

Holographic three flavor baryon in the Witten–Sakai–Sugimoto model with the D0–D4 background

Wenhe Cai^{1,a} , Si-wen Li^{2,b} 

¹ Department of Physics, Shanghai University, Shanghai 200444, China

² Department of Physics, Center for Field Theory and Particle Physics, Fudan University, Shanghai 200433, China

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Abstract With the construction of the Witten–Sakai–Sugimoto model in the D0–D4 background, we systematically investigate the holographic baryon spectrum in the case of three flavors. The background geometry in this model is holographically dual to $U(N_c)$ Yang–Mills theory in large N_c limit involving an excited state with a nonzero θ angle or glue condensate $\langle \text{Tr} \mathcal{F} \wedge \mathcal{F} \rangle = 8\pi^2 N_c \tilde{\kappa}$, which is proportional to the charge density of the smeared D0-branes through a parameter b or $\tilde{\kappa}$. The classical solution of baryon in this model can be modified by embedding the Belavin–Polyakov–Schwarz–Tyupkin instanton and we carry out the quantization of the collective modes with this solution. Then we extend the analysis to include the heavy flavor and find that the heavy meson is always bound in the form of the zero mode of the flavor instanton in strong coupling limit. The mass spectrum of heavy-light baryons in the situation with single- and double-heavy baryon is derived by solving the eigen equation of the quantized collective Hamiltonian. Afterwards we obtain that the constraint of stable baryon states has to be $1 < b < 3$ and the difference in the baryon spectrum becomes smaller as the D0 charge increases. It indicates that quarks or mesons can not form stable baryons if the θ angle or glue condensate is sufficiently large. Our work is an extension of the previous study of this model and also agrees with those conclusions.

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1 Introduction

In QCD, it is well-known that the spontaneous breaking of chiral symmetry dominates the light quark sector (u, d, s) while the heavy quark (c, b, t) is characterized by the heavy-quark symmetry [1–3]. As measured by [4,5], the origin of these symmetries relates to the chiral doubling in heavy-light mesons [6–9]. Recently, flurry of experiments report some new physics involving heavy-light multiquark states [10–19] which might be a priori outside many classifications the of quark model. Thus it would be urgent to formulate a non-perturbative model of QCD that includes both chiral and heavy quark symmetry. On the other hand, there have also been many researches about the spontaneous parity violation in QCD with the running of the RHIC in recent years [20–24]. People usually use a nonzero θ angle in the action to theoretically describe the P or CP violation. Accordingly, metastable state with nonzero θ angle might probably be created in the hot and dense situation in RHIC when the decon-

^a e-mail: whcai@shu.edu.cn

^b e-mail: siwenli@fudan.edu.cn

finement transition happens in QCD. After a very short time, these bubble forms with odd P or CP parity would decay into the true vacuum soon.¹ For comprehensive reviews, we refer the readers to [26–28] which is a proposal about the chiral magnetic effect (CME) as a test of such phenomena about P or CP violation in hot QCD. In this sense, the θ dependence of some observables in QCD or in the gauge theory would be theoretically interesting at least. So several present works have shown some properties of the θ dependence in the gauge field theory e.g. θ dependence of deconfinement transition [29,30], θ dependence in the spectrum of the glueball [31], θ dependence in the large N limit [32] and we strongly recommend [33] as an excellent review about the θ dependence in the gauge field theory.

Since QCD in the low energy region becomes non-perturbative, the framework based on the holographic construction by the gauge/gravity duality provides an approach to investigate the aspects of the strongly coupled gauge theory [34,35]. Among various approaches to the holographic duality of large N_c QCD, a top-down model proposed by Sakai and Sugimoto [37,38] as an extension of Witten's work [39] (WSS model) is the most successful one since it almost contains all necessary ingredients of QCD, e.g. baryons [40,41], chiral/deconfinement transitions [42–45], glueball spectrum and its interaction [46–50]. Specifically, flavors are introduced by embedding a stack of N_f pairs of D8 and anti D8-branes (D8/ $\overline{\text{D8}}$ -branes) as probes into the bubble geometry produced by N_c D4-branes, so this model describes the N_f massless quarks by constructing the configuration of N_c D4- and N_f D8/ $\overline{\text{D8}}$ -branes in type IIA string theory. The N_f chiral quark states in the fundamental representation are identified as the massless spectrum of the open strings stretched between the N_c D4- and N_f D8/ $\overline{\text{D8}}$ -branes. Particularly, there is a naturally geometrical description of the spontaneous breaking of the chiral symmetry in this model since the flavor branes are connected at the bottom of the bubble as illustrated in Fig. 1. So the separated D8/ $\overline{\text{D8}}$ -branes far away combining near the bottom of the bubble can be interpreted as the spontaneously breaking of $U_R(N_f) \times U_L(N_f)$ symmetry to $U_V(N_f)$ in the dual field theory. Additionally, baryon in this model could be identified as a D4-brane warped on S^4 , namely “baryon vertex”.² And it can be effectively described by the instanton configuration of the gauge field on the flavor branes [51]. Then in order to involve the topological θ term in the dual theory, it has been recognized that the instantonic D-brane (D-instanton) relates to the θ angle in holography by the construction of the string theory [52]. Hence the θ dependence could be introduced to the WSS model in this way, that is adding the D-instanton (D0-branes) to the N_c D4-

brane background geometry as in [53,54]. The systematical study of the WSS model in the D0–D4 brane background (i.e. D0–D4/D8 brane system) can be reviewed in [55,56] and we can accordingly investigate the properties of the θ dependence in QCD or Yang–Mills theory holographically e.g. [57–60].³

The purpose of this paper is to study the three-flavor baryon spectrum with θ dependence or glue condensation in a holographic approach i.e. using the WSS model in the D0–D4 brane background. Notice that we have studied the $N_f = 2$ two-flavor case with this model in [57,65], so it would be natural to extend the present analysis to the case of three flavors $N_f = 3$. The main content of this manuscript consists of two parts. In the first part, through the analysis in [66], we search for a instanton solution for the flavored gauge field in the situation with $N_f = 3$ by including the D0-branes. Then following [57] and employing the soliton picture, we derive the effective Hamiltonian for the collective modes of baryon. After quantization, the baryon spectrum can be obtained and all the calculations are done in the strong coupling limit. In the second part, we extend our analysis to involve the heavy flavor in the baryon spectrum. The heavy flavor could be introduced into this model by embedding one pair of probe flavor branes (named as “heavy flavor brane”) separated from the other N_f coincident flavor branes (named as “light flavor brane”) with a heavy-light string (HL-string) stretched between them [65,67–70] (as shown in Fig. 2). The low energy modes of HL-strings could be identified as the heavy-light mesons and they could be approximated in the bi-fundamental representation by the local vector fields in the vicinity of the light flavor brane. Due to the finite separation of the heavy and light flavor branes, the HL-string has nonzero vacuum expectation value (vev) and the heavy-light fields therefore get mass by the moduli span of the dilaton in the action. As a result, we can evaluate the effective action involving the heavy flavor from the dynamics of the light flavor brane since the baryon vertex lives inside the light flavor brane. And this setup also develops the approach of bound state in the context of the Skyrme model (e.g. [71]) in holography.

The outline of this paper is as follows. In Sect. 2, we review the D0–D4 system and its dual theory. In Sect. 3, we search for a instanton solution for the gauge fields on the embedded flavor branes in the case of three flavors. Afterwards the classical mass of the soliton is evaluated with the instanton solution. In Sect. 4, the collective modes and their quantization are systematically investigated and the baryon spectrum is then obtained by solving the eigen equation of the collective Hamiltonian. In Sect. 5, we start to consider the heavy

¹ The recently holographic approach [25] also supports this statement.

² To distinguish the N_c D4-branes, we use “D4'-brane” to denote a baryon vertex from now on.

³ There are also some other applications of the D0–D4 background in holography e.g. [61–64] where the setup may be a little different from the model we introduced in this paper.

flavor additional to the light flavor baryon spectrum. This section includes the effective action, quantization and single/double baryon spectra with the heavy flavor. In the Sect. 6, we briefly use our baryon spectra to fit the experimental data and give some discussion.

2 The D0–D4 background and the dual field theory

In this section, we will briefly review the D0–D4 background and its dual field theory by following [55,56]. In Einstein frame, the IIA supergravity solution of black coincident N_c D4-branes with N_0 smeared D0-branes is given as,

$$ds^2 = H_4^{-\frac{3}{8}} \left[-H_0^{\frac{7}{8}} f(U) d\tau^2 + H_0^{\frac{1}{8}} \delta_{\mu\nu} dx^\mu dx^\nu \right] + H_4^{\frac{5}{8}} H_0^{\frac{1}{8}} \left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right]. \tag{2.1}$$

Here τ represents the compactified direction on a cycle with the period β . Respectively the dilaton, Ramond–Ramond 2- and 4-form are given as,

$$e^{-\Phi} = g_s^{-1} \left(\frac{H_4}{H_0^3} \right)^{\frac{1}{4}}, \quad f_2 = \frac{(2\pi l_s)^7 g_s N_0}{\omega_4 V_4} \frac{1}{U^4 H_0^2} dU \wedge d\tau, \quad f_4 = \frac{(2\pi l_s)^3 N_c g_s}{\omega_4} \epsilon_4, \tag{2.2}$$

where g_s denotes the string coupling and,

$$H_4 = 1 + \frac{U_{Q4}^3}{U^3}, \quad H_0 = 1 + \frac{U_{Q0}^3}{U^3}, \quad f(U) = 1 - \frac{U_{KK}^3}{U^3}, \quad U_{Q0}^3 = \frac{1}{2} \left(-U_{KK}^3 + \sqrt{U_{KK}^6 + ((2\pi)^5 l_s^7 g_s \tilde{\kappa} N_c)^2} \right), \quad U_{Q4}^3 = \frac{1}{2} \left(-U_{KK}^3 + \sqrt{U_{KK}^6 + ((2\pi)^5 l_s^7 g_s N_c)^2} \right). \tag{2.3}$$

We use $d\Omega_4$, ϵ_4 and $\omega_4 = 8\pi^2/3$ to represent the line element, the volume form and the volume of a unit S^4 . V_4 represents the volume of the D4-branes and U_{KK} is the horizon of the radius coordinate. N_c, N_0 denotes the numbers of D4- and D0-branes respectively and D0-branes are smeared in the directions of x^0, x^1, x^2, x^3 as shown in Table 1. So the number density of the D0-branes can be defined as N_0/V_4 . According to [53], we have required that N_0 is order of N_c in order to take account of the full backreaction from the D0-branes. Therefore $\tilde{\kappa}$ would be order of $\mathcal{O}(1)$ in the large N_c limit which is defined as $\tilde{\kappa} = N_0/(N_c V_4)$.

