phase speed $v_{ph}=\mathrm{v}_A$ can propagate.

In the remainder of this section, the metrics for acoustic, magnetoacoustic, and Alfvén longitudinal waves (irrotational solutions) are derived.

3.1 Magnetoacoustic metric

Here, in the presence of initial material flow the magnetoacoustic metric using the magnetoacoustic wave is derived. For an irrotational flow the velocity is satisfied by a scalar field $\mathbf{v} = \nabla\varphi$. We consider small perturbations from equilibrium as

$$\rho(\mathbf{r}, t) = \rho_0(x, t) + \rho_0(x, t)\psi(\mathbf{r}, t), \quad (11)$$

$$\mathbf{v}(\mathbf{r}, t) = \mathrm{v}_0(x)\hat{x} + \nabla\varphi(\mathbf{r}, t), \quad (12)$$

$$\mathbf{B}(\mathbf{r}, t) = B_0\hat{x} + \mathbf{B}_1(\mathbf{r}, t), \quad (13)$$

where, equilibrium quantities indicated by subscript “0” are function of position x and time t, $\psi(\mathbf{r}, t)$, $\varphi(\mathbf{r}, t)$, and $\mathbf{B}_1(\mathbf{r}, t)$ are perturbed quantities [30,31]. Equilibrium quantities (ρ_0, v_0, $\mathbf{B}_0$, and p_0) are satisfied by

$$\frac{\partial\rho_0}{\partial t} + \mathrm{v}_0\frac{\partial\rho_0}{\partial x} + \rho_0\frac{\partial \mathrm{v}_0}{\partial x} = 0, \quad (14)$$

$$\rho_0\mathrm{v}_0\frac{\partial \mathrm{v}_0}{\partial x} = -c_0^2\frac{\partial\rho_0}{\partial x} \quad (15)$$

$$\nabla \times (\mathbf{v}_0 \times \mathbf{B}_0) = 0, \quad (16)$$

$$\nabla.\mathbf{B}_0 = 0, \quad (17)$$

$$p_0 = k\rho_0^{\gamma}. \quad (18)$$

Linearization of Eqs. (1)–(5) (products and squares of the small perturbations are neglected) and after some mathematical manipulations, give

$$\frac{\partial\psi}{\partial t} + \nabla\ln\rho_0.\nabla\varphi + \mathbf{v}_0.\nabla\psi + \nabla^2\varphi = 0, \quad (19)$$

$$\rho_0\nabla\left(\frac{\partial\varphi}{\partial t} + \mathbf{v}_0.\nabla\varphi + c_0^2\psi\right) = \frac{1}{\mu}(\nabla \times \mathbf{B}_1) \times \mathbf{B}_0, \quad (20)$$

$$\frac{\partial\mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{v}_0 \times \mathbf{B}_1) + \nabla \times (\mathbf{v}_1 \times \mathbf{B}_0), \quad (21)$$

$$\nabla.\mathbf{B}_1 = 0, \quad (22)$$

As we explained in the previous section, by choosing $\mathbf{v} = \nabla\varphi$ the parallel propagation of Alfvén waves along the magnetic field ($\mathbf{B}_0 = B_0\hat{x}$) is absent. Therefore, the first term on right hand of Eq. (21) is set to zero, $\nabla \times (\mathbf{v}_0 \times \mathbf{B}_1) = 0$. A combination of this assumption and the solenoidal condition for magnetic field ($\nabla.\mathbf{B}_0 = 0$), gives $\frac{\partial B_{1x}}{\partial x} = 0$. Thus the propagation of Alfvén waves along the background magnetic field is removed from our analysis. We assume the irrotational part of the vector $\mathbf{v}_0 \times \mathbf{B}_1 = \nabla\delta(x, y, z, t)$ in which $\delta(x, y, z, t)$ is a function.

The x and z components of Eq. (20) are

$$\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial t} + \mathbf{v}_0.\nabla\varphi + c_0^2\psi\right) = 0. \quad (23)$$

$$\rho_0\frac{\partial}{\partial z}\left(\frac{\partial\varphi}{\partial t} + \mathbf{v}_0.\nabla\varphi + c_0^2\psi\right) = \frac{B_0}{\mu}\left(\frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z}\right). \quad (24)$$

Here after, first, we focus our analysis on the derivation of acoustic waves with propagation along the background magnetic field directions ($\hat{x}$) and second, magnetoacoustic wave propagation in all directions except the magnetic field direction.

