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SU(3) symmetry breaking in charmed baryon decays

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Abstract We explore the breaking effects of the $SU(3)$ flavor symmetry in the singly Cabibbo-suppressed anti-triplet charmed baryon decays of $\mathbf{B}_c \to \mathbf{B}_n M$, with $\mathbf{B}_c = (\Xi_c^0, \Xi_c^+, \Lambda_c^+)$ and $\mathbf{B}_n(M)$ the baryon (pseudo-scalar) octets. We find that these breaking effects can be used to account for the experimental data on the decay branching ratios of $\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+, \Lambda^0 K^+)$ and $R'_{K/\pi} = \mathcal{B}(\Xi_c^0 \to \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$. In addition, we obtain that $\mathcal{B}(\Xi_c^0 \to \Xi^- K^+, \Sigma^-\pi^+) = (4.6\pm1.7, 12.8\pm3.1)\times10^{-4}$, $\mathcal{B}(\Xi_c^0 \to pK^-, \Sigma^+\pi^-) = (3.0\pm1.0, 5.2\pm1.6)\times10^{-4}$ and $\mathcal{B}(\Xi_c^+ \to \Sigma^{0(+)}\pi^{+(0)}) = (10.3 \pm 1.7) \times 10^{-4}$, which all receive significant contributions from the breaking effects, and can be tested by the BESIII and LHCb experiments.

1 Introduction

It is known that the theoretical approach based on the factorization and quantum chromodynamics (QCD) barely explains the charmed hadron decays [1]. This is due to the fact that the mass of the charm quark, $m_c \simeq 1.5$ GeV, is not as heavy as that of the bottom one, $m_b \simeq 4.8$ GeV, resulting in an undetermined correction to the heavy quark expansion, such that the alternative models have to take place for this correction [2–8]. On the other hand, the $SU(3)$ flavor ($SU(3)_f$) symmetry that works in the b-hadron decays [9–13] can be well applied to $D \to MM$ and $\mathbf{B}_c \to \mathbf{B}_n M$ [14–25], where $\mathbf{B}_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+)$ are the lowest-lying anti-triplet charmed baryon states, while $\mathbf{B}_n$ and M represent baryon and pseudoscalar meson states, respectively. Particularly, the $SU(3)_f$ symmetry has been extended to investigate the singly charmed baryon sextet states as well as the doubly and triply charmed baryon ones [23,24]. For $D \to MM$ decays, the measurements produce [26]

$$\mathcal{R}_{D^0(K/\pi)} \equiv \frac{\mathcal{B}(D^0 \to K^+K^-)}{\mathcal{B}(D^0 \to \pi^+\pi^-)} = 2.82 \pm 0.07,$$

$$\mathcal{B}_{D^0(2K_s^0)} \equiv \mathcal{B}(D^0 \to K_s^0 K_s^0) = (1.70 \pm 0.12) \times 10^{-4}, \tag{1}$$

in comparison with $(\mathcal{R}_{D^0(K/\pi)}, \mathcal{B}_{D^0(2K_s^0)}) \simeq (1, 0)$ given by the theoretical calculations based on the $SU(3)_f$ symmetry. The disagreements between the theory and experiment imply that the breaking effects of the $SU(3)_f$ symmetry cannot be ignored in the singly Cabibbo-suppressed (SCS) processes. We note that, in the literature, the $SU(3)_f$ breaking effects were used to relate $\mathcal{R}_{K/\pi}$ to the possible large difference of the CP violating asymmetries of $\Delta\mathcal{A}_{CP} \equiv \mathcal{A}_{CP}(D^0 \to K^+K^-) - \mathcal{A}_{CP}(D^0 \to \pi^+\pi^-)$ [15,27,28], which is recently measured to be $(-0.10 \pm 0.08 \pm 0.03)\%$ by LHCb [29].

For the two-body $\mathbf{B}_c \to \mathbf{B}_n M$ decays, both Cabibbo flavored (CF) and SCS decays are not well explained. In particular, the experimental measurements show that

$$\begin{aligned}
\mathcal{B}_{p\pi^0} &\equiv \mathcal{B}(\Lambda_c^+ \to p\pi^0) \\
&< 3 \times 10^{-4}\ (90\%\ \text{C.L.})\ [30,31], \\
\mathcal{R}'_{K/\pi} &\equiv \frac{\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)}{\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)} \\
&= 0.028 \pm 0.006 \simeq (0.6 \pm 0.2)s_c^2\ [26], \\
\mathcal{B}(\Lambda_c^+ &\to \Lambda^0 K^+) = (6.1 \pm 1.2) \times 10^{-4}\ [26], \\
\mathcal{B}(\Lambda_c^+ &\to \Sigma^0 K^+) = (5.2 \pm 0.8) \times 10^{-4}\ [26],
\end{aligned} \tag{2}$$

where $s_c \equiv \sin\theta_c = 0.2248$ [26] with θ_c the well-known Cabbibo angle. However, theoretical evaluations based on the $SU(3)_f$ symmetry lead to $\mathcal{B}_{p\pi^0} = (5.7\pm1.5)\times10^{-4}$ and $\mathcal{R}'_{K/\pi} \simeq 1.0s_c^2$ [21], and those in the factorization approach give $\mathcal{B}_{p\pi^0} = f_\pi^2/(2f_K^2)s_c^2\,\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0) = (5.5 \pm 0.3) \times 10^{-4}$ and $\mathcal{R}'_{K/\pi} = (f_K/f_\pi)^2 s_c^2 \simeq 1.4s_c^2$, where we have used the data of $\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0) = (3.16\pm0.16)\times10^{-2}$ [26]. In addition, the fitted results of $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 K^+, \Sigma^0 K^+) = (4.6 \pm 0.9, 4.0 \pm 0.8) \times 10^{-4}$ [22] are $(1.3 - 1.6)\sigma$ away from the data in Eq. (2). In this study, we will consider the breaking effects of the $SU(3)_f$ symmetry due to the fact of

