

Resolving the SELEX–LHCb double-charm baryon conflict: the impact of intrinsic heavy-quark hadroproduction and supersymmetric light-front holographic QCD

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Abstract In this paper we show that the intrinsic heavy-quark QCD mechanism for the hadroproduction of heavy hadrons at large x_F can resolve the apparent conflict between measurements of double-charm baryons by the SELEX fixed-target experiment and the LHCb experiment at the LHC collider. We show that in fact both experiments are compatible, and that both can be correct. The observed spectroscopy of double-charm hadrons is in agreement with the predictions of supersymmetric light front holographic QCD.

1 Introduction

The first experimental evidence for the existence of double-charm baryons was published by the SELEX collaboration 15 years ago [1–6]. By utilizing the Fermilab negative and positive charged beams at 600 GeV/c to produce charmed particles in a thin foil of copper or on a diamond target, the SELEX collaboration observed two different decay channels for the $|dcc\rangle$ state at a mass close to 3520 MeV/c².

The SELEX fixed-target experiment measured hadron production in the forward kinematic domain $x_F > 0.1$. The negative beam composition was about 50% Σ^- and 50% π^- , whereas the positive beams were composed of 50% protons. The experimental data recorded used both positive and negative beams: 67% of the events were induced by Σ^- , 13% by π^- , and 18% by protons. In the first observation using the sample of $\Lambda_c^+ \rightarrow pK^-\pi^+$ [7,8] SELEX found a signal of 15.9 events over 6.1 ± 0.1 background events in the channel $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^-\pi^+$ [1]. To complement this result, SELEX published an observation of 5.62 signal events over 1.38 ± 0.13 background events for the decay mode $\Xi_{cc}^+ \rightarrow pDK^-$ from a sample of $D^+ \rightarrow K^-\pi^+\pi^+$ decays [4].

Two charm quarks will not be produced at high x_F from DGLAP evolution [9–11] or perturbative gluon splitting $g \rightarrow g + g \rightarrow (\bar{c}c) + (\bar{c}c)$ [11–13]. Therefore, the observation by SELEX of a double-charm baryon $|qcc\rangle$ at a large mean value for x_F and a relatively small mean transverse momentum can raise skepticism. However, the $\Lambda_c(udc)$ and the $\Lambda_b(udb)$ were both discovered at the ISR at high x_F [14–16]. In addition, the NA3 experiment measured both single-quarkonium hadroproduction $\pi A \rightarrow J/\psi X$ [17] and double-quarkonium hadroproduction $\pi A \rightarrow J/\psi J/\psi X$ [18] at high x_F . In fact, all of the $\pi A \rightarrow J/\psi J/\psi X$ events were observed by NA3 with total $x_F > 0.4$.

The existence of heavy quarks at large light-front (LF) momentum fraction x in the proton's light-front wavefunction is in fact predicted by QCD if one analyzes the higher Fock states $|uudc\bar{c}\rangle$ and $|uudc\bar{c}\bar{c}\rangle$ in the hadronic eigenstate; i.e., Fock states where the heavy quark pairs are multi-connected to the valence quarks. LF wavefunctions, the eigensolutions of the QCD LF Hamiltonian, are defined at fixed LF time $\tau = t + z/c$, and are thus off-shell in invariant mass. For example, in QED, positronium has an analogous $|e^+e^-\mu^+\mu^-\rangle$ Fock state due to the insertion of light-by-light scattering in the positronium self-energy amplitude. In such an “intrinsic charm” Fock state $|uudc\bar{c}\rangle$, the maximum kinematic configuration occurs at minimum invariant mass, where all quarks are at rest in the hadron's rest frame; i.e., at equal rapidity in the moving hadron. Equal rapidity implies $x_i \propto (m^2 + \mathbf{k}_\perp^2)^{1/2}$ for each quark, so that the heavy quarks in the Fock state carry most of the hadron's LF momentum. The operator product expansion predicts that the probability of intrinsic heavy-quark Fock states $|uudQ\bar{Q}\rangle$ to scale as $1/m_Q^2$ due to the non-Abelian couplings of QCD [19,20].