First, combining Eqs. (19) and (23) gives

$$-\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial t} + \mathrm{v}_0\frac{\partial\varphi}{\partial x}\right) + \frac{\partial}{\partial x}\left(c_0^2\frac{\partial\varphi}{\partial x} - \mathrm{v}_0\left(\frac{\partial\varphi}{\partial t} + \mathrm{v}_0\frac{\partial\varphi}{\partial x}\right)\right) + c_0^2\nabla^2\varphi = 0. \quad (25)$$

Equation (25) is the well-known Klein–Gordon equation for acoustic waves. Eliminating $\partial\psi/\partial x$ between Eqs. (19) and (23) one obtains

$$\mathrm{v}_0\frac{\partial}{\partial x}\left(\frac{\partial\varphi}{\partial t} + \mathbf{v}_0.\nabla\varphi\right) + \mathrm{v}_0\psi\frac{\partial c_0^2}{\partial x} - c_0^2\left(\frac{\partial\psi}{\partial t} + \nabla\ln\rho_0.\nabla\varphi + \nabla^2\varphi\right) = 0. \quad (26)$$

Briefly, by choosing the irrotational solution for the velocity disturbance, the transversal Alfvén wave is absent, and the propagation of the longitudinal Alfvén wave parallel with the background magnetic field is also absent as expected. Our analysis shows that, along the magnetic field only the acoustic wave can propagate. Because our goal is to analyse the magnetoacoustic black hole, hereafter, we focus our analysis in all directions except the magnetic field direction. Second, by differentiating Eq. (26) with respect to z and substituting $\partial\psi/\partial z$ from Eq. (24) one finds

$$\frac{\partial}{\partial z}\left(\frac{1}{c_0^2}\frac{\partial}{\partial x}\frac{d\varphi}{dt}\right) = \frac{1}{\mathrm{v}_0}\nabla^2\frac{\partial\varphi}{\partial z} + \frac{1}{\mathrm{v}_0}\nabla\ln\rho_0.\nabla\frac{\partial\varphi}{\partial z} + \frac{1}{\mathrm{v}_0}\frac{\partial}{\partial t}\left(-\frac{B_0}{\mu\rho_0 c_0^2}(\nabla \times \mathbf{B}_1)_y - \frac{1}{c_0^2}\frac{\partial}{\partial z}\frac{d\varphi}{dt}\right) + \frac{B_0}{\mu\rho_0 c_0^4}\frac{\partial c_0^2}{\partial x}(\nabla \times \mathbf{B}_1)_y + \frac{\partial}{\partial z}\left(\frac{1}{c_0^4}\frac{\partial c_0^2}{\partial x}\frac{d\varphi}{dt}\right). \quad (27)$$

in which, $d/dt = \partial/\partial t + \mathbf{v}_0.\nabla$. Substituting $-\frac{\partial}{\partial t}(\nabla \times \mathbf{B}_1)_y = B_0\nabla^2\frac{\partial\varphi}{\partial z}$ from Eq. (21), into Eq. (27) and differentiating with respect to t using $\frac{\partial}{\partial t}(\nabla \times \mathbf{B}_1)_y = -B_0\nabla^2\frac{\partial\varphi}{\partial z}$ we derived a single equation for velocity potential (φ)

$$\frac{\partial}{\partial t}\left(\frac{1}{N}\left(\frac{1}{c_0^2}\frac{\partial}{\partial x}\frac{d\varphi}{dt}-\frac{1}{c_0^4}\frac{\partial c_0^2}{\partial x}\frac{d\varphi}{dt}-\frac{\nabla^2\varphi}{\mathrm{v}_0}-\frac{\nabla \ln\rho_0.\nabla\varphi}{\mathrm{v}_0}\right)\right)$$
$$+\frac{\partial}{\partial t}\left(\frac{1}{N}\left(\frac{1}{\mathrm{v}_0}\frac{\partial}{\partial t}\left(\frac{1}{c_0^2}\frac{d\varphi}{dt}\right)-\frac{\mathrm{v}_A^2}{c_0^2\mathrm{v}_0}\nabla^2\varphi\right)\right)=B_0\nabla^2\varphi, \tag{28}$$

where,

$$N=\frac{B_0}{\mu\mathrm{v}_0}\frac{\partial}{\partial t}\left(\frac{1}{\rho_0 c_0^2}\right)-\frac{B_0}{\mu\rho_0 c_0^4}\frac{\partial c_0^2}{\partial x} \tag{29}$$

$$=\frac{\mathrm{v}_A^2}{B_0 c_0^2}\left(\frac{\gamma}{\mathrm{v}_0}+\frac{\mathrm{v}_0}{c_0^2}\right)\frac{d\mathrm{v}_0}{dx}. \tag{30}$$

