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Lorentzian Goldstone modes shared among photons and gravitons

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Abstract It has long been known that photons and gravitons may appear as vector and tensor Goldstone modes caused by spontaneous Lorentz invariance violation (SLIV). Usually this approach is considered for photons and gravitons separately. We develop the emergent electrogravity theory consisting of the ordinary QED and the tensor-field gravity model which mimics the linearized general relativity in Minkowski spacetime. In this theory, Lorentz symmetry appears incorporated into higher global symmetries of the length-fixing constraints put on the vector and tensor fields involved, $A_{\mu}^2 = \pm M_A^2$ and $H_{\mu\nu}^2 = \pm M_H^2$ (M_A and M_H are the proposed symmetry breaking scales). We show that such a SLIV pattern being related to breaking of global symmetries underlying these constraints induces the massless Goldstone and pseudo-Goldstone modes shared by photon and graviton. While for a vector field case the symmetry of the constraint coincides with Lorentz symmetry $SO(1,3)$ of the electrogravity Lagrangian, the tensor-field constraint itself possesses much higher global symmetry $SO(7,3)$, whose spontaneous violation provides a sufficient number of zero modes collected in a graviton. Accordingly, while the photon may only contain true Goldstone modes, the graviton appears at least partially to be composed of pseudo-Goldstone modes rather than of pure Goldstone ones. When expressed in terms of these modes, the theory looks essentially nonlinear and contains a variety of Lorentz and CPT violating couplings. However, all SLIV effects turn out to be strictly cancelled in the lowest order processes considered in some detail. How this emergent electrogravity theory could be observationally different from conventional QED and GR theories is also briefly discussed.

1 Introduction

The extremely successful concept of the spontaneously broken internal symmetries in particle physics allows one to think that spontaneous violation of spacetime symmetries and, particularly, spontaneous Lorentz invariance violation (SLIV), could also provide some dynamical approach to quantum electrodynamics [1], gravity [2] and Yang–Mills theories [3] with photon, graviton and non-Abelian gauge fields appearing as massless Nambu–Goldstone (NG) bosons [4,5] (for some later developments, see [6–14]). In this connection, we recently suggested [15,16] an alternative approach to the emergent gravity theory in the framework of nonlinearly realized Lorentz symmetry for the underlying symmetric two-index tensor field in a theory, which mimics linearized general relativity in Minkowski space-time. It was shown that such a SLIV pattern, due to which a true vacuum in the theory is chosen, induces massless tensor Goldstone and pseudo-Goldstone modes, some of which can naturally be associated with the physical graviton.

This approach itself has had a long history, dating back to the model of Nambu [17] for QED with a nonlinearly realized Lorentz symmetry for the underlying vector field. This may indeed appear through the "length-fixing" vector field constraint

$$A_{\mu}^2 = n^2 M_A^2, \quad A_{\mu}^2 \equiv A_{\mu}A^{\mu},\ n^2 \equiv n_{\nu}n^{\nu} = \pm 1 \tag{1}$$

(where n_{ν} is a properly oriented unit Lorentz vector, while M_A is the proposed scale for Lorentz violation) much as it works in the nonlinear σ-model [18] for pions, $\sigma^2 + \pi^2 = f_{\pi}^2$, where f_{π} is the pion decay constant. Note that a correspondence with the nonlinear σ model for pions may appear rather suggestive in view of the fact that pions are the only presently known Goldstone particles whose theory, chiral dynamics [18], is given by the nonlinearly realized chiral $SU(2) \times SU(2)$ symmetry rather than by an ordinary linear σ model. The constraint (1) means in essence that the vector field A_{μ} develops some constant background value,

$$< A_{\mu}(x) > = n_{\mu} M_A, \tag{2}$$

and the Lorentz symmetry $SO(1,3)$ formally breaks down to $SO(3)$ or $SO(1,2)$ depending on the timelike ($n^2 > 0$)

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or spacelike ($n^2 < 0$) nature of SLIV. However, in sharp contrast to the nonlinear σ model for pions, the nonlinear QED theory, due to the starting gauge invariance involved, ensures that all the physical Lorentz violating effects turn out to be non-observable. It was shown [17], while only in the tree approximation and for the timelike SLIV ($n^2 > 0$), that the nonlinear constraint (1) implemented into the standard QED Lagrangian containing a charged fermion field $\psi(x)$

$$L_{QED} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \overline{\psi}(i\gamma\partial + m)\psi - eA_\mu\overline{\psi}\gamma^\mu\psi \quad (3)$$

as a supplementary condition appears in fact as a possible gauge choice for the vector field A_μ, while the S-matrix remains unaltered under such a gauge convention. Really, this nonlinear QED contains a plethora of Lorentz and CPT violating couplings when it is expressed in terms of the pure emergent photon modes (a_μ) according to the constraint condition (1)

$$A_\mu = a_\mu + n_\mu(M_A^2 - n^2a^2)^{\frac{1}{2}}, \quad n_\mu a_\mu = 0 \quad (a^2 \equiv a_\mu a^\mu). \quad (4)$$

For definiteness, one takes the positive sign for the square root (giving an effective Higgs mode) when expanding it in powers of a^2/M_A^2,

$$A_\mu = a_\mu + M_A n_\mu - \frac{n^2}{2M_A}a^2 n_\mu + O(1/M_A^2). \quad (5)$$

However, the contributions of all these Lorentz violating couplings to physical processes completely cancel out among themselves. So, SLIV is shown to be superficial as it affects only the gauge of the vector potential A_μ at least in the tree approximation [17].

Some time ago, this result was extended to the one-loop approximation and for both the timelike ($n^2 > 0$) and spacelike ($n^2 < 0$) Lorentz violation [19]. All the contributions to the photon–photon, photon–fermion and fermion–fermion interactions violating physical Lorentz invariance were shown to exactly cancel among themselves in the manner observed long ago by Nambu for the simplest tree-order diagrams. This means that the constraint (1), having been treated as a nonlinear gauge choice at the tree (classical) level, remains as a gauge condition when quantum effects are taken into account as well. So, in accordance with Nambu's original conjecture, one can conclude that physical Lorentz invariance is left intact at least in the one-loop approximation, provided we consider the standard gauge invariant QED Lagrangian (3) taken in flat Minkowski space-time. Later this result was confirmed for the spontaneously broken massive QED [20], non-Abelian theories [21] and tensor-field gravity [15,16]. The point is, however, that all these calculations represent somewhat "empirical" confirmation of gauge invariance of the nonlinear QED and other emergent theories rather than a theoretical one. Indeed, whether the constraint (1) amounts in general to a special gauge choice for a vector field is an open question unless the corresponding gauge function satisfying the constraint condition is explicitly constructed. We discuss this important issue in more detail in Sect. 4.

Let us note that, in principle, the vector field constraint (1) may be formally obtained in some limit from a conventional potential that could be included in the QED Lagrangian (3),

$$U(A) = \lambda_A(A_\mu^2 - n^2M_A^2)^2, \quad (6)$$

thus extending QED to the so-called bumblebee model [22]. Here $\lambda_A > 0$ stands for the coupling constant of the vector field, while values of $n^2 = \pm 1$ determine again its possible vacuum configurations. Indeed, one can readily see that the potential (6) inevitably causes spontaneous violation of Lorentz symmetry in an ordinary way, much as an internal symmetry violation is caused in the linear σ model for pions [18]. As a result, one has a massive "Higgs" mode (with mass $2\sqrt{2\lambda_A}M_A$) together with massless Goldstone modes associated with the photon components. However, as was argued in [23], the bumblebee model adding the potential terms (6) to the standard QED Lagrangian is generally unstable. Indeed, its Hamiltonian appears to be unbounded from below unless the phase space is constrained just by the nonlinear condition $A_\mu^2 = n^2M_A^2$. With this condition imposed, the Hamiltonian becomes positive, the massive Higgs mode never emerges, and the model is physically equivalent to the Nambu model [17]. Remarkably, this pure Goldstone theory limit can be reached when, just as in the σ model for pions, one goes from the linear model for the SLIV to the nonlinear one by taking the limit $\lambda_A \to \infty$. This immediately fixes in (6) the vector field square to its vacuum value, thus leading to the above constraint (1). As a matter of fact, the vector field theory turns out to be stable in this limit only.

Actually, for the tensor-field gravity we use a similar nonlinear constraint for a symmetric two-index tensor field,

$$H_{\mu\nu}^2 = \mathfrak{n}^2M_H^2, \quad H_{\mu\nu}^2 \equiv H_{\mu\nu}H^{\mu\nu}, \quad \mathfrak{n}^2 \equiv \mathfrak{n}_{\mu\nu}\mathfrak{n}^{\mu\nu} = \pm 1 \quad (7)$$

(where $\mathfrak{n}_{\mu\nu}$ is now a properly oriented unit Lorentz tensor, while M_H is the proposed scale for Lorentz violation in the gravity sector) which fixes its length in the same manner as it appears for the vector field (1). Again, the nonlinear constraint (7) may in principle appear from the standard potential terms added to the tensor-field Lagrangian

$$U(H) = \lambda_H(H_{\mu\nu}^2 - \mathfrak{n}^2M_H^2)^2 \quad (8)$$

in the nonlinear σ-model type limit when the coupling constant λ_H goes to infinity. Just in this limit the tensor field theory appears stable, though, due to the corresponding Higgs mode excluded, it does not lead to physical Lorentz violation [15,16].

