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The effective supergravity of little string theory

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Abstract In this work we present the minimal supersymmetric extension of the five-dimensional dilaton-gravity theory that captures the main properties of the holographic dual of little string theory. It is described by a particular gauging of $\mathcal{N} = 2$ supergravity coupled with one vector multiplet associated with the string dilaton, along the $U(1)$ subgroup of $SU(2)$ R-symmetry. The linear dilaton in the fifth coordinate solution of the equations of motion (with flat string frame metric) breaks half of the supersymmetries to $\mathcal{N} = 1$ in four dimensions. Interest in the linear dilaton model has lately been revived in the context of the clockwork mechanism, which has recently been proposed as a new source of exponential scale separation in field theory.

1 Introduction

Besides its own theoretical interest, little string theory provides a framework with interesting phenomenological consequences. It offers a way to address the hierarchy when the string scale is at the TeV scale [1–3], without postulating large extra dimensions (in string units) but instead an ultra-weak string coupling [4,5]. Recently, interest in the holographic dual to LST (the linear dilaton model) has been revived in the context of the so-called clockwork models [6–8] which address the exponential scale separation in field theory in a new way [9,10].

Little string theory (LST) corresponds to a non-trivial weak coupling limit of string theory in six dimensions with gravity decoupled and is generated by stacks of (Neveu–Schwarz) NS5-branes [11]. Its holographic dual corresponds to a seven-dimensional gravitational background with flat string-frame metric and the dilaton linear in the extra dimension [12]. Its properties can be studied in a simpler toy model by reducing the theory in five dimensions. Introducing back gravity weakly coupled, one has to compactify the extra dimension on an interval and place the Standard Model on one of the boundaries, in analogy with the Randall–Sundrum model [13] on a slice of a five-dimensional (5d) anti-de Sitter bulk [1].

Since we know that the bulk LST geometry preserves space-time supersymmetry, in this work we study the corresponding effective supergravity which in the minimal case is $\mathcal{N} = 2$. In principle, there should be a generalisation with more supersymmetries, or equivalently in higher dimensions. The $\mathcal{N} = 2$ gravity multiplet contains the graviton, a graviphoton and the gravitino (8 bosonic and 8 fermionic degrees of freedom), while the heterotic (or type I) string dilaton is in a vector multiplet containing a vector, a real scalar and a fermion. The corresponding supergravity action [15] admits a gauging of the $U(1)$ subgroup of the $SU(2)$ R-symmetry, that generates a potential for the single scalar field [15,16]. This potential depends on two parameters allowing a multiple of possibilities with critical or non critical points, or even flat potential with supersymmetry breaking. Here, we observe that the vanishing of one of the parameters generates the runaway dilaton potential of the non-critical string. This potential has no critical point with 5d maximal symmetry but it leads to the linear dilaton solution in the fifth coordinate that preserves 4d Poincaré symmetry. We show that this solution breaks one of the two supersymmetries, leading to $\mathcal{N} = 1$ in four dimensions.

The outline of the paper is the following. In Sect. 2, we review the gauged $\mathcal{N} = 2$ supergravity in five dimensions, based on the references [14–17], and specialize in the case of one vector multiplet using the results of the string effective action of Ref. [18]. In Sect. 3, we present the 5d graviton-dilaton toy model that describes the holographic dual of

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LST and identify it with a particular choice of the gauging of the $\mathcal{N}=2$ supergravity. We also show that the linear dilaton solution preserves half of the supersymmetries, i.e. $\mathcal{N}=1$ in four dimensions. In Sect. 4, we write the complete Lagrangian, including the fermion terms, depending on three constant parameters. In Sect. 5, we derive the spectrum classified using the 4d Poincaré symmetry and we conclude with some phenomenological remarks. Finally, there are three appendices containing our conventions, the equations of motion with the linear dilaton solution, and some explicit calculations that we use in the study of supersymmetry transformations.

