

Reconstructing warm inflation

Ramón Herrera^a

Instituto de Física, Pontificia Universidad Católica de Valparaíso, Casilla 4059, Valparaíso, Chile

Received: 16 January 2018 / Accepted: 15 March 2018 / Published online: 21 March 2018
© The Author(s) 2018

Abstract The reconstruction of a warm inflationary universe model from the scalar spectral index $n_S(N)$ and the tensor to scalar ratio $r(N)$ as a function of the number of e-folds N is studied. Under a general formalism we find the effective potential and the dissipative coefficient in terms of the cosmological parameters n_S and r considering the weak and strong dissipative stages under the slow roll approximation. As a specific example, we study the attractors for the index n_S given by $n_S - 1 \propto N^{-1}$ and for the ratio $r \propto N^{-2}$, in order to reconstruct the model of warm inflation. Here, expressions for the effective potential $V(\phi)$ and the dissipation coefficient $\Gamma(\phi)$ are obtained.

1 Introduction

It is well known that during the evolution of the early universe, it exhibited an accelerated expansion or an inflationary scenario commonly called the inflationary universe [1, 2]. A crucial characteristic of the inflationary universe is that this scenario explicates the Large-Scale Structure (LSS) of the universe, and also the source of the anisotropies observed in the Cosmic Microwave Background (CMB) radiation [3–6]. Although, inflation originally was proposed to solve some problems of the standard hot big-bang model such as; the flatness, horizon, among other [1, 2].

In the context of the different models that give account of the inflationary universe and its early evolution, we can distinguish the model of warm inflation. In the framework of warm inflation, the universe is described by a self-interacting radiation field and a field scalar or inflaton field. In contradiction to the standard cold inflation, the model of warm inflation has the attractive feature that it avoids the reheating period, because the radiation production takes place concurrently together with the inflationary expansion driven by the scalar field [7–11]. This is possible through a friction

term enclosed on the dynamical equations and this term describes the processes of the scalar field dissipating into a thermal bath with other fields. In this sense, the scenario of warm inflation ends whenever the universe stops inflating and softly goes into the radiation epoch of the standard big-bang model.

Another difference of warm inflation in relation to the cold inflation are the initial fluctuations essential for the LSS formation. In fact, during the development of warm inflation the thermal fluctuations have a fundamental role in the LSS formation and the density fluctuations from the scalar field arise from thermal rather than quantum fluctuations [12–16]. Thus, from the background dynamics and initial fluctuations, the stage of warm inflation differs substantially from the cold inflation (or the standard inflation) [17–27]. For a review of models of warm inflation, see e.g. Refs. [11, 17–36] and for a list of recent articles, see [37–45].

On the other hand, the reconstruction of the effective potential in the evolution of cold inflation from observational data such as the scalar spectrum, scalar spectral index n_S and the tensor to scalar ratio r , have been discussed by several authors [46–56]. Here, we mentioned that the reconstruction of inflationary potentials for the case of a single scalar field assuming the primordial scalar spectrum was first made in Ref. [46], in which the general slow-roll approximation and without considering any specific form of the scalar spectrum index were assumed. An interesting mechanism in order to construct the effective potential of inflation assuming the slow roll approximation, is through the parametrization of the cosmological parameters or attractors $n_S(N)$ and $r(N)$, where N corresponds to the number of e-folds. The observational tests from Planck data [57] are in good accord with the parametrization on the scalar spectral index given by $n_S \sim 1 - 2/N$ and the tensor to scalar ratio $r \propto N^{-2}$, assuming that the number $N \simeq 50 - 70$ at the end of the inflationary epoch. For large N ($N \gg 1$) the attractor $n_S(N) \sim 1 - 2/N$ together with different expressions for the tensor to scalar ratio $r(N)$

^ae-mail: ramon.herrera@pucv.cl

can be deduced from different models in the case of cold inflation such as; the T-model [58], E-model [59], Staronbisky R^2 -model [1], the chaotic model [60], the model of Higgs inflation with non minimal coupling [61–63] among other.

On the other hand, it is also possible to consider the slow-roll parameter ϵ and its parametrization in terms of N , in order to obtain the effective potential, scalar spectral index and the tensor to scalar ratio in models of cold inflation [52, 64–66]. In particular in Ref. [52] was studied different types of slow-roll parameter $\epsilon(N)$ and thereby reconstructing the effective potential. Also, from the two slow roll parameters $\epsilon(N)$ and $\eta(N)$ the effective potential was reconstructed in Ref. [67]. Analogously, in Refs. [68,69] related results are obtained for the reconstruction.

The objective of this article is to reconstruct the model of the warm inflation, considering the parametrization of the cosmological parameters as the scalar spectral index and the tensor to scalar ratio in terms of the number of e-folds. In this context, we analyze how the background dynamics in which there is a self-interacting scalar field and radiation affects the reconstruction of the effective potential and the dissipative coefficient from the attractors. Under a general formalism, we will build the potential and dissipative coefficient during the scenario of the weak and strong dissipative regimes from the attractors $n_S(N)$ and $r(N)$. Also, for the reconstruction the model of warm inflation we will consider the weak and strong dissipative regimes assuming the slow roll approximation.

In order to reconstruct analytical quantities for the potential and dissipative coefficient, we will study a concrete example for the cosmological parameters $n_S(N)$ and $r(N)$. Here, we will consider the attractors $n_s - 1 \propto 1/N$ and $r \propto 1/N^2$, for two regimes during the stages of warm inflation. In both scenarios, we will find the potential and dissipative coefficient together with the constraints on the different parameters assuming the condition for the weak regime and the strong regime, respectively.

The outline of the article is as follows: the next section presents a short review of the basic equations during the stage of warm inflation. In Sect. 3 we discuss the reconstruction in the framework of warm inflation. In Sects. 4 and 5 we obtain under a general formalism, explicit expressions for the effective potential and dissipative coefficient in terms of the number of e-folds N during the weak and strong dissipative regimes, respectively. In Sect. 6 we discuss a concrete example for our model, in which we consider the specific attractors for $n_S(N)$ and $r(N)$, in order to find the potential $V(\phi)$ and the coefficient $\Gamma(\phi)$. Here, we analyze the reconstruction in the weak and strong dissipative regimes, respectively. Finally, our conclusions are presented in Sect. 7. We chose units so that $c = \hbar = 8\pi G = 1$.

2 Warm inflation: basic relations

We start by writing down the Friedmann equation in the framework of the warm inflation, by considering a spatially flat Friedmann–Robertson–Walker (FRW) metric, together with a scalar field homogeneous and radiation. In this sense, the Friedmann equation is given by

$$H^2 = \frac{1}{3} \rho = \frac{1}{3} [\rho_\phi + \rho_\gamma], \quad (1)$$

where $H = \dot{a}/a$ denotes the Hubble parameter and the quantity a corresponds to the scale factor. During the scenario of warm inflation we assume a two-component system, a scalar field homogeneous $\phi = \phi(t)$ with an energy density ρ_ϕ and a radiation field of energy density ρ_γ . Here, the total energy density $\rho = \rho_\phi + \rho_\gamma$, where the energy density ρ_ϕ in terms of the scalar field is defined by $\rho_\phi = \dot{\phi}^2/2 + V$, where V denotes the effective potential. In the following, we will consider that the dots correspond to differentiation with respect to the time.