Usually, an emergent gauge field framework is considered either regarding emergent photons or regarding emergent gravitons. For the first time, we consider it regarding them both in the so-called electrogravity theory where together with the Nambu QED model [17] with its gauge invariant Lagrangian (3) we propose the linearized Einstein–Hilbert kinetic term for the tensor field preserving a diffeomorphism (diff) invariance. We show that such a combined SLIV pattern, conditioned by the constraints (1) and (7), induces the massless Goldstone modes which appear shared among photon and graviton. Note that one needs in common nine zero modes both for photon (three modes) and graviton (six modes) to provide all necessary (physical and auxiliary) degrees of freedom. They actually appear in our electrogravity theory due to spontaneous breaking of high symmetries of the constraints involved. While for the vector field case the symmetry of the constraint coincides with the Lorentz symmetry $SO(1,3)$, the tensor field constraint itself possesses a much higher global symmetry $SO(7,3)$, whose spontaneous violation provides a sufficient number of zero modes collected in a graviton. These modes are largely pseudo-Goldstone modes (PGMs) since $SO(7,3)$ is a symmetry of the constraint (7) rather than the electrogravity Lagrangian whose symmetry is only given by Lorentz invariance. The electrogravity theory we start with becomes essentially nonlinear, when expressed in terms of the Goldstone modes, and contains a variety of Lorentz (and CPT) violating couplings. However, as our calculations show, all SLIV effects turn out to be strictly cancelled in the low order physical processes involved once the tensor-field gravity part of the electrogravity theory is properly extended to general relativity (GR). This can be taken as an indication that in the electrogravity theory physical Lorentz invariance is preserved in this approximation. Thereby, the length-fixing constraints (1) and (7) put on the vector and tensor fields appear as the gauge fixing conditions rather than sources of the actual Lorentz violation just as it was in the pure nonlinear QED framework [17]. From this viewpoint, if this cancellation were to work in all orders, one could propose that emergent theories, like as the electrogravity theory, are not different from conventional gauge theories. We argue, however, that even in this case some observational difference between them could unavoidably appear, if gauge invariance were presumably broken by quantum gravity at the Planck scale order distances.

The paper is organized in the following way. In Sect. 2 we formulate the model for the tensor-field gravity and find corresponding massless Goldstone modes some of which are collected in the graviton. Then in Sect. 3 we consider in significant detail the combined electrogravity theory consisting of QED and tensor field gravity. In Sect. 4 we derive general Feynman rules for basic interactions in the emergent framework. The model appears to be in essence three-parametric containing the inverse Planck and SLIV scales, $1/M_P$, $1/M_A$ and $1/M_H$, respectively, as the perturbation parameters, so that the SLIV interactions are always proportional to some powers of them. Further, some lowest order SLIV processes, such as an elastic photon–graviton scattering and photon–graviton conversion are considered in detail. We show that all these effects, taken in the tree approximation, appear in fact to be vanishing so that the physical Lorentz invariance is ultimately restored. Finally, in Sect. 5 we present our conclusion.

2 Tensor-field gravity

We propose here, closely following Refs. [15,16], the tensor-field gravity theory which mimics linearized general relativity in Minkowski space-time. The corresponding Lagrangian for one real vector field A_μ (still representing all sorts of matter in the model)

$$\mathcal{L}(H,A)=\mathcal{L}(H)+\mathcal{L}(A)+\mathcal{L}_{\text{int}} \tag{9}$$

consists of the tensor field kinetic terms of the form

$$\mathcal{L}(H)=\frac{1}{2}\partial_\lambda H^{\mu\nu}\partial^\lambda H_{\mu\nu}-\frac{1}{2}\partial_\lambda H_{\text{tr}}\partial^\lambda H_{\text{tr}}-\partial_\lambda H^{\lambda\nu}\partial^\mu H_{\mu\nu}+\partial^\nu H_{\text{tr}}\partial^\mu H_{\mu\nu}, \tag{10}$$

(H_{tr} stands for the trace of $H_{\mu\nu}$, $H_{\text{tr}}=\eta^{\mu\nu}H_{\mu\nu}$), which is invariant under the diff transformations

$$\delta H_{\mu\nu}=\partial_\mu\xi_\nu+\partial_\nu\xi_\mu,\quad \delta x^\mu=\xi^\mu(x), \tag{11}$$

and the interaction terms

$$\mathcal{L}_{\text{int}}(H,A)=-\frac{1}{M_P}H_{\mu\nu}T^{\mu\nu}(A). \tag{12}$$

The $\mathcal{L}(A)$ and $T^{\mu\nu}(A)$ are the conventional free Lagrangian and energy-momentum tensor for a vector field

$$\mathcal{L}(A)=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu},\ T^{\mu\nu}(A)=-F^{\mu\rho}F^\nu_\rho+\frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}. \tag{13}$$

It is clear that, in contrast to the tensor field kinetic terms, the other terms in (9) are only approximately invariant under the diff transformations (11). They become more and more invariant when the tensor-field gravity Lagrangian (9) is properly extended to GR with higher terms in H-fields included.[1] Following the nonlinear σ-model for QED [17], we propose the SLIV condition (7) as some tensor field length-fixing constraint which is supposed to be substituted into the total

[1] Such an extension means that in all terms included in the GR action, particularly in the QED Lagrangian term, $(-g)^{1/2}g_{\mu\nu}g_{\lambda\rho}F^{\mu\lambda}F^{\nu\rho}$, one expands the metric tensors

$$g_{\mu\nu}=\eta_{\mu\nu}+H_{\mu\nu}/M_P,\quad g^{\mu\nu}=\eta^{\mu\nu}-H^{\mu\nu}/M_P+H^{\mu\lambda}H^\nu_\lambda/M_P^2+\cdots$$

taking into account the higher terms in H-fields.

Lagrangian $\mathcal{L}(H, A)$ prior to the variation of the action. This eliminates, as was mentioned above, a massive Higgs mode in the final theory, thus leaving only massless Goldstone modes, some of which are then collected in a graviton.

Let us first turn to the spontaneous Lorentz violation itself in a gravity sector, which is caused by the constraint (7), while such a violation in a QED sector is assumed to be determined by the constraint (1). The latter leads, as was mentioned above, only to two possible breaking channels of the starting Lorentz symmetry, namely to $SO(3)$ or $SO(1,2)$, depending on the timelike ($\mathfrak{n}^2 > 0$) or spacelike ($\mathfrak{n}^2 < 0$) nature of SLIV. For the tensor-field constraint (7) the choice turns out to be wider. Indeed, this constraint can be written in the more explicit form

$$H_{\mu\nu}^2 = H_{00}^2 + H_{i=j}^2 + (\sqrt{2}H_{i\neq j})^2 - (\sqrt{2}H_{0i})^2 = \mathfrak{n}^2 M_H^2 = \pm M_H^2 \tag{14}$$

(where the summation over all indices ($i, j = 1, 2, 3$) is imposed) and means in essence that the tensor field $H_{\mu\nu}$ develops the vacuum expectation value (VEV) configuration

$$< H_{\mu\nu}(x) >= \mathfrak{n}_{\mu\nu} M_H \tag{15}$$

determined by the matrix $\mathfrak{n}_{\mu\nu}$. The initial Lorentz symmetry $SO(1,3)$ of the Lagrangian $\mathcal{L}(H, A)$ given in (9) then formally breaks down at a scale M_H to one of its subgroups. If one assumes a "minimal" vacuum configuration in the $SO(1,3)$ space with the VEV (15) developed on a single $H_{\mu\nu}$ component, there are in fact the following three breaking channels:

$$\begin{aligned}
&(a)\quad \mathfrak{n}_{00} \neq 0, \quad SO(1,3) \to SO(3),\\
&(b)\quad \mathfrak{n}_{i=j} \neq 0, \quad SO(1,3) \to SO(1,2),\\
&(c)\quad \mathfrak{n}_{i\neq j} \neq 0, SO(1,3) \to SO(1,1),
\end{aligned} \tag{16}$$

for the positive sign in (14), and

$$(d)\mathfrak{n}_{0i} \neq 0, \quad SO(1,3) \to SO(2), \tag{17}$$

for the negative sign. These cases can be readily derived taking an appropriate exponential parametrization for the tensor field:

$$H_{\alpha\beta} = \left[e^{i\,\mathfrak{n}_{\mu\nu}\mathcal{J}^{\mu\sigma}\eta^{\nu\tau}h_{\sigma\tau}/M_H}\right]_{\alpha\beta}^{\gamma\delta}\mathfrak{n}_{\gamma\delta}M_H, \quad \left(\mathcal{J}^{\mu\nu}\right)_\alpha^\beta = i\left(\delta_\alpha^\mu\eta^{\nu\beta} - \delta_\alpha^\nu\eta^{\mu\beta}\right) \tag{18}$$

where the Lorentz generators $\mathcal{J}^{\mu\nu}$ are explicitly included.[2] Accordingly, there are only three Goldstone modes in the cases (a, b) and five modes in the cases (c)–(d). In order to associate at least one of the two transverse polarization states of the physical graviton with these modes, one could have any of the above-mentioned SLIV channels except for the case (a) where the only nonzero Goldstone modes are given by the tensor components h_{0i} ($i = 1, 2, 3$). Indeed, it is impossible for a graviton to have all vanishing spatial components, as in the case (a). However, these components may be provided by some accompanying pseudo-Goldstone modes, as we argue below. Apart from the minimal VEV configuration, there are many others as well. A particular case of interest is that of the traceless VEV tensor $\mathfrak{n}_{\mu\nu}$

$$\mathfrak{n}_{\mu\nu}\eta^{\mu\nu} = 0, \tag{19}$$

in terms of which the emergent gravity Lagrangian acquires an especially simple form (see below). It is clear that the VEV in this case can be developed on several $H_{\mu\nu}$ components simultaneously, which in general may lead to total Lorentz violation with all six Goldstone modes generated. For simplicity, we will use sometimes this form of vacuum configuration in what follows, while our arguments can be applied to any type of VEV tensor $\mathfrak{n}_{\mu\nu}$.

