Eur. Phys. J. C (2018) 78:364
https://doi.org/10.1140/epjc/s10052-018-5834-9

THE EUROPEAN
PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Estimating decay rate of $X^{\pm}(5568) \to B_s\pi^{\pm}$ while assuming them to be molecular states

Hong-Wei Ke[1,a], **Xue-Qian Li**[2,b]

[1] School of Science, Tianjin University, Tianjin 300072, China
[2] School of Physics, Nankai University, Tianjin 300071, China

Received: 26 February 2018 / Accepted: 21 April 2018 / Published online: 8 May 2018

Abstract Discovery of $X(5568)$ brings up a tremendous interest because it is very special, i.e. made of four different flavors. The D0 collaboration claimed that they observed this resonance through portal $X(5568) \to B_s\pi$, but unfortunately, later the LHCb, CMS, CDF and ATLAS collaborations' reports indicate that no such state was found. Almost on the Eve of 2017, the D0 collaboration reconfirmed existence of $X(5568)$ via the semileptonic decay of B_s. To further reveal the discrepancy, supposing $X(5568)$ as a molecular state, we calculate the decay rate of $X(5568) \to B_s\pi^+$ in an extended light front model. Numerically, the theoretically predicted decay width of $\Gamma(X(5568) \to B_s\pi^+)$ is 20.28 MeV which is consistent with the result of the D0 collaboration ($\Gamma = 18.6^{+7.9}_{-6.1}(stat)^{+3.5}_{-3.8}(syst)$ MeV). Since the resonance is narrow, signals might be drowned in a messy background. In analog, two open-charm molecular states DK and BD named as X_a and X_b, could be in the same situation. The rates of $X_a \to D_s\pi^0$ and $X_b \to B_c\pi^0$ are estimated as about 30 and 20 MeV respectively. We suggest the experimental collaborations round the world to search for these two modes and accurate measurements may provide us with valuable information.

1 Introduction

Following discovery of numbers of X, Y, Z particles [1–10], whose exotic behaviors cannot be interpreted by the regular $q\bar{q}'$ structures and must be attributed to a new type, either four-quark states or hybrids structures, the discussion on them becomes a hot topic of the hadron physics. For the four-quark states, there are several possibilities: molecular state which is made of two color-singlet mesons; tetraquark which consists of a color-anti-triplet diquark and a color-triplet anti-diquark, or a mixture of the previous two. All the possibilities are under intensive discussions from various angles.

Mostly, the observed exotic X, Y, Z states are composed of hidden charm or bottom flavors. In 2016 the D0 collaboration declared to have observed a new resonance $X(5568)$ at the $B_s\pi^{\pm}$ invariant mass spectrum with the mass and width being $(5567.8 \pm 2.9^{+0.9}_{-1.9})$ and $(21.9 \pm 6.4^{+5.0}_{-2.5})$ MeV [11]. Since the decay rate of $X(5568) \to B_s\pi^{\pm}$ is much larger than that determined by weak interactions, one can assure that this is a decay caused by strong interaction. Since for the strong interaction, flavor components do not change and the final state includes B_s whose quark-component is $(\bar{b}s)$ and π^+ made of $u\bar{d}$, so in the final state there are four different flavors which cannot be created from vacuum, thus one can confirm that $X(5568)$ is a four-quark state which consists of $\bar{b}su\bar{d}$ ingredients. Analysis implies $X(5568)$ to be an exotic state (if it indeed exists), but whether it is a molecule or a tetraquark would be another open question and need to be answered by precise measurements combining with careful theoretical studies. In this work, we investigate its inner structure via studying its decay behavior.

Unfortunately, the LHCb collaboration [12], the CMS collaboration of LHC [13], the CDF collaboration of Fermilab [14] and the ATLAS Collaboration of LHC [15] claimed that no such decay mode was detected. Of course, all experimentalists are very careful, so that they only offered upper bounds on the decay channel. Just on the Eve of new year, the D0 collaboration declared that $X(5568)$ was reconfirmed in the portal $X(5568) \to B_s\pi^{\pm}$ via a sequent semileptonic decay of $B_s^0 \to \mu^{\pm}D_s^{\mp}$ [16] and the result is consistent with the previous data which were obtained with $B_s \to J/\psi\phi$, but the measured width is slightly shifted to $18.6^{+7.9}_{-6.1}(stat)^{+3.5}_{-3.8}(syst)$ MeV. The acute discrepancy among the experimental groups stimulates a dispute. Because $X(5568)$ may be the first observed exotic state possessing four different flavors, studies on it (both theoretical

---

[a] e-mail: khw020056@hotmail.com
[b] e-mail: lixq@nankai.edu.cn

and experimental) are of obvious significance for getting a better understanding of the quark model.

In literature [17–31], there are different opinions which originate from different considerations. In various models, the spectrum of $X(5568)$ was computed to be compared with the measured value. Naively, by its decay width it seems to be a molecular state of BK [21,31] and its binding energy is about 205 MeV which is a bit too large for binding two mesons into a hadronic molecule based on our intuition, thus an alternative suggestion is that it is a tetraquark [22–24]. The authors of Ref. [32] regard that neither a molecular state nor a tetraquark can explain the data, so they consider that $B_s\pi$ is produced in an electroweak decay where an extra hadron is also created, but evades detection.