As previously mentioned in the framework of the warm inflation, the universe is filled with a self-interacting scalar field and radiation, and the basic equations for the densities ρ_ϕ and ρ_γ are given by [7,8]

$$\begin{aligned} \dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) &= -\Gamma \dot{\phi}^2, \quad \text{or equivalently} \\ \ddot{\phi} + [3H + \Gamma]\dot{\phi} &= -\frac{\partial V}{\partial \phi} = -V_{,\phi}, \end{aligned} \quad (2)$$

and

$$\dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma\dot{\phi}^2. \quad (3)$$

Here, we mention that the continuity equation for the total energy density ρ satisfies the standard relation $\dot{\rho} + 3H(\rho + p) = 0$.

In this context, the quantity Γ refers to the dissipation coefficient and considering the second law of thermodynamics the coefficient Γ is defined as positive [7,8,12–14]. In this sense, from Eqs. (2) and (3) we interpret that the coefficient Γ gives origin to the decay of the scalar field into radiation during the inflationary epoch of the universe. The parameter Γ can be considered to be a constant or a function of the scalar field ϕ , or the temperature of the thermal bath T , or both i.e., $\Gamma = \Gamma(\phi, T)$ [7,8].

In the following we will analyze the reconstruction of the model of warm inflation, assuming that the dissipation coefficient and the effective potential depend only of the scalar field, i.e., $\Gamma = \Gamma(\phi)$ and $V = V(\phi)$, respectively.

During the inflationary epoch, the energy density of the scalar field predominates over the energy density associated to the radiation field in warm inflation, wherewith $\rho_\phi > \rho_\gamma$.

Also, considering the set of slow-roll approximations in which $\dot{\phi}^2 \ll V$ and $\ddot{\phi} \ll (3H + \Gamma)\dot{\phi}$, then the Friedmann equation (1) can be rewritten as [7, 8, 12–14]

$$H^2 \simeq \frac{1}{3} \rho_\phi \simeq \frac{1}{3} V, \tag{4}$$

and from the equation of the scalar field (2) we have

$$\dot{\phi} \simeq -\frac{V_{,\phi}}{3H(1+R)}. \tag{5}$$

Here, the quantity R corresponds to the ratio between the coefficient Γ and the Hubble parameter and is defined as

$$R = \frac{\Gamma}{3H}. \tag{6}$$

Typically, during the scenario of warm inflation we can identify two regimes; called the weak dissipative regime, in which $R \ll 1$ (or equivalently $\Gamma \ll 3H$) and the strong dissipative regime where the ratio $R \gg 1$ ($\Gamma \gg 3H$).

On the other hand, following Refs. [7, 8, 12–14], we assume that during the inflationary expansion of the universe the radiation production is quasi-stable, in which $\dot{\rho}_\gamma \ll 4H\rho_\gamma$ and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$. In this form, from Eq. (3) we find that the energy density of the radiation field becomes

$$\rho_\gamma = C_\gamma T^4 \simeq \frac{\Gamma\dot{\phi}^2}{4H} = \frac{R}{4(1+R)^2} \frac{V_{,\phi}^2}{V}, \tag{7}$$

where C_γ is a constant and is defined as $C_\gamma = \pi^2 g_*/30$, in which g_* denotes the number of relativistic degrees of freedom [7, 8, 11]. Here, we have used Eq. (5).

Also, from Eq. (7), we find that temperature of the thermal bath can be written as

$$T = \left[\frac{R}{4C_\gamma(1+R)^2} \frac{V_{,\phi}^2}{V} \right]^{1/4}. \tag{8}$$

In order to have a measure of the inflationary expansion of the universe, we define the number of e-folding N between two different values of cosmological times t and t_e , where the time t_e corresponds to the end of inflation. Thus, the number of e-folds N assuming the slow roll approximation can be written as

$$N = \int_t^{t_e} H dt' = \int_\phi^{\phi_e} H \frac{d\phi'}{\dot{\phi}} \simeq \int_{\phi_e}^\phi \frac{V(1+R)}{V_{,\phi}} d\phi'. \tag{9}$$

Here, we have considered Eqs. (4) and (5).

On the other hand, due to the presence of the radiation field in the dynamics of warm inflation, the source of the density fluctuations correspond to thermal fluctuations [7–11].

In this sense, during the evolution of the expansion inflationary, the fluctuations of the scalar field are dominantly thermal rather than quantum [7, 8, 12–16]. Thus, in the scenario of warm inflation, the curvature and entropy perturbations coexist, since the mixture of the scalar field and radiation are generate at the perturbative levels. This happens because the model of warm inflation can be viewed as a model of two basic fields [70]. However, as was demonstrated in Ref. [11], during warm inflation the entropy perturbations on the large scales decay and only the curvature (adiabatic modes) survives [7, 8, 11–16]. In this context, the power spectrum of the curvature perturbations \mathcal{P}_S during warm inflation, assuming $\Gamma = \Gamma(\phi)$ and $V = V(\phi)$ together with the slow roll approximations becomes [11, 72–74]

$$\mathcal{P}_S \simeq \frac{H^3 T}{\dot{\phi}^2} \sqrt{(1+R)}. \tag{10}$$

The scalar spectral index n_S is defined as $n_S = d \ln \mathcal{P}_S / d \ln k$ and in terms of the slow roll parameters is given by [73]

$$n_S - 1 = -\frac{(9R+17)}{4(1+R)^2} \epsilon - \frac{(9R+1)}{4(1+R)^2} \beta + \frac{3}{2} \frac{1}{(1+R)} \eta, \tag{11}$$

where the slow roll parameters ϵ , η and β are given by

$$\epsilon = \frac{1}{2} \left(\frac{V_{,\phi}}{V} \right)^2, \quad \eta = \frac{V_{,\phi\phi}}{V}, \quad \text{and} \quad \beta = \frac{V_{,\phi} \Gamma_{,\phi}}{V\Gamma}. \tag{12}$$

Here, we mention that in the case in which $\Gamma = \Gamma(\phi, T)$ and $V = V(\phi, T)$, we should add two new parameters in Eq. (11) for the scalar spectral index n_S , see Ref. [73].

On the other hand, since the tensor perturbation do not couple to the thermal background, then this suggests that the tensor modes have an equivalent amplitude to the model of cold inflation, where the tensor spectrum \mathcal{P}_T is defined as $\mathcal{P}_T = 8H^2$. Thus, the tensor to scalar ratio r in the scenario of warm inflation can be written as

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} = \frac{8\dot{\phi}^2}{HT\sqrt{1+R}} = \frac{16\epsilon}{(1+R)^{5/2}} \frac{H}{T}. \tag{13}$$

Here, we have used Eq. (5). We note that the tensor to scalar ratio r in the model of warm inflation, see Eq. (13), cannot be written only in terms of the slow-roll parameter ϵ as it occurs in cold inflation, in which $r = 16\epsilon$. Also, we observe that Eq. (13) coincides with the ratio r obtained in Ref. [74].

In the following we will study the reconstruction of the effective potential V and the dissipative coefficient Γ in the scenario of warm inflation, considering an attractor point from scalar spectral index $n_S(N)$ and the tensor to scalar ratio $r(N)$ in the $r-n_S$ plane.