Aside from the pure Lorentz Goldstone modes, the question of the other components of the symmetric two-index tensor $H_{\mu\nu}$ naturally arises. Remarkably, they turn out to be pseudo-Goldstone modes (PGMs) in the theory. Indeed, although we only propose Lorentz invariance of the Lagrangian $\mathcal{L}(H, A)$, the SLIV constraint (7) formally possesses the much higher global accidental symmetry $SO(7,3)$ of the constrained bilinear form (14), which manifests itself when considering the $H_{\mu\nu}$ components as the "vector" ones under $SO(7,3)$. This symmetry is in fact spontaneously broken, side by side with Lorentz symmetry, at the scale M_H. Assuming again a minimal vacuum configuration in the $SO(7,3)$ space, with the VEV (15) developed on a single $H_{\mu\nu}$ component, we have either timelike ($SO(7,3) \to SO(6,3)$) or spacelike ($SO(7,3) \to SO(7,2)$) violations of the accidental symmetry depending on the sign of $\mathfrak{n}^2 = \pm 1$ in (14). According to the number of broken $SO(7,3)$ generators, just nine massless Goldstone modes appear in both cases. Together with an effective Higgs

[2] One may alternatively argue starting from the vector representation of the higher $SO(7,3)$symmetry determined by the constraint equation (14) itself (see below). Thereby, one has a standard parametrization

$$H_A = \left[e^{i\,\mathfrak{n}_M\mathcal{J}^{MN}h_N/M_H}\right]_A^B \mathfrak{n}_B M_H$$

where the "big" indices A, B, M, N correspond to the pairs of different values of the old indices $(\mu\nu)$ appearing in (14). Consequently, one has the equality $\mathfrak{n}_M\mathcal{J}^{MN}h_N = \mathfrak{n}_{\mu\nu}\mathcal{J}^{\mu\sigma}\eta^{\nu\tau}h_{\sigma\tau}$ when going to the standard Lorentz indices so that antisymmetry in the indices (M, N) goes to antisymmetry in the index pairs $(\mu\nu, \sigma\tau)$.

component, on which the VEV is developed, they complete the whole ten-component symmetric tensor field $H_{\mu\nu}$ of the basic Lorentz group as is presented in its parametrization (20). Some of them are true Goldstone modes of the spontaneous Lorentz violation, others are PGMs since, as was mentioned, an accidental $SO(7,3)$ symmetry is not shared by the whole Lagrangian $\mathcal{L}(H, A)$ given in (9). Notably, in contrast to the scalar PGM case [18], they remain strictly massless being protected by the starting diff invariance which becomes exact when the tensor-field gravity Lagrangian (9) is properly extended to GR[1]. Owing to this invariance, some of the Lorentz Goldstone modes and PGMs can then be gauged away from the theory, as usual.

Now, one can rewrite the Lagrangian $\mathcal{L}(H, A)$ in terms of the tensor Goldstone modes explicitly using the SLIV constraint (7). For this purpose, let us take the following handy parameterization for the tensor field $H_{\mu\nu}$:

$$H_{\mu\nu} = h_{\mu\nu} + \mathfrak{n}_{\mu\nu}(M_H^2 - \mathfrak{n}^2 h^2)^{\frac{1}{2}}, \\ \mathfrak{n} \cdot h = 0 \quad (\mathfrak{n} \cdot h \equiv \mathfrak{n}_{\mu\nu} h^{\mu\nu}). \tag{20}$$

Here $h_{\mu\nu}$ corresponds to the pure emergent modes satisfying the orthogonality condition, while the effective "Higgs" mode (or the $H_{\mu\nu}$ component in the vacuum direction) is given by the square root for which we take again the positive sign when expanding it in powers of h^2/M_H^2 ($h^2 \equiv h_{\mu\nu}h^{\mu\nu}$)

$$H_{\mu\nu} = h_{\mu\nu} + \mathfrak{n}_{\mu\nu} M_H - \frac{\mathfrak{n}^2 h^2}{2M_H} + O(1/M_H^2). \tag{21}$$

It should be particularly emphasized that the modes collected in the $h_{\mu\nu}$ are generally the Goldstone modes of the broken accidental $SO(7,3)$ symmetry of the constraint (7), thus containing the Lorentz Goldstone modes and PGMs put together. If Lorentz symmetry is completely broken then the pure Goldstone modes appear enough to be solely collected in a physical graviton. On the other hand, when one has a partial Lorentz violation, some PGMs should be added.

Putting then the parameterization (21) into the total Lagrangian $\mathcal{L}(H, A)$ given in (9), (10), and 12), one arrives at the truly emergent tensor-field gravity Lagrangian $\mathcal{L}(h, A)$, containing an infinite series in powers of the $h_{\mu\nu}$ modes. For the traceless VEV tensor $\mathfrak{n}_{\mu\nu}$ (19) it takes, without loss of generality, the especially simple form

$$\begin{aligned}\mathcal{L}(h, A) = {} & \frac{1}{2}\partial_\lambda h^{\mu\nu}\partial^\lambda h_{\mu\nu} - \frac{1}{2}\partial_\lambda h_{\mathrm{tr}}\partial^\lambda h_{\mathrm{tr}} - \partial_\lambda h^{\lambda\nu}\partial^\mu h_{\mu\nu} \\ & + \partial^\nu h_{\mathrm{tr}}\partial^\mu h_{\mu\nu} + \\ & - \frac{\mathfrak{n}^2}{M_H} h^2 \mathfrak{n}^{\mu\lambda}\left[\partial_\lambda\partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu\partial_\lambda h_{\mathrm{tr}}\right] \\ & + \frac{\mathfrak{n}^2}{8M_H^2}\left(\eta^{\mu\nu} - \frac{\mathfrak{n}^{\mu\lambda}\mathfrak{n}^{\nu\lambda}}{\mathfrak{n}^2}\right)\partial_\mu h^2 \partial_\nu h^2 \\ & + \mathcal{L}(A) + \frac{M_H}{M_P}\left[\mathfrak{n}_{\mu\nu} F^{\mu\rho} F^\nu_\rho\right] - \frac{1}{M_P} h_{\mu\nu} T^{\mu\nu} \\ & - \frac{1}{2M_H M_P} h^2 \left[\mathfrak{n}_{\mu\nu} F^{\mu\rho} F^\nu_\rho\right],\end{aligned} \tag{22}$$

written in the $O(h^2/M_H^2)$ approximation in which, besides the conventional graviton bilinear kinetic terms, there are also three- and four-linear interaction terms in powers of $h_{\mu\nu}$ in the Lagrangian. Some of the notations used are collected here:

$$h^2 \equiv h_{\mu\nu}h^{\mu\nu}, \quad h_{\mathrm{tr}} \equiv \eta^{\mu\nu}h_{\mu\nu}. \tag{23}$$

The bilinear vector field term

$$\frac{M_H}{M_P}\left[\mathfrak{n}_{\mu\nu} F^{\mu\rho} F^\nu_\rho\right] \tag{24}$$

in the third line in the Lagrangian (22) merits special notice. This term arises from the interaction Lagrangian $\mathcal{L}_{\mathrm{int}}$ (12) after application of the tracelessness condition (19) for the VEV tensor $\mathfrak{n}_{\mu\nu}$. It could significantly affect the dispersion relation for the vector field A (and any other matter as well) thus leading to an unacceptably large Lorentz violation if the SLIV scale M_H were comparable with the Planck mass M_P. However, this term can be gauged away [15,16] by an appropriate redefinition of the vector field by going to the new coordinates

$$x^\mu \to x^\mu + \xi^\mu. \tag{25}$$

In fact, with a simple choice of the parameter function $\xi^\mu(x)$ being linear in the 4-coordinate,

$$\xi^\mu(x) = \frac{M_H}{M_P}\mathfrak{n}^{\mu\nu}x_\nu, \tag{26}$$

the term (24) is cancelled by an analogous term stemming from the vector field kinetic term in $\mathcal{L}(A)$ given in (12). On the other hand, since the diff invariance is an approximate symmetry of the Lagrangian $\mathcal{L}(H, A)$ we started with (9), this cancellation will only be accurate up to the linear order corresponding to the tensor field theory. Indeed, a proper extension of this theory to GR[1] with its exact diff invariance will ultimately restore the usual dispersion relation for the vector (and other matter) fields. We will consider all that in significant detail in the next section.

Together with the Lagrangian one must also specify other supplementary conditions for the tensor field $h^{\mu\nu}$ (appearing eventually as possible gauge fixing terms in the emergent tensor-field gravity) in addition to the basic emergent "gauge" condition $\mathfrak{n}_{\mu\nu}h^{\mu\nu} = 0$ given above (20). The point is that the spin 1 states are still left in the theory being described by some of the components of the new tensor $h_{\mu\nu}$. This is certainly inadmissible.[3] Usually, the spin 1 states (and one of the spin

[3] Indeed, the spin-1 component must be necessarily excluded in the tensor $h_{\mu\nu}$, since the sign of the energy for the spin-1 component is always opposite to that for the spin-2 and spin-0 ones.

