Evading the non-continuity equation in the $f(R, T)$ cosmology

P. H. R. S. Moraes^{1,a}, R. A. C. Correa^{1,b}, G. Ribeiro^{1,2,c}

¹ Departamento de Física, ITA-Instituto Tecnológico de Aeronáutica, São José dos Campos, São Paulo 12228-900, Brazil

² Departamento de Física e Química, UNESP-Universidade Estadual Paulista “Júlio de Mesquita Filho”, Guaratinguetá, São Paulo 12516-410, Brazil

Received: 30 August 2017 / Accepted: 19 February 2018 / Published online: 7 March 2018

© The Author(s) 2018. This article is an open access publication

Abstract We present a new approach for the $f(R, T)$ gravity formalism, by thoroughly exploring the extra terms of its effective energy-momentum tensor $T_{\mu\nu}^{\text{eff}}$, which we name $\tilde{T}_{\mu\nu}$, so that $T_{\mu\nu}^{\text{eff}} = T_{\mu\nu} + \tilde{T}_{\mu\nu}$, with $T_{\mu\nu}$ being the usual energy-momentum tensor of matter. Purely from the Bianchi identities, we obtain the conservation of both parts of the effective energy-momentum tensor, rather than the non-conservation of $T_{\mu\nu}$, originally occurring in the $f(R, T)$ theories. In this way, the intriguing scenario of matter creation, which still lacks observational evidence, is evaded. One is left, then, with two sets of cosmological equations to be solved: the Friedmann-like equations along with the conservation of $T_{\mu\nu}$ and along with the conservation of $\tilde{T}_{\mu\nu}$. We present a physical interpretation for the conservation of $\tilde{T}_{\mu\nu}$, which can be related to the presence of stiff matter in the universe. The cosmological consequences of this approach are presented and discussed as well as the benefits of evading the matter energy-momentum tensor non-conservation.

1 Introduction

Alternative cosmological models have been constantly used to solve or at least evade the dark energy (DE) problem [1–3] – among other shortcomings – found in standard (Λ CDM) cosmological model. Higher dimensional [4–6] and $f(R)$ gravity theories [7–9], with $f(R)$ indicating an arbitrary function of the Ricci scalar R , emerge as optimistic scenarios from which healthy cosmological models can be derived. $f(R)$ gravity in higher dimensions can also generate well-behaved models [10–14].

In generalized and higher dimensional gravitational models, the extra terms of the field equations¹ can induce the effects of a cosmic acceleration, predicted by type Ia supernovae observations [15, 16] and cosmic microwave background temperature fluctuations [17]. Good agreement between theory and cosmological and astrophysical observations can also be obtained from such alternative theories [18–20].

Another reputed alternative gravity model was proposed in [21], named $f(R, T)$ theory of gravity, which presents in its field equations extra contributions from both geometry, through a general dependence on R , and matter, through a general dependence on T , the trace of the energy-momentum tensor (EMT). The T -dependence is motivated by the consideration of quantum effects or imperfect fluids.

Cosmological scenarios derived from the $f(R, T)$ theory of gravity have been continuously proposed. In [22], the authors have introduced the bulk viscosity in the $f(R, T)$ formalism within the framework of a flat Friedmann–Robertson–Walker (FRW) model. Since according to the authors describing $f(R, T)$ gravity the dependence on T of the gravitational part of the action might come from the consideration of imperfect fluids, the authors have investigated more realistic models, by explicitly taking into account dissipative processes due to viscosity. In this regard, it is known that when neutrinos decoupled in the early universe, matter could have behaved like a viscous fluid.

Moreover, in [23], a Little Rip model in $f(R, T)$ gravity was investigated. An analysis of $f(R, T)$ models through energy conditions can be found in [24]. A complete cosmological scenario was derived from the $f(R, T^\phi)$ formalism, which is nothing but the $f(R, T)$ theory in the presence of a scalar field, in [25]. Cosmological models from the simplest non-minimal matter–geometry coupling, that is, $f(R, T) = R + \alpha RT$ with constant α , and from the simplest non-trivial polynomial function of T in the $f(R, T)$

^a e-mail: moraes.phrs@gmail.com

^b e-mail: rafael.couceiro@ufabc.edu.br

^c e-mail: ribeiro.gabriel.fis@hotmail.com

¹ “Extra terms” when compared to standard gravity field equations.

formalism were presented, respectively, in [26,27]. $f(R, T)$ theories of gravity have also been expanded to five dimensions, as explored in [28–31]. Furthermore, an anisotropic cosmological model was built in $f(R, T)$ gravity in [32].