2 Gauged $\mathcal{N}=2$, $D=5$ supergravity

The references used in the following are [14–17], while our conventions may be found in the Appendix A. In $D=5$ spacetime dimensions, the pure $\mathcal{N}=2$ supergravity multiplet contains the graviton e_M^m, the gravitino $SU(2)$-doublet ψ_M^i, where i is the $SU(2)$ index, and the graviphoton, while the $\mathcal{N}=2$ Maxwell multiplet contains a real scalar ϕ, an $SU(2)$ fermion doublet λ^i and a gauge field. Upon coupling n Maxwell multiplets to pure $\mathcal{N}=2$, $D=5$ supergravity, the total field content of the coupled theory can be written as

$$\left\{e_M^m\,,\;\psi_M^i\,,\;A_M^I\,,\;\lambda^{ia}\,,\;\phi^x\right\},\tag{1}$$

where $I=0,1,\ldots,n$, $a=1,\ldots,n$ and $x=1,\ldots,n$. The real scalars ϕ^x can be seen as coordinates of an n-dimensional space $\mathcal{M}$ that has metric g_{xy} that is symmetric for our purposes, while the spinor fields λ^{ia} transform in the n-dimensional representation of $SO(n)$, which is the tangent space group of $\mathcal{M}$, so that

$$g_{xy}=f_x^a\,f_y^b\,\delta_{ab},\tag{2}$$

where f_x^a is the corresponding vielbein. The bosonic part of the Lagrangian is

$$\begin{aligned}e^{-1}\mathcal{L}_{bos}=&-\frac{1}{2}\mathcal{R}(\omega)-\frac{1}{2}g_{xy}(\partial_M\phi^x)(\partial^M\phi^y)\\&-\frac{1}{4}G_{IJ}F^I_{MN}F^{MNJ}\\&+\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{MNP\Sigma\Lambda}F^I_{MN}F^J_{P\Sigma}A^K_\Lambda,\end{aligned}\tag{3}$$

where $e=\det(e_M^m)$, ω is the spacetime spin-connection, G_{IJ} is the symmetric gauge kinetic metric, C_{IJK} are totally symmetric constants and the gravitational coupling κ has been set equal to 1. The supersymmetry transformations of the fermions of the theory are

$$\begin{aligned}\delta\psi_{Mi}&=D_M(\omega)\epsilon_i+\cdots\\\delta\lambda_i^a&=-\frac{1}{2}if_x^a(\partial\!\!\!/\phi^x)\epsilon_i+\cdots,\end{aligned}\tag{4}$$

where ϵ_i is the supersymmetry spinor parameter and the dots stand for terms that vanish in the vacuum.

In fact, the n-dimensional $\mathcal{M}$ can be seen as a hypersurface of an $(n+1)$-dimensional space $\mathcal{E}$ with coordinates

$$\xi^I=\xi^I(\phi^x,\mathcal{F}),\tag{5}$$

where $\mathcal{F}$ is the additional coordinate of $\mathcal{E}$ compared to $\mathcal{M}$. It can be shown that $\mathcal{F}$ is a homogeneous polynomial of degree three and, more precisely, that

$$\mathcal{F}=\beta^3C_{IJK}\xi^I\xi^J\xi^K,\tag{6}$$

where $\beta=\sqrt{2/3}$. It can also be shown that, on $\mathcal{M}$, the scalars ϕ^x satisfy the constraint

$$\mathcal{F}=1.\tag{7}$$

Moreover,

$$G_{IJ}=-\frac{1}{2}\partial_I\partial_J\ln\mathcal{F}|_{\mathcal{F}=1},\quad g_{xy}=G_{IJ}\,\partial_x\xi^I\partial_y\xi^J|_{\mathcal{F}=1},\tag{8}$$

where $\partial_I=\frac{\partial}{\partial\xi^I}$ and $\partial_x=\frac{\partial}{\partial\phi^x}$. Finally, we note that the symmetric third-rank tensor T_{xyz} on $\mathcal{M}$ is covariantly constant for the symmetric $\mathcal{M}$ that we will be concerned with and thus satisfies the algebraic constraint

$$T^w_{(xy}T_{zu)w}=\frac{1}{2}g_{(xy}g_{zu)}.\tag{9}$$

The gauging of the $U(1)$ subgroup of $SU(2)$ generates a scalar potential P, with

$$P=-P_0^2+P_aP^a,\tag{10}$$

where P_0 and P_a are functions of the scalars ϕ^x that satisfy the following constraints due to supersymmetry