0 states) are excluded by the conventional Hilbert–Lorentz condition,

$$\partial^{\mu} h_{\mu\nu} + q\partial_{\nu} h_{\text{tr}} = 0 \tag{27}$$

(q is an arbitrary constant, giving for $q = -1/2$ the standard harmonic gauge condition). However, as we have already imposed the emergent constraint (20), we cannot use the full Hilbert–Lorentz condition (27) eliminating four more degrees of freedom in $h_{\mu\nu}$. Otherwise, we would have an "over-gauged" theory with a non-propagating graviton. In fact, the simplest set of conditions which conform with the emergent condition $\mathfrak{n} \cdot h = 0$ in (20) turns out to be

$$\partial^{\rho}(\partial_{\mu} h_{\nu\rho} - \partial_{\nu} h_{\mu\rho}) = 0. \tag{28}$$

This set excludes only three degrees of freedom[4] in $h_{\mu\nu}$ and, besides, it automatically satisfies the Hilbert–Lorentz spin condition as well. So, with the Lagrangian (22) and the supplementary conditions (20) and (28) lumped together, one eventually arrives at a working model for the emergent tensor-field gravity [15,16]. Generally, from ten components of the symmetric two-index tensor $h_{\mu\nu}$ four components are excluded by the supplementary conditions (20) and (28). For a plane gravitational wave propagating in, say, the z direction another four components are also eliminated, due to the fact that the above supplementary conditions still leave freedom in the choice of a coordinate system, $x^{\mu} \rightarrow x^{\mu} + \xi^{\mu}(t - z/c)$, much as in standard GR. Depending on the form of the VEV tensor $\mathfrak{n}_{\mu\nu}$, caused by SLIV, the two remaining transverse modes of the physical graviton may consist solely of Lorentzian Goldstone modes or of pseudo-Goldstone modes, or include both of them. This theory, similar to the nonlinear QED [17], while suggesting an emergent description for the graviton, does not lead to physical Lorentz violation [15,16].

3 Electrogravity theory

3.1 Emergent photons and gravitons together

So far we considered the vector field A_{μ} as an unconstrained material field which the emergent gravitons interact with. Now, we propose that the vector field also develops the VEV through the SLIV constraint (1), thus generating the massless vector Goldstone modes associated with a photon. We also include the complex scalar field φ (taken to be massless, for simplicity) as an actual matter in the theory

$$\mathcal{L}(\varphi) = D_{\mu}\varphi \left(D_{\mu}\varphi\right)^{*}, \quad D_{\mu} = \partial_{\mu} + ieA_{\mu}. \tag{29}$$

[4] The solution for a gauge function $\xi_{\mu}(x)$ satisfying the condition (28) can generally be chosen as $\xi_{\mu} = \Box^{-1}(\partial^{\rho} h_{\mu\rho}) + \partial_{\mu}\theta$, where $\theta(x)$ is an arbitrary scalar function, so that only three degrees of freedom in $h_{\mu\nu}$ are actually eliminated.

So, the total starting electrogravity Lagrangian is, therefore, proposed to be

$$\mathcal{L}_{\text{tot}} = \mathcal{L}(A) + \mathcal{L}(H) + \mathcal{L}(\varphi) + \mathcal{L}_{\text{int}}(H, A, \varphi) \tag{30}$$

where the Lagrangians $\mathcal{L}(A)$ and $\mathcal{L}(H)$ were given above in Eqs. (13) and (10), while the gravity interaction part,

$$\mathcal{L}_{\text{int}}(H, A, \varphi) = -\frac{1}{M_P} H_{\mu\nu}[T^{\mu\nu}(A) + T^{\mu\nu}(\varphi)], \tag{31}$$

contains the tensor field couplings with canonical energy-momentum tensors of vector and scalar fields.

In the symmetry broken phase one goes to the pure Goldstone vector and tensor modes, a_{μ} and $h_{\mu\nu}$, respectively. Whereas the tensor modes $h_{\mu\nu}$, including their kinetic and interaction terms, have been thoroughly discussed in terms of Eq. (22), the vector modes a_{μ} are not yet properly exposed. Putting the parametrization (4) into the Lagrangian (13) one has from the vector field kinetic term (taken to the first order in a^2/M_A^2)

$$\begin{aligned}\mathcal{L}(A) \rightarrow \mathcal{L}(a) = &-\frac{1}{4} f_{\mu\nu} f^{\mu\nu} - \frac{1}{2}\delta(n \cdot a)^2 \\ &-\frac{1}{4}\frac{n^2}{M_A} f_{\mu\nu}(\partial^{\mu\nu} a^2).\end{aligned} \tag{32}$$

We have denoted the a_{μ} strength tensor by $f_{\mu\nu} = \partial_{\mu} a_{\nu} - \partial_{\nu} a_{\mu}$, while $\partial^{\mu\nu} = n^{\mu}\partial^{\nu} - n^{\nu}\partial^{\mu}$ is a new SLIV oriented differential tensor acting on the infinite series in a^2 coming from the expansion of the effective "Higgs" mode $(M_A^2 - n^2 a^2)^{\frac{1}{2}}$ in (4), from which we have only included the lowest order term $-n^2 a^2/2M_A$ throughout the Lagrangian $\mathcal{L}(a)$. We have also explicitly introduced in the Lagrangian the emergent orthogonality condition $n \cdot a = 0$, which can be treated as the gauge fixing term (when taking the limit $\delta \rightarrow \infty$). At the same time, the scalar field Lagrangian $\mathcal{L}(\varphi)$ in (30) is going now to

$$\mathcal{L}(\varphi) = \left|\left(\partial_{\mu} + iea_{\mu} + ieM_A n_{\mu} - ie\frac{n^2}{2M_A} a^2 n_{\mu}\right)\varphi\right|^2, \tag{33}$$

while the tensor field interacting terms (31) in $\mathcal{L}_{\text{int}}(H, A, \varphi)$ convert to

$$\begin{aligned}\mathcal{L}_{\text{int}} = &-\frac{1}{M_P}\left(h_{\mu\nu} + M_H \mathfrak{n}_{\mu\nu} - \frac{\mathfrak{n}^2}{2M_H} h^2 \mathfrak{n}_{\mu\nu}\right) \\ &\times \left[T^{\mu\nu}\left(a_{\mu} - \frac{n^2}{2M_A} a^2 n_{\mu}\right) + T^{\mu\nu}(\varphi)\right],\end{aligned} \tag{34}$$

where the vector field energy-momentum tensor is now solely a function of the Goldstone a_{μ} modes.

3.2 Constraints and zero mode spectrum

Before going any further, let us make some necessary comments. Note first of all that, apart from dynamics described by the total Lagrangian $\mathcal{L}_{\text{tot}}$, the vector and tensor field constraints (1) and (7) are also proposed. In principle, these constraints could be formally obtained from the conventional potential terms included in the total Lagrangian $\mathcal{L}_{\text{tot}}$, as was discussed in Sect. 1. The most general potential, where the vector and tensor field couplings possess the Lorentz and $SO(7,3)$ symmetry, respectively, must be solely a function of $A_\mu^2 \equiv A_\mu A^\mu$ and $H_{\mu\nu}^2 \equiv H_{\mu\nu} H^{\mu\nu}$. Indeed, it cannot include any contracted and intersecting terms like H_{tr}, $H^{\mu\nu} A_\mu A_\nu$ and others, which would immediately reduce the above symmetries to the common Lorentz one. So, one may only write

$$U(A,H) = \lambda_A (A_\mu^2 - n^2 M_A^2)^2 + \lambda_H (H_{\mu\nu}^2 - \mathfrak{n}^2 M_H^2)^2 + \lambda_{AH} A_\mu^2 H_{\rho\nu}^2 \quad (35)$$

where $\lambda_{A,H,AH}$ stand for the coupling constants of the vector and tensor fields, while the values of $n^2 = \pm 1$ and $\mathfrak{n}^2 = \pm 1$ determine their possible vacuum configurations. As a consequence, an absolute minimum of the potential (35) might appear for the couplings satisfying the conditions

$$\lambda_{A,H} > 0, \quad \lambda_A \lambda_H > \lambda_{AH}/4. \quad (36)$$

However, as in the pure vector field case discussed in Sect. 1, this theory is generally unstable with the Hamiltonian being unbounded from below unless the phase space is constrained just by the above nonlinear conditions (1) and (7). They in turn follow from the potential (35) when going to the nonlinear σ-model type limit $\lambda_{A,H} \to \infty$. In this limit, the massive Higgs mode disappears from the theory, the Hamiltonian becomes positive, and one arrives at the pure emergent electrogravity theory considered here.

We note again that the Goldstone modes appearing in the theory are caused by breaking of global symmetries related to the constraints (1) and (7) rather than directly to Lorentz violation. Meanwhile, for the vector field case symmetry of the constraint (1) coincides in fact with Lorentz symmetry whose breaking causes the Goldstone modes depending on the vacuum orientation vector n_μ, as can be clearly seen from an appropriate exponential parametrization for the starting vector field,

$$A_\alpha = \left[e^{i n_\mu \mathcal{J}^{\mu\nu} a_\nu / M_A} \right]_\alpha^\beta n_\beta M_A \quad (37)$$

where $n_\mu \mathcal{J}^{\mu\nu}$ just corresponds to the broken Lorentz generators. However, in the tensor field case, due to the higher symmetry $SO(7,3)$ of the constraint (7), there are much more tensor zero modes than would appear from SLIV itself. In fact, they complete the whole tensor multiplet $h_{\mu\nu}$ in the parametrization (20). However, as was discussed in the previous section, only a part of them are true Goldstone modes; the others are pseudo-Goldstone ones. In the minimal VEV configuration case, when these VEVs are developed only on the single A_μ and $H_{\mu\nu}$ components, one has several possibilities determined by the vacuum orientations n_μ and $\mathfrak{n}_{\mu\nu}$ in Eqs. (37), (16), (17) and (18), respectively. There appear 12 zero modes in total, three from Lorentz violation itself and nine from a violation of the $SO(7,3)$ symmetry, which is more than enough to have the necessary three photon modes (two physical and one auxiliary ones) and six graviton modes (two physical and four auxiliary ones). We list below all interesting cases classifying them according to the corresponding $n - \mathfrak{n}$ values.