If such a Fock state interacts in a hadronic collision, the comoving udc of the projectile proton can readily coalesce

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in approximate consistency with the complementary measurement of the double J/ψ production by the NA3 experiment. It is interesting to note that the intrinsic charm mechanism predicts $\langle x_F(\Xi_{cc}) \rangle = 0.33$, as shown in Ref. [30]. This is in excellent agreement with the value $\langle x_F(\Xi_{cc}^+) \rangle \sim 0.33$ measured by the SELEX experiment.

It should be emphasized that SELEX observed the weak decay of the 3520 MeV double-charm baryon in two different decay channels, namely $\Xi_{cc}^+(3519 \pm 1) \rightarrow \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+(3518 \pm 3) \rightarrow p D^+ \pi^-$ with statistical significances of 6.3σ and 4.8σ , respectively. The probability that these two signals are statistical fluctuations is extremely small.

3 Mass difference

In order to resolve the discrepancy between the results from SELEX and LHCb we will utilize the predictions of the supersymmetric light front holographic QCD (SUSY LFHQCD). This approach was developed by imposing the constraints from the superconformal algebraic structure on LFHQCD for massless quarks [31]. As has been shown in Refs. [31,32], supersymmetry holds to a good approximation, even if conformal symmetry is strongly broken by the heavy quark mass.

Note that the $3_C + \bar{3}_C$ diquark structure of the Ξ_{cc}^+ can be written explicitly as $[[dc]c]$ state [33,34]. The production of the double-charm baryon $\Xi_{[dc]c}^+$ with $[dc]$ in a spin-singlet state is natural in the SELEX fixed target experiment since it has acceptance at high x_F , i.e., in the realm of intrinsic charm; the $[dc]c$ configuration can easily re-coalesce from a higher Fock state of the proton such as $|uudc\bar{c}\bar{c}\bar{c}\rangle$. In contrast, the production of this state is likely to be suppressed in $q\bar{q} \rightarrow c\bar{c}c\bar{c}$ or $gg \rightarrow c\bar{c}c\bar{c}$ reactions at the LHCb. Thus LHCb has most likely observed the double-charm baryon state $|u(cc)\rangle$, as will be explained in the next section.¹ The mass difference between the $[[dc]c]$ and the $|u(cc)\rangle$ states is due to the hyperfine interaction between the quarks.

Supersymmetric light front holographic QCD, if extended to the case of two heavy quarks, predicts that the mass of the spin-1/2 baryon should be the same as the mass of $h_c(1P)(3525)$ meson [32]. This is well compatible with the SELEX measurement of $3520.2 \pm 0.7 \text{ MeV}/c^2$ for the $\Xi_{cc}^+(d[cc])$, although the uncertainty of SUSY LFHQCD predictions is at least of the order of 100 MeV. Indeed, the mass of the $|u(cc)\rangle$ state is predicted to be the same as that of the $\chi_{c2}(1P)(3556)$ meson, which is in turn lower than the LHCb result of $3621.40 \pm 0.72(\text{stat}) \pm 0.27(\text{sys}) \pm 0.14(\Lambda_c^+) \text{ MeV}/c^2$ for the Ξ_{cc}^{++} .

Supersymmetric LFHQCD is based on and best tested in the chiral limit of QCD, where all quarks are massless.

The mass difference between the $h_c(1P)$ and the $\chi_{c2}(1P)$ is mainly due to the hyperfine splitting between the two charm quarks, and hence very small. In the baryon there is also the larger spin-spin interaction between the c and a light quark. By comparing hadron masses with light and charmed quarks, one can estimate the strength of this additional, the supersymmetry-breaking interaction in the range $84\text{--}136 \text{ MeV}/c^2$ [32,35], which is well compatible with the mass difference between the SELEX and the LHCb states.