For a flow having a slight change in the speed ($d\mathrm{v}_0/dx \ll 1$), for high frequency waves (short period $\Delta t \ll 1$) the term $\Delta t(d\mathrm{v}_0/dx)$ becomes too small. In this case, the right hand side term $B_0\nabla^2\varphi$ of Eq. (28) can be negligible compared to the last term $\frac{\partial}{\partial t}\left(\frac{\mathrm{v}_A^2}{Nc_0^2\mathrm{v}_0}\nabla^2\varphi\right)$ of the left hand side. This leaves an equation for φ

$$-\frac{\partial}{\partial t}\left(\frac{\partial\varphi}{\partial t}+\mathrm{v}_0\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial x}\left(c_0^2\frac{\partial\varphi}{\partial x}-\mathrm{v}_0\left(\frac{\partial\varphi}{\partial t}+\mathrm{v}_0\frac{\partial\varphi}{\partial x}\right)\right)$$
$$+\mathrm{v}_A^2\frac{\partial^2\varphi}{\partial x^2}+(\mathrm{v}_A^2+c_0^2)\left(\frac{\partial^2\varphi}{\partial y^2}+\frac{\partial^2\varphi}{\partial z^2}\right)=0. \tag{31}$$

Equation (31) describes the propagation of acoustic, Alfvén, and magnetoacoustic waves in laboratory and astrophysical plasma. This equation is in the form of the well-known Klein-Gordon equation. As expected, in the case of unmagnetized fluid ($B_0 = 0$), Eq. (31) reduces to the acoustic wave equation for velocity potential. Usually, a d'Alembertian equation (for a minimally coupled massless scalar field) of motion was derived for velocity potential in a barotropic, inviscid, and rotational free flow [25].

Equation (31) can be reformulate in a tensorial form

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi\right)=0, \tag{32}$$

where, $g^{\mu\nu}$ and g are inverse metric tensor and its determinant, μ and ν runs from 0 (indicates the time coordinate) to 3 (denotes the spatial coordinate). The magnetoacoustic inverse metric tensor $g^{\mu\nu}$ and metric tensor $g_{\mu\nu}$ are obtained as

$$g^{\mu\nu}=\frac{1}{\rho_0 c_f^3}\begin{pmatrix}-1 & -\mathrm{v}_0 & 0 & 0\\ -\mathrm{v}_0 & c_f^2-\mathrm{v}_0^2 & 0 & 0\\ 0 & 0 & c_f^2 & 0\\ 0 & 0 & 0 & c_f^2\end{pmatrix},$$

$$g_{\mu\nu}=-\rho_0 c_f\begin{pmatrix}c_f^2-\mathrm{v}_0^2 & \mathrm{v}_0 & 0 & 0\\ \mathrm{v}_0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}, \tag{33}$$

where, $c_f^2 = \mathrm{v}_A^2 + c_0^2$. Using metric tensor, Eq. (33), the magnetoacoustic interval can be defined as

$$ds^2=g_{\mu\nu}dx^\mu dx^\nu=-\rho_0 c_f\left((c_f^2-\mathrm{v}_0^2)dt^2\right.$$
$$\left.+2\mathrm{v}_0 dt dx-dx^2-dy^2-dz^2\right). \tag{34}$$

Inserting the specific time interval $d\tau = dt + \frac{\mathrm{v}_0 dx}{c_f^2-\mathrm{v}_0^2}$ into Eq. (34), one obtains

$$ds^2=c_f\rho_0\left(-\left(1-\frac{\mathrm{v}_0^2}{c_f^2}\right)c_f^2 d\tau^2+\frac{dx^2}{\left(1-\frac{\mathrm{v}_0^2}{c_f^2}\right)}+dy^2+dz^2\right). \tag{35}$$

In the following section, the properties of acoustic, magnetoacoustic, and Alfvénic BHs are investigated.

4 Acoustic, magnetoacoustic, and Aflvénic black holes

4.1 Acoustic black hole

The theory of gravitational BHs has been developed into the supersonic flow by Unruh [26]. For a moving fluid medium at the horizon where speed of medium is closed to propagation speed of the acoustic signals " then nothing can fight its way back upstream and signals are trapped" [27]. In the case of unmagnetised gas($B_0 = 0$), Eq. (28) reduces to the Klein–Gordon equation, for propagation of acoustic waves in the presence of material flow. Using the resultant equation the acoustic metric can be derived [26].