In this work, accepting the D0 analysis that $X(5568)$ indeed exists, we would ask which structure is more preferred by the nature, it should be answered by fitting more data besides the mass spectrum, namely one needs to investigate its decay behaviors. Thus a careful computation on its decay rate is absolutely necessary even though such a calculation is somehow model-dependent. In fact, a few groups of authors assumed $X(5568)$ as a tetraquark and computed the rate of $X(5568) \to B_s\pi$ in terms of the QCD sum rules [33–35].

Different inner structures may result in different decay rates for a designated channel. Theoretically assigning the molecular structure to $X(5568)$, we can predict its decay rate to $B_s\pi$. Since strong interaction is blind to quark flavors, the running effective coefficients for b and c quarks do not deviate much from each other. By the heavy flavor symmetry, one believes that at the leading order, the binding energies for BK and DK are the same and the symmetry breaking should occur at $(\frac{1}{m_c} - \frac{1}{m_b})$ corrections. As noted, the binding energy for BD might be different from that of BK. Even though the SU(2) symmetry between c and s quarks is not a good one, the deviation does not prevent us to make a rough estimate on the binding energy. We will study their decays while they are supposed to be molecular states and the results can be a cross check for the mysterious $X(5568)$.

In order to explore the decay rates of a molecular state, we extend the light front quark model (LFQM) which has been successfully applied for calculating decay rates of regular mesons and baryons [36–56]. Using the method and the parameters obtained by fitting well measured data, we deduce the corresponding transition matrix element and estimate the decay widths of $X(5568) \to B_s\pi^+$. Then, we further estimate decay rates of $X_a \to D_s\pi^0$ and $X_b \to B_c\pi^0$ in terms of the same method where X_a and X_b are the molecular states consisting of DK and BD constituents respectively.

After the introduction we derive the amplitude for transition $X(5568) \to B_s\pi^+$, $X_a \to D_s\pi^0$ and $X_b \to B_c\pi^0$ in Sect. 2. Then we numerically evaluate their decay widths in Sect. 3. In the last section we discuss the numerical results and draw our conclusion. Some details about the approach are collected in the Appendix.

2 The strong decays $X(5568)$, X_a and X_b

2.1 The strong decays $X(5568) \to B_s\pi^+$

In this section we calculate the decay rate of $X(5568)^+ \to B_s\pi^+$, while assuming $X(5568)$ as a $B\bar{K}$ molecular state whose quantum number $I(J^P)$ is $0(0^+)$, in the light-front model. Because of successful applications of the method to study strong decay processes of molecular states [57] we apply the framework to the present case. The configuration of the concerned BK molecular state is $\frac{1}{\sqrt{2}}(B^0K^+ + B^+K^0)$ [19]. The Feynman diagrams for $X(5568)$ decaying into $B_s\pi^+$ by exchanging B^{*0} ($\bar{B}^{*0}$) or K^{*+} (K^{*-}) mesons are shown in Fig. 1.

Following Ref. [52], the hadronic matrix element corresponding to the diagrams in Fig. 1 is written as

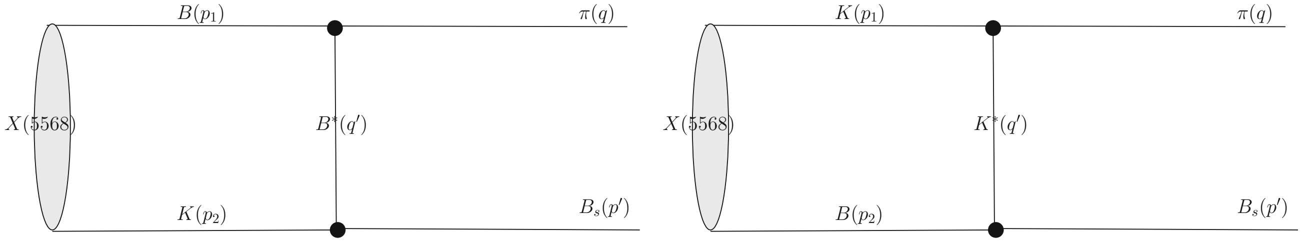
$$\mathcal{A}_1 = i\frac{1}{(2\pi)^4}\int d^4p_1 \frac{H_A(S^{(a)} + S^{(b)})}{N_1 N_1' N_2} \tag{1}$$

with

$$\begin{aligned} S^{(a)} &= -i g_{BB^*\pi} g_{KB^*B_s} g_{\alpha\beta}(p_1+q)^\alpha(2P-p_1-q)^\beta \\ &\quad \times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{B^*},q'), \\ S^{(b)} &= -2i g_{KK^*\pi} g_{K^*BB_s} g_{\alpha\beta}(p_1+q)^\alpha(2P-p_1-q)^\beta \\ &\quad \times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{K^*},q'), \end{aligned}$$