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$m_s \gg m_{u,d}$ in the $\mathbf{B}_c \to \mathbf{B}_n M$ decays, particularly, the SCS processes, in accordance with the $D \to MM$ ones. Our goal is to find out whether the data in Eq. (2) can be understood by introducing the breaking effects.

The paper is organized as follows. In Sect. 2, we provide the formalism, in which the amplitudes of the $\mathbf{B}_c \to \mathbf{B}_n M$ decays with and without the breaking effects of $SU(3)_f$ symmetry are presented. The numerical analysis is performed in Sect. 3. In Sect. 4, we discuss our results and give the conclusions.

2 Formalism

The two-body charmed baryon weak decays, such as $\Xi_c^0 \to \Xi^-\pi^+(\Xi^-K^+)$ and $\Lambda_c^+ \to p\pi^0$, proceed through the quark-level transitions of $c \to su\bar{d}$, $c \to ud\bar{d}$ and $c \to us\bar{s}$, with the effective Hamiltonian given by [32]

$$\mathcal{H}_{eff} = \sum_{i=+,-} \frac{G_F}{\sqrt{2}} c_i \left(V_{cs} V_{ud} O_i + V_{cd} V_{ud} O_i^d + V_{cs} V_{us} O_i^s \right), \tag{3}$$

where G_F is the Fermi constant, $c_\pm$ are the scale-dependent Wilson coefficients, and the CKM matrix elements $V_{cs}V_{ud} \simeq 1$ and $V_{cs}V_{us} \simeq -V_{cd}V_{ud} \simeq s_c$ correspond to the Cabibbo-favored (CF) and singly Cabibbo-suppressed (SCS) charmed hadron decays, respectively, while $O_\pm^{(d,s)}$ are the four-quark operators, written as

$$O_\pm = \frac{1}{2}\left[(\bar{u}d)(\bar{s}c) \pm (\bar{s}d)(\bar{u}c)\right],$$
$$O_\pm^q = \frac{1}{2}\left[(\bar{u}q)(\bar{q}c) \pm (\bar{q}q)(\bar{u}c)\right], \tag{4}$$

where $q = (d, s)$ and $(\bar{q}_1 q_2) = \bar{q}_1\gamma_\mu(1-\gamma_5)q_2$. With $q_i = (u, d, s)$ as the triplet of 3, the operator of $(\bar{q}^i q_k \bar{q}^j)c$ can be decomposed as the irreducible forms, that is, $(\bar{3} \times 3 \times \bar{3})c = (\bar{3} + \bar{3}' + 6 + \overline{15})c$. Accordingly, the operators $O_-^{(q)}$ and $O_+^{(q)}$ belong to 6 and $\overline{15}$, respectively, given by [17]

$$O_\mp \simeq \mathcal{O}_{6(\overline{15})} = \frac{1}{2}(\bar{u}d\bar{s} \mp \bar{s}d\bar{u})c,$$
$$O_\mp^q \simeq \mathcal{O}^q_{6(\overline{15})} = \frac{1}{2}(\bar{u}q\bar{q} \mp \bar{q}q\bar{u})c, \tag{5}$$

such that the effective Hamiltonian in Eq. (3) can be transformed into the tensor form of

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\left[c_- \frac{\epsilon^{ijl}}{2} H(6)_{lk} + c_+ H(\overline{15})_k^{ij}\right], \tag{6}$$

with the non-zero entries:

$$H_{22}(6) = 2,\ H_2^{13}(\overline{15}) = H_2^{31}(\overline{15}) = 1,$$
$$H_{23}(6) = H_{32}(6) = -2s_c,\ H_{12}^2(\overline{15}) = H_{21}^2(\overline{15}) = s_c, \tag{7}$$

where the notations of (i, j, k) are quark indices, to be connected to the initial and final states in the amplitudes. Note that $H_{23}(6)$ and $H_{32}(6)$ are derived from $\mathcal{O}_6^s$ and $\mathcal{O}_6^d$, respectively. The lowest-lying charmed baryon states $\mathbf{B}_c$ are an anti-triplet of $\bar{3}$ to consist of $(ds - sd)c$, $(us - su)c$ and $(ud - du)c$, presented as