The well-behaved cosmological models cited above, among others found in the literature, make reasonable and promising to consider the $f(R, T)$ gravity as a possible alternative to general relativity (GR), from which Λ CDM cosmological model is derived. Once the gravitational part of the action is generalized, including a general dependence not only on geometrical terms, as in $f(R)$ gravity, but also on material terms through the general dependence on T , the new terms of the derived field equations might be responsible for inducing the observed late-time acceleration of the universe expansion with no need of a cosmological constant (CC), which still lacks a convincing physical interpretation [33]. Moreover, as shown in [34], some specific functional forms for $f(R, T)$ may retrieve other cosmological models, as the Chaplygin gas and quintessence models, manifesting the generic aspect of such a theory of gravity, i.e., different cosmological models found in the literature can be obtained from different particular cases of $f(R, T)$.

$f(R, T)$ theories, as originally proposed, predict a non-conservation of the usual EMT of matter, which will be carefully described in Sect. 2 below. Such a non-conservation yields the motion of massive test particles to be non-geodesic, taking place in the presence of an extra force orthogonal to the four-velocity. In [35], the solar system's bounds on this extra force were found. In [36], it was argued that due to the coupling between matter and geometry predicted in $f(R, T)$ theories, there should be an energy flow between the gravitational field and matter. The $\nabla^\mu T_{\mu\nu} \neq 0$ issue was considered to be related to an irreversible matter creation process.

Anyhow, observational evidence of particle creation on a cosmological scale is still missing. Likewise there is no observational evidence of the predicted extra force. In order to confirm such an intriguing property predicted by $f(R, T)$ gravity (among other theories, such as $f(R, L_m)$ theory [37], with L_m being the matter lagrangian density), those theories should be tested in a non-usual aspect. If there is creation of matter throughout the universe history, some kind of signature should be imprinted in the cosmic microwave background anisotropy. The classical macroscopic theory predictions in structure formation with linear perturbations may also corroborate or dismiss the scenarios with EMT non-conservation.

Such a lack of observational evidence as regards matter creation led some authors to evade the non-continuity equation in $f(R, T)$ gravity. The vanishing of $\nabla^\mu T_{\mu\nu}$ was imposed in [38] and an alternative to the DE problem was obtained. Moreover, the approach has revealed some constraints to the functionality of $f(T)$ in $f(R, T)$. In [39]

the authors have reconstructed $f(R, T)$ gravity for a specific model that permitted the standard continuity equation to hold. The dynamics and stability of $f(R, T)$ theory for de Sitter and power-law expansions of the universe with EMT conservation were discussed in [40].

In the present approach we will also evade the EMT non-conservation predicted in $f(R, T)$ theories. However, there will be no imposition of conservation. The non-vanishing covariant derivative of $T_{\mu\nu}$ will be evaded purely from GR geometrical identities.

The present article is organized as follows. In Sect. 2 a review of $f(R, T)$ gravity is given. The equation for the non-conservation of $T_{\mu\nu}$ is indicated. In Sect. 3 we distinguish the presence of extra terms in the effective energy-momentum tensor (EEMT) in the $f(R, T)$ formalism and construct a scenario in which there is no violation of the continuity equation. General cosmological solutions for this approach are presented in Sect. 4. In Sect. 5 we physically interpret the extra terms of the EEMT. Section 6 is devoted to a discussion of the benefits of having the non-conservation of $T_{\mu\nu}$ evaded in a given theory. A rigorous discussion concerning the results of our approach is presented in Sect. 7.

2 A brief review of the $f(R, T)$ gravity

As recently proposed by Harko et al. [21], the $f(R, T)$ theory of gravity assumes the gravitational part of the action to depend on an arbitrary function of R and T . According to the authors, the dependence on T may be induced by the consideration of exotic fluids or quantum effects. The total action in such a theory is given by

$$S = \frac{1}{16\pi} \int d^4x f(R, T) \sqrt{-g} + \int d^4x L_m \sqrt{-g}. \quad (1)$$

In (1), $f(R, T)$ is an arbitrary function of R and T , and g is the metric determinant. Moreover, throughout this paper, we will assume units such that $c = G = 1$.

The usual EMT of matter is written as

$$T_{\mu\nu} = g_{\mu\nu} L_m - 2 \frac{\partial L_m}{\partial g^{\mu\nu}}. \quad (2)$$

We will assume the matter source is a perfect fluid (PF) in (2) and the universe is homogeneous and isotropic at cosmological scales, i.e., its geometry is described by an FRW metric. We will also consider a flat universe, in accordance with recent cosmic microwave background observations [17] and work with the case $f(R, T) = R + 2\lambda T$, with λ being a constant. By assuming $f(R, T) = R + 2\lambda T$, such an $f(R, T)$ functional form benefits from the fact that one can recover GR just by letting λ to be null.