where $N_1 = p_1^2 - m_1^2 + i\varepsilon$, $N_1' = q'^2 - m_{q'}^2 + i\varepsilon$, $N_2 = p_2^2 - m_2^2 + i\varepsilon$ and P stands for the momentum of $X(5568)$. The form factor $\mathcal{F}(m_i,k^2) = \frac{(m_i+\Lambda)^2 - m_i^2}{(m_i+\Lambda)^2 - k^2}$ is introduced to compensate the off-shell effect caused by the intermediate meson of mass m_i and momentum k. The concerned normalized wavefunction of the decaying meson with the assigned quantum numbers is included in the vertex function H which is invariant in the four-dimensional space-time. In fact, for a practical computation their exact forms are not necessary, because after integrating over dp_1^- the integral is reduced into a three-dimensional one, and then H is replaced by h whose explicit form is calculable in the light-front frame. In that frame the momentum p_i is written in terms of its components as $(p_i^-, p_i^+, p_{i\perp})$ and integrating out p_1^- with the method given in Ref. [50] one has

$$\int d^4p_1 \frac{H_A S}{N_1 N_1' N_2} \to -i\pi \int dx_1 d^2p_\perp \frac{h_A \hat{S}}{x_2 \hat{N}_1 \hat{N}_1'}, \tag{2}$$



(a) **(b)**

Fig. 1 Strong decays of $X(5568)$

with

$$\hat{N}_1 = x_1(M^2 - M_0{}^2),$$
$$\hat{N}_1' = x_2 q^2 - x_1 M_0{}^2 + x_1 M'^2 + 2p_\perp \cdot q_\perp,$$
$$h_A = \sqrt{x_1 x_2}(M^2 - M_0^2)h'_A$$

where M is the mass of the decaying meson and M' is the mass of the heavier one of the two produced mesons. In the expression, q is the four-momentum of the lighter meson of the decay products, while calculating the hadronic transition matrix element, we deliberately let q^2 vary within a reasonable range, then while obtaining the partial width of $X(5568) \to B_s\pi$, we set q^2 to be the on-shell mass of the produced pion as m_π^2. The factor $\sqrt{x_1 x_2}(M^2 - M_0^2)$ in h_A was introduced in literature [52]. The explicit expressions of the effective form factors h'_A are presented in the Appendix for readers' convenience.

Since we calculate the transition in the $q^+ = 0$ reference frame the zero mode contributions which come from the residues of virtual pair creation processes, were not included. To involve them, $p_{1\mu}$ and $p_{1\nu}$ in s^a must be replaced by appropriate expressions as discussed in Ref. [52], that is

$$p_{1\mu} \to \mathcal{P}_\mu A_1^{(1)} + q_\mu A_2^{(1)} \tag{3}$$

where $\mathcal{P} = P + P'$ and $q = P - P'$ with P and P' denoting the momenta of the concerned mesons in the initial and final states respectively.

For example, $S^{(a)}$ turns into a replaced form as

$$\hat{S}^{(a)} = \Big\{ - m_1{}^2 + (1 + A_1^{(1)} + A_1^{(2)})\, M^2 - M'^2 + 3\, A_1^{(1)}\, M'^2 - A_1^{(2)}\, M'^2 - N1 - A_1^{(1)}\, q^2 - A_1^{(2)}\, q^2 \Big\} \times \frac{-i g_{BB^*\pi} g_{KB^*B_s}}{m_{B^*}{}^2} \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{B^*}, q'). \tag{4}$$

Some notations such as $A_i^{(j)}$ and M_0' can be found in Ref. [52]. With the replacement the amplitude $\mathcal{A}$ can be calculated numerically.

2.2 The decay rate of $X_a \to D_s\pi^0$

Now we turn to study the decays of molecules with an open charm. The formulas are similar to that in the case of open-bottom molecules.

Due to the quark structure, decay $X_a \to D_s\pi^0$ realizes via strong interaction. The supposed molecular state DK (X_a) is structured as $\frac{1}{\sqrt{2}}(D^0K^+ + D^+K^0)$. The Feynman diagrams for $X_a \to D_s\pi^0$ are shown in Fig. 2. The corresponding $S^{(a)}$ and $S^{(b)}$ are

$$S^{(a)} = -i g_{DD^*\pi} g_{KD^*D_s} g_{\alpha\beta}(p_1 + q)^\alpha (2P - p_1 - q)^\beta \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q'),$$
$$S^{(b)} = -2i g_{KK^*\pi} g_{K^*DD_s} g_{\alpha\beta}(p_1 + q)^\alpha (2P - p_1 - q)^\beta \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{K^*}, q'),$$

2.3 The decay rate of $X_b \to B_c\pi^0$

The molecular state BD (X_b) is structured as $\frac{1}{\sqrt{2}}(\bar{D}^0B^- + D^-B^0)$. The Feynman diagrams for the decay $X_b \to B_c\pi^0$ is shown in Fig. 3. The corresponding $S^{(a)}$ and $S^{(b)}$ should be modified as

$$S^{(a)} = -i g_{BB^*\pi} g_{DB^*B_c} g_{\alpha\beta}(p_1 + q)^\alpha (2P - p_1 - q)^\beta \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{B^*}, q'),$$
$$S^{(b)} = -i g_{DD^*\pi} g_{D^*BB_c} g_{\alpha\beta}(p_1 + q)^\alpha (2P - p_1 - q)^\beta \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q').$$

3 Numerical results

3.1 For $X(5568) \to B_s\pi^+$

In this subsection we present our predictions on the decay rate of $X(5568) \to B_s\pi^+$ while all the input parameters are taken from relevant literatures.