3 Reconstruction

In this section and following Ref. [53], we explicate the procedure to follow in the reconstruction of the effective potential and the dissipation coefficient as a function of the scalar field ϕ in the framework of warm inflation, considering the scalar spectral index $n_S(N)$ and the tensor to scalar ratio $r(N)$ as attractors. Since we have two quantities $V(\phi)$ and $\Gamma(\phi)$ in the reconstruction, we need first of all to express the scalar spectral index and the tensor to scalar ratio in terms of the number of e-folds N . For this it is necessary to rewrite Eqs. (11) and (13) in terms of the potential and the dissipation coefficient as a function of the number of e-folds N and its derivatives. Thus, from these equations and giving $n_S = n_S(N)$ and $r = r(N)$ we should obtain the effective potential and the dissipation coefficient as a function of the number N . Posteriorly, from Eq. (9) we should find the e-folds N in terms of the scalar field in order to reconstruct the potential $V(\phi)$ and the coefficient $\Gamma(\phi)$, respectively.

We start by rewriting the index and ratio given by Eqs. (11) and (13) in terms of the number of e-folding N . In fact, the slow roll parameters can be rewritten in terms of the number of e-folds N , considering that

$$V_{,\phi} = \frac{dV}{d\phi} = \frac{V(1+R)}{V_{,\phi}} V_{,N},$$

then we get

$$V_{,\phi}^2 = V(1+R) V_{,N}, \quad \text{wherewith } V_{,N} = \frac{dV}{dN} > 0. \quad (14)$$

In the following, we will consider the subscript $V_{,N} = dV/dN$, $V_{,NN}$ to $V_{,NN} = d^2V/dN^2$, $\Gamma_{,N} = d\Gamma/dN$ etc.

Analogously for $V_{,\phi\phi}$ we have

$$V_{,\phi\phi} = \frac{1}{2V_{,N}} \left[(1+R)[V_{,N}^2 + V V_{,NN}] + V V_{,N} R_{,N} \right], \quad (15)$$

and for the quantity $\Gamma_{,\phi}$ we have

$$\Gamma_{,\phi} = \left[\frac{V(1+R)}{V_{,N}} \right]^{1/2} \Gamma_{,N}. \quad (16)$$

Thus, the slow roll parameters can be rewritten as

$$\epsilon = \frac{V_{,N}}{2V} (1+R), \quad \beta = \frac{\Gamma_{,N}}{\Gamma} (1+R), \quad (17)$$

and

$$\eta = \frac{1}{2V V_{,N}} \left[(1+R)[V_{,N}^2 + V V_{,NN}] + V V_{,N} R_{,N} \right], \quad (18)$$

respectively.

Equally, the temperature of the thermal bath T from Eq. (8) results

$$T = \left[\frac{R V_{,N}}{4C_\gamma (1+R)} \right]^{1/4}. \quad (19)$$

Here, we have used Eq. (14).

The relation between the e-folding N and the scalar field can be written as

$$\int \left[\frac{V_{,N}}{V(1+R)} \right]^{1/2} dN = \int d\phi. \quad (20)$$

In this form, the scalar spectral index n_S can be rewritten in terms of the e-folds N , considering the Eqs. (11), (17) and (18) such that

$$\begin{aligned} n_S - 1 = & -\frac{(9R+17)}{8(1+R)} \frac{V_{,N}}{V} - \frac{(9R+1)}{4(1+R)} \frac{\Gamma_{,N}}{\Gamma} \\ & + \frac{3}{4} \frac{1}{(1+R)} \frac{1}{V V_{,N}} \\ & \times \left[(1+R)[V_{,N}^2 + V V_{,NN}] + V V_{,N} R_{,N} \right]. \end{aligned} \quad (21)$$

For the tensor to scalar ratio we have

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S} = \frac{8 V_{,N}}{(1+R)^{3/2}} \frac{1}{\sqrt{3V} T}, \quad (22)$$

where the temperature of the thermal bath T is given by Eq. (19). Here, we have considered Eqs. (13) and (17).

In the following, we will restrict ourselves to the weak and strong dissipation regimes in order to reconstruct under a general formalism the model of warm inflation from the cosmological parameters $n_S(N)$ and $r(N)$.

4 The weak dissipative regime

We begin by considering the reconstruction for the case in which the model of warm inflation evolves in the weak scenario in which the dissipation coefficient $\Gamma \ll 3H$ (or equivalently $R \ll 1$). During this regime and considering Eq. (20), the relation between the e-folding N and the scalar field, is given by

$$\int \left[\frac{V_{,N}}{V} \right]^{1/2} dN = \int d\phi. \quad (23)$$

On the other, from Eq. (11) the spectral index n_S when $R \ll 1$ becomes

$$n_S - 1 = -\frac{17}{4} \epsilon - \frac{1}{4} \beta + \frac{3}{2} \eta, \quad (24)$$

and by considering the slow roll parameters given by Eqs. (17) and (18), then the scalar spectral index in this regime can be rewritten as

$$n_S - 1 = \frac{1}{4} \left[-\frac{11 V_{,N}}{2V} + 3 \frac{V_{,NN}}{V_{,N}} - \frac{\Gamma_{,N}}{\Gamma} \right] \tag{25}$$

$$= \frac{1}{4} \left[\ln \left(\frac{V_{,N}^3}{V^{11/2} \Gamma} \right) \right]_{,N}$$

From Eq. (13) the tensor to scalar ratio r during this scenario can be written as

$$r = 8 \left[\frac{4\sqrt{3} C_\gamma}{9} \frac{V_{,N}^3}{V^{3/2} \Gamma} \right]^{1/4} \tag{26}$$

Here, we have used that the temperature of the thermal bath is given by $T = \left[\frac{\Gamma V_{,N}}{4\sqrt{3} C_\gamma V^{1/2}} \right]^{1/4}$, during this scenario.

On the other hand, by combining Eqs. (25) and (26) we find that the effective potential in terms of the e-folds N can be written as

$$V(N) = V = \frac{r}{\tilde{C}_\gamma^{1/4}} \exp \left[\int (1 - n_S) dN \right], \tag{27}$$

and the coefficient dissipation $\Gamma(N)$ becomes

$$\Gamma(N) = \Gamma = \frac{\tilde{C}_\gamma}{r^4} \left[\frac{V_{,N}}{V^{1/2}} \right]^3$$

$$= \left[\frac{\tilde{C}_\gamma^{1/4}}{r} \right]^{5/2} \left[\frac{r_{,N}}{r} + (1 - n_S) \right]^3$$

$$\times \exp \left[\frac{3}{2} \int (1 - n_S) dN \right], \tag{28}$$

where the constant \tilde{C}_γ is defined as $\tilde{C}_\gamma = 8^5 \sqrt{3} C_\gamma / 18$.

Here, we mention that the Eqs. (23), (27) and (28) are the fundamental equations in order to reconstruct of the effective potential $V(\phi)$ and of the dissipation coefficient $\Gamma(\phi)$, during this regime from the attractors $n_S(N)$ and $r(N)$.