In the string frame, interchanging x^0, τ and taking the near horizon limit $\alpha' \rightarrow 0$ with fixed U/α' and U_{KK}/α' , we could obtain the D0–D4 bubble geometry which is,

$$ds^2 = \left(\frac{U}{R} \right)^{3/2} \left[H_0^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H_0^{-1/2} f(U) d\tau^2 \right] + H_0^{1/2} \left(\frac{R}{U} \right)^{3/2} \left[\frac{dU^2}{f(U)} + U^2 d\Omega_4^2 \right], \tag{2.4}$$

and the dilaton becomes,

$$e^\Phi = g_s \left(\frac{U}{R} \right)^{3/4} H_0^{3/4}, \tag{2.5}$$

where $\alpha' = l_s^2, R^3 = \pi g_s l_s^3 N_c$ is the limit of U_{Q4}^3 and l_s represents the length of the string. The spacetime ends at $U = U_{KK}$ in the bubble geometry (2.4) as shown in Fig. 1. The period β of τ must satisfy the following relation in order to avoid the conical singularity at U_{KK} , which is,

$$\beta = \frac{4\pi}{3} U_{KK}^{-1/2} R^{3/2} b^{1/2}, \quad b \equiv H_0(U_{KK}). \tag{2.6}$$

In the low-energy effective description, the dual theory is a five-dimensional $U(N_c)$ Yang–Mills theory which lives inside the worldvolume of D4-brane. Since one direction of the D4-brane is compactified on a cycle τ , the four-dimensional Yang–Mills coupling could be obtained as studied in [39], which is relating the D4-brane tension and the five-dimensional Yang–Mills coupling constant g_5 , then analyzing the relation of the compactified five-dimensional theory and the four dimensions on the τ direction. Thus the resultant four-dimensional Yang–Mills coupling is,

$$g_{YM}^2 = \frac{g_s^2}{\beta} = \frac{4\pi^2 g_s l_s}{\beta}, \tag{2.7}$$

b and R^3 can be accordingly evaluated as,

$$b = \frac{1}{2} \left[1 + \left(1 + C\beta^2 \right)^{1/2} \right], \quad C \equiv \left(2\pi l_s^2 \right)^6 \lambda^2 \tilde{\kappa} / U_{KK}^6, \quad R^3 = \frac{\beta \lambda l_s^2}{4\pi}, \tag{2.8}$$

where the 't Hooft coupling λ is defined as $\lambda = g_{YM}^2 N_c$. Hence it is natural to define a mass scale as $M_{KK} = 2\pi/\beta$ so that the Kaluza–Klein (KK) modes can be introduced. In order to break the supersymmetry in the low-energy theory, the anti-periodic condition has to be imposed on the fermions as in [37], thus the scalar and fermion become massive below the KK mass scale. Consequently, the massless modes of the open strings on the D4-branes, which is described by a pure Yang–Mills theory, dominate the dynamics in dual field theory. According to (2.6) and (2.8), we can obtain the following relations,

$$\beta = \frac{4\pi \lambda l_s^2}{9U_{KK}} b, \quad M_{KK} = \frac{9U_{KK}}{2\lambda l_s^2 b}. \tag{2.9}$$

Due to $b \geq 1$ and $U_{KK} \geq 2\lambda l_s^2 M_{KK}/9$, β can be solved by using (2.8) and (2.9),

$$\beta = \frac{4\pi\lambda l_s^2}{9U_{KK}} \frac{1}{1 - \frac{(2\pi l_s^2)^8}{81U_{KK}^8} \lambda^4 \tilde{\kappa}^2}, \quad b = \frac{1}{1 - \frac{(2\pi l_s^2)^8}{81U_{KK}^8} \lambda^4 \tilde{\kappa}^2}. \tag{2.10}$$

In the presence of the smeared D0-branes, let us consider a probe D4-brane whose effective action takes the following form,

$$S_{D_4} = -\mu_4 \text{Tr} \int d^4x d\tau e^{-\Phi} \sqrt{-\det(G + 2\pi\alpha' \mathcal{F})} + \mu_4 \int C_5 + \frac{1}{2} (2\pi\alpha')^2 \mu_4 \int C_1 \wedge \mathcal{F} \wedge \mathcal{F}, \tag{2.11}$$

where $\mu_4 = (2\pi)^{-4} l_s^{-5}$, and G is the induced metric on the world volume of the D4-brane. \mathcal{F} represents the gauge field strength on the D4-brane. We have used C_5 , C_1 to denote the Ramond–Ramond 5- and 1- form respectively and their field strengths are given in (2.2). Notice that the leading-order expansion of the first term in (2.11) with respect to sufficiently small \mathcal{F} forms the Yang–Mills action. In the bubble D0–D4 solution, we have $C_1 \sim \theta d\tau$ according to (2.2), thus D0-branes are indeed D-instantons (as shown in Table 1), so the last term in (2.11) could be integrated out as,⁴

$$\int_{S^1_\tau} C_1 \sim \theta \sim \tilde{\kappa}, \quad \int_{S^1_\tau \times \mathbb{R}^4} C_1 \wedge \mathcal{F} \wedge \mathcal{F} \sim \theta \int_{\mathbb{R}^4} \mathcal{F} \wedge \mathcal{F}. \tag{2.12}$$

A free parameter $\tilde{\kappa}$ (related to the θ angle in the dual field theory) has been introduced into the Witten–Sakai–Sugimoto model by this string theory background, hence this background is not dual to the vacuum state of the gauge field theory. As studied in [54–56], it implies some excited states in the dual field theory with a constant homogeneous field strength background may be described in the D0–D4 model, or equivalently, the dual field theory behaves like θ -dependent Yang–Mills (YM) theory. By following [55–57], we can evaluate the expectation value of $\text{Tr} \mathcal{F} \wedge \mathcal{F}$ as $\langle \text{Tr} \mathcal{F} \wedge \mathcal{F} \rangle = 8\pi^2 N_c \tilde{\kappa}$. So the deformed relations to the QCD variables in the presence of D0-branes are given as,

$$R^3 = \frac{\lambda l_s^2}{2M_{KK}}, \quad g_s = \frac{\lambda}{2\pi M_{KK} N_c l_s}, \quad U_{KK} = \frac{2}{9} M_{KK} \lambda l_s^2 b. \tag{2.13}$$

⁴ The explicit derivation of the θ angle and glue condensate in the D0–D4 background can be respectively reviewed in [54] and [55]. One can find that θ angle and glue condensate have a same origin in this holographic system although they are different physical concepts.

Table 1 The D-brane configurations: “=” denotes the smeared directions, “-” denotes the world volume directions.

	0	1	2	3	4(τ)	5(U)	6	7	8	9
N_0 smeared D0-branes	=	=	=	=	-					
N_c D4-branes	-	-	-	-	-					
N_f flavor branes D8/ $\overline{\text{D8}}$	-	-	-	-	-	-	-	-	-	-
Baryon vertex D4'-brane	-						-	-	-	-

3 The effective action from D-brane

3.1 D-brane setup

The chiral symmetry in the D0–D4 system is $U_R(N_f) \times U_L(N_f)$ which can be introduced by adding a stack of probe N_f D8-anti-D8 (D8/ $\overline{\text{D8}}$) branes. Usually they are named as flavor branes. The separated D8/ $\overline{\text{D8}}$ -branes far away combining near the bottom $U = U_{KK}$ can be geometrically interpreted as the spontaneously breaking of $U_R(N_f) \times U_L(N_f)$ symmetry to $U_V(N_f)$ in the dual field theory. This can be verified by the appearance of massless Goldstones [72]. The brane configurations are illustrated in Table 1.

Accordingly, the induced metric on the probe D8/ $\overline{\text{D8}}$ -branes is,

$$ds_{\text{D8}/\overline{\text{D8}}}^2 = \left(\frac{U}{R}\right)^{3/2} H_0^{-1/2} \left[f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0}{f(U)} U'^2 \right] d\tau^2 + \left(\frac{U}{R}\right)^{3/2} H_0^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + H_0^{1/2} \left(\frac{R}{U}\right)^{3/2} U'^2 d\Omega_4^2. \tag{3.1}$$

where U' denotes the derivative with respect to τ . The action of the D8/ $\overline{\text{D8}}$ -branes can be obtained as,

$$S_{\text{D8}/\overline{\text{D8}}} \propto \int d^4x dU H_0(U) U^4 \left[f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0}{f(U)} U'^2 \right]^{1/2}, \tag{3.2}$$

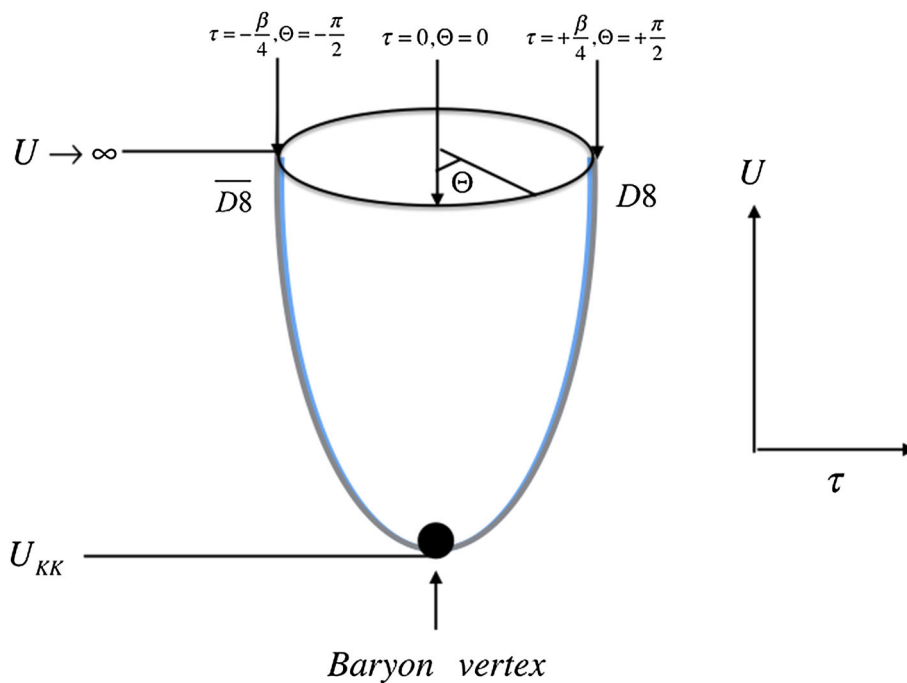
then the equation of motion for $U(\tau)$ can be derived as,

$$\frac{d}{d\tau} \left(\frac{H_0(U) U^4 f(U)}{\left[f(U) + \left(\frac{R}{U}\right)^3 \frac{H_0}{f(U)} U'^2 \right]^{1/2}} \right) = 0, \tag{3.3}$$

which can be interpreted as the conservation of the energy. With the initial conditions $U(\tau = 0) = U_0$ and $U'(\tau = 0) = 0$, the generic formula of the embedding function $\tau(U)$ can be solved as,

$$\tau(U) = E(U_0) \int_{U_0}^U dU \frac{H_0^{1/2}(U) \left(\frac{R}{U}\right)^{3/2}}{f(U) [H_0^2(U) U^8 f(U) - E^2(U_0)]^{1/2}}, \tag{3.4}$$

Fig. 1 The bubble configuration of the D0–D4 geometry in the $\tau - U$ plane where τ is compactified on S^1 . This background (cigar) is produced by N_c D4-branes with N_0 smeared D0-branes. The flavor D8/ $\overline{D8}$ -branes live at antipodal position of the cigar represented by the blue line. Particularly we will consider the case of $N_f = 3$ in this paper. The baryon spectrum is created by the baryon vertex (D4'-brane) living inside the flavor branes.



where $E(U_0) = H_0(U_0)U_0^4 f^{1/2}(U_0)$ and U_0 denotes the connected position of the D8/ $\overline{D8}$ -branes. Following [37, 55], we introduce the new coordinates (r, Θ) and (y, z) which satisfy,

$$y = r \cos \Theta, z = r \sin \Theta, \\ U^3 = U_{KK}^3 + U_{KK}r^2, \quad \Theta = \frac{2\pi}{\beta}\tau = \frac{3}{2} \frac{U_{KK}^{1/2}}{R^{3/2}H_0^{1/2}(U_{KK})}. \tag{3.5}$$

In this model, the probe D8/ $\overline{D8}$ -branes are embedded at $\Theta = \pm \frac{1}{2}\pi$ respectively i.e. $y = 0$ as illustrated in Fig. 1. Hence the embedding function of the flavor branes is $\tau(U) = \frac{1}{4}\beta$ so that we have $U^3 = U_{KK}^3 + U_{KK}z^2$ on the D8/ $\overline{D8}$ -branes.⁵ Therefore the induced metric on the flavor branes becomes,

$$ds_{D8/\overline{D8}}^2 = H_0^{1/2} \left(\frac{U}{R}\right)^{3/2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{4}{9} \frac{U_{KK}}{U} \left(\frac{U}{R}\right)^{3/2} H_0^{1/2} dz^2 + H_0^{1/2} \left(\frac{R}{U}\right)^{3/2} U^2 d\Omega_4^2. \tag{3.6}$$

With the approach presented in [51, 73], the baryon spectrum with two flavors in this system has been studied in [57–59, 61, 62]. Therefore it would be natural to extend the analysis into the three-flavor case (i.e. $N_f = 3$) in this paper.