$$\begin{aligned}&P_{0,x}=-\sqrt{2}\beta P_x\\&P_{0,x;y}+\beta\,T^z_{xy}P_{0,z}-\beta^2g_{xy}P_0=0,\end{aligned}\tag{11}$$

where the symbols "," and ";" denote differentiation and covariant differentiation respectively and $P_x=f_x^aP_a$. The functions P_0 and P_a also appear in the fermion transformations that get deformed due to the gauging, namely

$$
\begin{aligned}
\tilde{\delta}\psi_{Mi} &= D_M(\omega)\epsilon_i + \frac{ig}{2\sqrt{6}} P_0 \Gamma_M \varepsilon_{ij}\delta^{jk}\epsilon_k + \cdots \\
\tilde{\delta}\lambda_i^a &= -\frac{1}{2} i f_x^a (\partial\!\!\!/ \phi^x)\epsilon_i + \frac{g}{\sqrt{2}} P^a \varepsilon_{ij}\delta^{jk}\epsilon_k + \cdots ,
\end{aligned}
\tag{12}
$$

where $\tilde{\delta}$ denotes the supersymmetry transformation after the gauging (under which the deformed action is invariant), g is the $U(1)$ coupling constant, Γ_μ is the Γ-matrix in five spacetime dimensions and the dots stand again for terms that vanish in the vacuum.

Now let us consider the case in which there is only one real physical scalar s. In the following, we use t to denote the additional coordinate on $\mathcal{E}$, namely $\xi^I = \xi^I(s,t)$, $I = 0, 1$. The effective supergravity related to the 5-dimensional model for the gravity dual of LST is given by

$$
\mathcal{F} = ts^2 + as^3, \tag{13}
$$

where a is a constant parameter. Indeed in the graviton-dilaton system obtained from string compactifications in five dimensions, the first term corresponds to the tree-level contribution (identifying t with the inverse heterotic string coupling) and the second term to the one-loop correction [18].[1]

The solution of the constraint (7) is then

$$
t = \frac{1 - as^3}{s^2}. \tag{14}
$$

and the components of the gauge kinetic metric are

$$
G_{tt} = \frac{1}{2}s^4, \quad G_{st} = \frac{1}{2}as^4, \quad G_{ss} = \frac{1}{s^2} + \frac{1}{2}a^2 s^4. \tag{15}
$$

We then find that the scalar metric, the Christoffel symbols and the third-rank tensor (that have only one component each) are respectively

$$
g_{ss} = \frac{3}{s^2}, \quad f_s^a = \frac{\sqrt{3}}{s}, \quad \Gamma^s_{ss} = -\frac{1}{s}, \quad T_{sss} = \frac{3}{\beta}\frac{1}{s^3}, \tag{16}
$$

where we have used (9) to compute T_{sss}. The system (11) takes thus the form

$$
\begin{aligned}
&P_s = -\frac{\sqrt{3}}{2} P_0' \\
&P_0'' + \frac{2}{s}P_0' - \frac{2}{s^2}P_0 = 0,
\end{aligned}
\tag{17}
$$

[1] Note a change of notation between s and t compared to Ref. [18].

whose solution is

$$
\begin{aligned}
P_0 &= As + B\frac{1}{s^2} \\
P_s &= -\frac{\sqrt{3}}{2}\left(A - 2B\frac{1}{s^3}\right), \\
P^a &= f_s^a g^{ss} P_s = -\frac{A}{2}s + B\frac{1}{s^2}.
\end{aligned}
\tag{18}
$$

where A, B are constant parameters. Using (10) we then find the potential to be

$$
P = -3A\left(\frac{A}{4}s^2 + B\frac{1}{s}\right) \tag{19}
$$

so that the kinetic term and the potential for s take the form

$$
e^{-1}\mathcal{L}_{dilaton} = -\frac{1}{2}\frac{3}{s^2}(\partial_M s)(\partial^M s) + 3A\left(\frac{A}{4}s^2 + B\frac{1}{s}\right). \tag{20}
$$

Upon redefining

$$
\sqrt{3}\ln s = \Phi, \tag{21}
$$

we obtain the Lagrangian for the canonically normalized Φ

$$
\begin{aligned}
e^{-1}\mathcal{L}_{dilaton} = &-\frac{1}{2}(\partial_M\Phi)(\partial^M\Phi) \\
&+ 3g^2 A\left(\frac{A}{4}e^{\frac{2}{\sqrt{3}}\Phi} + Be^{-\frac{1}{\sqrt{3}}\Phi}\right).
\end{aligned}
\tag{22}
$$