5 The strong dissipative regime

Now, we assume that the model of warm inflation evolves in the strong dissipative regime in which the coefficient $\Gamma \gg 3H$. Thus, from Eq. (20) the relation between the e-folding N and the scalar field during this regime, is given by

$$\int \left[\frac{V_{,N}}{V R} \right]^{1/2} dN = \int \left[\frac{\sqrt{3} V_{,N}}{\sqrt{V} \Gamma} \right]^{1/2} dN = \int d\phi. \tag{29}$$

By considering Eq. (11) the spectral index in this regime becomes

$$n_S - 1 = -\frac{9}{4R} \epsilon - \frac{9}{4R} \beta + \frac{3}{2R} \eta, \tag{30}$$

and taking into account the slow roll parameters in this scenario ($R \gg 1$), then the scalar spectral index n_S can be rewritten as

$$n_S - 1 = \frac{3}{4} \left[-\frac{V_{,N}}{2V} + \frac{V_{,NN}}{V_{,N}} + \frac{R_{,N}}{R} - 3 \frac{\Gamma_{,N}}{\Gamma} \right]$$

$$= \frac{3}{4} \left[\ln \left(\frac{V_{,N} R}{V^{1/2} \Gamma^3} \right) \right]_{,N}, \tag{31}$$

o equivalently

$$n_S - 1 = \frac{3}{4} \left[\ln \left(\frac{V_{,N}}{3^{1/2} V \Gamma^2} \right) \right]_{,N}. \tag{32}$$

Here, we have considered Eqs. (4) and (6).

During this regime, the tensor to scalar ratio r results

$$r = \frac{8V_{,N}}{R^{3/2}} \frac{1}{\sqrt{3V} T} = \frac{1}{\Gamma^{3/2}} \left[\tilde{C}_\gamma V V_{,N}^3 \right]^{1/4}. \tag{33}$$

Here, we have used that the temperature of the thermal bath T in the strong regime is given by $T = (V_{,N}/4C_\gamma)^{1/4}$ and the constant \tilde{C}_γ is defined as $\tilde{C}_\gamma = 8^5 3 C_\gamma / 2$.

By combining Eqs. (32) and (33) we find that the effective potential $V(N)$ can be written as

$$V(N) = \frac{r}{3^{3/8} \tilde{C}_\gamma^{1/4}} \exp \left[\int (1 - n_S) dN \right]. \tag{34}$$

Curiously, we find that the potentials $V(N)$ in the weak and strong dissipative regime have the same structure, i.e., $V(N) \sim r \exp[\int (1 - n_S) dN]$, see Eqs. (27) and (34).

The dissipative coefficient Γ in terms of the e-folds N can be expressed as

$$\Gamma(N) = \frac{\tilde{C}_\gamma^{1/6}}{r^{2/3}} (V^{1/3} V_{,N})^{1/2}$$

$$= \frac{1}{3^{1/4}} \left[\frac{r_{,N}}{r} + (1 - n_S) \right]^{1/2} \exp \left[\frac{2}{3} \int (1 - n_S) dN \right]. \tag{35}$$

Thus, we find that the dissipation coefficient given by Eq. (35) is different from the obtained in the weak regime, see Eq. (28). Also, we observe that the coefficient $\Gamma(N)$ does not depend of the constant C_γ , during the strong dissipative regime.

Again, we refer to that the Eqs. (29), (34) and (35) are the fundamental expressions in order to reconstruct of the

effective potential $V(\phi)$ and $\Gamma(\phi)$ from the quantities $n_S(N)$ and $r(N)$, during the strong dissipative regime.

6 An example

In this section we apply the formalism of above to the two dissipative regimes (weak and strong) in the scenario of warm inflation, considering the simplest example for the cosmological quantities $n_S(N)$ and $r(N)$, in order to reconstruct analytically the effective potential $V(\phi)$ and dissipative coefficient $\Gamma(\phi)$. Following, Refs. [1, 53, 58] we consider that the spectral index is given by

$$n_S - 1 = -\frac{2}{N}, \tag{36}$$

and the tensor to scalar ratio as

$$r = \frac{1}{N(1 + \xi N)}, \tag{37}$$

where ξ corresponds to a constant. In this sense, if we consider that in particular the number N before the end of inflationary epoch at the horizon exit corresponds to $N \simeq 60$, then the scalar spectral index and the tensor to scalar ratio given by relations (36) and (37) are well corroborate by observational data if $\xi > -1/72$ for $r < 0.1$ [57] and $\xi > -4/315$ for the ratio $r < 0.07$ [75]. In particular from Ref. [76] in which the ratio $r < 0.04$ (at $1 - \sigma$ confidence level) we have $\xi > -0.0096$. Recently, in Ref. [77] was obtained different constraints on the parameter ξ from observational data. In the following we will assume that the number of e-folds N is large, for values of $N \sim \mathcal{O}(10^2)$.

As we mentioned before, the attractors given by Eqs. (36) and (37) in the limit $\xi N \gg 1$ (such that $r \propto 1/N^2$) in the framework of cold inflation can be obtained in the E-model [59] and also in the model of the Higgs inflation with the nonminimal coupling [61, 62], see also Ref. [63]. A generalization of the attractors $r(N)$ and $n_S(N)$ are given by $r = 12\sigma/N^2$ and $n_S - 1 = -2/N$ or also called σ attractor (or usually called α attractor) which was proposed in Ref. [78], see also Ref. [79].

6.1 The weak regime

By considering the spectral index given by Eq. (36) we find that $\exp[\int(1 - n_S)dN] = \frac{N^2}{\alpha}$, where the quantity α corresponds to the integration constant. From Eq. (27) we obtain that the effective potential in terms of the e-folding becomes

$$V(N) = \frac{1}{\alpha \tilde{C}_\gamma^{1/4}} \left[\frac{1}{\xi + 1/N} \right]. \tag{38}$$

Thus, we find that $V_{,N} = \alpha^{-1} \tilde{C}_\gamma^{-1/4} (\xi N + 1)^{-2}$, and then the quantity $V_{,N}/V^2 = \alpha \tilde{C}_\gamma/N^2$ suggests that the integration constant $\alpha > 0$, since \tilde{C}_γ and $V_{,N}$ are positives, see Eq. (14). Also, we mention that the potential given by Eq. (38), is similar to that found in Ref. [53] for cold inflation, where the reconstruction of $V(N)$ is only obtained from $n_S(N)$. In this sense, from the potential (38) we identify that $\alpha \tilde{C}_\gamma^{1/4}$ corresponds to α and the parameter $\xi \rightarrow \beta/\alpha$ from cold inflation [53].

By considering Eq. (28), we find that the dissipation coefficient in terms of the number N becomes

$$\Gamma(N) = \Gamma_0 N^{5/2} (1 + \xi N)^{-1/2}, \quad \text{where } \Gamma_0 = \frac{\tilde{C}_\gamma^{5/8}}{\alpha^{3/2}}. \tag{39}$$

From Eqs. (38) and (39) we find that the rate R as a function of the number of e-folds in this regime is given by

$$R(N) = \frac{\Gamma}{3H} \approx \frac{\Gamma}{\sqrt{3V}} = \frac{\tilde{C}_\gamma^{3/4}}{\sqrt{3}\alpha} N^2. \tag{40}$$

Here, we note that during the weak regime the ratio $R(N)$ does not depend of the constant ξ , when it is expressed in terms of the number of e-folds N . Also, in order to obtain a scenario of weak dissipation in which $R \ll 1$, we find a lower bound for the parameter α , given by $\alpha \gg \frac{\tilde{C}_\gamma^{3/4}}{\sqrt{3}} N^2$. In particular for large N in which $N = 60$, we obtain the lower limit $\alpha \gg 8^3 \times 41.338 \sim \mathcal{O}(10^7)$. Here, we have used $C_\gamma = 70$ [7, 8, 11].