4 The SELEX state at the LHCb

In the previous section we identified the SELEX state as a $[[dc]c]$ state and the LHCb state as a $|u(cc)\rangle$ state of the double charmed baryon. While the SELEX state is definitely a spin-1/2 state, both $J^P = 1/2^+$ and $J^P = 3/2^+$ are possible assignments for the LHCb state. As becomes clear in the following, $J^P = 1/2$ is favored by the LHCb mass measurement and the very suppressed radiative decay to $[[qc]c] + \gamma$. Based on the Heavy Quark Effective Theory (HQET), in Ref. [36] the baryon masses are estimated to be $3610 \text{ MeV}/c^2$ for the spin-1/2 state $|u(cc)\rangle_{1/2^+}$ and $3680 \text{ MeV}/c^2$ for the spin-3/2 state $|u(cc)\rangle_{3/2^+}$. For a qualitative estimate one can also compare with the nucleon states where the lowest mass $I = 1/2, J^P = 3/2^+$ state is the $N(1720)$ which is considerably more massive than the proton.

It is interesting to analyze the ability of the LHCb experiment to observe the $|dcc\rangle$ state, i.e. Ξ_{cc}^+ . Note that in a sample corresponding to 0.65 fb^{-1} of integrated luminosity at 7 TeV, in case of the decay process $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ the LHCb collaboration published an upper limit for the ratio $\sigma(\Xi_{cc}^+) \cdot Br(\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+) / \sigma(\Lambda_c^+)$ of 1.5×10^{-2} and 3.9×10^{-4} for the lifetimes 100 fs and 400 fs, respectively [37]. In case of the decay $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ analyzed in Ref. [22], one expects a larger lifetime, $\tau(\Xi_{cc}^{++}) / \tau(\Xi_{cc}^+) \approx 2.5 - 4$ (cf. Refs. [25,38]). Taking into account that the cuts were optimized for the lifetime of 333 fs and that the minimum lifetime reached by the LHCb is about three times larger than the lifetime $\tau(\Xi_{cc}^+) < 33 \text{ fs}$ measured by the SELEX at 90% confidence level, the LHCb provided an analysis which was outside the signal region for Ξ_{cc}^+ .

It is also of interest to analyze the expectation of the production ratio between states. As we discussed above, the production of the double-charm baryons at the LHCb is due to $q\bar{q} \rightarrow c\bar{c}c\bar{c}$ or $gg \rightarrow c\bar{c}c\bar{c}$ reactions with the following fragmentation of the cc -diquark into the double-charm baryons. Due to the Pauli principle the cc -diquark has to be a state with spin 1 or higher, leading to a state $|q(cc)\rangle$ with higher mass. The normalization of the fragmentation of the cc -diquark into the double-charm baryons is unknown. However, we are still able to provide some quantitative analysis. The fragmentation function is proportional to the wave function at the origin.

¹ We use square brackets [] for spin-0 and round brackets () for spin-1 internal states.

The color-anti-triplet wave function can be estimated on the basis of information about the color-singlet wave function, $|R(0)[cc]_3\rangle \sim |R(0)[c\bar{c}]_1\rangle$. It is clear that the $[[qc]c\rangle$ states cannot be produced through the fragmentation mechanism on the LHCb.

5 Suppression of the radiative decay

An important issue is the rate for the heavier $|u(cc)\rangle$ state to decay radiatively to the spin-1/2 ground state $[[uc]c]_{1/2^+}$. However, LHCb explicitly observed that the state they discovered decays weakly which means that the radiative decay of the LHCb state at 3621 MeV has to be strongly suppressed: The radiative lifetime has to be longer than approximately 50 fs in order that at least some of the LHCb 3621 MeV states would have survived and observed to decay weakly. A lifetime of 50 fs means that the transition rate $\Gamma(3621 \rightarrow 3520 + \gamma)$ has to be less than 0.01 eV. The energy of the photon emitted by the radiative transition is $\omega = 101$ MeV. The dependence of the transition rate on ω comes from (a) the phase space of the final state and (b) the dynamical suppression from the square of the matrix element of the electromagnetic current connecting the initial and final eigenstates.