3 The 5D dual of LST

The holographic dual of six-dimensional Little String Theory can be approximated by a five-dimensional model, in which the Lagrangian in the bulk takes the following form [1][2]

$$
e^{-1}\mathcal{L}_{LST} = -\widetilde{M}_5^3\mathcal{R} - \frac{1}{3}(\partial_M\tilde{\Phi})(\partial^M\tilde{\Phi}) - e^{\frac{2}{3}\frac{\tilde{\Phi}}{\widetilde{M}_5^{3/2}}}\Lambda \tag{23}
$$

in the Einstein frame, where $\tilde{\Phi}$ is the dilaton and Λ is a constant. Upon redefining

$$
\tilde{\Phi} = \sqrt{\frac{3}{2}}\Phi, \quad \widetilde{M}_5^3 = \frac{1}{2}M_5^3 \tag{24}
$$

and setting the gravitational coupling κ in five dimensions equal to one ($\kappa^2 = 1/M_5^3$, where M_5 is the Planck mass in five

[2] We neglect the remaining spectator five dimensions of the string background which play no role in the properties of the model relevant for our analysis.

dimensions), we obtain the Lagrangian for the canonically normalized dilaton Φ

$$e^{-1}\mathcal{L}_{LST} = -\frac{1}{2}\mathcal{R} - \frac{1}{2}(\partial_M\Phi)(\partial^M\Phi) - e^{\frac{2}{\sqrt{3}}\Phi}\Lambda. \tag{25}$$

We thus observe that the potential that arises from LST is equal to the potential in (22) for a scalar that belongs to a gauged $\mathcal{N} = 2, D = 5$ Maxwell multiplet coupled to supergravity, upon making the identification

$$\frac{3}{4}g^2A^2 = -\Lambda, \quad B = 0. \tag{26}$$

We then have

$$P_0 = Ae^{\frac{1}{\sqrt{3}}\Phi}, \quad P^a = -\frac{A}{2}e^{\frac{1}{\sqrt{3}}\Phi}. \tag{27}$$

Moreover, it is known that the dilaton potential in (25) exhibits a runaway behaviour and does not have a five-dimensional maximally symmetric vacuum, but has a four-dimensional Poincaré vacuum in the linear dilaton background

$$\Phi = Cy, \tag{28}$$

where $y > 0$ is the fifth dimension and C a constant parameter. The background bulk metric is then

$$ds^2 = e^{-\frac{2}{\sqrt{3}}Cy}(\eta_{\mu\nu}dx^\mu dx^\nu + dy^2), \tag{29}$$

where $\eta_{\mu\nu}$ is the Minkowski metric of four-dimensional space, under the fine-tuning condition (see Appendix B)

$$C = \frac{gA}{\sqrt{2}}. \tag{30}$$

To have at least one unbroken supersymmetry, the fermion transformations must vanish in the vacuum for at least one linear combination of the supersymmetry parameters. Using Eq. (27), the fermion transformations (12) take the following form on the four-dimensional brane (in the vacuum)[3]

$$\begin{aligned}\tilde{\delta}\psi_{\mu i} &= \frac{i}{2\sqrt{3}}\Gamma_\mu\left(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ij}\delta^{jk}\epsilon_k\right)\\ \tilde{\delta}\lambda_i &= -\frac{1}{2}e^{\frac{1}{\sqrt{3}}Cy}\left(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ij}\delta^{jk}\epsilon_k\right).\end{aligned} \tag{31}$$

Upon diagonalizing the second of Eq. (31) and using (30), we find that $\mathcal{N} = 2$ supersymmetry is partially broken to $\mathcal{N} = 1$, with

$$\tilde{\delta}(\lambda_1 + i\Gamma^5\lambda_2) = 0, \quad \tilde{\delta}(i\Gamma^5\lambda_1 + \lambda_2) \sim i\Gamma^5\epsilon_1 + \epsilon_2. \tag{32}$$

[3] The details of this calculation are given in the Appendix C.