The acoustic metric can be obtained by setting $\mathrm{v}_A = 0$ in Eq. (33)

$$g^s_{\mu\nu}=-\rho_0 c_0\begin{pmatrix}c_0^2-\mathrm{v}_0^2 & \mathrm{v}_0 & 0 & 0\\ \mathrm{v}_0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}. \tag{36}$$

Equivalently, the acoustic interval can be expressed as

$$ds^2=c_0\rho_0\left(-\left(1-\frac{\mathrm{v}_0^2}{c_0^2}\right)c_f^2 d\tau^2+\frac{dx^2}{\left(1-\frac{\mathrm{v}_0^2}{c_0^2}\right)}+dy^2+dz^2\right). \tag{37}$$

Combination of continuity and momentum equations (Eqs. 14 and 15) in stationary state, the relation between cross section S and velocity v is given by

$$\left(\frac{\mathrm{v}_0^2}{c_0^2}-1\right)\frac{d\mathrm{v}_0}{\mathrm{v}_0}=\frac{dS}{S}. \tag{38}$$

This relation shows that for ($dS<0$) a subsonic flow ($\mathrm{v}_0<c_0$) will be accelerated and a supersonic flow ($\mathrm{v}_0>c_0$) will be decelerated. If the nozzle is sufficiently narrow and with a slightly variable cross section the speed of flow exceeds to sound speed at the throat (sonic horizon). This shows the acoustic interval Eq. (37) interpreted acoustic BH which has a sonic horizon and trapped phonon in the surface gravity of acoustic BH. This means that, when the acoustic waves cross from upstream to downstream, the acoustic wave quanta (phonons) are captured in the horizon of the BH where they emit Hawking radiation made by phonons. In this regard, in many papers a Laval nozzle setup (Fig. 1) has been proposed to discuss the above mentioned acoustic BH. This setup uses, an axisymmetric sufficiently thin tube with slightly decreasing cross section ($S(x)$) that reaches its minimum cross section at the throat and then slightly increases that. An initial material flow ($\mathbf{v}_0=\mathrm{v}_0(x)\ \hat{x}$) along the tube axis is considered.

4.2 Magnetoacoustic black hole

The magnetoacoustic metric Eq. (35) is singular at the magnetoacoustic point, where $\mathrm{v}_0=c_f$, determines a magnetoacoustic horizon. The speed of super magnetoacoustic plasma flow reduces to local propagation speed of magnetoacoustic wave at horizon; then signal of magnetoacoustic wave is trapped and therefore it can be called magnetoacoustic BH. Similar to the HR emitted from acoustic and gravitational BHs, the magnetoacoustic BH also should emit HR. In this regard, we propose a setup (Fig. 2) to discuss the above mentioned BH. The setup consists of an axisymmetric sufficiently thin tube with slightly variable cross section ($S(x)$), a uniform force free magnetic field $\mathbf{B}_0=\mathrm{B}_0\hat{x}$ and an initial material flow ($\mathbf{v}_0=\mathrm{v}_0(x)\ \hat{x}$) along the tube axis (Fig. 2).

A similar treatment of sub and supersonic flow in tube configuration can be explained for sub and super-magnetoacoustic flow based on Eq. (38). In other words, the super-magnetoacoustic flow ($\mathrm{v}_0>c_f$) will be decelerated along the tube where its cross section slightly decreases. It is possible to release a super-magnetoacoustic flow in the tube, which its speed tends to the speed of magnetoacoustic wave ($\mathrm{v}_0=c_f$) at the horizon. The boundary between sub-magnetoacoustic and super-magnetoacoustic flow could be called the magnetoacoustic horizon, analogous to the sonic horizon in acoustic BHs. Phononic version of HR is an inevitable result of trapping acoustic wave at the acoustic horizon [24,32]. Under a likely scenario, the magnetoacoustic wave cannot escape from the magnetoacoustic horizon, therefore should emit HR made of magnephonon. Indeed, a magnephonon will be a quantum for magnetoacoustic wave, analogous to the phonon which is a quantum for acoustic wave.