For comparison, one can consider the measured radiative decay rate for $J/\psi \rightarrow \eta_c \gamma$ with photon energy $\omega = 115$ MeV [39]. The measured radiative decay rate for $J/\psi \rightarrow \eta_c \gamma$ is $\Gamma^{\text{exp}} = 1.13 \pm 0.35$ keV. Note that the spatial wavefunctions of the J/ψ and the η_c are almost identical. In contrast, the initial and final state wavefunctions are very different. In the case of transition between the double-charm baryons $|q(cc)\rangle_{3/2^+} \rightarrow [[qc]c]_{1/2^+} + \gamma$ the amplitude for radiative decay thus involves the matrix element $\langle f | \mathbf{J}_{\text{em}} | i \rangle$ of the electromagnetic current between highly orthogonal hadronic eigenstates. In particular, the emission of the $\omega = 101$ MeV photon also has to interchange one of the charm quarks in the spin-1 (cc) diquark with the light quark q to form the spin-0 $[qc]$ diquark.

The matrix element in the LF framework involves the overlap of the current with nearly orthogonal three-body light-front wavefunctions $\psi_i(x, \mathbf{k}_\perp)$ and $\psi_f(x, \mathbf{k}_\perp)$ at the small 101 MeV momentum transfer. The matrix element thus must vanish as $(\omega r_{cc})^3$, where r_{cc} characterizes the size of the radial wavefunctions. This gives a strong suppression of the rate of $(\omega r_{cc})^6$ relative to that of radiative charmonium decays.

According to Ref. [40], $r_{J/\psi} \sim 0.39$ fm $\simeq 2$ GeV $^{-1}$. As a first estimate, we will assume that the characteristic radial size of the charm diquark cc in the double-charm baryons is the same as that of the J/ψ , $r_{cc} \sim r_{J/\psi}$, resulting in $(\omega r_{cc})^6 \sim 6 \times 10^{-5}$. In comparison with the 1 keV $J/\psi \rightarrow \eta_c \gamma$ decay width, this would give a radiative transition rate for $|q(cc)\rangle_{3/2^+} \rightarrow [[qc]c]_{1/2^+} + \gamma$ of order

0.06 eV and thus a radiative transition lifetime of 10 fs. A radiative lifetime of this order would not prevent the LHCb from observing the weak decay of the $|q(cc)\rangle$ double-charm baryon at 3621 MeV. In this case LHCb might be able to observe some radiative events $[[qc]c]_{1/2^+} + \gamma$, where the $[[qc]c]_{1/2^+}$ at 3520 MeV decays weakly.

It is also possible that the 3621 MeV state observed by LHCb is a $J^P = 1/2^+$ double-charm baryon state $|q(cc)\rangle_{1/2^+}$, rather than $J^P = 3/2^+$ since we have assumed that it is a bound state of a spin-1/2 quark and a spin-1 (cc) diquark. In this case the spin-1 photon needs to be emitted with orbital angular momentum $L = 1$ to conserve parity in the radiative decay of the 3621 MeV state $|q(cc)\rangle_{1/2^+}$ to the 3520 MeV state $[[qc]c]_{1/2^+}$. This would give an additional factor of v^2 in the phase space for the radiative transition rate where $v \simeq \omega/M \sim 1/35$ is the recoil velocity of the 3520 MeV double-charm baryon. This additional suppression of the rate implies that the lifetime of the radiative transition would then be increased to be of order 12000 fs.

We have also done a comparison with the radiative transition rates between the $J^P = 1^-$ charmonium S states analyzed in the classic paper by Feinberg and Sucher [41], such as the estimated 10 keV transition rate between the $\psi(2S)$ (3686 MeV) and the J/ψ (3097 MeV). The transition energy in our case is 101 MeV, compared to the transition energy between the $\psi(2S)$ (3686 MeV) and the J/ψ (3097 MeV) of 589 MeV – a relative reduction in the photon transition energy of $101/589 \sim 1/6$. This factor enters at the third power in the matrix element of the spin current in Eq. (9) of Ref. [41] through the overlap of radial wavefunctions, i.e. the spherical Bessel function $j_0(kr/2)$. In addition, one has to take into account the additional suppression of electromagnetic transitions between the $q(cc)$ and $[qc]c$ configurations when extrapolating to double-charm baryons.

The above discussion is clearly only a first estimate. A rigorous treatment of the transition radiative decay rate between double-charm baryons with diquarks of different spin and composition is clearly necessary.