⁵ With the boundary condition as discussed in [37, 55], $\tau(U) = \frac{1}{4}\beta$ is indeed a solution of (3.3).

3.2 Yang–Mills and Chern–Simons action with generic flavors

Since the baryon vertex lives inside the flavor branes, the concern of this section is to study the effective dynamics of the baryons on the D8/ $\overline{D8}$ -branes. So let us consider the effective theory of N_f probe D8/ $\overline{D8}$ -branes in the background (2.4). The action of the flavor branes contains two terms which are Yang–Mills (YM) and Chern–Simons (CS) action. Respectively they are,

$$S_{YM}^{D8/\overline{D8}} = -2\tilde{T}U_{KK}^{-1} \int d^4x dz H_0^{1/2} \text{Tr} \\ \times \left[\frac{1}{4} \frac{R^3}{U} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + \frac{9}{8} \frac{U^3}{U_{KK}} \mathcal{F}_{\mu z} \mathcal{F}^{\mu z} \right], \\ S_{CS}^{D8/\overline{D8}} = \frac{N_c}{24\pi^2} \text{Tr} \int_{\mathbb{R}^{4+1}} \omega_5(\mathcal{A}) + S_{C_7}^{D8/\overline{D8}}, \tag{3.7}$$

where,⁶

$$\tilde{T} = \frac{(2\pi\alpha')^2}{3g_s} T_8 \omega_4 U_{KK}^{3/2} R^{3/2} = \frac{M_{KK}^2 \lambda N_c b^{3/2}}{486\pi^3}, \\ \omega_5(\mathcal{A}) = \mathcal{A}^2 - \frac{1}{2} \mathcal{A}^3 \mathcal{F} + \frac{1}{10} \mathcal{A}^5, \\ S_{C_7}^{D8/\overline{D8}} = 2\pi\alpha' \mu_8 \int_{D8/\overline{D8}} C_7 \wedge \text{Tr}[\mathcal{F}]. \tag{3.8}$$

Here \mathcal{A} is $U(N_f)$ gauge field and \mathcal{F} is its field strength. We can decompose the $U(N_f)$ gauge field \mathcal{A} and its gauge field

⁶ T_8 is the tension of the D8-branes.

\mathcal{F} strength into $U(1)$ part \hat{A} , \hat{F} and $SU(N_f)$ part A , F as,

$$A = \mathcal{A}_\mu dx^\mu + \mathcal{A}_z dz = A + \frac{1}{\sqrt{2N_f}} \hat{A} = A^s T_s + \frac{1}{\sqrt{2N_f}} \hat{A},$$

$$\mathcal{F} = dA + iA \wedge A = F + \frac{1}{\sqrt{2N_f}} \hat{F}, \tag{3.9}$$

where the index μ runs from 0 - 3 and T_s are hermitian generators of $SU(N_f)$ with the index $s = 1, 2, \dots, N_f^2 - 1$. In order to simplify the calculations, we can further define the dimensionless variables by the following replacement,

$$(x^\mu, z) \rightarrow (x^\mu/M_{KK}, zU_{KK}), (\mathcal{A}_\mu, \mathcal{A}_z) \rightarrow (\mathcal{A}_\mu U_{KK}, \mathcal{A}_z/M_{KK}). \tag{3.10}$$

On the other hand, in the strongly coupled limit, we also need to obtain the explicit expression with λ by the rescaling in [51, 57, 66] which is,

$$(x^0, x^M) \rightarrow (x^0, \lambda^{1/2} x^M),$$

$$(\mathcal{A}_0, \mathcal{A}_M) \rightarrow (\mathcal{A}_0, \lambda^{-1/2} \mathcal{A}_M),$$

$$(\mathcal{F}_{MN}, \mathcal{F}_{0M}) \rightarrow (\lambda^{-1} \mathcal{F}_{MN}, \lambda^{-1/2} \mathcal{F}_{0M}), \tag{3.11}$$

where the index M, N runs over 1, 2, 3, z and the $\mathcal{O}(\lambda^{-1})$ terms would be neglected as in [51, 57, 66]. So when (3.9) (3.10) and (3.11) is imposed, the action $S_{\text{YM}}^{\text{D8}/\overline{\text{D8}}}$ can be expanded as,

$$S_{\text{YM}}^{\text{D8}/\overline{\text{D8}}} = -aN_c b^{3/2} \int d^4 x dz \left[\frac{\lambda}{4} (F_{MN}^a)^2 - \frac{bz^2}{2} \left(\frac{5}{12} - \frac{1}{4b} \right) (F_{ij}^a)^2 + \frac{bz^2}{4} \left(1 + \frac{1}{b} \right) (F_{iz}^a)^2 - (F_{0M}^a)^2 + \frac{\lambda}{4} \hat{F}_{MN}^2 - \frac{bz^2}{2} \left(\frac{5}{12} - \frac{1}{4b} \right) \hat{F}_{ij}^2 + \frac{bz^2}{4} \left(1 + \frac{1}{b} \right) \hat{F}_{iz}^2 - \frac{1}{2} \hat{F}_{0M}^2 \right] + \mathcal{O}(\lambda^{-1}), \tag{3.12}$$

with the index $i, j = 1, 2, 3$.

Notice that the CS term $S_{C_7}^{\text{D8}/\overline{\text{D8}}}$ in (3.7) becomes $\mathcal{O}(\lambda^{-1})$ in the strongly coupled limit. Thus by using (3.9) (3.10) and

(3.11), we can obtain the explicit formula of the CS term (3.7) which is,

$$S_{\text{CS}}^{\text{D8}/\overline{\text{D8}}} = \frac{N_c}{24\pi^2} \int \omega_5^{SU(N_f)}(A) + \frac{N_c}{24\pi^2} \sqrt{\frac{2}{N_f}} \epsilon_{MNPQ} \int d^4 x dz \left[\frac{3}{8} \hat{A}_0 \text{Tr}(F_{MN} F_{PQ}) - \frac{3}{2} \hat{A}_M \text{Tr}(\partial_0 A_N F_{PQ}) + \frac{3}{4} \hat{F}_{MN} \text{Tr}(A_0 F_{PQ}) + \frac{1}{16} \hat{A}_0 \hat{F}_{MN} \hat{F}_{PQ} - \frac{1}{4} \hat{A}_M \hat{F}_{MN} \hat{F}_{PQ} + \text{total derivatives} \right] + \mathcal{O}(\lambda^{-1}), \tag{3.13}$$

with the index $M, N, P, Q = 1, 2, 3, z$ and $\epsilon_{123z} = 1$.

3.3 Classical soliton solution representing a baryon

With the action (3.12) (3.13), we can obtain the equations of motion (EOM) up to $\mathcal{O}(\lambda^{-1})$ for the gauge fields as,

$$D_M F_{0M} + \frac{1}{64\pi^2 a b^{3/2}} \sqrt{\frac{2}{N_f}} \epsilon_{MNPQ} \hat{F}_{MN} F_{PQ} + \frac{1}{64\pi^2 a b^{3/2}} \epsilon_{MNPQ} \left\{ F_{MN} F_{PQ} - \frac{1}{N_f} \text{Tr}(F_{MN} F_{PQ}) \right\} + \mathcal{O}(\lambda^{-1}) = 0,$$

$$D_N F_{MN} + \mathcal{O}(\lambda^{-1}) = 0,$$

$$\partial_M \hat{F}_{0M} + \frac{1}{64\pi^2 a b^{3/2}} \sqrt{\frac{2}{N_f}} \epsilon_{MNPQ} \left\{ \text{Tr}(F_{MN} F_{PQ}) + \frac{1}{2} \hat{F}_{MN} \hat{F}_{PQ} \right\} + \mathcal{O}(\lambda^{-1}) = 0,$$

$$\partial_N \hat{F}_{MN} + \mathcal{O}(\lambda^{-1}) = 0. \tag{3.14}$$

The first two equations in (3.14) are the EOMs for $SU(N_f)$ part while the last two equations are EOMs for the $U(1)$ part. Then let us find a static soliton solution of the EOMs in (3.14) which could be able to represent a baryon. Since the baryon can be identified as a $D4'$ -brane wrapped on S^4 which is equivalent to instanton configuration on the $D8/\overline{D8}$ -branes, the solution of leading part $D_N F_{MN} = 0$ carrying a unit baryon number could be obtained by the embedding of $SU(2)$ Belavin–Polyakov–Schwarz–Tyupkin (BPST) instanton solution in the flat space to $SU(N_f)$:

$$A_M^{\text{cl}}(x) = -if(x) g(x) \partial_M g(x)^{-1}, \tag{3.15}$$

where the function $f(x)$ and $g(x)$ are given as,

$$\begin{aligned}
 f(x) &= \frac{x^2}{x^2 + \rho^2}, \\
 x &= \sqrt{(x^M - X^M)^2}, \\
 g(x) &= \begin{pmatrix} g^{SU(2)}(x) & 0 \\ 0 & \mathbf{1}_{N_f-2} \end{pmatrix}, \\
 g^{SU(2)}(x) &= \frac{1}{x} \left[(z - Z) \mathbf{1}_2 + i(x^i - X^i) \sigma_i \right]. \tag{3.16}
 \end{aligned}$$