First, we need to calculate the corresponding amplitude which was deduced in last section. The formula include some parameters which need to be priori fixed. We use the central

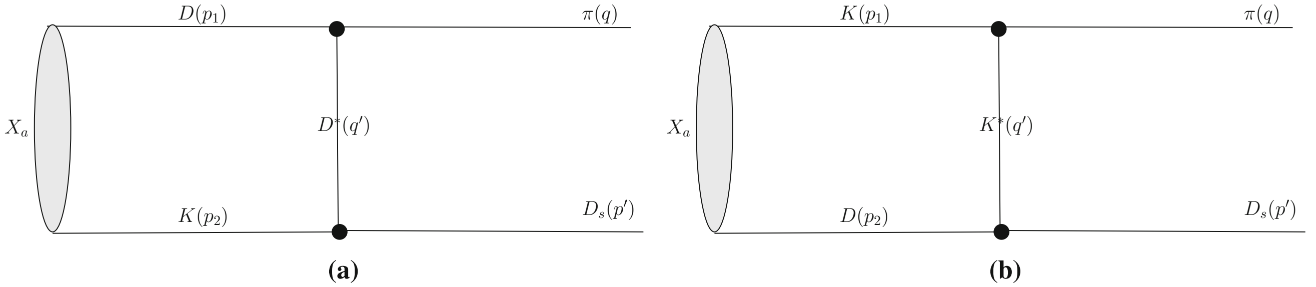


Fig. 2 Strong decays of X_a

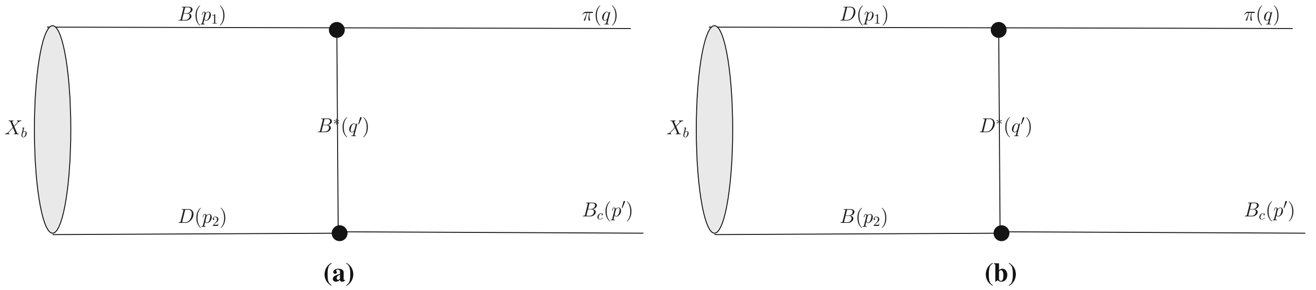


Fig. 3 Strong decays of X_b

value of the observed resonance peak 5.5678 GeV [11] as the mass of $X(5568)$. The masses of the involved mesons are set as $m_B = 5.279$ GeV, $m_{B_s} = 5.367$ GeV, $m_\pi = 0.139$ GeV, $m_{B^*} = 5.325$ GeV and $m_\rho = 0.775$ GeV according to the data book [58]. The coupling constant $g_{K^*K\pi}$ is 4.61 [59]. About the coupling constants $g_{B^*B\pi}$, $g_{K^*B_sB}$ and $g_{B^*B_sK}$ one cannot fix them from the corresponding physical processes at present but it is natural to conjecture that they would be equal to $g_{D^*D\pi}$, $g_{K^*D_sD}$ and $g_{D^*D_sK}$ respectively under the heavy quark limit and then they are set as 17.9 [59], 3.787 [60] and 2.02 [61] respectively. The cutoff parameter Λ in the vertex $\mathcal{F}$ was suggested to be 0.88 to 1.1 GeV [62]. In our calculation we vary it from 0.88 to 1.1 GeV to study how it affects the numerical results. β in the wavefunction is a free parameter, even though so far it cannot be precisely determined by phenomenological studies yet, its value can be roughly estimated to fall within a certain range. We observe that it should be close to the value for B meson which was fixed as 0.5329 GeV.

Since the amplitude is derived in the reference frame of $q^+ = 0$ ($q^2 < 0$) i.e. in the space-like region, we need to extend it to the time-like region by means of a normal procedure provided in literatures. In Ref. [52] a three-parameter form factor as

$$\mathcal{A}(q^2) = \frac{\mathcal{A}(0)}{\left[1 - a\left(\frac{q^2}{M_X^2}\right) - b\left(\frac{q^2}{M_X^2}\right)^2\right]}, \tag{5}$$

was employed in order to naturally extrapolate the formula from the space-like region to the time-like (physical) region.