$$\mathbf{B}_c = (\Xi_c^0, -\Xi_c^+, \Lambda_c^+), \tag{8}$$

together with the baryon and meson octets, given by

$$\mathbf{B}_n = \begin{pmatrix} \frac{1}{\sqrt{6}}\Lambda^0 + \frac{1}{\sqrt{2}}\Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}}\Lambda^0 - \frac{1}{\sqrt{2}}\Sigma^0 & n \\ \Xi^- & \Xi^0 & -\sqrt{\frac{2}{3}}\Lambda^0 \end{pmatrix},$$
$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 & \pi^- & K^- \\ \pi^+ & -\frac{1}{\sqrt{2}}\pi^0 & \bar{K}^0 \\ K^+ & K^0 & 0 \end{pmatrix}, \tag{9}$$

where we have removed the octet η_8 and singlet η_1 meson states to simplify our discussions. Subsequently, the amplitudes of $\mathbf{B}_c \to \mathbf{B}_n M$ can be derived as

$$\mathcal{A}(\mathbf{B}_c \to \mathbf{B}_n M) = \langle \mathbf{B}_n M | \mathcal{H}_{eff} | \mathbf{B}_c \rangle = \frac{G_F}{\sqrt{2}} T(\mathbf{B}_c \to \mathbf{B}_n M), \tag{10}$$

with $T(\mathbf{B}_c \to \mathbf{B}_n M) = T(\mathcal{H}_6) + T(\mathcal{H}_{\overline{15}})$, where $T(\mathcal{H}_{6,\overline{15}})$ are decomposed as [21–23]

$$\begin{aligned} T(\mathcal{H}_6) &= a_1 H_{ij}(6) T^{ik} (\mathbf{B}_n)_k^l (M)_l^j \\ &+ a_2 H_{ij}(6) T^{ik} (M)_k^l (\mathbf{B}_n)_l^j \\ &+ a_3 H_{ij}(6) (\mathbf{B}_n)_k^i (M)_l^j T^{kl}, \\ T(\mathcal{H}_{\overline{15}}) &= a_4 H_{li}^k(\overline{15}) (\mathbf{B}_c)^j (M)_j^i (\mathbf{B}_n)_k^l \\ &+ a_5 (\mathbf{B}_n)_j^i (M)_i^l H(\overline{15})_l^{jk} (\mathbf{B}_c)_k \\ &+ a_6 (\mathbf{B}_n)_l^k (M)_j^i H(\overline{15})_i^{jl} (\mathbf{B}_c)_k \\ &+ a_7 (\mathbf{B}_n)_i^l (M)_j^i H(\overline{15})_l^{jk} (\mathbf{B}_c)_k, \end{aligned} \tag{11}$$

with $T^{ij} \equiv (\mathbf{B}_c)_k \epsilon^{ijk}$. In Eq. (11), $a_{1,2,3}$ and $a_{4,5,6,7}$ are the $SU(3)$ parameters from $H(6)$ and $H(\overline{15})$, respectively, in which $c_\mp$ have been absorbed. We hence obtain [22]

$$T(\Lambda_c^+ \to p\pi^0) = -\sqrt{2}\left(a_2 + a_3 - \frac{a_6 - a_7}{2}\right) s_c,$$
$$T(\Xi_c^0 \to \Xi^-\pi^+) = 2\left(a_1 + \frac{a_5 + a_6}{2}\right),$$
$$T(\Xi_c^0 \to \Xi^- K^+) = -2\left(a_1 + \frac{a_5 + a_6}{2}\right) s_c, \tag{12}$$

based on the exact $SU(3)_f$ symmetry.

According to Refs. [21,22], the numerical analysis with the minimum χ^2 fit has well explained the ten observed $\mathbf{B}_c \to \mathbf{B}_n M$ decays by neglecting the terms associated with $a_{4,5,6,7}$ to reduce the parameters [20–23,25]. This reduction is due to the fact that the contributions to the branching rates from $H(\overline{15})$ and $H(6)$ lead to a small ratio of $\mathcal{R}(\overline{15}/6) = c_+^2/c_-^2 \simeq 17\%$ with $(c_+, c_-) = (0.76, 1.78)$ calculated at the scale $\mu = 1$ GeV in the NDR scheme [33,34]. There remain two measurements to be explained. In Eq. (2), the prediction for $\mathcal{B}_{p\pi^0}$ has the 2σ gap to reach the edge of the experimental upper bound. However, with $\mathcal{R}(\overline{15}/6)$ to be small, it is nearly impossible that, by restoring $a_{4,5,6,7}$ that have been ignored in the literature [20–23,25], one can accommodate the data of $\mathcal{B}_{p\pi^0}$ but without having impacts on the other decay modes, which are correlated with the same sets of parameters. Moreover, as seen from Eq. (12), there is no room for $\mathcal{R}'_{K/\pi}$ as it is fixed to be $(1.0)s_c^2$. On the other hand, the results for $D \to MM$ decays in Eq. (1) suggest some possible corrections from the breaking effects of the $SU(3)_f$ symmetry in the SCS processes. In the charm baryon decays, we consider the similar effects. Due to $m_s \gg m_{u,d}$, we present the matrix of $M_s = \epsilon(\lambda_s)^i_j$ [14] to break $SU(3)_f$, where $\epsilon \sim 0.2-0.3$ and λ_s is given by [14,15,18]