Here we have used $\mathbf{1}_{N_f}$, σ_i ($i = 1, 2, 3$) to represent the $N_f \times N_f$ identity matrix and three Pauli matrices respectively. The constants $X^M = \{X^i, Z\}$ denotes the position of the instanton and ρ represents its size. Notice that these constants have also been rescaled as in (3.10) (3.11). The field strengths of (3.15) are given as,

$$F_{ij}^{cl} = \frac{4\rho^2}{(x^2 + \rho^2)^2} \epsilon_{ijk} t_k, \quad F_{iz}^{cl} = \frac{4\rho^2}{(x^2 + \rho^2)^2} t_i, \tag{3.17}$$

where $t_i = \begin{pmatrix} \sigma_i & 0 \\ 0 & 0 \end{pmatrix}$ is the $SU(N_f)$ embedding of σ_i . Then the EOM of the $U(1)$ field \hat{A}_M gives the solution $\hat{A}_M = 0$ up to a gauge transformation. So the EOM for \hat{A}_0 becomes,

$$\partial_M^2 \hat{A}_0 + \sqrt{\frac{2}{N_f}} \frac{3\rho^4}{ab^{3/2}\pi^2 (x^2 + \rho^2)^4} = 0, \tag{3.18}$$

then we have its solution as,

$$\hat{A}_0^{cl} = \sqrt{\frac{2}{N_f}} \frac{1}{8ab^{3/2}\pi^2} \frac{1}{x^2} \left[1 - \frac{\rho^4}{(x^2 + \rho^2)^2} \right]. \tag{3.19}$$

The present solution \hat{A}_0^{cl} is regular at $x = 0$ and vanished at the infinity $x \rightarrow \infty$. Finally, let us solve the EOM for A_0 to find a solution. For a generic N_f , plugging (3.15) and (3.19) into the first equation in (3.14), we have,

$$D_M^2 A_0 + \frac{3}{2ab^{3/2}\pi^2} \frac{\rho^4}{(x^2 + \rho^2)^2} \begin{pmatrix} \mathbf{1}_2 - \frac{2}{N_f} \mathbf{1}_2 & 0 \\ 0 & -\frac{2}{N_f} \end{pmatrix} = 0. \tag{3.20}$$

Then A_0 can be accordingly solved as,

$$A_0^{cl} = \frac{1}{16ab^{3/2}\pi^2} \frac{1}{x^2} \left[1 - \frac{\rho^4}{(x^2 + \rho^2)^2} \right] \begin{pmatrix} \mathbf{1}_2 - \frac{2}{N_f} \mathbf{1}_2 & 0 \\ 0 & -\frac{2}{N_f} \end{pmatrix}. \tag{3.21}$$

So the mass M of the static soliton solution can be evaluated by the relation $S_{\text{onshell}} = -\int dt M$ with the solution (3.15) (3.19) and (3.21). It gives,

$$\begin{aligned}
 M &= aN_c b^{3/2} \int d^4 x dz \left[\frac{1}{4} (F_{MN}^a)^2 - \frac{bz^2}{2} \left(\frac{5}{12} - \frac{1}{4b} \right) (F_{ij}^a)^2 \lambda^{-1} \right. \\
 &\quad \left. + \frac{bz^2}{4} \left(1 + \frac{1}{b} \right) (F_{iz}^a)^2 \lambda^{-1} - (F_{0M}^a)^2 \lambda^{-1} - \frac{\lambda^{-1}}{2} \hat{F}_{0M}^2 \right] \\
 &\quad - \frac{N_c}{24\pi^2} \epsilon_{MNPQ} \int d^4 x dz \left[\sqrt{\frac{2}{N_f}} \frac{3}{8} \hat{A}_0 \text{Tr}(F_{MN} F_{PQ}) \right. \\
 &\quad \left. + \frac{3}{4} \text{Tr}(A_0 F_{MN} F_{PQ}) \right] + \mathcal{O}(\lambda^{-1}) \\
 &= 8ab^{3/2}\pi^2 N_c \lambda + 8ab^{3/2}\pi^2 N_c \left[\frac{3-b}{12} (2Z^2 + \rho^2) \right. \\
 &\quad \left. + \frac{1}{320ab^3\pi^4\rho^2} \right] + \mathcal{O}(\lambda^{-1}). \tag{3.22}
 \end{aligned}$$

For the stable solution, we have to minimize M in order to determine the values of ρ and Z . They are,

$$\rho_{min}^2 = \frac{\sqrt{15}}{20} \frac{1}{\sqrt{3-b}} \frac{1}{ab^{3/2}\pi^2}, \quad Z_{min} = 0. \tag{3.23}$$

So the mass formula (3.22) and size of the instanton (3.23) are all independent of N_f while they depends on b which relates to the number density of D0-branes. We can further rewrite the expression in terms of the original parameters which means ρ has to be rescaled as $\rho \rightarrow \lambda^{1/2}\rho$, hence the minimum value of the mass of the soliton is given by,

$$M_{min} = 8ab^{3/2}\pi^2 N_c + \sqrt{\frac{3-b}{15}} N_c. \tag{3.24}$$

Note that the size ρ of the soliton becomes parametrically small for large λ with respect to the scale of the curvature. Hence it implies the soliton energy density is concentrated in a small region of the space which motivates our approach of the BPST instanton solution. Although this is an approximation based on the assumption that the CS term and the curvature (which are crucial in determining the size ρ of the soliton) do not alter the soliton field significantly, the numerical computation in [74] strongly supports its validity. Therefore the self-dual instanton configuration would be a good approximation to describe the holographic baryons in the limitation of the small-instanton size. Additionally it is also worth noticing that the contributions of higher order derivative terms in the DBI action, which has been dropped off in (3.7), might become important in the limitation of the small-instanton size as mentioned in [51]. We leave this issue for future study and continue analysis based on the YM action (3.7) in the rest of this paper.

4 Dynamics of the collective modes

4.1 Collectivization

In this section, let us briefly outline the collective mode of baryon and its effective dynamics. In order to derive the Lagrangian of the collective modes, we require the moduli of the solution to be time-dependent, i.e.

$$X^\alpha, a^s \rightarrow X^\alpha(t), a^s(t), \quad s = 1, 2, \dots, 8 \tag{4.1}$$

with $X^\alpha = \{X^M, \rho\}$ and a^s refers to the $SU(3)$ orientation. So the $SU(3)$ gauge field is assumed to transform as,

$$A(t, x) \rightarrow V(t, x) \left(A^{\text{cl}}(t, x) - id \right) V(t, x)^{-1}, \tag{4.2}$$

where A^{cl} refers to the classical solution (3.15) (3.21) with time-dependent X^α and $V(t) \in SU(3)$. Using (3.19), the $U(1)$ gauge field with time-dependent X^α are given as,

$$\hat{A}_M(t, x) = 0, \quad \hat{A}_0(t, x) = \hat{A}_0^{\text{cl}}(x, X^\alpha(t)). \tag{4.3}$$

Accordingly, the gauge field strength can be obtained as,

$$\begin{aligned} F_{MN} &= V(t, x) F_{MN}^{\text{cl}} V(t, x)^{-1}, \\ F_{0M} &= V(t, x) \left(\dot{X}^\alpha \frac{\partial}{\partial X^\alpha} A_M^{\text{cl}} - D_M^{\text{cl}} \Phi - D_M^{\text{cl}} A_0^{\text{cl}} \right) V(t, x)^{-1}, \end{aligned} \tag{4.4}$$

where “ D ” denotes the covariant derivative operator, Φ is defined as $\Phi = -iV^{-1}\partial_0 V$ and “ $\dot{}$ ” represents the derivative respected to time. Then the first equation in (3.14) implies,

$$D_M^{\text{cl}} \left(\dot{X}^N \frac{\partial}{\partial X^N} A_M^{\text{cl}} + \dot{\rho} \frac{\partial}{\partial \rho} A_M^{\text{cl}} - D_M^{\text{cl}} \Phi \right) = 0. \tag{4.5}$$

For the case of $N_f = 3$, the solution for Φ can be written as,

$$\Phi(t, x) = -\dot{X}^N(t) A_N^{\text{cl}}(x, X^\alpha(t)) + \chi^s(t) \Phi_s(x, X^\alpha(t)), \tag{4.6}$$

where the explicit form of $\Phi_s(x, X^\alpha(t))$ can be found in [66]. With the boundary condition $\Phi_I(x, X^\alpha(t)) \rightarrow T_I$ and $A_M^{\text{cl}}(t, x) \rightarrow 0$ as $z \rightarrow +\infty$, we can further obtain,

$$\chi^s(t) = -2i \text{Tr} \left[T_s \mathbf{a}(t)^{-1} \cdot \dot{\mathbf{a}}(t) \right], \tag{4.7}$$

where $\mathbf{a}(t) \equiv a^s(t) T_s$. Finally we find that F_{0M} is given by,

$$F_{0M} = V(t, x) \left(\dot{X}^N F_{MN}^{\text{cl}} + \dot{\rho} \frac{\partial}{\partial \rho} A_M^{\text{cl}} - \chi^s D_M^{\text{cl}} \Phi_s - D_M^{\text{cl}} A_0^{\text{cl}} \right) V(t, x)^{-1}. \tag{4.8}$$

And the whole gauge field can be therefore expressed as,

$$A(t, x) = V(t, x) \left[A^{\text{cl}}(x, X^\alpha(t)) + \Phi(t, x) dt \right] V(t, x)^{-1}. \tag{4.9}$$

4.2 Quantization

The effective potential $U(X^\alpha)$ of the collective modes can be obtained by the relation $S_{\text{YM}}^{\text{D8/D8}} + S_{\text{CS}}^{\text{D8/D8}} = -\int dt U(X^\alpha)$. Inserting (4.4) (4.8) into action (3.12) (3.13), we have,

$$\begin{aligned} U(X^\alpha) &= M - ab^{3/2} N_c \\ &\int d^3x dz \text{Tr} \left(\dot{X}^N F_{MN}^{\text{cl}} + \dot{\rho} \frac{\partial}{\partial \rho} A_M^{\text{cl}} - \chi^I D_M^{\text{cl}} \Phi_I \right)^2 \\ &- L_{\text{CS}} + \mathcal{O}(\lambda^{-1}), \end{aligned} \tag{4.10}$$

where the L_{CS} term is defined as,

$$S_{\text{CS}}^{\text{D8/D8}}[A] - S_{\text{CS}}^{\text{D8/D8}}[A^{\text{cl}}] = \int dt L_{\text{CS}}. \tag{4.11}$$

However, if we use the Chern–Simons term (3.13) for the case of $N_f > 2$, it fails to give the exact transformation law under the gauge or chiral transformations. Particularly, (3.13) is unable to reproduce the important constraint of the hypercharge,

$$J_8 = \frac{N_c}{2\sqrt{3}}. \tag{4.12}$$

This issue has been revisited in [75] where the contribution at the boundary was added to (3.13). Resultantly, a new Chern–Simons term was proposed in [75] whose formula is,⁷

$$S_{\text{CS}}^{\text{new}} = S_{\text{CS}} + \int_{N_5} \frac{1}{10} \text{Tr} \left(h^{-1} dh \right)^5 + \int_{\partial M_5} \alpha_4(dh, A), \tag{4.13}$$

where the gauged 4-form α_4 is given in [75] and S_{CS} refers to the CS term (3.7). Here N_5 denotes a 5-dimensional manifold whose boundaries are defined as $\partial N_5 = \partial M_5 = M_{4+\infty} - M_{4-\infty}$ with the asymptotic gauge field on the flavor branes,

$$A|_{z \rightarrow \pm\infty} = \bar{A}^\pm = h^\pm (d + A) h^{\pm-1}, \tag{4.14}$$

where $h|_{\partial M_5} = (h^+, h^-)$ and \bar{A}^\pm denotes the external gauge field. Notice that A is required to be well defined gauge field throughout M_5 and produces no-boundary contributions. So it implies that the information about the topology is kept at

⁷ Since only the first term in (3.7) survives in the large λ limit, we consider the boundary contributions based on this term in the following calculation.

the holographic boundary $z \rightarrow \pm\infty$. Hence we can work in the gauge $A_z = 0$ for the instanton profile,

$$(h^+, h^-) = \left(P e^{-\int_{-\infty}^{+\infty} A_z dz}, 1 \right). \tag{4.15}$$

Note that the CS term in (4.11) has to be replaced by the new CS term in (4.13) where the new contribution can be accordingly identified as the constraint of the hypercharge (4.12).