In this form, during the stage of warm inflation we obtain a lower bound for the integration constant α , considering the condition of the weak dissipative regime $\Gamma \ll 3H$. Also, we mention that this lower bound for the integration constant $\alpha \gg \frac{\tilde{C}_\gamma^{3/4}}{\sqrt{3}} N^2$, can not be obtained in the case of the reconstruction of the standard cold inflation from the background level, and it is only possible to say that $\alpha > 0$ [53].

On the other hand, from Eq. (23) we obtain that the relation between N and ϕ is given by the integral

$$\int \sqrt{\frac{1}{N(1 + \xi N)}} dN = \int d\phi. \tag{41}$$

Analogously as it occurs in the model of cold inflation, this integral depends on the sign of the constant ξ . In the case in which $\xi > 0$, we find that the integral given by Eq. (41) becomes

$$N = \xi^{-1} \sinh^2 \left[\frac{\sqrt{\xi}}{2} (\phi - \phi_0) \right], \tag{42}$$

where ϕ_0 denotes an integration constant. In this form, considering Eq. (42) the potential given by Eq. (38) can be written in terms of the scalar field as

$$V(\phi) = V_0 \tanh^2 \left[\frac{\sqrt{\xi}}{2} (\phi - \phi_0) \right],$$

$$\text{where } V_0 = \frac{1}{\alpha \xi \tilde{C}_\gamma^{1/4}}. \tag{43}$$

This effective potential corresponds to the potential studied in the T-model [58], see also Ref. [53] from the reconstruction. From the point of view of the reconstruction, we noted that in the case of the weak dissipative regime, the equation that relates $N = N(\phi)$ (see Eq. (23)) is equivalent to the model of cold inflation, since during the weak dissipative regime ($3H \gg \Gamma$) the expression to find $N = N(\phi)$ is the same, however, the initial fluctuations are of a different nature (thermal and quantum) and hence n_S and the ratio r . In this sense, we notice that utilizing the specific attractors for n_S and r given by Eqs. (36) and (37), the reconstruction in the model of warm inflation during the stage of weak dissipative regime coincides with the stage of cold inflation and it is a mere coincidence. Besides, we mention that this potential can also adapted to provide the Starobinsky model and α -attractor model, see Ref. [53].

From Eqs. (39) and (42) we obtain that the dissipation coefficient in terms of the scalar field is given by

$$\Gamma(\phi) = \frac{\Gamma_0}{\xi^{5/2}} \tanh \left[\frac{\sqrt{\xi}}{2} (\phi - \phi_0) \right] \sinh^4 \left[\frac{\sqrt{\xi}}{2} (\phi - \phi_0) \right]. \tag{44}$$

This suggests that, in order to obtain during the weak dissipative regime the associated effective potential to the T-model or classes of inflationary models, with an attractor point given by Eqs. (36) and (37), then necessarily the dissipative coefficient should be given by expression (44). The dissipation coefficient for large $\sqrt{\xi}\phi$ can be approximated to

$$\Gamma(\phi) \simeq \frac{\Gamma_0}{16\xi^{5/2}} e^{2\sqrt{\xi}(\phi-\phi_0)} \left(1 - 4e^{-\sqrt{\xi}(\phi-\phi_0)} \right) \sim e^{2\sqrt{\xi}(\phi-\phi_0)}, \tag{45}$$

and the effective potential (43) to the Starobinsky model or the α -attractor model [53].

On the other hand, an important situation occurs when the integration constant ξ is negative, since in particular for $N = 60$ and $\xi = -1/72$, the tensor to scalar ratio r takes the upper bound from Planck $r = 0.1$ [57] and $\xi = -4/315$ for $r = 0.07$ [75]. In this sense, assuming the case in which the constant $\xi < 0$ and considering that $|\xi^{-1}| > N$, then

the integration given by Eq. (41) becomes

$$N = -\xi^{-1} \sin^2 \left[\frac{\sqrt{-\xi}}{2} (\phi - \phi_0) \right], \tag{46}$$

and we find that the effective potential in terms of the scalar field is similar to the found in Ref. [53] and becomes

$$V(\phi) = -V_0 \tan^2 \left[\frac{\sqrt{-\xi}}{2} (\phi - \phi_0) \right]. \tag{47}$$

However, the dissipative coefficient Γ as a function of the scalar field can be written as

$$\Gamma(\phi) = \frac{\Gamma_0}{(-\xi)^{5/2}} \tan \left[\frac{\sqrt{-\xi}}{2} (\phi - \phi_0) \right] \times \sin^4 \left[\frac{\sqrt{-\xi}}{2} (\phi - \phi_0) \right]. \tag{48}$$

Finally in the situation in which the constant $\xi = 0$, the relation between the number of e-folds and the scalar field results $N = (\phi - \phi_1)^2/4$ and the potential is given by

$$V(\phi) = \frac{1}{4\alpha \tilde{C}_\gamma^{1/4}} (\phi - \phi_0)^2, \tag{49}$$

corresponding to chaotic potential [53]. The dissipation coefficient as a function of ϕ in this case becomes

$$\Gamma(\phi) = \frac{\Gamma_0}{32} (\phi - \phi_0)^5, \tag{50}$$

with a dependency power-law in which $\Gamma \propto \phi^5$. Here, we have used Eq. (39).

We emphasize that the reconstruction of the effective potentials given by Eqs. (43), (47) and (49) in the weak dissipative regime are the same as those found in Ref. [53] for the case of cold inflation only assuming the scalar spectral index $n_S(N) = 1 - 2/N$.

In Fig. 1 we show the evolution of the ratio $R = \frac{\Gamma}{3H}$ versus the number of e-folds N (left panel) and the dependence of the dissipative coefficient Γ on the scalar field (right panel) during the weak dissipative regime $R \ll 1$. The left panel shows the condition of the weak dissipative regime in which $\Gamma \ll 3H$, for three different values of α . In order to write down the rate $R = \Gamma/3H$ in terms of the e-folding N during this regime, we consider Eq. (40). The right panel shows the evolution of the dissipation coefficient Γ as a function of the scalar field. Also, in order to write down the coefficient Γ in terms of the scalar field, we consider Eqs. (44), (48) and (50) for three different values of $\xi \gtrless 0$, in which we have fixed $8^{-3}\alpha = 413.380 \sim \mathcal{O}(10^6)$. In both panels we have considered $C_\gamma = 70$. From the left panel, we observe that the condition for the weak dissipative regime ($R \ll 1$) is satisfied for the values of the integration constant $8^{-3}\alpha \gg 41.338 \sim$