4.3 Alfvénic black hole

In the limit of $\mathrm{c}_0<<\mathrm{v}_\mathrm{A}$ (zero β plasma condition), the magnetoacoustic wave (Eq. 31) reduces to an Alfvén wave in the presence of initial material flow. The resultant Alfvén wave equation is in the form of Klein–Gordon equation. Immediately, the Alfvénic metric can be derived from magnetoacoustic metric (Eqs. 33 and 35) by setting c_0 tends to 0,

$$g^A_{\mu\nu}=-\rho_0\mathrm{v}_A\begin{pmatrix}\mathrm{v}_A^2-\mathrm{v}_0^2 & \mathrm{v}_0 & 0 & 0\\ \mathrm{v}_0 & -1 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}. \tag{39}$$

Equivalent Alfvénic interval is given by

$$ds^2=\mathrm{v}_A\rho_0\left(-\left(1-\frac{\mathrm{v}_0^2}{\mathrm{v}_A^2}\right)\mathrm{v}_A^2d\tau^2+\frac{dx^2}{\left(1-\frac{\mathrm{v}_0^2}{\mathrm{v}_A^2}\right)}+dy^2+dz^2\right). \tag{40}$$

The Alfvénic metric (Eq. 40) is singular at the location of Alfvénic point where $\mathrm{v}_0=\mathrm{v}_\mathrm{A}$. This singular behaviour of Alfvénic metric leads to an Alfvénic BH. We propose a setups that is illustrated in Fig. 3, the condition for occurrence of an Alfvénic BH can be discussed. For compressional (longitudinal) Alfvén wave, the plasma density is nearly constant. As a result of mass continuity, the flux, $S(x)\mathrm{v}_0(x)$, is constant in the tube cross section. Consider a super-Alfvénic flow in a tube with increasing cross section (Fig. 3), speed of the flow decreases along the tube axis and reaches to Alfvén velocity $\mathrm{v}_0=\mathrm{v}_A$ at Alfvénic point and then transformed to sub-Alfvénic flow. Therefore, we call Alfvénic horizon to be the interface between super-Alfvénic flow and sub-Alfvénic flow. In horizon of the Alfvénic BH, the Alfvén wave is trapped and it is expected to be radiated by Alphonon. Alphonon is introduced as a quantum particle for Alfvén wave packet.

5 Hawking temperature

Hawking temperature is an important characteristic of BHs. Unruh showed that fluid flows mimic BHs. Hawking temperature (T_H) for acoustic BH was obtained

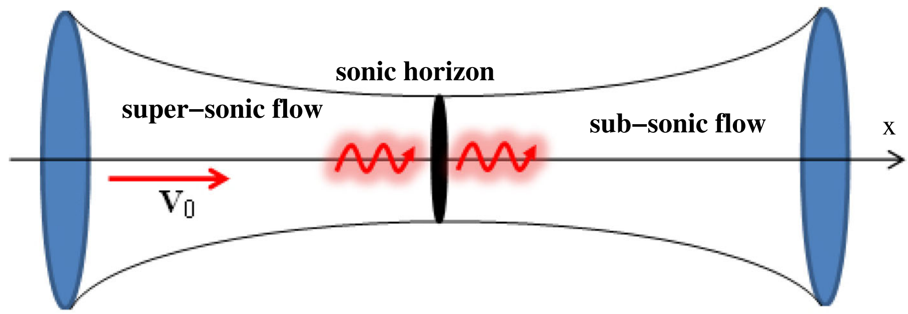

Fig. 1 A schematic presentation of a acoustic black hole in Laval nozzle tube. The speed of super sonic flow reduces to local propagation speed of acoustic wave at sonic horizon; then acoustic signals are trapped

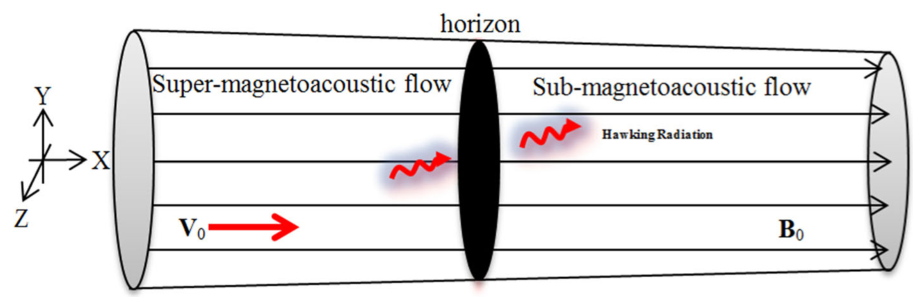

Fig. 2 Sketch of a tube with a slightly variable cross section embedded in a slightly uniform magnetic field. Super-magnetoacoustic flow, $v_0 > c_f$, sub-magnetoacoustic flow, $v_0 < c_f$, and magnetoacoustic wave trapped in the horizon are presented