We thus identify $\lambda_1 + i\Gamma^5\lambda_2$ with the fermion residing in a multiplet of the unbroken $\mathcal{N} = 1$ supersymmetry and $i\Gamma^5\lambda_1 + \lambda_2$ with the Goldstino of the broken $\mathcal{N} = 1$ supersymmetry. To determine the dependence of ϵ_i on y, we consider the fifth component of the first of the Eq. (12) in the vacuum[4]

$$\tilde{\delta}\psi_{5i} = \partial_5\epsilon_i + \frac{igA}{2\sqrt{6}}\Gamma_5\varepsilon_{ij}\delta^{jk}\epsilon_k, \tag{33}$$

which gives

$$\epsilon_1 = e^{\frac{C}{2\sqrt{3}}y}\tilde{\epsilon}, \quad \epsilon_2 = -e^{\frac{C}{2\sqrt{3}}y}\, i\Gamma_5\,\tilde{\epsilon}, \tag{34}$$

where $\tilde{\epsilon}$ is a constant symplectic spinor. The above relations are consistent with the direction of the unbroken supersymmetry $\epsilon_2 = -i\Gamma_5\epsilon_1$ from Eq. (32).

4 Final Lagrangian

The Lagrangian of ungauged $\mathcal{N} = 2$, $D = 5$ supergravity is

$$\begin{aligned}e^{-1}\mathcal{L} = &-\frac{1}{2}\mathcal{R}(\omega) - \frac{1}{2}g_{xy}(\partial_M\phi^x)(\partial^M\phi^y)\\ &-\frac{1}{4}G_{IJ}F^I_{MN}F^{MNJ}\\ &-\frac{1}{2}\bar{\psi}^i_M\Gamma^{MNP}D_N\psi_{Pi}\\ &-\frac{1}{2}\bar{\lambda}^{ia}(\not{D}\delta^{ab} + \Omega^{ab}_x\not{\partial}\phi^x)\lambda^b_i\\ &-\frac{1}{2}i\bar{\lambda}^{ia}\Gamma^M\Gamma^N\psi_{Mi}f^a_x\partial_N\phi^x\\ &+\frac{1}{4}h^a_I\bar{\lambda}^{ia}\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}F^I_{\Lambda P}\\ &+\frac{1}{4}i\Phi_{Iab}\bar{\lambda}^{ia}\Gamma^{MN}\lambda^b_iF^I_{MN}\\ &+\frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{MNP\Sigma\Lambda}F^I_{MN}F^J_{P\Sigma}A^K_\Lambda\\ &-\frac{3i}{8\sqrt{6}}h_I\left[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^I_{P\Sigma}+2\bar{\psi}^{Mi}\psi^N_iF^I_{MN}\right]\\ &+ \text{(4-fermion terms)},\end{aligned} \tag{35}$$

where Ω^{ab}_x is the spin-connection of the scalar manifold and h_I, h^x_I and Φ_{Ixy} are functions of the scalars that will be defined later.

Upon gauging $U(1)$, the Lagrangian acquires the additional terms

[4] The details of this calculation are given in the Appendix C.

$$e^{-1}\mathcal{L}' = -g^2 P - \frac{i\sqrt{6}}{8} g \bar{\psi}^i_M \Gamma^{MN} \psi^j_N \delta_{ij} P_0 - \frac{g}{\sqrt{2}} \bar{\lambda}^{ia} \Gamma^M \psi^j_M \delta_{ij} P_a + \frac{ig}{2\sqrt{6}} \bar{\lambda}^{ia} \lambda^{jb} \delta_{ij} P_{ab}, \tag{36}$$

and the derivatives become

$$D_M \lambda^{ia} + \Omega^{ab}_x \partial_M \phi^x \lambda^{bi} \Rightarrow (\tilde{\mathcal{D}}_M \lambda^a)^i \equiv D_M \lambda^{ia} + \Omega^{ab}_x \partial_M \phi^x \lambda^{bi} + g \upsilon_I A^I_M \delta^{ij} \lambda^a_j, \tag{37}$$

where υ_I is an arbitrary constant vector and

$$P_{ab} \equiv \frac{1}{2}\delta_{ab} P_0 + 2\sqrt{2} T_{abc} P^c. \tag{38}$$

Using (16) and (27) we find that for a single scalar

$$P_{aa} = \frac{1}{2} P_0 + 2\sqrt{2}(f^a_s)^{-3} T_{sss} P^a = -\frac{A}{2} e^{\frac{1}{\sqrt{3}}\Phi}. \tag{39}$$