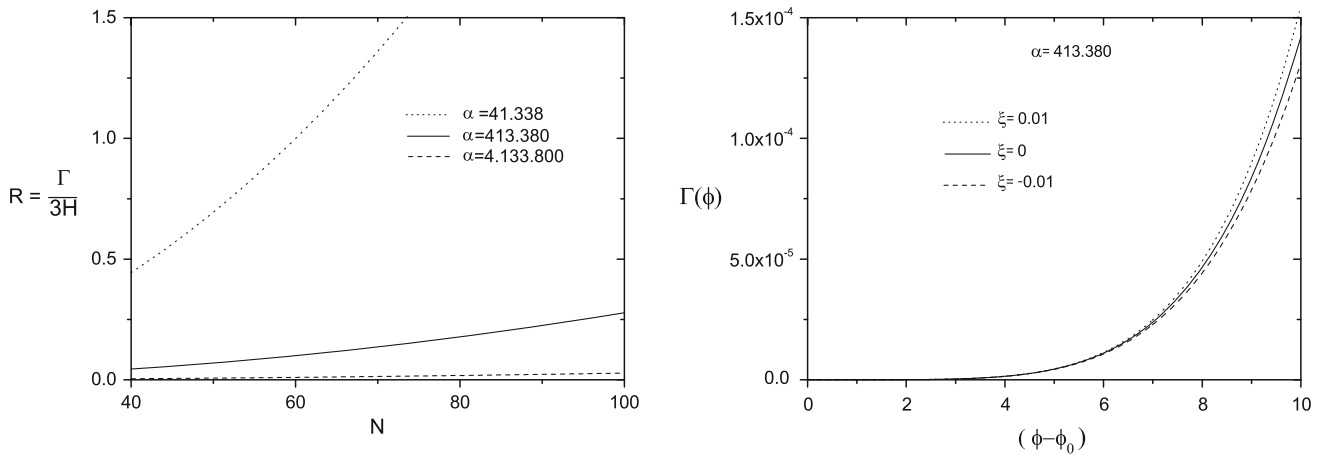


Fig. 1 The dependence of the ratio $R = \frac{\Gamma}{3H}$ versus the number of e-folds N (left panel) and the dependence of the dissipation coefficient Γ versus the scalar field (right panel) during the weak dissipative regime. From Eq. (40) we plot $R = R(N)$ in which the dotted solid and

dashed lines correspond to three different values of α (left panel). From Eqs. (44), (48) and (50) we plot $\Gamma = \Gamma(\phi)$ for three different values of $\xi \gtrless 0$, in which we have fixed $\alpha = 413.380$ (right panel). Also, in these plots we have used $C_\gamma = 70$

$\mathcal{O}(10^5)$. Also, we consider in this plot the limit case of the weak scenario in which the rate $R = 1$, corresponding to the values $N = 60$ and $8^{-3}\alpha = 41.338$, respectively (dotted line).

From right panel, we note that the behaviors of the different parameters $\Gamma = \Gamma(\phi)$ for the values of $\xi \gtrless 0$ are similar. Also, we mention that from the relation given by Eq. (37) and considering values $\xi > 0$ together with large N i.e., $N \sim \mathcal{O}(10^2)$, the tensor to scalar ratio $r \sim 0$.

6.2 The strong regime

By assuming the strong dissipative scenario in which $\Gamma \gg 3H$ and from Eq. (34) we obtain that the effective potential in terms of the e-folds N results

$$V(N) = \frac{1}{3^{3/8}\alpha \bar{C}_\gamma^{1/4}} \left[\frac{1}{\xi + 1/N} \right], \tag{51}$$

and this potential is similar to Eq. (38) for the weak regime. We emphasize that the integration constant $\alpha > 0$. From Eq. (35), we obtain that the dissipation coefficient in terms of the e-folds is given by

$$\Gamma(N) = \bar{\Gamma}_0 N^{5/6} (1 + \xi N)^{-1/2}, \quad \text{where } \bar{\Gamma}_0 = \frac{1}{3^{1/4}\alpha^{2/3}}. \tag{52}$$

As, we mentioned before this coefficient does not depend of the parameter C_γ during the strong regime.

Now, from Eqs. (34) and (35) we obtain that the ratio R during the strong dissipative regime is given by

$$R = \frac{\Gamma}{3H} \approx \frac{\Gamma}{\sqrt{3V}} = \frac{\bar{C}_\gamma^{1/8}}{3^{9/16}\alpha^{1/6}} N^{1/3}, \tag{53}$$

and this ratio does not depend of the parameter ξ , in analogy to the weak regime. From the condition of the strong regime in which $R \gg 1$, we find an upper bound for the parameter α given by $\frac{\bar{C}_\gamma^{3/4} N^2}{3^{27/8}} \gg \alpha$. For the case in which $N = 60$, we obtain that the upper limit for α is given by $7 \times 10^6 \sim \mathcal{O}(10^7) \gg \alpha$.

Nevertheless, from these solutions we find a transcendental equation from Eq. (29) to express the number of e-folds in function of the scalar field. Hence, in order to obtain analytical expressions for $V(\phi)$ and $\Gamma(\phi)$ and therefore the reconstructions, we can study the potential and the dissipation coefficient in the limits $N \gg 1/\xi$ and $N \ll 1/\xi$.

We start with the limit $\xi N \gg 1$. For large N and in particular for $N = 60$, we find a lower bound for the constant ξ given by $\xi \gg 1/60 \simeq 0.017$. On the other hand, the tensor to scalar ratio $r(N)$ given by Eq. (37) in this limit is approximately

$$r(N) = \frac{1}{N(1 + \xi N)} \approx \frac{1}{\xi N^2}, \tag{54}$$

with ξ a positive quantity. Here, we note that the attractor given by Eq. (54) corresponds to the σ -attractor, in which $\xi = (12\sigma)^{-1}$ [78,79] or also to T -model when ξ takes the value $\xi = 1/12$ in cold inflation [58]. Also, we note that from Eqs. (36) and (37) we get $2\xi = (1 + n_s)[(1 + n_s)/2r - 1]$ and considering the limit $\xi N \gg 1$, we have $(1 - n_s) \gg 4r$ and then the ratio $r \ll 0.008$ in this limit.

In this context, in which $\xi N \gg 1$, the effective potential $V(N)$ given by Eq. (51) results

$$V(N) \approx \frac{1}{3^{3/8} \alpha \bar{C}_\gamma^{1/4} \xi} = Cte. \tag{55}$$

Thus, the universe presents an exponential expansion, since $H \propto V^{1/2} = \text{constant}$ and from Eq. (29) we find $\phi = \phi_0 = Cte.$, because $d\phi/dN = 0$. This suggests that the reconstruction does not work during the strong regime when $r \propto N^{-2}$ and $n_S - 1 \propto N^{-1}$.

On the other hand, now we consider the limit in which $\xi N \ll 1$ where $r(N) \approx 1/N$. We note this the attractor for large N and in particular for $N = 60$ results $r(N = 60) \approx 1/60 \simeq 0.02$, wherewith still this attractor is well supported by the Planck data.

From Eq. (51) we find that the effective potential $V(N)$ becomes

$$V(N) \approx \frac{1}{3^{3/8} \alpha \bar{C}_\gamma^{1/4}} N. \tag{56}$$

Considering that the integration constant $\alpha \ll 7 \times 10^6$, we find a lower bound for the effective potential for large N (in particular $N = 60$) given by $V(N = 60) \gg 10^{-7}$ (in units of m_p^4 , with m_p the Planck mass).