Table 1 The amplitude of $X(5568) \to B_s\pi^+$ with three parameters ($\Lambda = 0.88$ GeV,)

β (GeV^{-1})	$A(0)$	a	b
0.2	$6.64i$	9.61	15.95
0.3	$8.71i$	9.62	15.93
0.4	$9.87i$	9.41	15.48
0.5329	$10.37i$	8.97	14.53
0.6	$10.36i$	8.72	13.97
0.7	$10.17i$	8.32	13.12

The resultant form factors are listed in Table 1 and the dependence of the corresponding decay width $\Gamma(X(5568) \to B_s\pi^+)$ on β is illustrated in Fig. 4. By the results, we notice that the model parameter β affects the numerical results within a tolerable range. We also explore the change of the decay width for different Λ values when one sets $\beta = 0.5329$ GeV. Since the channel $X(5568) \to B_s\pi^+$ is the dominant portal the theoretical estimation supports the allegation that $X(5568)$ is a molecular BK state, especially when $\Lambda = 0.88$GeV and $\beta = 0.5329$ GeV the estimated decay width $\Gamma(X(5568) \to B_s\pi^+)$ is close to the experimentally measured total width (Table 2).

3.2 $X_a \to D_s\pi^0$

Let us turn to discuss the decays of an open-charm molecular state via strong interaction.

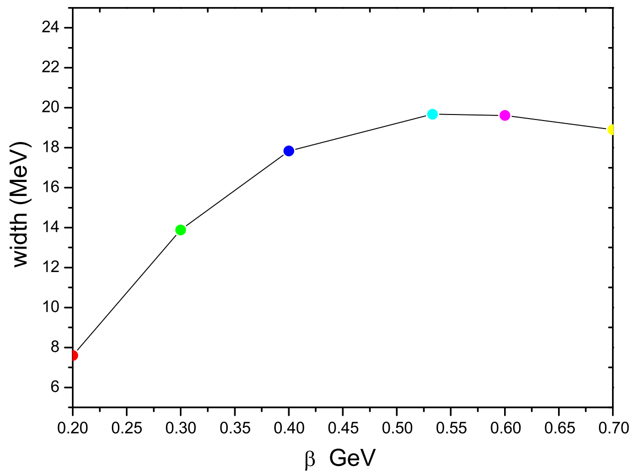


Fig. 4 The dependence of $\Gamma(X(5568) \to B_s\pi^+)$ on β

Table 2 The decay rate of $X(5568) \to B_s\pi^+$ ($\beta = 0.5329$ GeV)

Λ(GeV)	0.88	0.95	1.0	1.05	1.1
Width (MeV)	20.28	25.40	30.68	36.20	42.03

Table 3 The decay widths of $X_a \to D_s\pi^0$ ($\beta = 0.4395$ GeV)

Mass (GeV)	2.164	2.184	2.204	2.224	2.244	2.264
Width (MeV)	21.60	27.47	33.29	39.58	46.27	53.43

As the reduced mass of the DK system is slightly smaller than that for the BK system, the corresponding kinematic energy may be larger. Thus, with the same potential, naively, one would expect a smaller binding energy than that for the BK system (but not much because the reduced mass is closer to the mass of the lighter constituent, i.e. the K-meson) where the binding energy is determined to be 205 MeV as it is considered as an X(5568) molecule. In our concrete numerical computations, we let the adopted binding vary from 100 to 200 MeV.

The masses $m_D = 1.8696$ GeV, $m_{D^*} = 2.010$ GeV and $m_{D_s} = 1.968$ GeV are taken from the Databook [58]. A naive consideration suggests that the parameter β is close to that for D_s which is 0.4395 GeV, meanwhile we set the cutoff parameter Λ to be 0.88 GeV which was obtained in previous works. The mass variation covers a range from 2.164 to 2.264 GeV corresponding to the variation of binding energy from 100 to 200 MeV. The results are shown in Table 3.

3.3 $X_b \to B_c\pi^0$

Since the D meson is heavier than K meson, assuming the same arguments on the reduced mass, the binging energy of the bound state of BD (X_b) might be larger than 205 MeV which is the binding energy of BK. In our calculation (Table 4) we let it vary from 160 to 240 MeV, which is a typical energy range (close to Λ_{QCD}) for binding two mesons into a compact system. $m_{B_c} = 6.2756$ GeV is taken from Ref. [58] and the parameter β adopted for a molecule with open bottom and charm should be close to B_c. Although one cannot fix it yet from a reliable source at present, we set it to be a value between 0.631 and 1.257 GeV which are the β parameters for J/ψ and Υ respectively, namely we interpolate the β value for X_b to be 0.944 GeV. The cutoff parameter Λ is set as 0.88 GeV. If the mass of BD molecular state is close to 6.929 GeV its width is estimated to be around 20 MeV.

Table 4 The decay widths of $X_b \to B_c\pi^0$ ($\beta = 0.944$ GeV)

Mass (GeV)	6.909	6.929	6.949	6.969	6.989
Width (MeV)	16.51	17.67	18.90	20.20	21.58

4 Conclusion and discussions

Supposing $X(5568)$ to be a molecular state made by B and K mesons (BK), we calculate the decay rate of $X(5568) \to B_s\pi^+$ in the light front model. Inside the four-quark molecule, the two constituents interact by exchanging corresponding mesons (scalar and/or vector). In this phenomenological study, the model parameters Λ and β are not fully determined yet at present, so we vary them within a reasonable range in the numerical computations. Numerically when $\Lambda = 0.88$ GeV and $\beta = 0.5329$ GeV are chosen, we obtain the rate of $X(5568) \to B_s\pi$ as 20.28 MeV which is consistent with the new data measured by the D0 collaboration $\Gamma = 18.6^{+7.9}_{-6.1}(stat)^{+3.5}_{-3.8}$ MeV. The consistency somewhat supports the allegation that $X(5568)$ is a molecular state composing of B and K mesons.