Consequently,

$$e^{-1}\mathcal{L}' = \frac{3g^2A^2}{4} e^{\frac{2}{\sqrt{3}}\Phi} - \frac{i\sqrt{6}}{8} gA\, e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\psi}^i_M \Gamma^{MN} \psi^j_N \delta_{ij} + \frac{gA}{2\sqrt{2}} e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \Gamma^M \psi^j_M \delta_{ij} - \frac{igA}{4\sqrt{6}} e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \lambda^j \delta_{ij}. \tag{40}$$

In addition, after the gauging, the following equations hold [15]

$$P_0 = 2h^I \upsilon_I, \quad P^a = \sqrt{2} h^{Ia} \upsilon_I, \tag{41}$$

so using (27) we find that

$$h^I = \frac{A}{2} \upsilon^I e^{\frac{1}{\sqrt{3}}\Phi}, \quad h^{Ia} = -\frac{A}{2\sqrt{2}} \upsilon^I e^{\frac{1}{\sqrt{3}}\Phi}, \tag{42}$$

where we have assumed that $\upsilon^I \upsilon_I = 1$ for simplicity. It thus follows that

$$h_I \equiv G_{IJ} h^J = \frac{A}{2} G_{IJ} \upsilon^J e^{\frac{1}{\sqrt{3}}\Phi}, \quad h^a_I \equiv G_{IJ} h^{Ia} = -\frac{A}{2\sqrt{2}} G_{IJ} \upsilon^J e^{\frac{1}{\sqrt{3}}\Phi}, \tag{43}$$

where we have used the fact that G_{IJ} raises and lowers I, J indices. Moreover,

$$\Phi_{Iab} \equiv \Phi_{Ixy} f^x_a f^y_b \equiv \sqrt{\frac{2}{3}} \left(\frac{1}{4} g_{xy} h_I + T_{xyz} h^z_I \right) f^x_a f^y_b, \tag{44}$$

using which we find that for a single scalar

$$\Phi_{Iaa} = -\frac{A}{8}\sqrt{\frac{2}{3}} G_{IJ} \upsilon^J e^{\frac{1}{\sqrt{3}}\Phi}. \tag{45}$$

Using (15), we find that the final Lagrangian $\tilde{\mathcal{L}} = \mathcal{L} + \mathcal{L}'$ takes the form