$$\lambda_s = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \tag{13}$$

which transforms as an octet of 8, such that its coupling to $H(6)$ is in the form of $8 \times 6 = \bar{3} + 6 + \overline{15} + 24$, and $\bar{3}$ is for the simplest break effects to be confined in the SCS processes [18,35]. Note that from $\frac{1}{8}\Big(\delta^i_l(\lambda_s)^n_j H(6)_{kn} - \delta^i_l(\lambda_s)^n_k H(6)_{jn} + \delta^i_k(\lambda_s)^n_j H(6)_{ln} - \delta^i_k(\lambda_s)^n_l H(6)_{jn}\Big)$ and the nonzero entry of $H(\bar{3})^1 = s_c$ from the coupling of $H(6)_{23}$ and $H(6)_{32}$ [14], one can trace back to the break effect between SCS $c \to us\bar{s}$ and $c \to ud\bar{d}$ transitions. As a result, the $SU(3)_f$ symmetry breaking gives rise to the new T-amplitudes, given by

$$\begin{aligned} T(\mathcal{H}_3) = &\, v_1(\mathbf{B}_c)_i H(\bar{3})^i (\mathbf{B}_n)^j_k (M)^k_j \\ &+ v_2(\mathbf{B}_c)_i H(\bar{3})^j (\mathbf{B}_n)^i_k (M)^k_j \\ &+ v_3(\mathbf{B}_c)_i H(\bar{3})^j (\mathbf{B}_n)^k_j (M)^i_k, \end{aligned} \tag{14}$$

where $v_{1,2,3}$ are the parameters related to the $SU(3)_f$ breaking. It is interesting to note that the v_1 terms associated with $(\mathbf{B}_c)_i H(\bar{3})^i$ in Eq. (14) occur in some of the $\Xi_c^{0,+}$ decays, but disappear in all Λ_c^+ modes. By adding $T(\mathcal{H}_3)$ into $T(\mathbf{B}_c \to \mathbf{B}_n M)$ in Eq. (10), the full expansions of $T(\mathbf{B}_c \to \mathbf{B}_n M)$ are given in Table 1, to be used to calculate the decay widths, given by [26]

$$\Gamma(\mathbf{B}_c \to \mathbf{B}_n M) = \frac{|\vec{p}_{cm}|}{8\pi m^2_{\mathbf{B}_c}} |\mathcal{A}(\mathbf{B}_c \to \mathbf{B}_n M)|^2, \tag{15}$$

where $|\vec{p}_{cm}| = \sqrt{[(m^2_{\mathbf{B}_c} - (m_{\mathbf{B}_n} + m_M)^2][(m^2_{\mathbf{B}_c} - (m_{\mathbf{B}_n} - m_M)^2]}/(2m_{\mathbf{B}_c})$.

3 Numerical analysis

In the numerical analysis, we examine $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 K^+, \Sigma^0 K^+, p\pi^0)$ and $\mathcal{R}'_{K/\pi}$ by including the breaking effects of the $SU(3)_f$ symmetry to see if one can explain their data in Eq. (2). The theoretical inputs for the CKM matrix elements are given by [26]

$$(V_{cs}, V_{ud}, V_{us}, V_{cd}) = (1 - \lambda^2/2, 1 - \lambda^2/2, \lambda, -\lambda), \tag{16}$$

with $\lambda = 0.2248$ in the Wolfenstein parameterization. We perform the minimum χ^2 fit, in terms of the equation of [22]

$$\chi^2 = \sum_i \left(\frac{\mathcal{B}^i_{th} - \mathcal{B}^i_{ex}}{\sigma^i_{ex}}\right)^2 + \sum_j \left(\frac{\mathcal{R}^j_{th} - \mathcal{R}^j_{ex}}{\sigma^j_{ex}}\right)^2, \tag{17}$$

with $\mathcal{B} = \mathcal{B}(\Lambda_c^+ \to \mathbf{B}_n M)$ and $\mathcal{R} = \mathcal{B}(\Xi_c^0 \to \mathbf{B}_n M)/\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$, where the subscripts th and ex are denoted as the theoretical inputs from the amplitudes in Table 1 and the experimental data points in Table 2, respectively, while $\sigma_{i,j}$ correspond to the 1σ errors. By following Refs. [21–23], we extract the parameters, which are in fact complex numbers, given by

$$a_1, a_2 e^{i\delta_{a_2}}, a_3 e^{i\delta_{a_3}}, v_1 e^{i\delta_{v_1}}, v_2 e^{i\delta_{v_2}}, v_3 e^{i\delta_{v_3}}, \tag{18}$$

where $a_{4,5,...,7}$ have been ignored as discussed in Sect. 3. Since only the relative phases contribute to the branching ratios, a_1 is set to be real without losing generality. However, we take v_i to be real numbers in order to fit 8 parameters with the 9 data points in Table 2. In the calculation, δ_{a_i} (i=2 or 3) from $a_i e^{\delta_{a_i}}$ is a fitting parameter, which can absorb the phase of δ_{v_i} from the interference in the data fitting. Note that δ_{a_i} $(i = 2, 3)$ have been fitted with the imaginary parts [22]. As a result, we may set $\delta_{v_{2,3}}$ along with the overall phase of δ_{v_1} to be zero for the estimations of the decay branching ratios due to the $SU(3)$ breaking effects. We will follow Ref. [25] to test our assumption, where a similar global fit in the approach of the $SU(3)_f$ symmetry has been done to extract $a_2 e^{\delta_{a_2}}$ by freely rotating the angle of δ_{a_2} from $-180°$ to $180°$ to estimate the uncertainties of the branching ratios. Subsequently, the fit with the breaking effects in the $SU(3)_f$ symmetry yields