The assumptions above yield, for the variation of the action (1) with respect to the metric, the following field equations:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} + \lambda[2(T_{\mu\nu} + pg_{\mu\nu}) + Tg_{\mu\nu}], \quad (3)$$

for which $G_{\mu\nu}$ is the usual Einstein tensor and p is the pressure of the universe.

A notorious feature about $f(R, T)$ theory is the non-nullity of the covariance divergence of $T_{\mu\nu}$. Recently, the authors in [41, 42] have recalculated $\nabla^\mu T_{\mu\nu}$, by arguing that Harko et al. [21] missed an essential term, which has consequences in the equation of motion of test particles. From such an argumentation, the corrected relation for $\nabla^\mu T_{\mu\nu}$ when $f(R, T) = R + 2\lambda T$ is

$$\nabla^\mu T_{\mu\nu} = \frac{2\lambda}{2\lambda - 8\pi} \left[\nabla^\mu (2T_{\mu\nu} + pg_{\mu\nu}) + \frac{1}{2} g_{\mu\nu} \nabla^\mu T \right]. \quad (4)$$

Note that, as required, the case $\lambda = 0$ retrieves GR in both (3) and (4).

3 Evading the non-continuity equation in the $f(R, T) = R + 2\lambda T$ gravity

The *lhs* of the field equations (3) is exactly the same as in GR, i.e., given uniquely by the Einstein tensor. The *rhs* clearly presents some extra terms when compared to standard gravity. In order to obtain an accelerated expanding universe in standard cosmology, one has to assume that $\sim 70\%$ of the universe composition is in the form of some exotic fluid dubbed DE, which may enter the GR field equations in the form of a CC EMT. The extra terms in (3) may play the role of the CC, however, evading the DE problem quoted above.

As carefully highlighted in the previous section, Eq. (2) refers to the usual EMT of matter. By keeping that in mind, we write

$$G_{\mu\nu} = 8\pi T_{\mu\nu}^{\text{eff}}, \quad (5)$$

with $T_{\mu\nu}^{\text{eff}} = T_{\mu\nu} + \tilde{T}_{\mu\nu}$, and

$$\tilde{T}_{\mu\nu} \equiv \frac{\lambda}{8\pi} [2(T_{\mu\nu} + pg_{\mu\nu}) + Tg_{\mu\nu}] \quad (6)$$

being the extra contribution of the EEMT.

Considering a PF in Eq. (2) yields $T_{\mu\nu} = \text{diag}(\rho, -p, -p, -p)$, with $T = \rho - 3p$ and ρ being the matter-energy density of the universe. Yet, from (6), we have

$$\tilde{T}_{00} = \frac{\lambda}{8\pi} (3\rho - p), \quad (7)$$

$$\tilde{T}_{11} = -\frac{\lambda}{8\pi} (\rho + p), \quad (8)$$

$\tilde{T}_{22} = \tilde{T}_{33} = \tilde{T}_{11}$ and $\tilde{T} = -(\lambda/2\pi)p$. The fluid described by Eq. (6) permeates the universe along with the usual PF which is often considered as the (only) matter source of the universe. Effectively, there is one fluid permeating the universe, whose density is given by the sum of the densities of both fluids presented above, i.e., $T_{\mu\nu}$ and $\tilde{T}_{\mu\nu}$. We shall revisit this question later on.

The non-null components of (5) are

$$3\left(\frac{\dot{a}}{a}\right)^2 = (8\pi + 3\lambda)\rho - \lambda p, \quad (9)$$

$$2\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 = \lambda\rho - (8\pi + \lambda)p, \quad (10)$$

with dots representing time derivatives. As required, $\lambda = 0$ yields the standard Friedmann equations.

When one breaks the EEMT into two parts, one being the usual EMT of matter, the other coming from the dependence of the action on T , a remarkable feature is easily seen: what makes $\nabla^\mu T_{\mu\nu} \neq 0$ in (4) is the indistinct presence of $\tilde{T}_{\mu\nu}$ defined in Eq. (6). However, the application of the Bianchi identities $\nabla^\mu G_{\mu\nu} = 0$ in (5) yields $\nabla^\mu [8\pi(T_{\mu\nu} + \tilde{T}_{\mu\nu})] = 0$, or

$$\nabla^\mu T_{\mu\nu} = 0 \quad (11)$$

and

$$\nabla^\mu \tilde{T}_{\mu\nu} = 0. \quad (12)$$

Therefore one has two sets of three equations with three unknowns to be solved: (9), (10), (11) and (9), (10), (12).

Note that by approaching the $f(R, T)$ gravity from such a perspective indeed evades the non-continuity equation $\nabla^\mu T_{\mu\nu} \neq 0$ originally predicted in the formalism.