Then motion of the collective coordinates can be characterized by the effective Lagrangian of the collective modes in the moduli space,

$$L = \frac{m_X}{2} g_{\alpha\beta} \dot{X}^\alpha \dot{X}^\beta - U(X^\alpha) + \mathcal{O}(\lambda^{-1}). \tag{4.16}$$

The derivative term in (4.16) refers to the line element in the moduli space which denotes the kinetics of the collective modes. Integrating over (x, z) , we obtain the following Hamiltonian associated to (4.16) which is,

$$H = M_0 + H_X + H_Z + H_\rho + \mathcal{O}(\lambda^{-1}), \tag{4.17}$$

where,

$$\begin{aligned} H_X &= \frac{1}{2m_X} P_X^2 + M_0, \quad H_Z = \frac{1}{2m_Z} P_Z^2 + \frac{1}{2} m_Z \omega_Z^2 Z^2, \\ H_\rho &= -\frac{1}{2m_y} P_y^2 + \frac{1}{2} m_y \omega_y^2 \rho^2 + \frac{Q}{\rho^2} \\ &\quad + \frac{2}{m_y \rho^2} \left(\sum_{s=1}^3 J_s^2 + 2 \sum_{s=4}^7 J_s^2 \right), \end{aligned} \tag{4.18}$$

with the constraint of the hypercharge (4.12) and,

$$\begin{aligned} m_X = m_z &= \frac{1}{2} m_y = 8ab^{3/2} \pi^2 N_c, \\ \omega_Z &= \sqrt{\frac{3-b}{3}}, \quad \omega_\rho = \omega_y = \sqrt{\frac{3-b}{12}}, \\ M_0 &= 8ab^{3/2} \pi^2 \lambda N_c, \quad Q = \frac{N_c}{40ab^{3/2} \pi^2}, \\ \rho^2 &= \sum_{a=1}^{a=\eta+1} (y^a)^2. \end{aligned} \tag{4.19}$$

Then quantization of the Hamiltonian would be nothing but replacing the momentum by its derivative operator. Specifically, we need

$$\begin{aligned} P_X^2 &= -\frac{1}{2m_X} \sum_{i=1}^3 \frac{\partial^2}{\partial X_i^2}, \quad P_Z^2 = -\frac{1}{2m_Z} \frac{\partial^2}{\partial Z^2}, \\ P_y^2 &= -\frac{1}{2m_y} \frac{1}{\rho^\eta} \partial_\rho (\rho^\eta \partial_\rho), \end{aligned} \tag{4.20}$$

for the Hamiltonian (4.17) and (4.18).

4.3 Baryon spectrum

The baryon spectrum can be obtained by solving the eigen equation of the Hamiltonian for the collective modes. Let us consider the solution for the Hamiltonian (4.17) in the (p, q) representation for the $SU(3)_I$ and $SU(3)_J$. In this representation, a state takes the following relation of the quantum number,

$$\begin{aligned} \sum_{s=1}^8 (J_s)^2 &= \frac{1}{3} (p^2 + q^2 + pq) + p + q, \\ \sum_{s=1}^3 (J_s)^2 &= j(j+1). \end{aligned} \tag{4.21}$$

Then the radius part of the Hamiltonian H_ρ can be rewritten as,

$$H_\rho = -\frac{1}{2m_y} \frac{1}{\rho^\eta} \partial_\rho (\rho^\eta \partial_\rho) + \frac{1}{2} m_y \omega_y^2 \rho^2 + \frac{K}{m_y \rho^2}, \tag{4.22}$$

where,

$$\begin{aligned} K &= \frac{N_c^2}{15} + \frac{4}{3} (p^2 + q^2 + pq) \\ &\quad + 4(p+q) - 2j(j+1), \end{aligned} \tag{4.23}$$

and we have used the definition of J_8 as (4.12). So the eigenfunction of (4.22) takes the following forms,

$$H_\rho \psi(\rho) = E_\rho \psi(\rho),$$

with

$$\begin{aligned} \psi(\rho) &= e^{-v/2} v^{\mathcal{B}} \gamma(v), \\ v &= m_y \omega_\rho \rho^2, \\ \mathcal{B} &= \frac{1}{4} \sqrt{(\eta-1)^2 + 8K} - \frac{1}{4} (\eta-1). \end{aligned} \tag{4.24}$$

The function $\gamma(v)$ needs to be solved by the following hypergeometrically differential equation,

$$\begin{aligned} \left[v \frac{d^2}{dv^2} + \left(2\mathcal{B} + \frac{\eta+1}{2} - v \right) \frac{d}{dv} \right. \\ \left. + \left(\frac{E_\rho}{2\omega_\rho} - \mathcal{B} - \frac{\eta+1}{4} \right) \right] \gamma(v) = 0. \end{aligned} \tag{4.25}$$

The normalizable regular solution for (4.25) has to satisfy,

$$\frac{E_\rho}{2\omega_\rho} - \mathcal{B} - \frac{\eta + 1}{4} = n_\rho = 0, 1, 2, 3, \dots \tag{4.26}$$

so the eigenvalues are given as,

$$E_\rho = \omega_\rho \left[2n_\rho + \frac{\sqrt{(\eta - 1)^2 + 8K}}{2} + 1 \right]. \tag{4.27}$$

On the other hand, the Z part Hamiltonian H_Z takes the same form as the Hamiltonian of a harmonic oscillator. Therefore the eigenvalues of H_Z can be immediately obtained as,

$$E_Z = \omega_Z \left(n_Z + \frac{1}{2} \right), \quad n_Z = 0, 1, 2, 3, \dots \tag{4.28}$$

Combining (4.27) with (4.28), we finally have the baryon spectrum as,

$$M = M_0 + \left(\frac{3 - b}{2} \right)^{1/2} \left[\sqrt{\frac{(\eta - 1)^2}{24} + \frac{K}{3}} + \sqrt{\frac{2}{3}} (n_Z + n_\rho + 1) \right] M_{KK}. \tag{4.29}$$

Particularly, we put $N_c = 3$ into the constraint (4.12) which implies,

$$p + 2q = 3 \times (\text{integer}). \tag{4.30}$$

So the low-energy baryon states satisfying the constraint (4.12) and (4.30) takes the following values for the quantum number,

$$\begin{aligned} (p, q) &= (1, 1), j = \frac{1}{2}, K = \frac{111}{10}, \text{ (octet)} \\ (p, q) &= (3, 0), j = \frac{3}{2}, K = \frac{171}{10}, \text{ (decuplet)} \\ (p, q) &= (0, 3), j = \frac{1}{2}, K = \frac{231}{10}. \text{ (anti - decuplet)} \end{aligned} \tag{4.31}$$

5 Involving the heavy flavor

5.1 The heavy flavor brane

The holographic baryon in the D0–D4/D8 construction has been discussed in the previous sections, let us extend the analysis to involve the heavy flavor in this section.

Following [65,67–70], the heavy flavor could be introduced into this system by embedding a pair of flavor brane which is separated from the other N_f coincident flavor branes

with a string stretched between them as illustrated in Fig. 2. The N_f coincident flavor branes, mentioned in the previous sections, are now named as “light flavor branes (L-brane)” while the separated flavor brane is named as “heavy flavor branes (H-brane)”. So the string stretched between L- and H-branes is therefore named as “HL-string” which produces massive multiplets. The low-energy modes of the HL-string, which corresponds to the heavy-light mesons, could be approximated in bi-fundamental representation by the local vector fields in the vicinity of the light flavor branes. The heavy-light fields become massive due to the nonzero vacuum expectation value (vev) of the HL-string. And their mass term in the action comes from the moduli span by the dilaton fields. For the heavy flavor branes, we should choose another solution from (3.4) denoted by $\tau_H(U)$ as the embedding function with $U_0 = U_H \neq U_{KK}$ since they must be embedded at the non-antipodal position of the background. Accordingly, there is a finite separation at $\tau_H(U_0) = 0$ between the H- and L-branes as shown in Fig. 2.

5.2 The effective heavy-light action

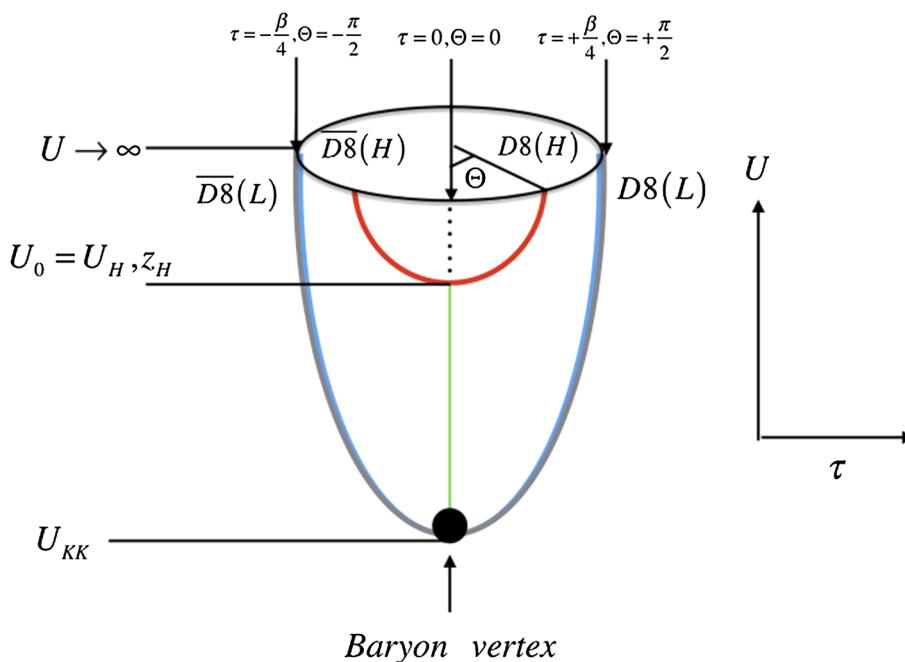
By involving the heavy-light interaction, the subject of this subsection is to study the effective low-energy dynamics of the baryons on the L-branes since the baryon vertex lives inside the L-brane. The lowest modes of the HL-string consist of longitudinal modes Φ_a and the transverse modes φ^I in the L-brane world volume. These fields acquire a nonzero vev at finite brane separation thus introduce the mass to the vector field [76]. These fields are always mentioned as “bi-local fields”, however we are going to approximate them by the local vector fields near the worldvolume of the L-branes so that their dynamics could be described by the DBI action. Hence this construction is distinct from the approaches in [77–83].