$$\begin{aligned} e^{-1}\tilde{\mathcal{L}} = &-\frac{1}{2}\mathcal{R}(\omega) - \frac{1}{2}(\partial_M \Phi)(\partial^M \Phi) \\ &- \frac{1}{8} e^{\frac{4}{\sqrt{3}}\Phi} F^0_{MN} F^{MN0} - \frac{1}{4} a e^{\frac{4}{\sqrt{3}}\Phi} F^0_{MN} F^{MN1} \\ &- \frac{1}{4}\left(e^{-\frac{2}{\sqrt{3}}\Phi} + \frac{1}{2} a^2 e^{\frac{4}{\sqrt{3}}\Phi} \right) F^1_{MN} F^{MN1} \\ &- \frac{1}{2} \bar{\psi}^i_M \Gamma^{MNP} \mathcal{D}_N \psi_{Pi} - \frac{1}{2} \bar{\lambda}^i \tilde{\not{\mathcal{D}}} \lambda_i \\ &- \frac{i}{2} (\partial_N \Phi)\, \bar{\lambda}^i \Gamma^M \Gamma^N \psi_{Mi} \\ &- \frac{A\tilde{\upsilon}}{16\sqrt{2}} e^{\frac{5}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \Gamma^M \Gamma^{\Lambda P} \psi_{Mi} F^0_{\Lambda P} \\ &- \frac{A}{8\sqrt{2}} \left(\frac{1}{2} a\tilde{\upsilon}\, e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1 e^{-\frac{1}{\sqrt{3}}\Phi} \right) \\ &\times \bar{\lambda}^i \Gamma^M \Gamma^{\Lambda P} \psi_{Mi} F^1_{\Lambda P} \\ &- \frac{iA\tilde{\upsilon}}{64} \sqrt{\frac{2}{3}} e^{\frac{5}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \Gamma^{MN} \lambda_i F^0_{MN} \\ &- \frac{iA}{32}\sqrt{\frac{2}{3}} \left(\frac{1}{2} a\tilde{\upsilon}\, e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1 e^{-\frac{1}{\sqrt{3}}\Phi} \right) \\ &\times \bar{\lambda}^i \Gamma^{MN} \lambda_i F^1_{MN} \\ &+ \frac{e^{-1}}{6\sqrt{6}} C_{IJK} \epsilon^{MNP\Sigma\Lambda} F^I_{MN} F^J_{P\Sigma} A^K_\Lambda \\ &- \frac{3iA\tilde{\upsilon}}{32\sqrt{6}} e^{\frac{5}{\sqrt{3}}\Phi} \\ &+ 3\left[\bar{\psi}^i_M \Gamma^{MNP\Sigma} \psi_{Ni} F^0_{P\Sigma} + 2\bar{\psi}^{Mi} \psi^N_i F^0_{MN} + 4 \right] \\ &- \frac{3iA}{16\sqrt{6}} \left(\frac{1}{2} a\tilde{\upsilon}\, e^{\frac{5}{\sqrt{3}}\Phi} + \upsilon^1 e^{-\frac{1}{\sqrt{3}}\Phi} \right) \\ &\times \left[\bar{\psi}^i_M \Gamma^{MNP\Sigma} \psi_{Ni} F^1_{P\Sigma} + 2\bar{\psi}^{Mi} \psi^N_i F^1_{MN} \right] \\ &+ \frac{3g^2A^2}{4} e^{\frac{2}{\sqrt{3}}\Phi} - \frac{i\sqrt{6}}{8} gA\, e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\psi}^i_M \Gamma^{MN} \psi^j_N \delta_{ij} \\ &+ \frac{gA}{2\sqrt{2}} e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \Gamma^M \psi^j_M \delta_{ij} - \frac{igA}{4\sqrt{6}} e^{\frac{1}{\sqrt{3}}\Phi}\, \bar{\lambda}^i \lambda^j \delta_{ij} \\ &+ \text{(4-fermion terms)}. \end{aligned} \tag{46}$$

where A^0_M and A^1_M correspond to the graviphoton and the gauge field of the vector multiplet respectively and we have set $\tilde{\upsilon} = \upsilon^0 + a\upsilon^1$. Since the parameter A appears only through the combination gA in the additional terms $\mathcal{L}'$

induced by the gauging, we choose to set $A = 1$. Moreover, at tree-level we may set $a = 0$, as discussed in Sect. 2. The final Lagrangian then takes the form