4 Cosmological scenario

The evaluation of (11) yields the well-known continuity equation of cosmology:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0. \quad (13)$$

By recombining Eqs. (9), (10) and (13), it is straightforward to obtain the following differential equation for the scale factor:

$$L(\lambda) \left[\frac{\ddot{a}}{a} + \frac{1}{2} \left(\frac{\dot{a}}{a} \right)^2 \right] + \lambda \left[\frac{8\pi}{3} \left(\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} \right) - \lambda \frac{\ddot{a}}{a} \right]$$

$$+(8\pi + 4\lambda) \left[(8\pi + 3\lambda) \frac{\ddot{a}}{a} + 4\pi \left(\frac{\dot{a}}{a} \right)^2 \right] = 0, \quad (14)$$

with $L(\lambda) \equiv \lambda^2 - (8\pi + 3\lambda)(8\pi + \lambda)$.

On the other hand, by developing (12) yields

$$\dot{\rho} = \dot{p}, \quad (15)$$

in which a constant of integration has been discarded.

Equation (15), if integrated, reveals the presence of stiff matter (SM) in the universe. Such a prediction of the present model shall be revisited later on.

By recombining Eqs. (9), (10) and (15), one obtains

$$8\pi \left(\frac{\ddot{a}}{a} - \frac{\dot{a}}{a} \right) - 3\lambda \frac{\ddot{a}}{a} = \frac{L(\lambda)}{8\pi - 2\lambda} \left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 \right]. \quad (16)$$

Substituting Eq. (16) into (14), we have

$$\frac{\ddot{a}}{a} + \frac{\Lambda_2}{\Lambda_1} \left(\frac{\dot{a}}{a} \right)^2 = 0, \quad (17)$$

where we are using the definitions

$$\Lambda_1 \equiv L(\lambda) \left(1 + \frac{\lambda}{24\pi + 6\lambda} \right) + (8\pi + 3\lambda)(8\pi + 4\lambda), \quad (18)$$

$$\Lambda_2 \equiv L(\lambda) \left(\frac{1}{2} + \frac{\lambda}{24\pi + 6\lambda} \right) + 4\pi(8\pi + 4\lambda). \quad (19)$$

After straightforward manipulations, we can put Eq. (17) in the form

$$\frac{d}{dt} (\ln \dot{a} + \varsigma \ln a) = 0, \quad (20)$$

with $\varsigma \equiv \Lambda_2/\Lambda_1$. Hence, by solving the above equation and applying some manipulations, the scale factor can be written as

$$a(t) = a_0 t^\gamma, \quad (21)$$

where a_0 is an arbitrary constant and $\gamma \equiv 1/(1 + \varsigma)$.

From Eq. (21) we can calculate the Hubble parameter $H = \dot{a}/a$ and the deceleration parameter $q = -\ddot{a}a/\dot{a}^2$.

We can plot q as a function of γ in order to visualize the range of values of γ that yield a value of q in accordance with observational data on the Hubble parameter [43]. This can be checked in Fig. 1 below.

From [43], $q = -0.18_{-0.12}^{+0.12}$, which means that the acceptable values for γ are $1.064 \leq \gamma \leq 1.429$.

Being in the possession of (21) we can also obtain the solutions for the matter-energy density of the universe. We can solve the Friedmann-like equations of the model for ρ

Fig. 1 The deceleration parameter q as a function of the free parameter γ

Fig. 2 Evolution of the matter-energy density of the universe in redshift for $\lambda = 0.857$. The thin (blue) line stands for the usual PF while the dotted (green) line represents the SMF. Effectively, the universe behaves as composed of one fluid, whose density is given by the sum of those contributions, as in the thick (red) curve

by using Eqs. (13) and (15), obtaining, in this way, solutions for the usual PF and for the SM fluid (SMF), respectively. Effectively, we have one fluid whose matter-energy density is given by the sum of those, as it can be checked in Fig. 2 below, in which the quantities are plotted against the redshift z , such that $a = 1/(1 + z)$.

In the same way, in possession of (21) we can obtain the solutions for the pressure of the universe. In Fig. 3 we plot these solutions in z .

5 The arising of stiff matter

Equation (15), after having been integrated, describes an SMF. It is not the first time that $f(R, T)$ gravity reveals the possibility of the existence of SM. In [29], a particular case of the cosmological solutions is the one of an SM dominated scenario. In [44, 45] the presence of SMF permeating the universe was also predicted in the $f(R, T)$ scenario.

Fig. 3 Evolution of the pressure of the universe in redshift for $\lambda = 0.857$. The thin (blue) line stands for the usual PF while the dotted (green) line represents the SMF. Effectively, the universe behaves as composed of one fluid, whose pressure is given by the sum of those contributions, as in the thick (red) curve

Some early universe models indicate that there may have existed a phase prior to the radiation-dominated era in which our universe dynamics was dominated by a gas of baryons with equation of state (EoS) $p = \rho$, which interacted through a vector-meson field [46].