As long as $X(5568)$ is a molecular state of BK one can expect two similar states of DK and BD which are named as X_a and X_b in this work. The widths of X_a and X_b are estimated in the same theoretical framework as roughly 30 and 20 MeV respectively. The results do not sensitively depend on the choices of the binding energies. It is worth of putting effort to search for $X_a \to D_s\pi^0$ and $X_b \to B_c\pi^0$ reactions in sensitive experimental facilities. It is of obvious theoretical significance, namely a definite conclusion would help to clarify if such molecular states are favored by the Nature.

$X(5568)$ is indeed facing an eccentric situation, namely, the D0 collaboration reconfirmed their observation of $X(5568)$ at the channel $B_s\pi^\pm$ whereas LHCb, CMS, ATLAS and CDF collaborations all gave negative reports. The sharp discrepancy might be due to a wrong experimental treatment, but there is still a slim possibility that both measurements are reasonable because all the measurements with negative conclusion only gave upper bounds of the rate. Actually, one should make a theoretical investigation towards the mysterious exotic hadron, i.e independent of the experimental data anyway. As a matter of fact, from the theoretical aspect, there is no rule to forbid existence of a four-quark state with four different flavors such as $X(5568)$. Following the lessons we learned from the structures of X, Y, Z exotic states, it is natural to assume a molecular state composed of $\bar{b}$, s, u, $\bar{d}$ whose main decay portal is $B_s\pi$. In this work we used the LFQM to calculate the decay rate of such a molecule ($X(5568)$) into $B_s\pi$, while another group [31] has also calculated this rate based on the molecule assumption in terms of the Bethe–Salpeter equation. Their results are qualitatively consistent with ours and the data measured by the D0 collaboration. Interesting, some theoretical groups calculated the decay rate based on the tetraquark assumption and obtained results of the same order of magnitude. All these theoretical studies indicate that $X(5568)$ still may exist, i.e the possibility cannot be simply negated. However, the discrepancy between the D0 collaborations with the others persists and must be taken serious, a reasonable interpretation might be needed. We believe that this mist would be clarified by the efforts of both theorists and experimentalists soon.

Acknowledgements This work is supported by the National Natural Science Foundation of China (NNSFC) under the Contract nos. 11375128 and 11675082.

Appendix A: The vertex function of molecular state

The wavefunction of a molecular state with total spin J and momentum P is [57]

$$|X(P, J, J_z)\rangle = \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}\, 2(2\pi)^3\delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \times \sum_{\lambda_1} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) \times \mathcal{F} \mid B(p_1, \lambda_1)K(p_2, \lambda_2)\rangle. \quad \text{(A1)}$$

For 0^+ molecular state of BK

$$\Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) = C_0\varphi(x, p_\perp) \equiv h'_{C_0} \quad \text{(A2)}$$

where C_0 is the normalization constants which can be fixed by normalizing the state [52]

$$\langle X(P', J', J'_z)|X(P, J, J_z)\rangle = 2(2\pi)^3 P^+\delta^3(\tilde{P}' - \tilde{P})\delta_{JJ'}\delta_{J_Z J_{Z'}}, \quad \text{(A3)}$$

and let the normalization $\int \frac{dxd^2p_\perp}{2(2\pi)^3}\varphi'^*_{L',L'_Z}(x, p_\perp)\varphi_{L,L_Z}(x, p_\perp) = \delta_{L,L'}\delta_{L_Z,L'_Z}$ hold.

C_0 is fixed by calculating Eq. (A3)

$$\int \frac{dxd^2p_\perp}{2(2\pi)^3} C_0^2\varphi^*(x, p_\perp)\varphi(x, p_\perp) = 1, \quad \text{(A4)}$$

then $C_0 = 1$. It is noted that $P^2 = M_0^2$, $p1 \cdot P = e_1M_0$ and $p2 \cdot P = e_2M_0$ are used as discussed in Ref. [52] and $\varphi = 4(\frac{\pi}{\beta^2})^{3/4}\sqrt{\frac{e_1e_2}{x_1x_2M_0}}\exp(\frac{-\mathbf{p}^2}{2\beta^2})$.

All other notations can be found in Refs. [42–45].

Appendix B: The effective vertices

The effective vertices can be found in [59],

$$\mathcal{L}_{B^*B\pi} = \frac{g_{B^*B\pi}}{2\sqrt{2}}(iB^{*\mu\dagger}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\partial_\mu\bar{B} - iB^{*\mu\dagger}\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\bar{B} + h.c.), \quad \text{(B1)}$$

$$\mathcal{L}_{K^*K\pi} = \frac{g_{K^*K\pi}}{\sqrt{2}}(iK^{*\mu\dagger}\boldsymbol{\tau}\cdot\boldsymbol{\pi}\partial_\mu\bar{K} - iK^{*\mu\dagger}\boldsymbol{\tau}\cdot\partial_\mu\boldsymbol{\pi}\bar{K} + h.c.), \quad \text{(B2)}$$

$$\mathcal{L}_{B^*B_sK} = g_{B^*B_sK}(iB^{*\mu\dagger}K\partial_\mu\bar{B}_s^0 - iB^{*\mu\dagger}\partial_\mu K\bar{B}_s^0 + h.c.), \quad \text{(B3)}$$

$$\mathcal{L}_{K^*B_sB} = g_{K^*B_sB}(iK^{*\mu\dagger}B\partial_\mu\bar{B}_s^0 - iK^{*\mu\dagger}\partial_\mu B\bar{B}_s^0 + h.c.), \quad \text{(B4)}$$

where $\boldsymbol{\tau}$ is usual Pauli matrix. For more details please refer to Ref. [59].