(1) For the timelike–timelike SLIV, when both $n_0 \neq 0$ and $\mathfrak{n}_{00} \neq 0$, the photon is determined by the space Goldstone components a_i $(i = 1, 2, 3)$ of the partially broken Lorentz symmetry $SO(1,3) \to SO(3)$, while the space–space components h_{ij} needed for physical graviton and its auxiliary components can be only provided by the pseudo-Goldstone modes following from the timelike symmetry breaking $SO(7,3) \to SO(6,3)$ related to the tensor-field constraint (7).

(2) Another interesting case seems to be the timelike–spacelike SLIV, when $n_0 \neq 0$ and $\mathfrak{n}_{i=j} \neq 0$ (one of the diagonal space components of the unit tensor $\mathfrak{n}_{\mu\nu}$ is nonzero). Now, Lorentz symmetry is broken up to the plane rotations $SO(1,3) \to SO(2)$, so that the five true Goldstone bosons appear shared among photon and graviton in the following way. The photon is given again by three space components a_i, while the graviton is determined by two space–space components, h_{12} and h_{13} (if the VEV was developed along the direction $\mathfrak{n}_{11}$), as directly follows from the parametrization Eqs. (37) and (18). Thus, again one necessary component h_{23} for physical graviton, as well as its gauge degrees of freedom, should be provided by the proper pseudo-Goldstone modes following from the spacelike symmetry breaking $SO(7,3) \to SO(7,2)$ related to the tensor-field constraint (7).

(3) For the similar timelike–spacelike SLIV case, when $n_0 \neq 0$ and $\mathfrak{n}_{i\neq j} \neq 0$ (one of the nondiagonal space components of the unit tensor $\mathfrak{n}_{\mu\nu}$ is nonzero), the Lorentz symmetry appears to be fully broken so that the photon has the same three space components a_i, while the graviton physical components are given by the tensor field space components h_{ij}. This is the only case when all physical components of both photon and graviton are provided by the true SLIV Goldstone modes, whereas some gauge degrees of freedom for a graviton are given by the PGM states stemming from the spacelike symmetry breaking $SO(7,3) \to SO(7,1)$ related to the tensor field constraint (7).

(4) Using the parametrization equations (37) and (18) one can readily consider all other possibilities as well; particularly, the spacelike–timelike (nonzero n_i and $\mathfrak{n}_{00}$), spacelike–spacelike diagonal (nonzero n_i and $\mathfrak{n}_{i=j}$) and spacelike–spacelike nondiagonal (nonzero n_i and $\mathfrak{n}_{i\neq j}$) cases. In all these cases, while the photon may only contain true Goldstone modes, some pseudo-Goldstone modes appear to be necessary so as to be collected in the graviton together with some true Goldstone modes.

3.3 Emergent electrogravity interactions

To proceed, one should eliminate, first of all, the large terms of the false Lorentz violation proportional to the SLIV scales M_A and M_H in the interaction Lagrangians (33) and (34). Arranging the phase transformation for the scalar field in the following way:

$$\varphi \to \varphi \exp[-ieM_A n_\mu x^\mu] \tag{38}$$

one can simply cancel that large term in the scalar field Lagrangian (33), thus arriving at

$$\mathcal{L}(\varphi) = \left| \left(D_\mu - ie\frac{n^2}{2M_A} a^2 n_\mu \right) \varphi \right|^2 \tag{39}$$

where the covariant derivative D_μ is read from now on as $D_\mu = \partial_\mu + iea_\mu$. Another unphysical set of terms, like the already discussed term (24), may appear from the gravity interaction Lagrangian L_{int} (34) where the large SLIV entity $M_H \mathfrak{n}_{\mu\nu}$ couples to the energy-momentum tensor. They also can be eliminated by going to the new coordinates (25), as was mentioned in the previous section.

For infinitesimal translations $\xi_\mu(x)$ the tensor field transforms according to (11), while the scalar and vector fields transform as

$$\delta\varphi = \xi_\mu \partial^\mu \varphi, \quad \delta a_\mu = \xi_\lambda \partial^\lambda a_\mu + \partial_\mu \xi_\nu a^\nu, \tag{40}$$

respectively. One can see, therefore, that the scalar field transformation has only the translation part, while the vector one has an extra term related to its nontrivial Lorentz structure. For the constant unit vector n_μ this transformation looks as

$$\delta n_\mu = \partial_\mu \xi_\nu n^\nu, \tag{41}$$

having no translation part. Using all that and also expecting that the phase parameter ξ_λ is in fact linear in coordinate x_μ (which allows one to drop its high-derivative terms), we can easily calculate all scalar and vector field variations, such as

$$\delta\left(D_\mu \varphi\right) = \xi_\lambda \partial^\lambda (D_\mu \varphi) + \partial_\mu \xi_\lambda D^\lambda \varphi, \quad \delta f_{\mu\nu} = \xi_\lambda \partial^\lambda f_{\mu\nu} + \partial_\mu \xi^\lambda f_{\lambda\nu} + \partial_\nu \xi^\lambda f_{\mu\lambda} \tag{42}$$

and others. This finally leads to the total variations of the above Lagrangians. Whereas the pure tensor-field Lagrangian $\mathcal{L}(H)$ (10) is invariant under diff transformations, $\delta\mathcal{L}(H) = 0$, the interaction Lagrangian $\mathcal{L}_{\text{int}}$ in (30) is only approximately invariant, being compensated (in the lowest order in the transformation parameter ξ_μ) by kinetic terms of all the fields involved. However, this Lagrangian becomes increasingly invariant once our theory is extending to GR[1].

In contrast, the vector and scalar field Lagrangians acquire some nontrivial additions,

$$\begin{aligned}
\delta\mathcal{L}(A) &= \xi_\lambda \partial_\lambda \mathcal{L}(A) - \frac{1}{2}\left(\partial_\mu \xi_\lambda + \partial_\lambda \xi_\mu\right) \\
&\quad \times \left[f^{\mu\nu} f^\lambda_\nu + \frac{n^2}{M_A}\left(f^\lambda_\nu \partial^{\mu\nu} a^2 + \frac{1}{2} f_{\rho\nu}\partial^{\rho\nu}\left(a^\mu a^\lambda\right)\right)\right], \\
\delta\mathcal{L}(\varphi) &= \xi_\lambda \partial_\lambda \mathcal{L}(\varphi) + \left(\partial_\mu \xi_\nu + \partial_\nu \xi_\mu\right) \\
&\quad \times \left[\left(\mathfrak{D}^\mu \varphi\right)^* \mathfrak{D}^\nu \varphi + \frac{a^\mu a^\nu n^2}{2M_A} n_\lambda J_\lambda \right],
\end{aligned} \tag{43}$$

where J_μ stands for the conventional vector field source current

$$J_\mu = ie[\varphi^* D_\mu \varphi - \varphi\left(D_\mu \varphi\right)^*], \tag{44}$$

while $\mathfrak{D}_\nu \varphi$ is the SLIV extended covariant derivative for the scalar field

$$\mathfrak{D}_\nu \varphi = D_\nu \varphi - ie\frac{n^2}{2M_A} a^2 n_\nu \varphi. \tag{45}$$

The first terms in the variations (43) are unessential, since they simply show that these Lagrangians transform, as usual, like scalar densities under diff transformations.

Combining these variations with $\mathcal{L}_{\text{int}}$ (34) in the total Lagrangian (30) one finds after simple, though long, calculations that the largest Lorentz violating terms it contains,

$$\begin{aligned}
&-\left(\frac{M_H}{M_P}\mathfrak{n}_{\mu\nu} - \frac{\partial_\mu \xi_\lambda + \partial_\lambda \xi_\mu}{2}\right) \\
&\quad \times \left[-f^{\mu\nu} f^\lambda_\nu - \frac{n^2}{M_A} f^\nu_\lambda \partial^{\mu\lambda} a^2 + 2\mathfrak{D}^\nu \varphi \left(\mathfrak{D}^\mu \varphi\right)^* \right],
\end{aligned} \tag{46}$$

will immediately cancel if the transformation parameter is chosen exactly as in Eq. (26). So, with this choice we finally have for the modified interaction Lagrangian

$$\begin{aligned}
\mathcal{L}'_{\text{int}}(h, a, \varphi) &= -\frac{1}{M_P} h_{\mu\nu} T^{\mu\nu}(a, \varphi) + \frac{1}{M_P M_A}\mathcal{L}_1 \\
&\quad + \frac{1}{M_P M_H}\mathcal{L}_2 + \frac{M_H}{M_P M_A}\mathcal{L}_3
\end{aligned} \tag{47}$$

where

$$\begin{aligned}
\mathcal{L}_1 &= n^2 h_{\mu\nu}\left[f^\nu_\lambda \partial^{\mu\lambda} a^2 - n^\mu J^\nu + \eta^{\mu\nu}\left(-\frac{1}{4} f_{\lambda\rho}\partial^{\lambda\rho} a^2 + n^\lambda J_\lambda \right)\right], \\
\mathcal{L}_2 &= \frac{1}{2}\mathfrak{n}^2 h^2 \mathfrak{n}_{\mu\nu}\left[-f^{\mu\lambda} f^\nu_\lambda + 2D^\nu \varphi \left(D^\mu \varphi\right)^* \right], \\
\mathcal{L}_3 &= n^2 \mathfrak{n}_{\mu\lambda}\left[\frac{1}{2} f_{\rho\nu}\partial^{\rho\nu}\left(a^\mu a^\lambda\right) - (a^\mu a^\lambda) n^\nu J_\nu \right].
\end{aligned} \tag{48}$$

Thereby, apart from a conventional gravity interaction part given by the first term in (47), there are Lorentz violating couplings in $\mathcal{L}_{1,2,3}$ being properly suppressed by corresponding mass scales. Note that the coupling presented in $\mathcal{L}_3$ between the vector and scalar fields is solely induced by the tensor-field SLIV. Remarkably, this coupling may be in principle of the order of a normal gravity coupling or even stronger, if $M_H > M_A$. However, appropriately simplifying this coupling (and using also a full derivative identity) one arrives at

$$\mathcal{L}_3 \sim n^2 \left(\mathfrak{n}_{\mu\lambda} a^\mu a^\lambda\right) n^\rho \left[\partial^\nu f_{\nu\rho} - J_\rho\right], \tag{49}$$

which, after applying of the vector field equation of motion, turns it into zero. We consider it in more detail in the next section where we calculate some tree level processes.