The conjecture of a primordial SM era firstly appeared in [47]. The existence of SM may explain the baryon asymmetry and the density perturbations of the right amplitude for the large scale structure formation in the universe, as shown in [48]. It can play an important role in the spectrum of inflationary gravitational waves [49] and characterize the EoS of compact astrophysical objects [50]. A thermodynamics analysis of an FRW universe with bulk viscous SMF was made in [51]. An article presenting a broad study of cosmology with an SM era was recently written [52]. SMF has also appeared in anisotropic cosmological models [46, 53, 54]. Moreover, SM occurs in the relativistic scalar fields approach when their kinetic energy dominates the potential energy. In fact, a primordial SM era is fundamental for any model based on a relativistic scalar field [55].

Our purpose in Sect. 3 was to evade the EMT non-conservation issue surrounding $f(R, T)$ gravity theory. By doing this, Eqs. (11) and (12) have arisen. By developing them, we obtained Eqs. (13) and (15). Equation (13) stands for the usual conservation of matter. On the other hand, Eq. (15) predicts the existence of SM in the universe.

As in another SM cosmological models (check [52], for instance), everything happens as if the universe were composed of two non-interacting fluids, in our case, one respecting Eq. (13) and another submissive to Eq. (15). Effectively, we have just one fluid whose energy density is given by the sum of the solutions of (13) and (15).

6 The importance of maintaining the usual energy-momentum tensor conserved

Despite the lack of observational evidence as regards corroborating scenarios with continuum or episodic creation of particles in cosmological scales, some efforts on this subject have been made, as can be checked, for instance, in [56, 57].

One might wonder how the evasion of $\nabla_\mu T^{\mu\nu} \neq 0$, which was described above in Sect. 3, can coexist with $f(R, T)$ solutions whose usual EMT of matter is not conserved. We argue about this issue in the following.

Non-minimal matter–curvature coupling is thought to be the mechanism responsible for gravitationally induce particle production in $f(R, T)$ or $f(R, L_m)$ theories [36, 37, 58]. The coupling of matter with higher order derivative curvature terms can be interpreted as an exchange of energy and momentum between them, which induces a gravitational particle production. In other words, the matter–curvature coupling generates an irreversible flow from the gravitational field to matter constituents being created, and the second law of thermodynamics requires that the geometric curvature transforms into matter (check [58]).

In $f(R, L_m)$ theory, the functional form $f(R, L_m) = f_1(R) + [1 + \lambda f_2(R)]L_m$, with $f_1(R)$ and $f_2(R)$ being arbitrary functions of R , was considered in the study of gravitationally induced particle creation in Ref. [58]. Recalling the argumentations above, it is clear that such a model indeed describes a non-minimal coupling of matter and geometry, through the product $f_2(R)L_m$, justifying, at a theoretical level, a scenario with matter creation. Such a coupling is prevented when $\lambda = 0$.

Departing from the case in Ref. [58], the functional form used in the present article, $f(R, T) = R + 2\lambda T$, does not predict a non-minimal matter–geometry coupling (recall that there is no product between material terms, proportional to T , and geometrical terms, proportional to R). In this way, there is no mechanism able to explain geometric curvature transforming into matter.

Therefore, when there is no explicit coupling between matter and geometry, as in the $f(R, T) = R + 2\lambda T$ theory, one is capable of escaping the argument of creation of matter in the universe through the approach above, which is worth considering, since so far there is no observational evidence of such a phenomenon.

Another importance in evading $\nabla_\mu T^{\mu\nu} \neq 0$ in the $f(R, T)$ theory arises when one considers an astrophysical approach for the theory, as the evaluation of the Tolman–Oppenheimer–Volkoff equation for compact astrophysical objects [59], as we argue below.

While in a cosmological scenario one interprets the usual EMT non-conservation as due to the creation of matter through the universe evolution (despite the absence of observational corroboration), on an astrophysical level, such

as the hydrostatic equilibrium configurations of compact objects, there is no mechanism able to physically interpret $\nabla_\mu T^{\mu\nu} \neq 0$. This can easily be seen: the Tolman–Oppenheimer–Volkoff equation is constructed from a static spherically symmetric metric. Naturally, in a static scenario it is not possible to create (or destroy) matter. Since, in order to construct the Tolman–Oppenheimer–Volkoff equation in $f(R, T)$ gravity, one needs the equation for the covariant derivative of the usual EMT, one could make use of the present approach to evade the lack of physical interpretation quoted above.