References

1. K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. **98**, 082001 (2007). arXiv:hep-ex/0507019
2. S.K. Choi et al. (Belle Collaboration), Phys. Rev. Lett. **94**, 182002 (2005)
3. S.K. Choi et al. (BELLE Collaboration), Phys. Rev. Lett. **100**, 142001 (2008). arXiv:0708.1790 [hep-ex]
4. B. Aubert et al. (BaBar Collaboration), Phys. Rev. Lett. **95**, 142001 (2005). https://doi.org/10.1103/PhysRevLett.95.142001. arXiv:hep-ex/0506081
5. M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. **112**, 132001 (2014). arXiv:1308.2760 [hep-ex]
6. M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. **111**, 242001 (2013). arXiv:1309.1896 [hep-ex]
7. M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. **110**, 252001 (2013). arXiv:1303.5949 [hep-ex]
8. M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. **113**, 212002 (2014). https://doi.org/10.1103/PhysRevLett.113.212002. arXiv:1409.6577 [hep-ex]
9. Z.Q. Liu et al. (Belle Collaboration), Phys. Rev. Lett. **110**, 252002 (2013). arXiv:1304.0121 [hep-ex]
10. Belle Collaboration, arXiv:1105.4583 [hep-ex]

11. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. Lett. **117**(2), 022003 (2016). arXiv:1602.07588 [hep-ex]
12. R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. **117**(15), 152003 (2016). https://doi.org/10.1103/PhysRevLett.117.152003. arXiv:1608.00435 [hep-ex] [Addendum: Phys. Rev. Lett. **118**(10), 109904 (2017). https://doi.org/10.1103/PhysRevLett.118.109904]
13. A.M. Sirunyan et al. (CMS Collaboration), arXiv:1712.06144 [hep-ex]
14. T.A. Aaltonen et al. (CDF Collaboration), arXiv:1712.09620 [hep-ex]
15. M. Aaboud et al. (ATLAS Collaboration), arXiv:1802.01840 [hep-ex]
16. V.M. Abazov et al. (D0 Collaboration), arXiv:1712.10176 [hep-ex]
17. F.K. Guo, U.G. Meißer, B.S. Zou, Commun. Theor. Phys. **65**(5), 593 (2016). https://doi.org/10.1088/0253-6102/65/5/593. arXiv:1603.06316 [hep-ph]
18. Z. Yang, Q. Wang, U.G. Meißer, Phys. Lett. B **767**, 470 (2017). https://doi.org/10.1016/j.physletb.2017.01.023. arXiv:1609.08807 [hep-ph]
19. R. Chen, X. Liu, Phys. Rev. D **94**(3), 034006 (2016). https://doi.org/10.1103/PhysRevD.94.034006. arXiv:1607.05566 [hep-ph]
20. M. Albaladejo, J. Nieves, E. Oset, Z.F. Sun, X. Liu, Phys. Lett. B **757**, 515 (2016). https://doi.org/10.1016/j.physletb.2016.04.033. arXiv:1603.09230 [hep-ph]
21. S.S. Agaev, K. Azizi, H. Sundu, Eur. Phys. J. Plus **131**(10), 351 (2016). https://doi.org/10.1140/epjp/i2016-16351-8. arXiv:1603.02708 [hep-ph]
22. F. Stancu, J. Phys. G **43**(10), 105001 (2016). https://doi.org/10.1088/0954-3899/43/10/105001. arXiv:1603.03322 [hep-ph]
23. Z.G. Wang, Commun. Theor. Phys. **66**(3), 335 (2016). https://doi.org/10.1088/0253-6102/66/3/335. arXiv:1602.08711 [hep-ph]
24. Y.R. Liu, X. Liu, S.L. Zhu, Phys. Rev. D **93**(7), 074023 (2016). arXiv:1603.01131 [hep-ph]
25. W. Wang, R. Zhu, Chin. Phys. C **40**(9), 093101 (2016). https://doi.org/10.1088/1674-1137/40/9/093101. arXiv:1602.08806 [hep-ph]
26. J.R. Zhang, J.L. Zou, J.Y. Wu, Chin. Phys. C **42**(4), 043101 (2018). https://doi.org/10.1088/1674-1137/42/4/043101. arXiv:1705.03741 [hep-ph]
27. X.W. Kang, J.A. Oller, Phys. Rev. D **94**(5), 054010 (2016). https://doi.org/10.1103/PhysRevD.94.054010. arXiv:1606.06665 [hep-ph]