4 The lowest order SLIV processes

4.1 Preamble

The emergent gravity Lagrangian in (22) taken alone or considered together with the material vector and scalar fields presents in fact a highly nonlinear theory which contains lots of Lorentz and CPT violating couplings. Nevertheless, as shown in [15,16] in the lowest order calculations, they all are cancelled and do not manifest themselves in physical processes. This may mean that the length-fixing constraints (7) put on the tensor fields appear as gauge fixing conditions rather than a source of an actual Lorentz violation.

However, as was mentioned in Sect. 1, one cannot be sure that these calculations, as well as the similar calculations in the Nambu model itself, fully confirm gauge invariance of the emergent theory considered. Indeed, whether the constraint (1) in QED amounts in general to a special gauge choice for a vector field $A_\mu(x)$ is an open question unless the corresponding gauge function $\omega(x)$ satisfying the constraint condition

$$[A_\mu(x) + \partial_\mu \omega(x)]^2 = n^2 M_A^2 \tag{50}$$

is explicitly constructed for an arbitrary $A_\mu(x)$. The original Nambu argument [17] was related to the observation that for the positive n^2 the constraint equation (50) is mathematically equivalent to a classical Hamilton–Jacobi equation for a massive charged particle,

$$[\partial_\mu S(x) + eA_\mu(x)]^2 = m^2, \tag{51}$$

where $S(x)$ is an action of a system, while e and m stand for the particle charge and mass, respectively. Comparison of Eqs. (50) and (51) shows the correspondence $\omega(x) = S(x)/e$ and $n^2 M_A^2 = m^2/e^2$. Thus, the constraint equation (50) should have a solution inasmuch as there is a solution to the classical problem described by Eq. (51). This conclusion was actually confirmed by Nambu for the timelike SLIV ($n^2 = +1$) in the lowest order calculation of the physical processes in [17] and then was extended to the one-loop approximation and for both the timelike ($n^2 > 0$) and spacelike ($n^2 < 0$) Lorentz violation in [19]. Thus, the status of the constraint (1) as a special gauge choice in QED is only partially confirmed by some low order calculations rather than having a serious theoretical reason.

The same may be said of the emergent tensor-field gravity. Its diff gauge invariance could only be fully approved if the corresponding gauge function $\xi_\mu(x)$ satisfying the constraint condition (7)

$$[H_{\mu\nu}(x) + \partial_\mu \xi_\nu(x) + \partial_\nu \xi_\mu]^2 = \mathfrak{n}^2 M_H^2 \tag{52}$$

is explicitly constructed for an arbitrary $H_{\mu\nu}(x)$. However, in contrast to the above nonlinear QED case where at least some heuristic argument could be applied, one cannot be sure that there exists a solution to Eq. (52) in a general case. So, the only way to answer this question is to explicitly check it in physical processes that in the lowest approximation has been done in [15,16]. Again, though the result appears positive, one cannot be sure that this will work in all orders.

The present electrogravity theory, in contrast to the pure QED and tensor-field gravity theories, contains both the photon and the graviton as the emergent gauge fields. This adds new variety of Lorentz and CPT violating couplings (47), being expressed in terms of tensor and vector Goldstone modes. In general, one cannot be sure that, even though both the emergent QED and the tensor-field gravity taken separately preserve Lorentz invariance (in the low order processes), the combined electrogravity theory does not lead to physical Lorentz violation as well. However, as shown by our calculations given below, just this appears to be the case. All Lorentz violation effects turn out again to be strictly cancelled among themselves at least in the lowest order SLIV processes in the electrogravity theory. Thus, similar to emergent vector field theories, both Abelian [17,19,20] and non-Abelian [21], as well as in the pure tensor-field gravity [15,16], such a cancellation may only mean that at least in the lowest approximation the SLIV constraints (1, 7) amount to a special gauge choice in the otherwise diff and Lorentz invariant emergent electrogravity theory presented here.

We will consider the lowest order SLIV processes, once the corresponding Feynman rules are properly established. For simplicity, both in the above Lagrangians and in forthcoming calculations, we continue to use the tracelessness of the VEV tensor $\mathfrak{n}_{\mu\nu}$ (19), while our results remain true for any type of vacuum configuration caused by SLIV.

4.2 Feynman rules

Though the Feynman rules and processes related to the nonlinear QED, as well as with emergent gravity with the matter scalar fields, are thoroughly discussed in our previous works [15,20], there are many new Lorentz and CPT breaking interactions in the total interaction Lagrangian (47). We present below some basic Feynman rules which are needed for calculations of different SLIV processes just appearing in the emergent electrogravity.

(i) The first and most important is the graviton propagator which only conforms with the emergent gravity Lagrangian (22) and the gauge conditions (20) and (28),

$$\begin{aligned} -iD_{\mu\nu\alpha\beta}(k) = &\frac{1}{2k^2}\left(\eta_{\beta\mu}\eta_{\alpha\nu}+\eta_{\beta\nu}\eta_{\alpha\mu}-\eta_{\alpha\beta}\eta_{\mu\nu}\right) \\ &-\frac{1}{2k^4}\left(\eta_{\beta\nu}k_\alpha k_\mu+\eta_{\alpha\nu}k_\beta k_\mu \right.\\ &\left.+\eta_{\beta\mu}k_\alpha k_\nu+\eta_{\alpha\mu}k_\beta k_\nu\right) \\ &-\frac{1}{k^2(\mathfrak{n}kk)}\left(k_\alpha k_\beta \mathfrak{n}_{\mu\nu}+k_\nu k_\mu \mathfrak{n}_{\alpha\beta}\right) \\ &+\frac{1}{k^2(\mathfrak{n}kk)^2}\left[\mathfrak{n}^2-\frac{2}{k^2}(k\mathfrak{n}\mathfrak{n}k)\right]k_\mu k_\nu k_\alpha k_\beta \\ &+\frac{1}{k^4(\mathfrak{n}kk)}\left(\mathfrak{n}_{\mu\rho}k^\rho k_\nu k_\alpha k_\beta+\mathfrak{n}_{\nu\rho}k^\rho k_\mu k_\alpha k_\beta\right. \\ &\left.+\mathfrak{n}_{\alpha\rho}k^\rho k_\nu k_\mu k_\beta+\mathfrak{n}_{\beta\rho}k^\rho k_\nu k_\alpha k_\mu\right) \end{aligned} \tag{53}$$

(where $(\mathfrak{n}kk) \equiv \mathfrak{n}_{\mu\nu}k^\mu k^\nu$ and $(k\mathfrak{n}\mathfrak{n}k) \equiv k^\mu \mathfrak{n}_{\mu\nu}\mathfrak{n}^{\nu\lambda}k_\lambda$). It automatically satisfies the orthogonality condition $\mathfrak{n}^{\mu\nu}D_{\mu\nu\alpha\beta}(k) = 0$ and on-shell transversality $k^\mu k^\nu D_{\mu\nu\alpha\beta}(k, k^2 = 0) = 0$. This is consistent with the corresponding polarization tensor $\epsilon_{\mu\nu}(k, k^2 = 0)$ of the free tensor fields, being symmetric, transverse ($k^\mu\epsilon_{\mu\nu} = 0$), traceless ($\eta^{\mu\nu}\epsilon_{\mu\nu}(k) = 0$) and also orthogonal to the vacuum direction, $\mathfrak{n}^{\mu\nu}\epsilon_{\mu\nu}(k) = 0$. As one can see, only standard terms given by the first bracket in (53) contribute when the propagator is sandwiched between the conserved energy-momentum tensors of matter fields, and the result is always Lorentz invariant.

We will also need the photon propagator,

$$ik^2D_{\mu\nu} = \eta_{\mu\nu} - \frac{n_\mu k_\nu + n_\nu k_\mu}{n\cdot k} + \frac{n^2}{(n\cdot k)^2}k_\mu k_\nu, \tag{54}$$

which in accordance with the vector field Lagrangian (32) possesses the following properties: $n^\mu D_{\mu\nu} = 0$ and $k^\mu D_{\mu\nu}(k^2 = 0) = 0$.