7 Discussion and conclusions

There are plenty of motivations for the search of alternative cosmological models nowadays. That happens because although the Λ CDM cosmological model is able to provide good agreement between theoretical predictions and observations, it is surrounded by shortcomings and drawbacks (check [1–3]).

In this article we have taken the $f(R, T)$ theory of gravity into account. Besides the references mentioned above, $f(R, T)$ gravity has shown to be an alternative to dark matter [60] and to respect solar system tests [61]. Moreover, it has been shown in [62] that wormholes in $f(R, T)$ gravity do not need to be filled by exotic matter.

Our goal in the present article was to surpass the non-conservation of the usual EMT of matter, which is originally predicted in the $f(R, T)$ theory. In order to do so, our proposal was to establish an alternative form of treating the EEMT of the $f(R, T) = R + 2\lambda T$ gravity, by distinguishing the presence of a fluid described by Eq. (6). Such an approach implies the continuity equations (11) and (12).

Indeed, the achievement of an $f(R, T)$ formalism able to conserve the EMT is the main difference of the present research article when compared to the literature. The motivation to conserve the EMT in a gravity theory was rigorously discussed in the previous section and will be revisited below. Anyhow, it is important, here, to remark that Alvarenga et al. have worked out a particular $f(T)$ function that conserves the EMT in $f(R, T)$ gravity, namely $f(T) = \alpha T^{\frac{3\omega+1}{2(\omega+1)}}$ [41]. Differently, we have assumed here that our functional form for $f(T)$ is linear in T and we have figured out the mechanism which allows the conservation of the EMT for this $f(T)$ case, which is the most popular of the $f(T)$ models, as can be checked, for instance, in several astrophysical and cosmological applications, like in Refs. [21, 28, 39, 58, 63–66].

The present motivation for working with $f(R, T) = R + 2\lambda T$ lies mainly in the remarkable outcomes that have been presented in the literature from such an assumption. Besides the references quoted above, Alhamzawi and Alhamzawi

have shown that such a model can give a considerable contribution to gravitational lensing effects [67]. Moreover, in [59] the $f(R, T) = R + 2\lambda T$ model increased the predicted maximum mass of neutron stars and, in view of recent astrophysical observations [68, 69], such an increase is very welcome. The extra terms of the field equations of the same $f(R, T)$ model were shown to be an alternative for galactic dark matter effects [60].

The integration of Eq. (15) has revealed the presence of SMF permeating the universe along with the usual PF described by Eq. (13). The existence of SMF was already predicted by $f(R, T)$ models in [29, 44, 45] and in [70].

We also remark that two-fluid cosmological models have been proposed a long time ago [71–73]. For recent references on this subject, see [74, 75]. Even three-fluid cosmological models have been proposed [76].

In the present article, we were capable of evading the non-conservation of the usual EMT of matter in $f(R, T)$ gravity. It has been shown that the presence of SM in the universe makes it possible to conserve the EMT of both fluids in the present two-fluid cosmological model.

The main motivation for such an evasion is the lack of observational evidence of particle creation in cosmology and the lack of interpretation for the astrophysical static case (hydrostatic equilibrium equation).

In contrast, there is some evidence that in the low redshift regime, neutron stars and even other compact astrophysical objects in the universe can have SM in their core (check [50] and [77]). There may have been even a SM era in the post-inflationary universe, as rigorously analyzed in [52]. It is argued that this phase occurred in the very early universe, before the known radiation era. It has been shown that in certain conditions of internal energy, SM leads to the standard Big-Bang scenario [78]. The SM models also offer an approach to gravitational particles creation, which may occur for certain temperatures of the universe [79].

One might wonder if the MEMT non-conservation evasion of the present approach conflicts with the original predictions of $f(R, T)$ gravity. The answer is no. In this article we have shown that when there is no product of R and $f(T)$, one is able to break the EEMT of $f(R, T)$ cosmology in two terms. Then, by applying the Bianchi identities only, we have shown that both of them are conservative.

Cosmologically, the present model has shown to be very healthy and well behaved. Our cosmological solutions have been constructed from the scale factor of Eq. (21), which was obtained from the method presented in Sect. 4.

In particular, we have derived a deceleration parameter which is in agreement with observations [43] for a range of values for γ (see Eq. (21)), namely $1.064 \leq \gamma \leq 1.429$. In this way, besides solving the EMT non-conservation issue, our model was capable of predicting the cosmic acceleration [15, 16], with no need for a CC, which certainly

sustains the viability of $f(R, T)$ gravity as an alternative to GR.

From Eq. (21), we were also able to plot the matter-energy density of the model. The value chosen for λ is in agreement with observations (it respects the range of values for γ mentioned above). Figure 2 indicates that, for low redshift, indeed, the SM contribution to the total matter-energy density is small when compared to the usual perfect fluid.