$$\begin{aligned} (a_1, a_2, a_3) &= (0.252 \pm 0.005, 0.127 \pm 0.009, 0.091 \\ &\qquad \pm 0.015)\,\text{GeV}^3, \\ (\delta_{a_2}, \delta_{a_3}) &= (73.0 \pm 27.3, 40.2 \pm 4.7)^\circ, \\ (v_1, v_2, v_3) &= (0.090 \pm 0.032, -0.037 \pm 0.013, 0.025 \\ &\qquad \pm 0.012)\,\text{GeV}^3, \end{aligned} \tag{19}$$

with $\chi^2/d.o.f = 3.0/1$, where $d.o.f$ represents the degree of freedom. Note that $a_{1,2,3}$ and their phases are nearly the same as those without the breaking of $SU(3)_f$ [22]. With the parameters in Eq. (19), we obtain the branching ratios of the CF and SCS $\mathbf{B}_c \to \mathbf{B}_n M$ decays, shown in Table 3.

Table 1 Amplitudes of $T(\mathbf{B}_c \to \mathbf{B}_n M)$, where T-amps refers to $T(\mathbf{B}_c \to \mathbf{B}_n M)$ and CF (SCS) represents Cabbibo favored (singly Cabbibo-suppressed)

CF mode	T-amp	SCS mode	T-amp
$\Xi_c^0 \to \Sigma^+ K^-$	$2\left(a_2 + \frac{a_4+a_7}{2}\right)$	$\Xi_c^0 \to \Sigma^+ \pi^-$	$2\left(a_2 + v_1 + v_3 + \frac{a_4+a_7}{2}\right)s_c$
$\Xi_c^0 \to \Sigma^0 \bar{K}^0$	$-\sqrt{2}\left(a_2 + a_3 - \frac{a_6-a_7}{2}\right)$	$\Xi_c^0 \to \Sigma^- \pi^+$	$2\left(a_1 + v_1 + v_2 + \frac{a_5+a_6}{2}\right)s_c$
$\Xi_c^0 \to \Xi^0 \pi^0$	$-\sqrt{2}\left(a_1 - a_3 - \frac{a_4-a_5}{2}\right)$	$\Xi_c^0 \to \Sigma^0 \pi^0$	$\left(a_2 + a_3 - 2v_1 - v_2 - v_3 - \frac{a_4-a_5+a_6-a_7}{2}\right)s_c$
$\Xi_c^0 \to \Xi^- \pi^+$	$2\left(a_1 + \frac{a_5+a_6}{2}\right)$	$\Xi_c^0 \to \Xi^- K^+$	$-2\left(a_1 - v_1 - v_2 + \frac{a_5+a_6}{2}\right)s_c$
$\Xi_c^0 \to \Lambda^0 \bar{K}^0$	$-\sqrt{\frac{2}{3}}\left(2a_1 - a_2 - a_3 + \frac{2a_5-a_6-a_7}{2}\right)$	$\Xi_c^0 \to p K^-$	$-2\left(a_2 - v_1 - v_3 + \frac{a_4+a_7}{2}\right)s_c$
		$\Xi_c^0 \to \Xi^0 K^0$	$-2\left(a_1 - a_2 - a_3 - v_1 + \frac{a_5-a_7}{2}\right)s_c$
		$\Xi_c^0 \to n \bar{K}^0$	$2\left(a_1 - a_2 - a_3 + v_1 + \frac{a_5-a_7}{2}\right)s_c$
		$\Xi_c^0 \to \Lambda^0 \pi^0$	$\sqrt{\frac{1}{3}}\left(-a_1 - a_2 + 2a_3 + v_2 + v_3 + \frac{a_4-a_5-a_6-a_7}{2}\right)s_c$
$\Xi_c^+ \to \Sigma^+ \bar{K}^0$	$-2\left(a_3 - \frac{a_4+a_6}{2}\right)$	$\Xi_c^+ \to \Sigma^0 \pi^+$	$-\sqrt{2}\left(a_1 - a_2 + v_2 - v_3 + \frac{a_4-a_5+a_6+a_7}{2}\right)s_c$
$\Xi_c^+ \to \Xi^0 \pi^+$	$2\left(a_3 + \frac{a_4+a_6}{2}\right)$	$\Xi_c^+ \to \Sigma^+ \pi^0$	$\sqrt{2}\left(a_1 - a_2 + v_2 - v_3 - \frac{a_4+a_5+a_6-a_7}{2}\right)s_c$
		$\Xi_c^+ \to \Xi^0 K^+$	$2\left(a_2 + a_3 - v_2 + \frac{a_6-a_7}{2}\right)s_c$
		$\Xi_c^+ \to p \bar{K}^0$	$2\left(a_1 - a_3 - v_3 + \frac{a_4-a_5}{2}\right)s_c$
		$\Xi_c^+ \to \Lambda^0 \pi^+$	$\sqrt{\frac{2}{3}}\left(a_1 + a_2 - 2a_3 - v_2 - v_3 - \frac{3a_4+a_5+a_6+a_7}{2}\right)s_c$
$\Lambda_c^+ \to \Sigma^0 \pi^+$	$-\sqrt{2}\left(a_1 - a_2 - a_3 - \frac{a_5-a_7}{2}\right)$	$\Lambda_c^+ \to \Sigma^+ K^0$	$2\left(a_1 - a_3 + v_3 - \frac{a_4-a_5}{2}\right)s_c$
$\Lambda_c^+ \to \Sigma^+ \pi^0$	$\sqrt{2}\left(a_1 - a_2 - a_3 - \frac{a_5-a_7}{2}\right)$	$\Lambda_c^+ \to \Sigma^0 K^+$	$\sqrt{2}\left(a_1 - a_3 + v_3 - \frac{a_4+a_5}{2}\right)s_c$
$\Lambda_c^+ \to \Xi^0 K^+$	$-2\left(a_2 - \frac{a_4+a_7}{2}\right)$	$\Lambda_c^+ \to p \pi^0$	$\sqrt{2}\left(a_2 + a_3 + v_2 - \frac{a_6-a_7}{2}\right)s_c$
$\Lambda_c^+ \to p \bar{K}^0$	$-2\left(a_1 - \frac{a_5+a_6}{2}\right)$	$\Lambda_c^+ \to n \pi^+$	$2\left(a_2 + a_3 + v_2 + \frac{a_6-a_7}{2}\right)s_c$
$\Lambda_c^+ \to \Lambda^0 \pi^+$	$-\sqrt{\frac{2}{3}}\left(a_1 + a_2 + a_3 - \frac{a_5-2a_6+a_7}{2}\right)$	$\Lambda_c^+ \to \Lambda^0 K^+$	$\sqrt{\frac{2}{3}}\left(a_1 - 2a_2 + a_3 - 2v_2 + v_3 - \frac{3a_4-a_5+2a_6+2a_7}{2}\right)s_c$