(ii) Next is the three-graviton vertex hhh, again from the Lagrangian (22), with graviton polarization tensors (and 4-momenta) given by $\epsilon^{\alpha\alpha'}(k_1)$, $\epsilon^{\beta\beta'}(k_2)$ and $\epsilon^{\gamma\gamma'}(k_3)$ we have

$$\begin{aligned}\Gamma_{3h}^{\alpha\alpha'\beta\beta'\gamma\gamma'} = &\frac{i}{2M_H}[\left(\eta^{\beta\gamma}\eta^{\beta'\gamma'}+\eta^{\beta\gamma'}\eta^{\beta'\gamma}\right)P^{\alpha\alpha'}(k_1) \\ &+\left(\eta^{\alpha\gamma}\eta^{\alpha'\gamma'}+\eta^{\alpha\gamma'}\eta^{\alpha'\gamma}\right)P^{\beta\beta'}(k_2) \\ &+\left(\eta^{\beta\alpha}\eta^{\beta'\alpha'}+\eta^{\beta\alpha'}\eta^{\beta'\alpha}\right)P^{\gamma\gamma'}(k_3)\end{aligned} \tag{55}$$

where the momentum tensor $P^{\mu\nu}(k)$ is

$$P^{\mu\nu}(k) = \mathfrak{n}^{\nu\rho}k_\rho k^\mu + \mathfrak{n}^{\mu\rho}k_\rho k^\nu - \eta^{\mu\nu}(\mathfrak{n}kk). \tag{56}$$

Note that all 4-momenta at the vertices are taken ingoing throughout.

(iii) Next, we address the contact tensor–tensor–vector–vector interaction coupling $hhaa$ coming from the Lagrangian $\mathcal{L}_2$ in (48). However, it would be useful to give first the standard tensor–vector–vector vertex haa with tensor and vector field polarizations, $\epsilon^{\alpha\alpha'}$ and $\xi^{\mu,\nu}$, respectively,

$$\Gamma_{st}^{\alpha\alpha'\mu\nu} = -\frac{i}{M_P}T^{\alpha\alpha'\mu\nu}(a_\lambda) \tag{57}$$

where $T^{\alpha\alpha'\mu\nu}(a_\lambda)$ stands for the conserved energy-momentum tensor of the Goldstone vector field a_λ,

$$\begin{aligned}T^{\alpha\alpha'\mu\nu} = &\frac{1}{2}\left(\eta^{\alpha\mu'}\eta^{\alpha'\nu'}+\eta^{\alpha\nu'}\eta^{\alpha'\mu'}\right) \\ &\times\left[\left(k_{2\mu'}\eta^\lambda_\nu - k_2^\lambda\eta_{\mu'\nu}\right)\left(k_{1\nu'}\eta_{\lambda\mu}-k_{1\lambda}\eta_{\mu\nu'}\right)\right. \\ &\left.+\left(k_{2\nu'}\eta^\lambda_\nu - k_2^\lambda\eta_{\nu'\nu}\right)\left(k_{1\mu'}\eta_{\lambda\mu}-k_{1\lambda}\eta_{\mu\mu'}\right)\right],\end{aligned} \tag{58}$$

being properly conserved,

$$(k_1-k_2)_\alpha T^{\alpha\alpha'\mu\nu}\xi_\mu(k_1)\xi_\nu(k_2) = 0. \tag{59}$$

One can see that this tensor is symmetric both in the (α, α') and (μ, ν) indices, though these pairs are not interchangeable. Using all that we are ready now to give the contact tensor–tensor–vector–vector interaction vertex,

$$\Gamma_{2h2a}^{\beta\beta'\gamma\gamma'\mu\nu} = \frac{i\mathfrak{n}^2}{2M_PM_H}\left(\eta^{\beta\gamma}\eta^{\beta'\gamma'}+\eta^{\beta\gamma'}\eta^{\beta'\gamma}\right)\mathfrak{n}_{\alpha\alpha'}T^{\alpha\alpha'\mu\nu}(a_\lambda), \tag{60}$$

with the corresponding tensor field ($\beta\beta'$ and $\gamma\gamma'$) and vector field (μ, ν) polarization indices.

(iv) We have also to derive the four-linear tensor–vector interaction vertex $haaa$ coming from the Lagrangian $\mathcal{L}_1$ in (48). Note that the last term in it which is proportional to h_{tr} will not contribute in the processes with graviton on external lines, since its polarization tensor is traceless. For the other terms one has the vertex

$$\begin{aligned}\Gamma_{h3a}^{\alpha\alpha'\mu\nu\lambda} = &\frac{in^2}{M_PM_A}\left(\eta^{\alpha\mu'}\eta^{\alpha'\nu'}+\eta^{\alpha\nu'}\eta^{\alpha'\mu'}\right) \\ &\times\left[\eta_{\nu\lambda}\left((n\cdot k_1)(p+k_1)_{\mu'}\eta_{\nu'\mu}\right.\right. \\ &\left.+n_{\mu'}(p+k_1)^\rho\left(k_{1\nu'}\eta_{\rho\mu}-k_{1\rho}\eta_{\nu'\mu}\right)\right) \\ &+\eta_{\mu\lambda}\left((n\cdot k_2)(p+k_2)_{\mu'}\eta_{\nu'\nu}\right. \\ &\left.+n_{\mu'}(p+k_2)^\rho\left(k_{2\nu'}\eta_{\rho\nu}-k_{2\rho}\eta_{\nu'\nu}\right)\right) \\ &+\eta_{\nu\mu}\left((n\cdot k_3)(p+k_3)_{\mu'}\eta_{\nu'\lambda}\right. \\ &\left.\left.+n_{\mu'}(p+k_3)^\rho\left(k_{3\nu'}\eta_{\rho\lambda}-k_{3\rho}\eta_{\nu'\lambda}\right)\right)\right]\end{aligned} \tag{61}$$

where the polarization $\epsilon^{\alpha\alpha'}(p)$ stands interacting tensor field, while polarizations $\xi_{1\mu}(k_1)$, $\xi_{2\nu}(k_2)$, $\xi_{3\lambda}(k_3)$ for interacting vector fields.

(v) For the three-vector Goldstone mode interaction aaa we have the well-known vertex [20] following for the pure vector field Lagrangian (32),

$$\begin{aligned}\Gamma_{3a}^{\mu\nu\lambda} = &-i\frac{n^2}{M_A}\left[(n\cdot k_1)\eta_{\nu\lambda}k_{1\mu}+(n\cdot k_2)\eta_{\mu\lambda}k_{2\nu}\right. \\ &\left.+(n\cdot k_3)\eta_{\mu\nu}k_{3\lambda}\right],\end{aligned} \tag{62}$$

and the new one coming from the Lagrangian $\mathcal{L}_3$ in (48),

$$\Gamma'^{\mu\nu\lambda}_{3a} = i\frac{n^2 M_H}{M_P M_A}\left[(n\cdot k_1)\mathfrak{n}_{\nu\lambda}k_{1\mu} + (n\cdot k_2)\mathfrak{n}_{\mu\lambda}k_{2\nu} + (n\cdot k_3)\mathfrak{n}_{\mu\nu}k_{3\lambda}\right]. \quad (63)$$

(**vi**) And finally, let us give also the vector–scalar–scalar interaction $a\varphi\varphi^*$ stemming from the same Lagrangian $\mathcal{L}_3$,

$$\Gamma'^{\mu\lambda}_{a2\varphi} = i\frac{en^2 M_H}{M_P M_A} l_{\mu\lambda} n^\nu J_\nu, \quad (64)$$

where J_ν is the conserved scalar field current discussed in the previous section.

These are rules that are actually needed to calculate the lowest order SLIV processes mentioned above. Note also that some of these processes could in principle appear in the pure nonlinear QED [20] or in the nonlinear tensor-field gravity [15,16] where, as is well known, all the physical Lorentz violation effects are eventually vanishing. Therefore, we consider the SLIV contributions which only appear in the combined nonlinear vector–tensor electrogravity theory presented here.

4.3 Elastic photon–graviton scattering

This SLIV part of this process, $\gamma + g \to \gamma + g$, may only be of order of $1/M_P M_H$ due to the emergent nature of the graviton. There are in fact two matrix elements: the first one is related to the contact diagram with the *hhaa* vertex (60),

$$\mathcal{M}_{\rm con} = \frac{i\mathfrak{n}^2}{M_P M_H}(\epsilon_1\cdot\epsilon_2)\,\mathfrak{n}_{\alpha\alpha'}T^{\alpha\alpha'\mu\nu}\xi_{3\mu}\xi_{4\nu}, \quad (65)$$

while the second one is related to the pole diagram with the longitudinal graviton exchange between the Lorentz violating h^3 (55) and the standard *haa* (57) vertices,

$$\mathcal{M}_{pole} = \epsilon_1^{\gamma\gamma'}\epsilon_2^{\beta\beta'}\Gamma_{3h}^{\alpha\alpha'\beta\beta'\gamma\gamma'}D_{\alpha\alpha'\lambda\rho}(q)\Gamma_{st}^{\lambda\rho\mu\nu}\xi_{3\mu}\xi_{4\nu}, \quad (66)$$

with the graviton and photon polarizations $\epsilon_{1,2}$ and $\xi_{3,4}$, respectively (q is the momentum of the propagating graviton).

Note now that all the terms in the propagator $D_{\alpha\alpha'\lambda\rho}$ which are proportional to the propagating momentum will render the energy-momentum tensor to zero, thus there are left only a few terms in the pole matrix element $\mathcal{M}_2$. Using also the fact that in the vertex Γ_{3h} there survives one term only when the transversality and tracelessness of a graviton is used we arrive at

$$\mathcal{M}_{pole} = \frac{i\mathfrak{n}^2}{M_H}(\epsilon_1\cdot\epsilon_2)\left[2\mathfrak{n}^{\alpha\rho}q_\rho q^{\alpha'} - \eta^{\alpha\alpha'}(\mathfrak{n}qq)\right]\frac{i}{q^2}\cdot \times\left[\frac{\eta_{\alpha\lambda}\eta_{\alpha'\rho}+\eta_{\alpha'\lambda}\eta_{\alpha\rho}-\eta_{\alpha\alpha'}\eta_{\lambda\rho}}{2} - \frac{q_{\alpha'}q_\alpha\mathfrak{n}_{\lambda\rho}}{(\mathfrak{n}qq)}\right]\frac{-i}{M_P}T^{\lambda\rho\mu\nu}\xi_{3\mu}\xi_{4\nu}, \quad (67)$$

which after evident simplifications is exactly cancelled with the contact matrix element $\mathcal{M}_1$ given above in (65),

$$\mathcal{M}_{\rm tot} = \mathcal{M}_{\rm con} + \mathcal{M}_{\rm pole} = 0. \quad (68)$$

Thereby, physical Lorentz invariance is left intact in the emergent graviton–photon scattering.