In Fig. 3, it is remarkable that the usual PF pressure is negative. While in standard cosmology, this is usually put in by hand as the equation of state $p \sim -\rho$, a negative pressure was a solution of the present model. Moreover, it might well be the mechanism responsible for inducing the cosmic acceleration ($q < 0$) in the model.

To finish we would like to highlight that the non-conservation of the EMT issue in the $f(R, T)$ gravity still lacks further investigation. In a cosmological level, it may well be related to the creation (or destruction) of matter particles through the evolution of the universe [36, 56, 80], although no observation of such a phenomenon on cosmological scales has been made so far. On the other hand, in the $f(R, T)$ gravity applications regarding the astrophysics of compact objects, such as the hydrostatic equilibrium configurations of neutron stars, quark stars and white dwarfs [59, 81], the latter interpretation is not suitable, since those objects are not expected to create matter, specially in a static analysis. In this way, the non-conservation of the EMT in the $f(R, T)$ gravity should not rule out the theory, since further observations may, somehow, corroborate the creation of particles even on cosmological scales. For now, anyway, the evading of the EMT non-conservation is worth considering.

We would also like to remark that the present approach can be applied to other functional forms for $f(R, T)$ in which geometry and matter are minimally coupled. These further applications may yield new two-fluid cosmological models.

Acknowledgements PHRSM would like to thank São Paulo Research Foundation (FAPESP), Grant 2015/08476-0, for financial support. RACC thanks FAPESP for financial support.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. M. Padmanabhan, Gen. Relativ. Gravit. **40**, 529 (2008)
2. J. Yoo, Y. Watanabe, Int. J. Mod. Phys. D **21**, 1230002 (2012)
3. F. Wang, J.M. Yang, Eur. Phys. J. C **45**, 815 (2006)
4. J.M. Overduin, P.S. Wesson, Phys. Rep. **283**, 303 (1997)
5. P.H.R.S. Moraes, Int. J. Mod. Phys. D **25**, 1650009 (2016)
6. B. Guo et al., Eur. Phys. J. C **75**, 63 (2015)
7. S. Capozziello, Int. J. Mod. Phys. D **11**, 483 (2002)
8. A.A. Starobinsky, JETP Lett. **86**, 157 (2007)
9. H.W. Lee, Eur. Phys. J. C **71**, 1748 (2011)
10. Y.-B. Wu et al., Eur. Phys. J. C **74**, 2791 (2014)
11. S. Chakraborty, S. SenGupta, Eur. Phys. J. C **75**, 538 (2015)
12. Z.-G. Xu et al., Eur. Phys. J. C **75**, 368 (2015)
13. D. Bazeia et al., Eur. Phys. J. C **75**, 569 (2015)
14. R.A.C. Correa, P.H.R.S. Moraes, A. de Souza Dutra, R. da Rocha, Phys. Rev. D **92**, 126005 (2015)
15. A.G. Riess et al., Astron. J. **116**, 1009 (1998)
16. G. Perlmutter et al., Astrophys. J. **517**, 565 (1999)
17. G. Hinshaw et al., Astrophys. J. **208**, 19 (2013)
18. M. Demianski, Astron. Astrophys. **454**, 55 (2006)
19. M. Lubini et al., Eur. Phys. J. C **71**, 1834 (2011)
20. X.-M. Deng, Y. Xie, Eur. Phys. J. C **75**, 539 (2015)
21. T. Harko et al., Phys. Rev. D **84**, 024020 (2011)
22. C.P. Singh, P. Kumar, Eur. Phys. J. C **74**, 11 (2014)
23. M.J.S. Houndjo et al., Eur. Phys. J. Plus **129**, 171 (2014)
24. M. Sharif et al., Eur. Phys. J. Plus **128**, 123 (2013)
25. P.H.R.S. Moraes, J.R.L. Santos, Eur. Phys. J. C **76**, 60 (2016)
26. P.H.R.S. Moraes, P.K. Sahoo, Eur. Phys. J. C **77**, 480 (2017)
27. P.H.R.S. Moraes, G. Ribeiro, R.A.C. Correa, Astrophys. Space Sci. **361**, 227 (2016)
28. P.H.R.S. Moraes, Astrophys. Space Sci. **352**, 273 (2014)
29. P.H.R.S. Moraes, Eur. Phys. J. C **75**, 168 (2015)
30. P.H.R.S. Moraes, R.A.C. Correa, Astrophys. Space Sci. **361**, 91 (2016)
31. R.A.C. Correa, P.H.R.S. Moraes, Eur. Phys. J. C **76**, 100 (2016)
32. P.K. Sahoo et al., Eur. Phys. J. Plus **131**, 18 (2016)
33. S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989)
34. M. Jamil et al., Eur. Phys. J. C **72**, 1999 (2012)
35. X.-M. Deng, Y. Xie, Int. J. Theor. Phys. **54**, 1739 (2015)
36. T. Harko, Phys. Rev. D **90**, 044067 (2014)
37. O. Bertolami et al., Phys. Rev. D **75**, 104016 (2007)
38. S. Chakraborty, Gen. Relativ. Gravit. **45**, 2039 (2013)
39. M. Sharif, M. Zubair, Astrophys. Space Sci. **349**, 529 (2014)
40. E.H. Baffou et al., Astrophys. Space Sci. **355**, 2197 (2014)
41. F.G. Alvarenga et al., Phys. Rev. D **87**, 103526 (2013)
42. O.J. Barrientos, G.F. Rubilar, Phys. Rev. D **90**, 028501 (2014)
43. S. Kumar, MNRAS **422**, 2532 (2012)
44. M. Farasat Shamir, Eur. Phys. J. C **75**, 354 (2015)
45. K.L. Mahanta, Astrophys. Space Sci. **353**, 683 (2014)
46. S.K. Rama, Phys. Rev. D **79**, 124031 (2009)
47. Y.B. Zel'dovich, MNRAS **160**, 1P (1972)
48. M. Joyce, T. Prokopec, Phys. Rev. D **57**, 6022 (1998)
49. V. Sahni et al., Phys. Rev. D **65**, 023518 (2001)
50. V. Suleimanov et al., Astrophys. J. **742**, 122 (2011)
51. T.K. Mathew et al., Eur. Phys. J. C **74**, 3188 (2014)
52. P.-H. Chavanis, Phys. Rev. D **92**, 103004 (2015)
53. M.K. Mak, T. Harko, Phys. Rev. D **71**, 104022 (2005)
54. C.-M. Chen et al., Phys. Rev. D **64**, 044013 (2001)
55. B. Li et al., Phys. Rev. D **89**, 083536 (2014)
56. V. Singh, C.P. Singh, Int. J. Theor. Phys. **55**, 1257 (2016)
57. J. Quintin et al., Phys. Rev. D **90**, 063507 (2014)
58. T. Harko et al., Eur. Phys. J. C **75**, 386 (2015)
59. P.H.R.S. Moraes, J.D.V. Arbañil, M. Malheiro, JCAP **06**, 005 (2016)
60. R. Zaregonbadi et al., Phys. Rev. D **94**, 084052 (2016)
61. H. Shabani, M. Farhoudi, Phys. Rev. D **90**, 044031 (2014)
62. P.H.R.S. Moraes, R.A.C. Correa, R.V. Lobato, JCAP **07**, 029 (2017)
63. M. Zubair, I. Noureen, Astrophys. Space Sci. **75**, 265 (2015)
64. Z. Yousaf et al., Eur. Phys. J. Plus **132**, 268 (2017)
65. M. Sharif, Z. Yousaf, Astrophys. Space Sci. **354**, 471 (2014)
66. H. Shabani, A.H. Ziaie, Eur. Phys. J. C **77**, 31 (2017)