28. W. Chen, H.X. Chen, X. Liu, T.G. Steele, S.L. Zhu, Phys. Rev. Lett. **117**(2), 022002 (2016). https://doi.org/10.1103/PhysRevLett.117.022002. arXiv:1602.08916 [hep-ph]
29. C.J. Xiao, D.Y. Chen, Eur. Phys. J. A **53**(6), 127 (2017). https://doi.org/10.1140/epja/i2017-12310-x. arXiv:1603.00228 [hep-ph]
30. J.X. Lu, X.L. Ren, L.S. Geng, Eur. Phys. J. C **77**(2), 94 (2017). https://doi.org/10.1140/epjc/s10052-017-4660-9. arXiv:1607.06327 [hep-ph]
31. Z.Y. Wang, J.J. Qi, C. Wang, X.H. Guo, arXiv:1802.04566 [hep-ph]
32. T.J. Burns, E.S. Swanson, Phys. Lett. B **760**, 627 (2016). arXiv:1603.04366 [hep-ph]
33. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **93**(11), 114007 (2016). https://doi.org/10.1103/PhysRevD.93.114007. arXiv:1603.00290 [hep-ph]
34. J.M. Dias, K.P. Khemchandani, A. Martínez Torres, M. Nielsen, C.M. Zanetti, Phys. Lett. B **758**, 235 (2016). https://doi.org/10.1016/j.physletb.2016.05.015. arXiv:1603.02249 [hep-ph]
35. Z.G. Wang, Eur. Phys. J. C **76**(5), 279 (2016). https://doi.org/10.1140/epjc/s10052-016-4133-6. arXiv:1603.02498 [hep-ph]
36. H.M. Choi, Phys. Rev. D **75**, 073016 (2007). arXiv:hep-ph/0701263
37. W. Jaus, Phys. Rev. D **41**, 3394 (1990)
38. W. Jaus, Phys. Rev. D **44**, 2851 (1991)
39. W. Jaus, Phys. Rev. D **60**, 054026 (1999)
40. C.R. Ji, P.L. Chung, S.R. Cotanch, Phys. Rev. D **45**, 4214 (1992)
41. H.-Y. Cheng, C.-K. Chua, C.-W. Hwang, Phys. Rev. D **70**, 034007 (2004). arXiv:hep-ph/0403232
42. H.W. Ke, X.Q. Li, Z.T. Wei, Phys. Rev. D **77**, 014020 (2008). arXiv:0710.1927 [hep-ph]
43. Z.T. Wei, H.W. Ke, X.Q. Li, Phys. Rev. D **80**, 094016 (2009). arXiv:0909.0100 [hep-ph]
44. H.-W. Ke, X.-H. Yuan, X.-Q. Li, Z.-T. Wei, Y.-X. Zhang, Phys. Rev. D **86**, 114005 (2012). arXiv:1207.3477 [hep-ph]
45. H.W. Ke, X.Q. Li, Phys. Rev. D **84**, 114026 (2011). arXiv:1107.0443 [hep-ph]
46. H.W. Ke, X.Q. Li, Z.T. Wei, Phys. Rev. D **80**, 074030 (2009). arXiv:0907.5465 [hep-ph]
47. H.W. Ke, X.Q. Li, Z.T. Wei, Eur. Phys. J. C **69**, 133 (2010). arXiv:0912.4094 [hep-ph]
48. H.W. Ke, X.H. Yuan, X.Q. Li, Int. J. Mod. Phys. A **26**, 4731 (2011). arXiv:1101.3407 [hep-ph]
49. H.W. Ke, X.Q. Li, Eur. Phys. J. C **71**, 1776 (2011). arXiv:1104.3996 [hep-ph]
50. H.Y. Cheng, C.Y. Cheung, C.W. Hwang, Phys. Rev. D **55**, 1559 (1997). arXiv:hep-ph/9607332
51. G. Li, F.L. Shao, W. Wang, Phys. Rev. D **82**, 094031 (2010). arXiv:1008.3696 [hep-ph]
52. H.Y. Cheng, C.K. Chua, C.W. Hwang, Phys. Rev. D **69**, 074025 (2004)
53. C.W. Hwang, Z.T. Wei, J. Phys. G **34**, 687 (2007)
54. C.D. Lu, W. Wang, Z.T. Wei, Phys. Rev. D **76**, 014013 (2007). arXiv:hep-ph/0701265
55. H.-W. Ke, X.-Q. Li, Y.-L. Shi, Phys. Rev. D **87**, 054022 (2013). arXiv:1301.4014 [hep-ph]
56. H.W. Ke, X.Q. Li, Z.T. Wei, X. Liu, Phys. Rev. D **82**, 034023 (2010). arXiv:1006.1091 [hep-ph]
57. H.W. Ke, Z.T. Wei, X.Q. Li, Eur. Phys. J. C **73**(10), 2561 (2013). arXiv:1307.2414 [hep-ph]
58. K.A. Olive et al. (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014)
59. G.Q. Feng, Z.X. Xie, X.H. Guo, Phys. Rev. D **83**, 016003 (2011)
60. Z.G. Wang, S.L. Wan, Phys. Rev. D **74**, 014017 (2006). arXiv:hep-ph/0606002
61. H.Y. Cheng, C.K. Chua, A. Soni, Phys. Rev. D **71**, 014030 (2005). arXiv:hep-ph/0409317
62. C. Meng, K.-T. Chao, Phys. Rev. D **75**, 114002 (2007). arXiv:hep-ph/0703205