By keeping these in mind, let us start with the effective action of the L-branes involving the heavy flavor. The generic expansion of the DBI for a D8-branes in the leading order can be written as,

$$\begin{aligned} S_{\text{DBI}}^{\text{D8/D8}} &= -\frac{T_8 (2\pi\alpha')^2}{4} \int_{\text{D8/D8}} d^9\xi \sqrt{-\det G} e^{-\Phi} \text{Tr} \\ &\quad \left\{ \mathcal{F}_{ab} \mathcal{F}^{ab} - 2D_a \varphi^I D_a \varphi^J + [\varphi^I, \varphi^J]^2 \right\}. \\ &\equiv S_{\text{YM}}^{\text{D8/D8}} + S_\Psi \end{aligned} \tag{5.1}$$

where φ^I is the transverse mode and the index a, b runs over the L-brane. Let us define $\varphi^I \equiv \Psi$ to omit the index since there is only one transverse coordinate to D8-brane. The field Ψ is traceless in adjoint representation additional to the adjoint gauge field \mathcal{A}_a . While $S_{\text{YM}}^{\text{D8/D8}}$ refers to the Yang–Mills action in (3.7), the worldvolume field should be

Fig. 2 Various brane configuration in the $\tau - U$ plane where τ is compactified on S^1 . The bubble background (cigar) is produced by N_c D4-branes with N_0 smeared D0-branes. The $N_f = 3$ light (chiral) D8/ $\overline{D8}$ -branes (L) at antipodal position of the cigar are represented by the blue line. A pair of heavy D8/ $\overline{D8}$ -brane (H) is separated from the light branes which is represented by the red line. The massive state is produced by the strings stretched between the light and heavy branes which is represented by the green line (i.e. the HL-string) in this figure. The baryon spectrum is produced by the baryon vertex which lives inside the L-branes.



combined in a superconnection according to string theory since the pair of the H-brane is separated from the N_f L-branes with a HL-string stretched between them [84]. Hence we use the following matrix-valued 1-form for the gauge field by involving the heavy flavor,

$$\mathcal{A}_a = \begin{pmatrix} \mathcal{A}_a & \Phi_a \\ -\Phi_a^\dagger & 0 \end{pmatrix}. \tag{5.2}$$

Here \mathcal{A}_a is $(N_f + 1) \times (N_f + 1)$ matrix-valued field while Ψ and \mathcal{A}_a are $N_f \times N_f$ valued fields. The Φ_a multiplet is massless if all the flavor branes are coincident, otherwise Φ_a could be massive field. The gauge field strength corresponding to (5.2) is,

$$\mathcal{F}_{ab} = \begin{pmatrix} \mathcal{F}_{ab} - \Phi_{[a} \Phi_{b]}^\dagger & \partial_{[a} \Phi_{b]} + \mathcal{A}_{[a} \Phi_{b]} \\ \partial_{[a} \Phi_{b]}^\dagger + \Phi_{[a}^\dagger \mathcal{A}_{b]} & -\Phi_{[a}^\dagger \Phi_{b]} \end{pmatrix}. \tag{5.3}$$

Therefore the Yang–Mills action $S_{\text{YM}}^{\text{D8}/\overline{\text{D8}}}$ involving the heavy flavor takes the same formula as in (3.7) but replacing the gauge field strength by (5.3), i.e. $\mathcal{F}_{ab} \rightarrow \mathcal{F}_{ab}$. Using the dimensionless variables defined in (3.10) and rescaling (3.11), the Yang–Mills Lagrangian associated to action $S_{\text{YM}}^{\text{D8}/\overline{\text{D8}}}$ is calculated as,

$$\mathcal{L}_{\text{YM}}^{\text{D8}/\overline{\text{D8}}} = \mathcal{L}_{\text{YM}}^L + a N_c b^{3/2} \lambda \mathcal{L}_0^H + a N_c b^{3/2} \mathcal{L}_1^H + \mathcal{O}(\lambda^{-1}), \tag{5.4}$$

where $\mathcal{L}_{\text{YM}}^L$ refers to the Lagrangian associated to the action $S_{\text{YM}}^{\text{D8}/\overline{\text{D8}}}$ in (3.7) and,

$$\begin{aligned} \mathcal{L}_0^H &= - \left(D_M \Phi_N^\dagger - D_N \Phi_M^\dagger \right) \left(D_M \Phi_N - D_N \Phi_M \right) \\ &\quad + 2 \Phi_M^\dagger \mathcal{F}^{MN} \Phi_N, \\ \mathcal{L}_1^H &= 2 \left(D_0 \Phi_M^\dagger - D_M \Phi_0^\dagger \right) \left(D_0 \Phi_M - D_M \Phi_0 \right) \\ &\quad - 2 \Phi_0^\dagger \mathcal{F}^{0M} \Phi_M - 2 \Phi_0^\dagger \mathcal{F}^{0M} \Phi_M + \tilde{\mathcal{L}}_1^H, \\ \tilde{\mathcal{L}}_1^H &= b z^2 \left(\frac{5}{6} - \frac{1}{2b} \right) \left(D_i \Phi_j - D_j \Phi_i \right)^\dagger \left(D_i \Phi_j - D_j \Phi_i \right) \\ &\quad - b z^2 \left(1 + \frac{1}{b} \right) \left(D_i \Phi_z - D_z \Phi_i \right)^\dagger \left(D_i \Phi_z - D_z \Phi_i \right) \\ &\quad - b z^2 \left(\frac{5}{3} - \frac{1}{b} \right) \Phi_i^\dagger \mathcal{F}^{ij} \Phi_j \\ &\quad + b z^2 \left(1 + \frac{1}{b} \right) \left(\Phi_z^\dagger \mathcal{F}^{zi} \Phi_i + c.c. \right). \end{aligned} \tag{5.5}$$

Note the derivative is defined as $D_M \Phi_N = \partial_M \Phi_N + \mathcal{A}_{[M} \Phi_{N]}$. On the other hand, the action S_Ψ in (5.1) is collected as,

$$\begin{aligned} S_\Psi &= - \frac{T_8 (2\pi\alpha')^2}{4} \int d^9 \xi \sqrt{-\det G} e^{-\Phi} \text{Tr} \\ &\quad \times \left\{ -2 D_a \varphi^i D_a \varphi^i + [\varphi^i, \varphi^j]^2 \right\} \\ &= \tilde{T}_8 \int d^4 x dz \sqrt{-\det G} e^{-\Phi} \text{Tr} \\ &\quad \times \left\{ \frac{1}{2} D_a \Psi D_a \Psi - \frac{1}{4} [\Psi, \Psi]^2 \right\}, \end{aligned} \tag{5.6}$$

with $D_a\Psi = \partial_a\Psi + i[A_a, \Psi]$. According to [84,85], the extrema of the potential contribution or $[\Psi, [\Psi, \Psi]] = 0$ in (5.6) can define the moduli of Ψ . So we can choose the moduli solution of Ψ with a finite vev v as,

$$\Psi = \begin{pmatrix} -\frac{v}{N_f} \mathbf{1}_{N_f} & 0 \\ 0 & v \end{pmatrix}, \tag{5.7}$$

Imposing the solution (5.7) into (5.6), we have

$$S_\Psi = -aN_c b^{3/2} \int d^4x dz z m_H^2 \Phi_M^\dagger \Phi_M + \mathcal{O}(\lambda^{-1}), \tag{5.8}$$

where $m_H = \frac{1}{\sqrt{6}} \frac{N_f+1}{N_f} v b^{1/4}$. Notice that we have used the dimensionless v by the replacement $v \rightarrow \frac{M_{KK}^{1/2}}{U_{KK}^{1/2}} v$ in (5.8).

Finally, we still need to derive the formula of the CS action involving the heavy flavor of the L-brane. Similarly, let us replace the gauge field strength $\mathcal{F}_{ab} \rightarrow \mathcal{F}_{ab}$ in $S_{CS}^{D8/\overline{D8}}$ (3.7), then we obtain,

$$\begin{aligned} \mathcal{L}_{CS}^{D8/\overline{D8}} &= \mathcal{L}_{CS}^L(A) + \mathcal{L}_{CS}^H + \mathcal{O}(\lambda^{-1}), \\ \mathcal{L}_{CS}^H &= -\frac{iN_c}{24\pi^2} (d\Phi^\dagger A d\Phi + d\Phi^\dagger dA\Phi + \Phi^\dagger dAd\Phi) \\ &\quad - \frac{iN_c}{16\pi^2} (d\Phi^\dagger A^2\Phi + \Phi^\dagger A^2 d\Phi + \Phi^\dagger AdA\Phi \\ &\quad + \Phi^\dagger dAA\Phi) - \frac{5iN_c}{48\pi^2} \Phi^\dagger A^3\Phi + \mathcal{O}(\Phi^4, A), \end{aligned} \tag{5.9}$$

where $\mathcal{L}_{CS}^L(A)$ refers to the Lagrangian associated to action $S_{CS}^{D8/\overline{D8}}$ in (3.7). So the action for heavy-light interaction can be collected through (5.4) (5.5) (5.8) (5.9).

5.3 The double limit

Since calculating the complete contributions from the heavy meson field Φ_M is difficult, we will work in the limit of $\lambda \rightarrow \infty$ followed by $m_H \rightarrow \infty$ (i.e. the ‘‘double limit’’) as in the previous works [65,69,70]. The leading contributions come from the presented $S_{YM}^{D8/\overline{D8}}$ in (2.5) which is of order λm_H^0 , while the heavy-light interaction Lagrangian \mathcal{L}_1^H in (5.5) and \mathcal{L}_{CS}^H in (5.9) contribute to the subleading order $\lambda^0 m_H$. The double limit implies that we assume the heavy meson field Φ_M is very massive, or equivalently, the separation of the H- and L-branes is very large as shown in Fig. 2. the straight pending string accordingly takes a value at $z = z_H$ which satisfies,

$$\begin{aligned} m_H &= \frac{1}{\pi l_s^2} \lim_{z_H \rightarrow \infty} \int_0^{z_H} dz \sqrt{-g_{00}g_{zz}} \\ &\simeq \frac{1}{\pi l_s^2} U_{KK}^{1/3} z_H^{2/3} + \mathcal{O}(z_H^0). \end{aligned} \tag{5.10}$$

Let us rewrite (5.10) with the dimensionless variables by the replacement $m_H \rightarrow m_H M_{KK}$, $z_H \rightarrow z_H U_{KK}$ for convenience, then using (2.13) we get,

$$\frac{m_H}{\lambda} = \frac{2b}{9\pi} z_H^{2/3}. \tag{5.11}$$

In the double limit, we follow [65,69] by redefining $\Phi_M = \phi_M e^{\pm im_H x^0}$ with ‘‘-’’ for particle and ‘‘+’’ for anti-particle. So the derivative term of Φ_M can be replaced by $D_0\Phi_M \rightarrow (D_0 \pm im_H)\Phi_M$. Then we collect the terms of order $\lambda^0 m_H$ from the heavy-light action which are,

$$\begin{aligned} \mathcal{L}_{m_H} &= \mathcal{L}_{1,m} + \mathcal{L}_{CS,m}, \\ \mathcal{L}_{1,m} &= ab^{3/2} N_c \left[4im_H \phi_M^\dagger D_0\phi_M \right. \\ &\quad \left. - 2im_H (\phi_0^\dagger D_M\phi_M - c.c.) \right], \\ \mathcal{L}_{CS,m} &= \frac{m_H N_c}{16\pi^2} \epsilon_{MNPQ} \phi_M^\dagger \mathcal{F}_{NP} \phi_Q = \frac{m_H N_c}{8\pi^2} \phi_M^\dagger \mathcal{F}_{MN} \phi_N. \end{aligned} \tag{5.12}$$

5.4 The equations of motion and the zero mode of vector

Let us consider the equations of motion of the heavy meson field Φ_M . The equations read from the Lagrangians in (5.4) (5.5) (5.8) (5.9),

$$D_M D_M \Phi_N - D_N D_M \Phi_M + 2\mathcal{F}_{NM} \Phi_M + \mathcal{O}(\lambda^{-1}) = 0. \tag{5.13}$$

Notice that Φ_M is independent on Φ_0 , thus the equation of motion for Φ_0 is,

$$\begin{aligned} D_M (D_0\Phi_M - D_M\Phi_0) - \mathcal{F}^{0M} \Phi_M \\ - \frac{1}{64\pi^2 ab^{3/2}} \epsilon_{MNPQ} \mathcal{K}_{MNPQ} + \mathcal{O}(\lambda^{-1}) = 0, \end{aligned} \tag{5.14}$$

where the 4-form \mathcal{K}_{MNPQ} is given by,

$$\begin{aligned} \mathcal{K}_{MNPQ} &= \partial_M \mathcal{A}_N \partial_P \Phi_Q + \mathcal{A}_M \mathcal{A}_N \partial_P \Phi_Q + \partial_M \mathcal{A}_N \mathcal{A}_P \Phi_Q \\ &\quad + \frac{5}{6} \mathcal{A}_M \mathcal{A}_N \mathcal{A}_P \Phi_Q. \end{aligned} \tag{5.15}$$

The equation of motion (5.14) suggests a simplification $D_M\Phi_M = 0$ in (5.12) is considerable which would imply that the transverse mode Φ_M is covariant.