67. A. Alhamzawi, R. Alhamzawi, *Int. J. Mod. Phys. D* **25**, 1650020 (2016)
68. P.B. Demorest et al., *Nature* **467**, 1081 (2010)
69. J. Antoniadis et al., *Science* **340**, 448 (2013)
70. C.P. Singh, V. Singh, *Gen. Relativ. Gravit.* **46**, 1696 (2014)
71. K. Dunn, *Gen. Relativ. Gravit.* **21**, 137 (1989)
72. A.A. Coley, B.O.J. Tupper, *J. Math. Phys.* **27**, 406 (1986)
73. A.A. Coley, J. Wainwright, *Class. Quantum Gravity* **9**, 651 (1992)
74. T. Harko, F.S.N. Lobo, *Phys. Rev. D* **83**, 124051 (2011)
75. S.Z.W. Lip, *Phys. Rev. D* **83**, 023528 (2011)
76. M. Tsamparlis, A. Paliathanasis, *Class. Quantum Gravity* **29**, 015006 (2012)
77. A. Drago et al., *Phys. Rev. D* **89**, 043014 (2014)
78. P.H. Chavanis, *Phys. Rev. D* **92**, 103004 (2012)
79. J. Lankinen, L. Vilja, *JCAP* **08**, 025 (2017)
80. P. Kumar, C.P. Singh, *Astrophys. Space Sci.* **357**, 120 (2015)
81. G.A. Carvalho, R.V. Lobato, P.H.R.S. Moraes, J.D.V. Arbañil, E. Otoniel, R.M. Marinho Jr., M. Malheiro, *Eur. Phys. J. C* **77**, 871 (2017)