4 Discussions and conclusions

As seen from Tables 2 and 3, the breaking effects associated with v_2 and v_3 on the branching ratios of the SCS $\Lambda_c^+ \to \mathbf{B}_n M$ decays are at most around 30%, which is close to the naive estimation of $(f_K/f_\pi)^2 \simeq 40\%$. In particular, we get $\mathcal{B}(\Lambda_c^+ \to \Lambda^0 K^+, \Sigma^0 K^+) = (6.1 \pm 0.9, 5.2 \pm 0.7) \times 10^{-4}$, which explain the data in Eq. (2) well and alleviate the $(1.3 - 1.6)\sigma$ deviations by the fit with the exact $SU(3)_f$ symmetry [22]. Meanwhile, the branching ratios for the CF modes are fitted to be the same as those without the breaking except $\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$, which is slightly different in order to account for the recent observational value given by BESIII [36].

Moreover, the fitted value of $\mathcal{R}'_{K/\pi} = (0.6 \pm 0.2)s_c^2 = 0.03 \pm 0.01$ explains the data very well for the first time. This leads to the prediction of $\mathcal{B}(\Xi_c^0 \to \Xi^- K^+) = (4.6 \pm 1.7) \times 10^{-4}$, with $v_1 + v_2$ as the destructive contribution to reduce the value of $(7.6 \pm 0.4) \times 10^{-4}$ under the exact $SU(3)_f$ symmetry, whereas $\mathcal{B}(\Xi_c^0 \to \Sigma^- \pi^+) = (12.8 \pm 3.1) \times 10^{-4}$ receives the constructive contribution from $v_1 + v_2$, with $T(\Xi_c^0 \to \Xi^- K^+, \Sigma^- \pi^+) = \mp[a_1 \mp (v_1 + v_2)]s_c$. Since

Table 2 The data of the $\mathbf{B}_c \to \mathbf{B}_n M$ decays, together with the reproduction with the exact (broken) $SU(3)_f$ symmetry in the 3rd (4th) column

(Branching) Ratios	Data [26,30,36]	Exact [22]	Broken
$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$	5.2 ± 0.8	4.0 ± 0.8	5.2 ± 0.7
$10^4\mathcal{B}(\Lambda_c^+ \to \Lambda^0 K^+)$	6.1 ± 1.2	4.6 ± 0.9	6.1 ± 0.9
$\mathcal{R}'_{K/\pi} = \frac{\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)}{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)}$	$(0.6 \pm 0.2)s_c^2$	$(1.0)s_c^2$	$(0.6 \pm 0.2)s_c^2$
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+)$	1.29 ± 0.07	1.3 ± 0.2	1.3 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \pi^0)$	1.24 ± 0.10	1.3 ± 0.2	1.3 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$	0.59 ± 0.09	0.5 ± 0.1	0.6 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0)$	3.16 ± 0.16	3.3 ± 0.2	3.2 ± 0.1
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \pi^+)$	1.30 ± 0.07	1.3 ± 0.2	1.3 ± 0.1
$\mathcal{R}''_{K/\pi} = \frac{\mathcal{B}(\Xi_c^0 \to \Lambda^0 \bar{K}^0)}{\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)}$	0.42 ± 0.06	0.5 ± 0.1	0.5 ± 0.1