In combination with the constraint equation (5.14), we find the equation of motion (5.13) are equivalent to the vector equation of zero-mode in the fundamental representation. To show this, let us recall the self-dual gauge field strength (3.17). By defining $f_{MN} = \partial_{[M}\phi_{N]} + \mathcal{A}_{[M}\phi_{N]}$, the Lagrangian \mathcal{L}_0^H in (5.5) can be rewritten in the compact form,

$$\begin{aligned} \mathcal{L}_0^H &= -f_{MN}^\dagger f_{MN} + 2\phi_M^\dagger \mathcal{F}_{MN} \phi_N \\ &= -f_{MN}^\dagger f_{MN} + 2\epsilon_{MNPQ} \phi_M^\dagger D_N D_P \phi_Q \\ &= -f_{MN}^\dagger f_{MN} + f_{MN}^\dagger \star f_{MN} \\ &= -\frac{1}{2} (f_{MN} - \star f_{MN})^\dagger (f_{MN} - \star f_{MN}), \end{aligned} \tag{5.16}$$

where \star denotes the Hodge dual. Hence the equations of motion (5.13) can be replaced by,

$$\begin{aligned} f_{MN} - \star f_{MN} &= 0, \\ D_M \phi_M &= 0, \end{aligned} \tag{5.17}$$

or equivalently,

$$\sigma_M D_M \psi = 0, \quad \text{with } \psi = \bar{\sigma}_M \phi_M. \tag{5.18}$$

Therefore the solution for ϕ_M in (5.17) can be given as,

$$\phi_M = \bar{\sigma}_M \xi \frac{\rho}{(x^2 + \rho^2)^{3/2}} \equiv \bar{\sigma}_M f(x) \xi, \tag{5.19}$$

Here ξ denotes a two-component spinor and the (5.19) agrees with the results in [65,69]. The interplay of (5.18) is remarkable because it illustrates that a heavy vector meson binds to an flavored bulk instanton in holography which concludes that the zero mode of a vector can be equivalently described by a spinor.

5.5 Quantization of the collective modes

The quantization of the leading λN_c contribution has been discussed in Section 3.2 where the classical moduli of the bound instanton is quantized by slowly translating and rotating the bound states. The quantization involving the heavy flavor follows the similar procedures by replacing $\mathcal{A} \rightarrow \mathcal{A}$. So we have,

$$\begin{aligned} \Phi_M &\rightarrow V[a^s(t)] \Phi_M [X^0(t), Z(t), \rho(t), \chi(t)], \\ \Phi_0 &= 0, \end{aligned} \tag{5.20}$$

where X^0, Z denotes the center in the x^i and z directions respectively. For the $N_f = 3$ case, a^s represents the $SU(3)$ gauge rotation. Then the time-dependent configuration could

be introduced in the heavy-light effective action as described earlier.

Due to the additional CS terms in (4.13), it picks up the collective instanton in quantization by defining,

$$\begin{aligned} h^- &= \text{diag}(a^s(t)^{-1}, 1), \\ h^+ &= h_0 \text{diag}(a^s(t)^{-1}, 1). \end{aligned} \tag{5.21}$$

Note that the field \mathcal{A} consists of the instanton solution A and the zero-mode solution Φ which carries the same topological number. Then inserting (5.21) into the new CS terms (4.13), we obtain,

$$S_{\text{CS}}^{\text{new}} = S_{\text{CS}} - \frac{iN_c}{48\pi^2} \int_{M_4} dt \text{Tr} [\mathbf{a}(t)^{-1} \cdot \dot{\mathbf{a}}(t)] (h_0^{-1} dh_0)^3, \tag{5.22}$$

Here the heavy-light contributions S_{CS} refers those in (5.9) while the new contribution from the second term can be identical to the coupling with the hypercharge as (4.12).

In order to obtain the quantized Lagrangian of the collective modes in $\lambda^0 m_H$ order, we recall (5.12) by imposing (3.17) and (5.20) which becomes,

$$\begin{aligned} \mathcal{L}_m &= ab^{3/2} N_c \left[16im_H \xi^\dagger \partial_t \xi f^2 - 16m_H \xi^\dagger \xi f^2 \frac{2\sqrt{6} + 1}{6} A_0 \right. \\ &\quad \left. - m_H f^2 \xi^\dagger \sigma_\mu \Phi \bar{\sigma}_\mu \xi + m_H \xi^\dagger \xi f^2 \frac{3}{ab^{3/2}\pi^2} \frac{\rho^2}{(x^2 + \rho^2)^2} \right]. \end{aligned} \tag{5.23}$$

The second term comes from the A_0 coupling and it would simply the expression for the zero-modes by,

$$\xi^\dagger \sigma_\mu \Phi \bar{\sigma}_\mu \xi = a_8 \frac{8\xi^\dagger \xi}{\sqrt{3}}. \tag{5.24}$$

The last term in (5.23) originates from the CS term in (5.12) with,

$$\frac{im_H N_c}{8\pi^2} \phi_M^\dagger F_{MN} \phi_N = \frac{3im_H N_c}{\pi^2} \frac{f^2 \rho^2}{(x^2 + \rho^2)^2} \xi^\dagger \xi. \tag{5.25}$$

There are additional terms to (5.23) due to the $\xi^\dagger \xi$ coupling to the $U(1)$ gauge field A_0 which induces a Coulomb-like backreaction of the form $(\xi^\dagger \xi)^2$ as we have indicated in [65]. By keeping this in mind and using the normalization $\int d^4x f^2 = 1$, the explicit form of $\mathcal{A}_0^{\text{cl}}$ and the rescaling $\xi \rightarrow \xi/\sqrt{16ab^{3/2}N_c m_H}$, we finally obtain,

$$\mathcal{L} = \mathcal{L}^L [a_I, X^\alpha] + i\xi^\dagger \partial_t \xi + \frac{\sqrt{6} + 2}{48\pi^2 ab^{3/2} \rho^2} \xi^\dagger \xi - \frac{13 (\xi^\dagger \xi)^2}{288\pi^2 ab^{3/2} \rho^2 N_c} + a^8 \frac{N_c}{2\sqrt{3}} \left(1 - \frac{\xi^\dagger \xi}{N_c} \right), \quad (5.26)$$

where $\mathcal{L}^L [a_I, X^\alpha]$ refers to the Lagrangian (4.16) of light flavors. The (5.26) can be understood as Lagrangian density of heavy-light degrees of freedom supplemented by the constraint of hypercharge with the excitations of the vacuum, namely,

$$\mathcal{L} = \mathcal{L}^L [a_I, X^\alpha] + i\xi^\dagger \partial_t \xi + \frac{\sqrt{6} + 2}{48\pi^2 ab^{3/2} \rho^2} \xi^\dagger \xi - \frac{13 (\xi^\dagger \xi)^2}{288\pi^2 ab^{3/2} \rho^2 N_c}, \quad (5.27)$$

with the hypercharge constraint,

$$J^8 = \frac{N_c}{2\sqrt{3}} \left(1 - \frac{\xi^\dagger \xi}{N_c} \right), \quad (5.28)$$

5.6 The heavy-light baryon spectrum

The heavy-light Hamiltonian associated to (5.27) takes the following form,

$$\mathcal{H} = \mathcal{H}^L [a_I, X^\alpha] - \frac{\sqrt{6} + 2}{48\pi^2 ab^{3/2} \rho^2} \xi^\dagger \xi + \frac{13 (\xi^\dagger \xi)^2}{288\pi^2 ab^{3/2} \rho^2 N_c}, \quad (5.29)$$

with the quantization rule for the spinor ξ ,

$$\xi_i \xi_j^\dagger + \xi_j^\dagger \xi_i = \delta_{ij}. \quad (5.30)$$

Here $\mathcal{H}^L [a_I, X^\alpha]$ refers to the Hamiltonian (4.17). Let us use U and Λ to represent the rotation of a spinor and a vector respectively i.e. $\xi \rightarrow U\xi$, $\phi_M \rightarrow \Lambda_{MN}\phi_N$, so we have the transformation $U^{-1}\bar{\sigma}_M U = \Lambda_{MN}\bar{\sigma}_M$ which implies the rotation of the spinor ξ is equivalent to a spatial rotation of the heavy vector meson field ϕ_M . Notice that the parity of ξ is negative which is opposite to ϕ_M .

The spectrum of (5.29) follows the same discussion in Section 3. Since the Hamiltonian (4.17) contains two terms which are proportional to ρ^{-2} , the heavy-light spectrum can be obtained by modifying Q if comparing (5.29) with Hamiltonian (4.17), which is,

$$Q = \frac{N_c}{40ab^{3/2}\pi^2} \rightarrow \frac{N_c}{40ab^{3/2}\pi^2} \times \left[1 - \frac{5\sqrt{6} + 10}{6N_c} \xi^\dagger \xi + \frac{65}{36N_c^2} (\xi^\dagger \xi)^2 \right]. \quad (5.31)$$

Let us use \mathbf{J} and \mathbf{I} to denote the total spin and isospin, the relation of them is given by,

$$\mathbf{J} = -\mathbf{I}_{SU(2)} + \mathbf{S}_\xi = -\mathbf{I}_{SU(2)} + \xi^\dagger \frac{\boldsymbol{\tau}}{2} \xi. \quad (5.32)$$

Here $\mathbf{I}_{SU(2)}$ refers to the first three generators in the induced representation for a general $SU(3)$ group. The quantum states for a single bound state i.e. $N_Q \equiv \xi^\dagger \xi = 1$ and IJ^π assignments are labeled by,

$$|N_Q, p, q, j, n_Z, n_\rho \rangle \quad \text{with} \quad IJ^\pi = \frac{l}{2} \left(\frac{l}{2} \pm \frac{1}{2} \right)^\pi. \quad (5.33)$$

Here $n_Z, n_\rho = 0, 1, 2, \dots$ respectively denotes the number of quanta associated to the collective motion and the radial breathing of the instanton core. According to (4.29) and (5.31), the spectrum of the bound heavy-light state in the D0–D4/D8 system is given as,

$$M_{N_Q} = M_0 + N_Q m_H + \left(\frac{3-b}{2} \right)^{1/2} \left[\sqrt{\frac{2}{3}} (n_\rho + n_z + 1) + \sqrt{\frac{49}{24} + \frac{\mathbf{K}}{3}} \right] M_{KK}, \quad (5.34)$$

where,

$$\mathbf{K} = \frac{2N_c^2}{5} \left(1 - \frac{5\sqrt{6} + 10}{6} \frac{N_Q}{N_c} + \frac{65}{36} \frac{N_Q^2}{N_c^2} \right) - \frac{N_c^2}{3} \left(1 - \frac{N_Q}{N_c} \right)^2 + \frac{4}{3} (p^2 + q^2 + pq) + 4(p + q) - 2j(j + 1), \quad (5.35)$$

and $M_0 = \frac{\lambda N_c b^{3/2}}{27\pi} M_{KK}$, M_{KK} is the Kaluza–Klein mass.