The dissipation coefficient from Eq. (52) is given by

$$\Gamma(N) \approx \bar{\Gamma}_0 N^{5/6}. \tag{57}$$

Here, we observe that the potential $V(N)$ and the dissipation coefficient $\Gamma(N)$ do not depend on the parameter ξ , since $r(N) \approx 1/N$.

From Eq. (29) we find that the relation between the number of e-folds and the scalar field in the limit $\xi N \ll 1$ can be written as

$$\frac{1}{\tilde{\alpha}} \int N^{-2/3} dN = \int d\phi, \quad \text{where } \tilde{\alpha} = \frac{\bar{C}_\gamma^{1/16}}{3^{9/32} \alpha^{1/12}}. \tag{58}$$

Integrating we have

$$N \approx N_0 (\phi - \phi_0)^3, \tag{59}$$

where $N_0 = (\tilde{\alpha}/3)^3$ and ϕ_0 corresponds to an integration constant. Thus, the reconstruction of the effective potential as a function of the scalar field results

$$V(\phi) \approx V_0 (\phi - \phi_0)^3, \quad \text{where } V_0 = \frac{N_0}{3^{3/8} \alpha \bar{C}_\gamma^{1/4}}. \tag{60}$$

Therefore, in this limit ($\xi N \ll 1$) the effective potential corresponds to a cubic potential.

Similarly, the dissipation coefficient from Eqs. (56) and (59) in this limit becomes

$$\Gamma(\phi) \approx \Gamma_0 (\phi - \phi_0)^{5/2}, \quad \text{where } \Gamma_0 = \bar{\Gamma}_0 N_0^{5/6}, \tag{61}$$

resulting in a power law dissipative coefficient in which $\Gamma \propto \phi^{5/2}$.

In this sense, we observed that considering the attractor $r(N) \approx 1/N$ (together with $n_S - 1 = -2/N$), the effective potential and the dissipation coefficient present a power law behavior during the strong regime, and its dependencies with the scalar field (reconstruction) are given by $V(\phi) \sim \phi^3$ and $\Gamma(\phi) \sim \phi^{5/2}$, respectively.

7 Conclusions

In this paper we have studied the reconstruction from recent cosmological observations in the framework of the warm inflation. Under a general formalism of reconstruction, we have found expressions for the effective potential and dissipative coefficient in the context of the slow roll approximation, motivated by the cosmological observations of the scalar spectral index n_S and tensor to scalar ratio r . In this general analysis we have obtained from the cosmological quantities $n_S(N)$ and $r(N)$ (where N corresponds to the number of e-folds), integrable expressions for the effective potential and dissipative coefficient. For warm inflation and its reconstruction, we have considered two different regimes, called the weak and strong dissipative regimes.

As a concrete example and in order to obtain the reconstructions for the effective potential $V(\phi)$ and dissipation coefficient $\Gamma(\phi)$, we have considered the attractors $n_S - 1 = -2/N$ and $r = (N[1 + \xi N])^{-1}$. Here, we have applied our general results considering the weak and strong dissipative regimes for these attractors.

For the weak regime in which $\Gamma \ll 3H$ (or equivalently $R \ll 1$) and considering the example or the attractors given by Eqs. (36) and (37), we have obtained a lower bound for the integration constant α given by $\alpha \gg \frac{\bar{C}_\gamma^{3/4}}{\sqrt{3}} N^2$, from the condition of weak dissipative regime i.e., $R(N) \ll 1$. In particular for the case in which $N = 60$ (large N), we have found that the lower bound for the integration constant α given by $\alpha \gg 2 \times 10^7 \sim \mathcal{O}(10^7)$. Also, we have obtained that during the weak regime the reconstruction on the effective potentials are given by Eqs. (43), (47) and (49), and it coincides with the obtained in the case of cold inflation [53]. Similarly, we have obtained that the construction of the dissipative coefficients $\Gamma(\phi)$ depends on the sign of the parameter $\xi \gtrless 0$. In particular for the case $\xi = 0$ where the potential corresponds to the chaotic potential, we have found that the dissipative coefficient $\Gamma(\phi) \propto \phi^5$.

For the case of the strong dissipative regime ($R \gg 1$) we have obtained the potential and dissipative coefficient in terms of the number of e-folds. During this regime, we have found that the potential $V(N)$ has the same structure that in the weak regime. However, we could not find analytical

solutions in order to obtain the number of e-fold in terms of the scalar field in form to obtain the reconstruction of $V(\phi)$ and $\Gamma(\phi)$. In this sense, we have analyzed the potential and the dissipation coefficient in the limits $N \gg 1/\xi$ and $N \ll 1/\xi$, in order to obtain analytical solutions. In the case in which $r \propto N^{-2}$ (limit $N \gg 1/\xi$), we have obtained that the potential $V(N) = \text{constant}$, and the reconstruction does not work. For the case in which $r \propto N^{-1}$ (limit $N \ll 1/\xi$) we have obtained that the potential and the dissipative coefficient in terms of the scalar field are given by $V(\phi) \propto \phi^3$ and $\Gamma(\phi) \propto \phi^{5/2}$, respectively.

Finally in this paper, we have not addressed the reconstruction of warm inflation in which the effective potential and dissipative coefficient also depend of the temperature of the thermal bath T , i.e., $V(\phi, T)$ and $\Gamma(\phi, T)$ [17–32, 37–45, 73]. We hope to return to this point in the near future.