4.4 Photon–graviton conversion

This SLIV process $\gamma + g \to \gamma + \gamma$ appears in the order of $1/M_A M_P$ (now, due to the emergent nature of photon). Again, this process in the tree approximation is basically related to the interplay between the contact and pole diagrams.

The contact *haaa* diagram being determined by the interaction vertex (61) has a matrix element

$$\mathcal{M}_{con} = \epsilon_{\alpha\alpha'}(p)\Gamma_{h3a}^{\alpha\alpha'\mu\nu\lambda}\xi_{1\mu}(k_1)\xi_{2\nu}(k_2)\xi_{3\lambda}(k_3) \quad (69)$$

where the polarization $\epsilon_{\alpha\alpha'}$ belongs to the graviton, while the polarizations $\xi_{1\mu}(k_1)$, $\xi_{2\nu}(k_2)$, $\xi_{3\lambda}(k_3)$ belong to the photons (p and k_1 are incoming and k_2 and k_3 outgoing momenta).

In turn, the pole diagrams with the longitudinal photon exchange between the Lorentz violating a^3 (62) and the standard *haa* (57) vertices consist in fact of three diagrams differing from each other by the interchangeable external photon legs. Their total matrix element is

$$\mathcal{M}_{pole} = \epsilon_{\alpha\alpha'}\Gamma_{st}^{\alpha\alpha'\mu\nu}D_{\nu\lambda}(q)\Gamma_{3a}^{\lambda\rho\sigma}(\xi_{1\mu}\xi_{2\rho}\xi_{3\sigma} + \xi_{2\mu}\xi_{1\rho}\xi_{3\sigma} + \xi_{3\mu}\xi_{1\rho}\xi_{2\sigma}), \quad (70)$$

where q is the propagating momentum, while the momenta of the graviton and photons, p and $k_{1,2,3}$, refer to the polarizations $\epsilon_{\alpha\alpha'}$ and $\xi_{1,2,3}$.

Using again the orthogonality properties and mass shell conditions for polarizations of the photons and graviton one can split the contact amplitude (69) into three terms which exactly cancel the corresponding terms in the pole amplitude (70). So, we will not have any physical Lorentz violation in this process as well.[5]

4.5 Elastic photon–scalar scattering

One can also consider a new type of vector field scattering process on the charged scalar, $\gamma + s \longrightarrow \gamma + s$, appearing at the $eM_H/M_P M_A$ order due to the emergent nature of both

[5] Note that together with the pure QED a^3 vertex (62) we could also use the new a^3 vertex (63) in the above pole diagrams. This would give some new contribution into this process with the lesser order $M_H/M_A M_P^2$. One may expect, however, that such a contribution will be cancelled by the corresponding contact term appearing in the same order when going to GR (see the footnote[1]).

photon and graviton.[6] Again, there are contact and pole diagrams for this process which cancel each other. The contact diagram corresponds to the vertex $a\varphi\varphi^*$ (64) appearing from the Lagrangian $\mathcal{L}_3$ in (48) and leads to the matrix element

$$\mathcal{M}_{con} = i\frac{en^2 M_H}{M_P M_A}\left(\mathfrak{n}_{\mu\lambda}\xi_{1\mu}\xi_{2\lambda}\right) n_\nu J_\nu. \tag{71}$$

Meanwhile, for the pole diagram with the longitudinal photon exchange between the Lorentz violating aaa vertex (63) and the standard scalar field current (44) in $\mathcal{L}_3$ one has, using the mass shell properties of the vector field polarization,

$$\mathcal{M}_{\rm pole} = i\frac{n^2 M_H}{M_P M_A}\left(\mathfrak{n}_{\mu\lambda}\xi_{1\mu}\xi_{2\lambda}\right)(n\cdot q)q_\nu D_{\nu\rho}(q)\left(ieJ_\rho\right). \tag{72}$$

This amplitude, when applying an explicit form of propagator and the current conservation $q_\rho J_\rho = 0$, is exactly cancelled with the contact one (71). Therefore, we show once again that there is no real physical SLIV effect in the theory considered.

4.6 Other processes

Many other tree level Lorentz violating processes related to gravitons and vector fields (interacting with each other and the matter scalar field in the theory) appear in higher orders in the basic SLIV parameters $1/M_H$ and $1/M_A$, by iteration of the couplings presented in our basic Lagrangians (22) and (47) or from further expansions of the effective vector and tensor-field Higgs modes (5) and (21) inserted into the starting total Lagrangian (30). Again, their amplitudes are essentially determined by an interrelation between the longitudinal graviton and photon exchange diagrams and the corresponding contact interaction diagrams, which appear to cancel each other, thus eliminating physical Lorentz violation in the theory.

Most likely, the same conclusion could be expected for SLIV loop contributions as well. Actually, as in the massless QED case considered earlier [19], the corresponding one-loop matrix elements in our emergent electrogravity theory could either vanish by themselves or amount to the differences between pairs of similar integrals whose integration variables are shifted relative to each other by some constants (being in general arbitrary functions of the external 4-momenta of the particles involved) which, in the framework of dimensional regularization, could lead to their total cancellation.

So, the emergent electrogravity theory considered here is likely to eventually possess physical Lorentz invariance provided that the underlying gauge and diff invariance in the theory remains unbroken.

5 Conclusion

We have developed an emergent electrogravity theory consisting of the ordinary QED and the tensor-field gravity model (which mimics the linearized general relativity in Minkowski spacetime) where both photons and gravitons emerge as states solely consisting of massless Goldstone and pseudo-Goldstone modes. This appears due to spontaneous violation of Lorentz symmetry incorporated into global symmetries of the length-fixing constraints put on the starting vector and tensor fields, $A_\mu^2 = \pm M_A^2$ and $H_{\mu\nu}^2 = \pm M_H^2$ (M_A and M_H are the proposed symmetry breaking scales). While for the vector field case the symmetry of the constraint coincides with Lorentz symmetry $SO(1,3)$ of the electrogravity Lagrangian, the tensor field constraint itself possesses the much higher global symmetry $SO(7,3)$, whose spontaneous violation provides a sufficient number of zero modes collected in a graviton. Accordingly, while the photon may only contain true Goldstone modes, the graviton appears at least partially composed from pseudo-Goldstone modes rather than from pure Goldstone ones. Thereby, the SLIV pattern related to breaking of the constraint symmetries, due to which the true vacuum in the theory is chosen, induces a variety of zero modes shared among photon and graviton.

This theory looks essentially nonlinear and contains a variety of Lorentz and CPT violating couplings, when expressed in terms of the pure tensor Goldstone modes. Nonetheless, all the SLIV effects turn out to be strictly cancelled in the lowest order processes considered. This can be taken as an indication that in the electrogravity theory physical Lorentz invariance is preserved in this approximation. Thereby, the length-fixing constraints (1) and (7) put on the vector and tensor fields appear as gauge fixing conditions rather than sources of the actual Lorentz violation in the gauge and diff invariant Lagrangian (30) we started with. In fact, some Lorentz violation through deformed dispersion relations for the material fields involved would appear in the interaction sector (34), which only possesses an approximate diff invariance. However, a proper extension of the tensor-field theory to GR, with its exact diff invariance, ultimately restores the normal dispersion relations and, therefore, the SLIV effects are cancelled at least in the lowest order considered. If this cancellation were to work in all orders, one could propose that emergent theories, like as the electrogravity theory, are not differed from conventional gauge theories. Accordingly, spontaneous Lorentz violation caused by the vector and tensor-field constraints (1) and (7) appear hidden in the gauge degrees of freedom, and only results in a noncovariant gauge choice in an otherwise gauge invariant emergent electrogravity theory.

[6] Note that a similar SLIV process can independently appear in the pure nonlinear scalar QED including the Lagrangians (32) and (33). However, it was shown [20] that the corresponding Lorentz violation terms are strictly cancelled in this scattering process.

From this standpoint, the only way for physical Lorentz violation to occur would be if the above gauge invariance were slightly broken at distances of the order of the Planck scale, which could be presumably caused by quantum gravity. This is in fact a place where the emergent vector- and tensor-field theories may drastically differ from conventional QED, Yang–Mills and GR theories where gauge symmetry breaking could hardly induce physical Lorentz violation. In contrast, in emergent electrogravity such breaking could readily lead to many violation effects including deformed dispersion relations for all matter fields involved. Another basic distinction of emergent theories with non-exact gauge invariance is a possible origin of a mass for graviton and other gauge fields (namely, for the non-Abelian ones, see [21]), if they, in contrast to the photon, are partially composed of pseudo-Goldstone modes rather than of pure Goldstone ones. Indeed, these PGMs are no longer protected by gauge invariance and may properly acquire tiny masses, which still do not contradict experiment. This may lead to a massive gravity theory where the graviton mass emerges dynamically, thus avoiding the notorious discontinuity problem [24].

So, while emergent theories with an exact local invariance are physically indistinguishable from conventional gauge theories, there are some principal distinctions when this local symmetry is slightly broken, which could eventually allow us to differentiate between the two types of theory in an observational way. We may return to a more detailed consideration of this interesting point elsewhere.

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