The heavy baryon also includes anti-heavy quarks. So in order to amount an anti-heavy-light meson, we return to the preceding arguments of the reduction $\Phi_M = \phi_M e^{+im_H x^0}$. Most of the calculations would be unchanged as we have indicated in the previous articles except for pertinent minus signs to the effective Lagrangian. If we bind one heavy-light and one anti-heavy-light meson in the form of a zero-mode, the effective Lagrangian now reads,

$$\mathcal{L} = \mathcal{L}^L [a_I, X^\alpha] + i\xi_Q^\dagger \partial_t \xi_Q + \frac{\sqrt{6} + 2}{48\pi^2 ab^{3/2} \rho^2} \xi_Q^\dagger \xi_Q - i\xi_Q^\dagger \partial_t \xi_{\bar{Q}} - \frac{\sqrt{6} + 2}{48\pi^2 ab^{3/2} \rho^2} \xi_Q^\dagger \xi_{\bar{Q}} + \frac{13 (\xi_Q^\dagger \xi_Q - \xi_{\bar{Q}}^\dagger \xi_{\bar{Q}})^2}{288\pi^2 ab^{3/2} \rho^2 N_c}. \quad (5.36)$$

Therefore the mass spectrum of baryons with N_Q heavy-quarks and $N_{\bar{Q}}$ anti-heavy quarks can be obtained by following the similar analysis with the substitution $N_Q \rightarrow N_Q - N_{\bar{Q}}$ in the order of $\lambda^0 m_H$, so its explicit form is,

$$M_{N_Q} = M_0 + (N_Q - N_{\bar{Q}}) m_H + \left(\frac{3-b}{2}\right)^{1/2} \times \left[\sqrt{\frac{2}{3}}(n_\rho + n_z + 1) + \sqrt{\frac{49}{24} + \frac{\mathbf{K}}{3}} \right] M_{KK}, \tag{5.37}$$

where

$$\mathbf{K} = \frac{2N_c^2}{5} \left[1 - \frac{5\sqrt{6} + 10}{6} \frac{N_Q - N_{\bar{Q}}}{N_c} + \frac{65}{36} \frac{(N_Q - N_{\bar{Q}})^2}{N_c^2} \right] - \frac{N_c^2}{3} \left(1 - \frac{N_Q - N_{\bar{Q}}}{N_c} \right)^2 + \frac{4}{3} (p^2 + q^2 + pq) + 4(p + q) - 2j(j + 1). \tag{5.38}$$

6 Summary and discussion

In this paper, we have investigated the baryon spectrum in the Witten–Sakai–Sugimoto model with the D0–D4 background in the situation of three flavors. The baryon states are created by the baryon vertex which could be treated as the instanton configuration of the gauge fields on the flavor branes, so we particularly study the quantization of the collective modes with the instanton solution. Then the baryon spectrum can be obtained by solving the Hamiltonian of the collective modes. One of the key importance here is that the original CS term has to be modified in order to reproduce the hypercharge constraint as discussed in [66,75]. Afterwards we extend our analysis to include the heavy flavor. By employing the mechanism proposed in [68–70], the heavy flavor is introduced by embedding an extra pair of flavor branes (heavy flavor brane) which is separated from the other coincident flavor branes (light flavor branes) with a string stretched between them. We derive the effective dynamics of the baryons involving the heavy flavor in the double limits and find that heavy meson binds in the zero mode of the flavor instanton. The quantization in the presence of heavy flavors follows the similar procedures as discussed in the sector of light flavors and we finally obtain the baryon spectra for the single- and double-heavy baryon all in the case of $N_f = 3$.

Since the smeared D0-branes are D-instantons in the background geometry, this model is holographically dual to the confined Yang–Mills theory with an excitation of nonzero glue condensate $\langle \text{Tr} \mathcal{F} \wedge \mathcal{F} \rangle = 8\pi^2 N_c \tilde{\kappa}$. Therefore

the baryon spectrum depends on such excitations through a parameter b and it allows to describe the influence of some metastable states with odd P or CP parity which may probably be produced in the hot and dense situation in RHIC. We notice that the constraint of the parameter b has to satisfy $1 \leq b < 3$ and the difference in the spectra becomes larger as the true vacuum appears in the dual field theory. Specifically, our results would return to those in [66,70] if $b = 1$ i.e. no D0-branes, and the spectrum would become complex if $b > 3$. Accordingly, it concludes that baryon can not stably exist if the glue condensate is sufficiently large, which is in agreement with the previous studies of this model [57–59,65].

Last but not the least, let us briefly discuss the comparison with experimental data by using our holographic baryon spectra. Notice that we will set $N_c = 3, \eta = 8$ in our spectrum to fit three flavor QCD. In the sector of light flavors, the spectrum (4.29) shows the following mass difference between the octet and the decuplet baryon states, and that between the octet and the anti-decuplet states all with the same (n_ρ, n_z) :

$$\begin{aligned} M_{10} - M_8 &\simeq 0.27309\sqrt{3-b}M_{KK}, \\ M_{10^*} - M_8 &\simeq 0.51264\sqrt{3-b}M_{KK}, \end{aligned} \tag{6.1}$$

which indicates the metastable states of baryon or the dependence of the D0 charge. To realize the experimental data,

$$\begin{aligned} M_{10}^{\text{exp}} - M_8^{\text{exp}} &\simeq 292\text{MeV}, \\ M_{10^*}^{\text{exp}} - M_8^{\text{exp}} &\simeq 590\text{MeV}, \end{aligned} \tag{6.2}$$

with taking account into the value of M_{KK} which is determined from ρ meson mass $M_{KK} = 949\text{MeV}$ [37], we can fit the experimental data by setting $b = 1.7305$. However b in this method has been identified as an arbitrarily phenomenal parameter. Strictly speaking, the stable baryon state corresponds to $b = 1$. So in this sense, the above baryon spectrum gives,

$$\begin{aligned} M_{10} - M_8 &\simeq 367\text{MeV}, \\ M_{10^*} - M_8 &\simeq 688\text{MeV}, \end{aligned} \tag{6.3}$$

which is not very close to the experimental data. Since all the quarks are massless in the light sector, the present three-flavor holographic baryon spectrum (4.29) might not be very realistic which therefore induces that (6.3) is a little far away from the experimental data.

Nonetheless, let us continue the analysis in the presence of the heavy flavor and find a way to fit the experiments since the mass term has been introduced in the heavy-light sector. The lowest heavy baryon states with one heavy quark are characterized by $n_\rho, n_z = 0, 1, (p, q, j) = (0, 1, 0)$,

$N_Q = 1$ for $\bar{\mathbf{3}}$ representation and $(p, q, j) = (2, 0, 1)$ for $\mathbf{6}$ representation. The parity and spin of $\bar{\mathbf{3}}$ states is $\frac{1}{2}^+$, so we can identify these states as Σ_Q^* , $\Xi_Q(\mathbf{6})^*$, Ω_Q^* respectively. Then the mass spectra are give by (5.34) as,

$$\begin{aligned} M_{\bar{\mathbf{3}}}(b) &\simeq M_0 + m_H \\ &\quad + \left(1.29 + \frac{n_\rho + n_Z + 1}{\sqrt{3}}\right) \sqrt{3 - b} M_{KK}, \\ M_{\mathbf{6}}(b) &\simeq M_0 + m_H \\ &\quad + \left(1.53 + \frac{n_\rho + n_Z + 1}{\sqrt{3}}\right) \sqrt{3 - b} M_{KK}, \end{aligned} \quad (6.4)$$

or equivalently,

$$\begin{aligned} M_{\bar{\mathbf{3}}}(b) - M(b)_{p=q=1, N_Q=0, j=1/2} - m_H \\ &\simeq -0.403\sqrt{3 - b} M_{KK}, \\ M_{\mathbf{6}}(b) - M(b)_{p=q=1, N_Q=0, j=1/2} - m_H \\ &\simeq -0.167\sqrt{3 - b} M_{KK}, \end{aligned} \quad (6.5)$$

which implies the mass splitting $M_{\mathbf{6}}(b) - M_{\bar{\mathbf{3}}}(b) \simeq 0.237\sqrt{3 - b} M_{KK}$ become large as those metastable states decay to the true vacuum in the dual field theory. Since there are also heavy baryon states with two heavy quarks, we can further evaluate the masses of such states by following the same construct with $N_Q = 2$ and $J^\pi = \frac{1}{2\sqrt{3}}$ in (5.34). So the lowest states of heavy baryon binding double heavy mesons are now characterized by $n_\rho, n_Z = 0, 1, (p, q, j) = (1, 0, 0)$ for the $\mathbf{3}$ -plet representation which can be identified as Ξ_{QQ} with u, d light quark, and Ω_{QQ} with s quark. Afterwards, their masses are given as,

$$\begin{aligned} M_{\mathbf{3}}^3(b) - M(b)_{p=q=1, N_Q=0, j=1/2} - 2m_H \\ &\simeq -0.489\sqrt{3 - b} M_{KK}. \end{aligned} \quad (6.6)$$

For the baryon states binding N_Q heavy quarks (Q) and $N_{\bar{Q}}$ anti-heavy quarks (\bar{Q}), the dependence of the D0 charge could be obtained by following the same analysis above. For example, the lowest states are characterized by $N_Q = N_{\bar{Q}} = 1, n_\rho, n_Z = 0, 1, (p, q, j) = (1, 1, \frac{1}{2})$ for the $\mathbf{8}$ -plet representation with J^π assignments $\frac{1}{2}^-, \frac{3}{2}^-$ and $(p, q, j) = (3, 0, \frac{3}{2})$ for $\mathbf{10}$ -plet representation $\frac{5}{2}^-, \frac{3}{2}^-, \frac{1}{2}^-$. So the mass splitting is given as,

$$M_{\mathbf{10}}^{10}(b) - M_{\mathbf{8}}^8(b) = 0.273\sqrt{3 - b} M_{KK}. \quad (6.7)$$

Finally, we expect this model could capture the qualitative behavior of $\bar{\kappa}$ (or θ angle) in QCD-like or Yang–Mills theory when the heavy-light interaction is involved since the baryon spectra demonstrate the behavior of baryons as discussed in two-flavor case [55, 58]. While this model theoretically

describes the influence of the glue condensate in the baryon states, it has to be further confirmed with more experimental data.

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