Acknowledgements The author thanks Prof. Øyvind Grøn by the comment on the tensor to scalar ratio. This work was supported by Proyecto VRIEA-PUCV N₀ 123.748/2017.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. A.A. Starobinsky, Phys. Lett. B **91**, 99 (1980)
2. A.H. Guth, Phys. Rev. D **23**, 347 (1981)
3. A.A. Starobinsky, JETP Lett. **30**, 682 (1979)
4. V.F. Mukhanov, G.V. Chibisov, JETP Lett. **33**, 532 (1981)
5. V.F. Mukhanov, G.V. Chibisov, Pisma Zh. Eksp. Teor. Fiz. **33**, 549 (1981)
6. D. Larson et al., Astrophys. J. Suppl. **192**, 16 (2011)
7. A. Berera, Phys. Rev. Lett. **75**, 3218 (1995)
8. A. Berera, Phys. Rev. D **55**, 3346 (1997)
9. A. Berera, Phys. Rev. D **54**, 2519 (1996)
10. A. Taylor, A. Berera, Phys. Rev. D **69**, 083517 (2000)
11. L.M.H. Hall, I.G. Moss, A. Berera, Phys. Rev. D **69**, 083525 (2004)
12. I.G. Moss, Phys. Lett. B **154**, 120 (1985)
13. A. Berera, L.Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995)
14. A. Berera, Nucl. Phys. B **585**, 666 (2000)
15. R. Herrera, JCAP **1705**(05), 029 (2017)
16. M. Motaharfar, E. Massaelli, H.R. Sepangi, Phys. Rev. D **96**(10), 103541 (2017)
17. A. Berera, Phys. Rev. D **55**, 3346 (1997)
18. J. Mimoso, A. Nunes, D. Pavon, Phys. Rev. D **73**, 023502 (2006)
19. R. Herrera, S. del Campo, C. Campuzano, JCAP **10**, 009 (2006)
20. S. del Campo, R. Herrera, D. Pavon, Phys. Rev. D **75**, 083518 (2007)
21. S. del Campo, R. Herrera, Phys. Lett. B **653**, 122 (2007)
22. M.A. Cid, S. del Campo, R. Herrera, JCAP **0710**, 005 (2007)
23. J.C.B. Sanchez, M. Bastero-Gil, A. Berera, K. Dimopoulos, Phys. Rev. D **77**, 123527 (2008)
24. S. del Campo, R. Herrera, Phys. Lett. B **665**, 100 (2008)
25. R. Herrera, M. Olivares, Int. J. Mod. Phys. D **21**, 1250047 (2012)
26. R. Herrera, M. Olivares, N. Videla, Phys. Rev. D **88**, 063535 (2013)
27. S. Bartrum, M. Bastero-Gil, A. Berera, R. Cerezo, R.O. Ramos, J.G. Rosa, Phys. Lett. B **732**, 116 (2014)
28. A. Berera, I.G. Moss, R.O. Ramos, Rep. Prog. Phys. **72**, 026901 (2009)
29. S. del Campo, R. Herrera, JCAP **0904**, 005 (2009)
30. R. Herrera, Phys. Rev. D **81**, 123511 (2010)
31. R. Herrera, E. San Martin, Eur. Phys. J. C **71**, 1701 (2011)
32. M. Bastero-Gil, A. Berera, R.O. Ramos, JCAP **1107**, 030 (2011)
33. M. Bastero-Gil, A. Berera, Phys. Rev. D **71**, 063515 (2005)
34. I.G. Moss, T. Yeomans, JCAP **1108**, 009 (2011)
35. R. Herrera, E. San Martin, Int. J. Mod. Phys. D **22**, 1350008 (2013)
36. R. Herrera, M. Olivares, N. Videla, Int. J. Mod. Phys. D **23**(10), 1450080 (2014)
37. M. Bastero-Gil, A. Berera, N. Kronberg, JCAP **1512**(12), 046 (2015)
38. R. Herrera, N. Videla, M. Olivares, Eur. Phys. J. C **76**(1), 35 (2016)
39. M. Bastero-Gil, A. Berera, R.O. Ramos, J.G. Rosa, Phys. Rev. Lett. **117**, 151301 (2016)
40. M. Bastero-Gil, A. Berera, R.O. Ramos, J.G. Rosa, [arXiv:1711.09023](https://arxiv.org/abs/1711.09023) [hep-ph]
41. R. Herrera, N. Videla, M. Olivares, Eur. Phys. J. C **75**(5), 205 (2015)
42. Y. Gim, W. Kim, JCAP **1611**(11), 022 (2016)
43. M. Benetti, R.O. Ramos, Phys. Rev. D **95**(2), 023517 (2017)
44. A. Jawad, S. Chaudhary, N. Videla, Eur. Phys. J. C **77**(11), 808 (2017)
45. M. Bastero-Gil, S. Bhattacharya, K. Dutta, M.R. Gangopadhyay, [arXiv:1710.10008](https://arxiv.org/abs/1710.10008) [astro-ph.CO]
46. H.M. Hodges, G.R. Blumenthal, Phys. Rev. D **42**, 3329 (1990)
47. F. Lucchin, S. Matarrese, Phys. Rev. D **32**, 1316 (1985)
48. R. Easther, Class. Quantum Gravity **13**, 1775 (1996)
49. J. Martin, D. Schwarz, Phys. Lett. B **500**, 1–7 (2001)
50. X.Z. Li, X.Z. Zhai, Phys. Rev. D **67**, 067501 (2003)
51. R. Herrera, R.G. Perez, Phys. Rev. D **93**(6), 063516 (2016)
52. V. Mukhanov, Eur. Phys. J. C **73**, 2486 (2013)
53. T. Chiba, PTEP **2015**(7), 073E02 (2015)
54. T. Miranda, J.C. Fabris, O.F. Piattella, JCAP **1709**(09), 041 (2017)
55. A. Achúcarro, R. Kallosh, A. Linde, D.G. Wang, Y. Welling, [arXiv:1711.09478](https://arxiv.org/abs/1711.09478) [hep-th]
56. S.D. Odintsov, V.K. Oikonomou, Ann. Phys. **388**, 267 (2018)
57. P.A.R. Ade et al. [Planck Collaboration], Astron. Astrophys. **594**, A20 (2016)
58. R. Kallosh, A. Linde, JCAP **1307**, 002 (2013)
59. R. Kallosh, A. Linde, JCAP **1310**, 033 (2013)
60. A.D. Linde, Phys. Lett. B **129**, 177 (1983)
61. D. Kaiser, Phys. Rev. D **52**, 42954306 (1995)
62. F. Bezrukov, M. Shaposhnikov, Phys Lett B **659**, 703 (2008)
63. R. Kallosh, A. Linde, D. Roest, Phys. Rev. Lett. **112**, 011303 (2014)
64. Q.G. Huang, Phys. Rev. D **76**, 061303 (2007)
65. J. Lin, Q. Gao, Y. Gong, Mon. Not. R. Astron. Soc. **459**(4), 4029 (2016)
66. Q. Gao, Sci. China Phys. Mech. Astron. **60**(9), 090411 (2017)
67. D. Roest, JCAP **1401**, 007 (2014)
68. J. Garcia-Bellido, D. Roest, Phys. Rev. D **89**(10), 103527 (2014)
69. P. Creminelli, S. Dubovsky, D. Lpez Nacir, M. Simonovic, G. Trevisan, G. Villadoro, M. Zaldarriaga, Phys. Rev. D **92**(12), 123528 (2015)
70. A. Starobinsky, J. Yokoyama, Density fluctuations in Brans–Dicke inflation. In: Nakao, K. et al. (eds.) Proceedings of the Fourth Workshop on General Relativity and Gravitation, p. 381. Kyoto University, Kyoto (1995). [arXiv:gr-qc/9502002](https://arxiv.org/abs/gr-qc/9502002)
71. A.A. Starobinsky, S. Tsujikawa, J. Yokoyama, Nucl. Phys. B **610**, 383 (2001)
72. I.G. Moss, C. Xiong, JCAP **0704**, 007 (2007)
73. I.G. Moss, C. Xiong, JCAP **0811**, 023 (2008)
74. M. Bastero-Gil, A. Berera, Int. J. Mod. Phys. A **24**, 2207 (2009)

75. P.A.R. Ade et al. [BICEP2 and Keck Array Collaborations], Phys. Rev. Lett. **116**, 031302 (2016)
76. J.R. Gott, W.N. Colley, [arXiv:1707.06755](https://arxiv.org/abs/1707.06755) [astro-ph.CO]
77. Ø. Gron, Universe **4**(2), 15 (2018)
78. R. Kallosh, A. Linde, D. Roest, JHEP **1311**, 198 (2013)
79. R. Jinno, K. Kaneta, Phys. Rev. D **96**(4), 043518 (2017)