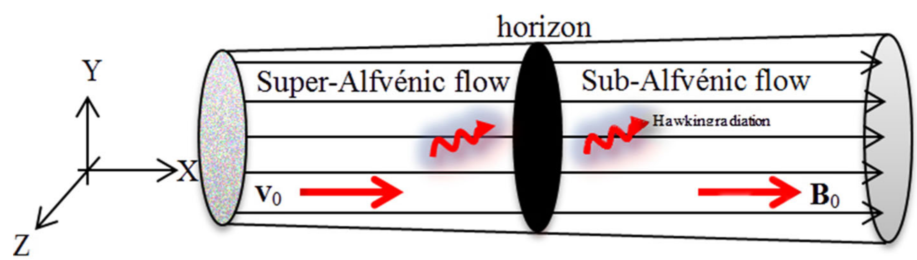

Fig. 3 A schematic presentation of a Alvénic black hole. Sketch of a tube with a slightly variable cross section embedded in a uniform magnetic field. The speed of super Alfvénic flow reduces to local propagation speed of Alfvénic wave at horizon; then Alfvénic signals are trapped

$$T_{\mathrm{H-acoustic}} = \frac{\hbar}{2\pi k c_s} \frac{d(v_0^2 - c_s^2)}{dx}\bigg|_{\mathtt{horizon}}, \tag{41}$$

where, $\hbar = h/2\pi$, h is the plank constant and k is the Boltzmann constant. Since the Hawking temperature is independent of metric conformal factor. It will therefore be as following for the magnetoacoustic BH,

$$T_{\mathrm{H-magnetoacoustic}} = \frac{\hbar}{2\pi k c_f} \frac{d(v_0^2 - c_f^2)}{dx}\bigg|_{\mathtt{horizon}}. \tag{42}$$

In the limit of v_A tends to zero, Eq. (42) then reduces to Hawking temperature for acoustic BH. Although, in the zero β plasma condition the above mentioned equation can describe the Hawking temperature for Alfvénic BH,

$$T_{\mathtt{H-Alfvénic}} = \frac{\hbar}{2\pi k v_A} \frac{d(v_{0^2} - v_{A^2})}{dx}\bigg|. \tag{43}$$

At the horizon where the tube cross section radius is equal to R, Eq. (42) can be simplified as

$$T_{H-magnetoacoustic} \approx \frac{\hbar\, c_f}{2\pi k\, R}, \tag{44}$$

where, the term $\frac{1}{c_f}\frac{d}{dx}(c_f - v_0)$ is approximately equal to $\frac{1}{R}$. For a plasma with a ratio of $\chi = c_0/v_A$ in which χ is a positive number, Eq. (44) gives

$$T_{H-magnetoacoustic} \approx 2.66 \times 10^4 \left(1 + \chi^2\right)^{0.5} \frac{B_0}{R\sqrt{n}}, \tag{45}$$

where, n is the number density of plasma and all units are in SI. For a typical plasma with magnetic field strength B_0=1 Tesla, number density $n = 10^{18}\, m^{-3}$, $R = 1$ mm, and $\chi^2 \ll 1$, the Hawking temperature is estimated about 0.0266 K. For a typical natural fluid in a Lavel nozzle experiment, the Hawking temperature was estimated about 10^{-6} K [27].

6 Conclusion

In this study, we introduced the magnetoacoustic and Alfvénic analogue BHs. In the horizon of magnetoacoustic and Alfvénic BHs, the magnetoacoustic and Alfvén waves are trapped, respectively, and should emit magnephonons and Alphonons version of HR at Hawking temperature. The next logical step is to investigate the physical properties of both magnephonon and Alphonon quasi-particles based on quantum approach. As stated in the literature, for acoustic BH in a natural fluid, the Hawking temperature is a function of sound speed and geometry of nozzle setup at the horizon. However, Hawking temperature for magnetoacoustic BH is related to the sound speed with additional positive terms that depends on the magnetic field and density. Magnephonons and Alphonons are particles (quanta) corresponding to magnetoacoustic and Alfvén waves, respectively. The idea for definition of acoustic, magnetoacoustic, and Alfvénic BHs can be applied to theoretical and/or experimental prediction of new non-gravitational BHs. Perhaps, the study of new BHs could help us observe HR.

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