We also note that LHCb may be able to detect radiative transitions involving double-charm baryons which have higher masses and higher spin. The observation of the 3520 MeV state $[[uc]c]_{1/2^+}$ with a significant transverse momentum kick from photon emission from a heavy double charm state would be an important confirmation of our picture. Because of this, we suggest that LHCb conduct such a search.

Still, there might be intrinsic charm in the wave function at LHC. To analyze this opportunity we will follow a similar discussion given in Ref. [30]. The contribution from the double intrinsic charm should be suppressed due to the kinematics of the LHCb experiment: Making the naive assumption that the momentum is split evenly between all final states and taking into account that the hadron identification efficiency

for pions and kaons is degraded above $100 \text{ GeV}/c$ [42, 43], the analysis loses sensitivity around $p(\Xi_{cc}^{++}) \approx 600 \text{ GeV}/c$, i.e. $x_F \approx 0.15$ and $x_F \approx 0.09$ for the 8 and 13 GeV analysis, respectively. This range of values for x_F corresponds to the rapidity region $2 < y < 5$ in which the LHCb detector operates [44]. Note that in contrast to SELEX, LHCb is a collider experiment where the acceptance excludes the detection of events close to the beam axis.

6 Summary

Using both theoretical and experimental arguments, we have shown that the SELEX and the LHCb results for the production of doubly charmed baryons can both be correct. We have compared the data for double J/ψ production observed by the NA3 experiment and the SELEX result for Ξ_{cc}^+ production at high Feynman- x_F . We have found that the NA3 data strongly complement the SELEX production rate for the spin-1/2 $|[dc]c\rangle$ state. In contrast, LHCb has most likely discovered the heavier $|u(cc)\rangle$ produced by gluon-gluon fusion $gg \rightarrow c\bar{c}c\bar{c}$ at $x_F \sim 0$. The application of supersymmetric algebra to hadron spectroscopy, together with the intrinsic heavy-quark QCD mechanism for the hadroproduction of heavy hadrons at large x_F , can thus resolve the apparent conflict between measurements of double-charm baryons by the SELEX fixed-target experiment and the LHCb experiment at the LHC collider. The mass difference of the two double-charm baryons reflects the distinct spins of the underlying diquarks.

An important conclusion from our study is that the natural kinematic domain for producing novel hadronic bound states, such as multi-heavy quark hadrons and the tetraquarks predicted by superconformal algebra, is large Feynman x_F . In this domain, the constituents of the higher Fock states of the projectile, which are comoving at the same rapidity, can coalesce to produce a wide variety of color-singlet hadrons.

Our paper shows why the state $|[qc]c\rangle_{1/2^+}$ is favorably produced at high x_F , within the kinematics of the SELEX acceptance, and conversely, why its production is unfavorable in the LHC acceptance. There are also very strong upper limits for the production of double-charm baryons in forward photoproduction from duality which explains why the FOCUS experiment does not observe any signal for double charm baryons [45].

These observations indicate the importance of the high x_F domain and fixed-target LHC experiments such as SMOG@LHCb [46, 47] and AFTER@LHC [48, 49] for observing the hadroproduction of exotic heavy quark states. As explained above, one also can understand why the radiative transition $3621|(cc)q\rangle \rightarrow 3520|[qc]c\rangle + \gamma$ is strongly suppressed due to the very small overlap of their respective radial wavefunctions.

The SELEX experiment measured the lifetime $\tau(\Xi_{cc}^+) < 33 \text{ fs}$ at 90% confidence level. Using the ratio $\tau(\Xi_{cc}^{++})/\tau(\Xi_{cc}^+) \approx 2.5-4$ we can assume $\tau(\Xi_{cc}^{++}) \approx 100 \text{ fs}$. This value stands in contrast to the minimum values theoretically predicted as $\tau(\Xi_{cc}^+) \approx 53 \text{ fs}$ and $\tau(\Xi_{cc}^{++}) \approx 185 \text{ fs}$ [38]. Therefore, definitive measurements of the lifetime will provide another test for the SELEX data.

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