Table 3 The branching ratios of the $\mathbf{B}_c \to \mathbf{B}_n M$ decays, where the numbers with the dagger (†) correspond to the reproductions of the experimental data input, instead of the predictions

CF mode	Exact [22]	Broken	SCS mode	Exact [22]	Broken
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^+ K^-)$	3.5 ± 0.9	3.8 ± 0.6	$10^4\mathcal{B}(\Xi_c^0 \to \Sigma^+ \pi^-)$	2.0 ± 0.5	5.2 ± 1.6
$10^3\mathcal{B}(\Xi_c^0 \to \Sigma^0 \bar{K}^0)$	4.7 ± 1.2	5.2 ± 0.8	$10^4\mathcal{B}(\Xi_c^0 \to \Sigma^- \pi^+)$	9.0 ± 0.4	12.8 ± 3.1
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^0 \pi^0)$	4.3 ± 0.09	4.4 ± 0.4	$10^4\mathcal{B}(\Xi_c^0 \to \Sigma^0 \pi^0)$	3.2 ± 0.3	7.7 ± 2.2
$10^3\mathcal{B}(\Xi_c^0 \to \Xi^- \pi^+)$	15.7 ± 0.7	15.2 ± 0.7	$10^4\mathcal{B}(\Xi_c^0 \to \Xi^- K^+)$	7.6 ± 0.4	4.6 ± 1.7
$10^{\mathcal{B}}(\Xi_c^0 \to \Lambda^0 \bar{K}^0)$	8.3 ± 0.9	7.8 ± 0.5	$10^4\mathcal{B}(\Xi_c^0 \to pK^-)$	2.1 ± 0.5	3.0 ± 1.0
			$10^4\mathcal{B}(\Xi_c^0 \to \Xi^0 K^0)$	6.3 ± 1.2	4.1 ± 0.7
			$10^4\mathcal{B}(\Xi_c^0 \to n\bar{K}^0)$	7.9 ± 1.4	12.7 ± 2.4
			$10^4\mathcal{B}(\Xi_c^0 \to \Lambda^0 \pi^0)$	0.2 ± 0.2	0.3 ± 0.1
$10^3\mathcal{B}(\Xi_c^+ \to \Sigma^+ \bar{K}^0)$	8.0 ± 3.9	7.8 ± 2.7	$10^4\mathcal{B}(\Xi_c^+ \to \Sigma^0 \pi^+)$	18.5 ± 2.2	10.3 ± 1.7
$10^3\mathcal{B}(\Xi_c^+ \to \Xi^0 \pi^+)$	8.1 ± 4.0	7.9 ± 2.7	$10^4\mathcal{B}(\Xi_c^+ \to \Sigma^+ \pi^0)$	18.5 ± 2.2	10.3 ± 1.7
			$10^4\mathcal{B}(\Xi_c^+ \to \Xi^0 K^+)$	18.0 ± 4.7	24.3 ± 4.1
			$10^4\mathcal{B}(\Xi_c^+ \to p\bar{K}^0)$	20.3 ± 4.2	16.1 ± 2.8
			$10^4\mathcal{B}(\Xi_c^+ \to \Lambda^0 \pi^+)$	1.6 ± 1.2	2.4 ± 1.0
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^0 \pi^+)$	$(1.3 \pm 0.2)^\dagger$	$(1.3 \pm 0.1)^\dagger$	$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^+ K^0)$	8.0 ± 1.6	10.4 ± 1.5
$10^2\mathcal{B}(\Lambda_c^+ \to \Sigma^+ \pi^0)$	$(1.3 \pm 0.2)^\dagger$	$(1.3 \pm 0.1)^\dagger$	$10^4\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$	$(4.0 \pm 0.8)^\dagger$	$(5.2 \pm 0.7)^\dagger$
$10^2\mathcal{B}(\Lambda_c^+ \to \Xi^0 K^+)$	$(0.5 \pm 0.1)^\dagger$	$(0.6 \pm 0.1)^\dagger$	$10^4\mathcal{B}(\Lambda_c^+ \to p\pi^0)$	5.7 ± 1.5	5.4 ± 1.0
$10^2\mathcal{B}(\Lambda_c^+ \to p\bar{K}^0\,)$	$(3.3 \pm 0.2)^\dagger$	$(3.2 \pm 0.1)^\dagger$	$10^4\mathcal{B}(\Lambda_c^+ \to n\pi^+)$	11.3 ± 2.9	10.7 ± 1.9
$10^2\mathcal{B}(\Lambda_c^+ \to \Lambda^0 \pi^+)$	$(1.3 \pm 0.2)^\dagger$	$(1.3 \pm 0.1)^\dagger$	$10^4\mathcal{B}(\Lambda_c^+ \to \Lambda^0 K^+)$	$(4.6 \pm 0.9)^\dagger$	$(6.1 \pm 0.9)^\dagger$

there are other similar interferences between a_i and v_i, which come from $T(\Xi_c^0 \to pK^-, \Sigma^+\pi^-) = \mp[a_2 \mp (v_1 + v_3)]s_c$ and $T(\Xi_c^+ \to \Sigma^0\pi^+, \Sigma^+\pi^0) = \mp\sqrt{2}[(a_1 - a_2) + (v_2 - v_3)]s_c$, it is predicted that $\mathcal{B}(\Xi_c^0 \to pK^-, \Sigma^+\pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $\mathcal{B}(\Xi_c^+ \to \Sigma^0\pi^+(\Sigma^+\pi^0)) = (10.3 \pm 1.7) \times 10^{-4}$. It is interesting to note that the important roles of the terms associated with v_1 in the T amplitudes are also projected in the $\Xi_c^{0,+}$ modes, particularly, $\Xi_c^0 \to \Xi^- K^+$ and $\Xi_c^0 \to \Sigma^0\pi^0$. Clearly, these SCS Ξ_c decays all contain sizable $SU(3)_f$ breaking effects, and can be treated as golden modes to test the $SU(3)_f$ symmetry.

In our calculation, we treat v_3 as the norm in $T(\Lambda_c^+ \to \Sigma^0 K^+) \simeq \sqrt{2}(a_1 - a_3 + v_3)s_c$ of Table I, such that δ_{v_3} is allowed to rotate from $-90°$ to $50°$ without letting $\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+)$ exceed the data. Since the allowed range for δ_{v_3} is large, it is clear that its value is insensitive to the data. On the other hand, in order to explain the experimental data of $\mathcal{R}'_{K/\pi}$ with the smallest corrections from $v_i e^{\delta_{v_i}}$, we assume maximumly destructive interferences between $a_i e^{\delta_{a_i}}$ and $v_i e^{\delta_{v_i}}$. Explicitly, in $T(\Xi_c^0 \to \Xi^- K^+) \simeq -2(a_1 - v_1 - v_2)s_c$ for $\mathcal{R}'_{K/\pi}$, we can take $\delta_{v_1} = \delta_{v_2} = \delta_{a_1} = 0$ as an overall phase in $T(\Xi_c^0 \to \Xi^- K^+)$. Consequently, we are able to assume real values for v_i (i = 1,2,3) without loss of generality. Finally, we remark that, even with the breaking effects, we are still unable to fit the data of $\Lambda_c^+ \to p\pi^0$ in Eq. (1) as our result for its branching ratio of $(5.4 \pm 1.0) \times 10^{-4}$, which is close to $(5.5 \pm 0.3) \times 10^{-4}$ from the factorization approach [22], is lower than the current experimental upper bound of 3×10^{-4} [30,31]. However, it is possible that $H(\overline{15})$ would be non-negligible in $\Lambda_c^+ \to p\pi^0$. For example, with $T(\Lambda_c^+ \to p\pi^0) = \sqrt{2}(a_2 + a_3 + v_2 - (a_6 - a_7)/2)s_c$ in Table 1, the contribution from $(a_6-a_7)/2$ of $H(\overline{15})$ might be comparable with that from $a_2 + a_3 + v_2$ of $H(6)$, while $a_{2,3}$ and v_2 of Eq. (19) are taken to be small. In particular, with $(a_6-a_7)/2$ to be around 25% of $a_2+a_3+v_2$, $\mathcal{B}(\Lambda_c^+ \to p\pi^0)$ can be reduced to be within the experimental upper bound due to the destructive interference. In this case, there is a corresponding constructive interference in $\Lambda_c^+ \to n\pi^+$, leading to $\mathcal{B}(\Lambda_c^+ \to n\pi^+) \sim 17 \times 10^{-4}$, which breaks the relation of $\mathcal{A}(\Lambda_c^+ \to n\pi^+) = \sqrt{2}\mathcal{A}(\Lambda_c^+ \to p\pi^0)$ [8]. Clearly, in order to confirm the importance of $H(\overline{15})$, both experimental observations of $\Lambda_c^+ \to p\pi^0$ and $\Lambda_c^+ \to n\pi^+$ are needed.

In sum, we have studied the singly Cabibbo-suppressed charmed baryon decays. We have shown that the breaking effect of the $SU(3)_f$ symmetry can be used to understand the experimental data of $\mathcal{B}(\Lambda_c^+ \to \Sigma^0 K^+, \Lambda^0 K^+)$ and $R'_{K/\pi} = \mathcal{B}(\Xi_c^0 \to \Xi^- K^+)/\mathcal{B}(\Xi_c^0 \to \Xi^-\pi^+)$. With these effects, we have obtained that $\mathcal{B}(\Xi_c^0 \to \Xi^- K^+, \Sigma^-\pi^+) = (4.6 \pm 1.7, 12.8 \pm 3.1) \times 10^{-4}$, $\mathcal{B}(\Xi_c^0 \to pK^-, \Sigma^+\pi^-) = (3.0 \pm 1.0, 5.2 \pm 1.6) \times 10^{-4}$ and $\mathcal{B}(\Xi_c^+ \to \Sigma^0\pi^+(\Sigma^+\pi^0)) = (10.3 \pm 1.7) \times 10^{-4}$, which are quite different from those predicted by the approach with the exact $SU(3)_f$ symmetry. However, even with the breaking effects, our result for the branching ratio of $\Lambda_c^+ \to p\pi^0$ is still higher than the current experimental upper bound, which clearly requires a close examination by a future dedicated experiment.

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