$$
\begin{aligned}
e^{-1}\tilde{\mathcal{L}} = &-\frac{1}{2}\mathcal{R}(\omega) - \frac{1}{2}(\partial_M\Phi)(\partial^M\Phi)\\
&-\frac{1}{8}e^{\frac{4}{\sqrt{3}}\Phi}F^0_{MN}F^{MN0} - \frac{1}{4}e^{-\frac{2}{\sqrt{3}}\Phi}F^1_{MN}F^{MN1}\\
&-\frac{1}{2}\bar{\psi}^i_M\Gamma^{MNP}\mathcal{D}_N\psi_{Pi} - \frac{1}{2}\bar{\lambda}^i\tilde{\not{\mathcal{D}}}\lambda_i\\
&-\frac{i}{2}(\partial_N\Phi)\,\bar{\lambda}^i\Gamma^M\Gamma^N\psi_{Mi}\\
&-\frac{\upsilon^0}{16\sqrt{2}}\,e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^0_{\Lambda P}\\
&-\frac{\upsilon^1}{8\sqrt{2}}e^{-\frac{1}{\sqrt{3}}\Phi}\bar{\lambda}^i\Gamma^M\Gamma^{\Lambda P}\psi_{Mi}\,F^1_{\Lambda P}\\
&-\frac{i\upsilon^0}{64}\sqrt{\frac{2}{3}}e^{\frac{5}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^0_{MN}\\
&-\frac{i\upsilon^1}{32}\sqrt{\frac{2}{3}}e^{-\frac{1}{\sqrt{3}}\Phi}\bar{\lambda}^i\Gamma^{MN}\lambda_i\,F^1_{MN}\\
&+\frac{1}{6\sqrt{6}}\,e^{\frac{5}{\sqrt{3}}\Phi}\,C_{IJK}\epsilon^{MNP\Sigma\Lambda}F^I_{MN}F^J_{P\Sigma}A^K_\Lambda\\
&-\frac{3i\upsilon^0}{32\sqrt{6}}e^{\frac{5}{\sqrt{3}}\Phi}\\
&\times\left[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^0_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_i\,F^0_{MN}\right]\\
&-\frac{3i\upsilon^1}{16\sqrt{6}}e^{-\frac{1}{\sqrt{3}}\Phi}\\
&\times\left[\bar{\psi}^i_M\Gamma^{MNP\Sigma}\psi_{Ni}F^1_{P\Sigma} + 2\bar{\psi}^{Mi}\psi^N_i\,F^1_{MN}\right]\\
&+\frac{3g^2}{4}e^{\frac{2}{\sqrt{3}}\Phi} - \frac{ig\sqrt{6}}{8}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\psi}^i_M\Gamma^{MN}\psi^j_N\delta_{ij}\\
&+\frac{g}{2\sqrt{2}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\Gamma^M\psi^j_M\delta_{ij} - \frac{ig}{4\sqrt{6}}\,e^{\frac{1}{\sqrt{3}}\Phi}\,\bar{\lambda}^i\lambda^j\delta_{ij}\\
&+(\text{4-fermion terms}).
\end{aligned}
\tag{47}
$$

This Lagrangian has three free parameters: g, υ^0 and υ^1.

5 Spectrum and concluding remarks

The spectrum of the above model can be decomposed using the 4d Poincaré invariance of the linear dilaton vacuum solution and should form obviously $\mathcal{N} = 1$ supermultiplets. It is known that every 5d field should give rise to a 4d zero mode and a continuum starting from a mass gap fixed by the linear dilaton coefficient $C = g/\sqrt{2}$. Using the results of Ref. [1] and the correspondence (24), one finds that the parameter α of [1] is given by $\alpha = \sqrt{3}C$ and that the mass gap M_{gap} is

$$M_{\text{gap}} = \frac{\sqrt{3}}{2\sqrt{2}}g. \tag{48}$$

The continuum becomes an ordinary discrete Kaluza–Klein (KK) spectrum on top of the mass gap, when the fifth coordinate y is compactified on an interval [1], allowing to introduce the Standard Model (SM) on one of the boundaries. This spectrum is valid for the graviton, dilaton and their superpartners by supersymmetry. Notice that the 5d graviton zero-mode has five polarisations that correspond to the 4d graviton, a KK vector and the radion. For the rest of the fields, special attention is needed because of the gauging that breaks half of the supersymmetry around the linear dilaton solution.

Indeed, one of the 4d gravitini acquires a mass fixed by g, giving rise to a massive spin-3/2 multiplet together with two spin-1 vectors. These are the 5d graviphoton and the additional 5d vector that have non-canonical, dilaton dependent, kinetic terms, as one can see from the Lagrangian (47). Using the background (28), (29), one finds that the y-dependence of the vector kinetic terms at the end of the first line of (47) is $\exp\{\pm\sqrt{3}C\}$ with the plus (minus) sign corresponding to the 5d graviphoton $I = 0$ (extra vector $I = 1$). It follows that they both acquire a mass given by the mass gap.

We conclude with some comments on some possible phenomenological implications of the above lagrangian. One has to dimensionally reduce it from $D = 5$ to $D = 4$, upon compactification of the y-coordinate. Moreover, one has to introduce the SM, possibly on one of the boundaries, a radion stabilization mechanism and the breaking of the leftover supersymmetry. An interesting possibility is to combine all of them along the lines of the stabilisation proposal of [3] based on boundary conditions.

There are several possibilities for Dark Matter (DM) candidates in this gravitational sector. There are two gravitini that, upon supersymmetry breaking can recombine to form a Dirac gravitino [19] or remain two different Majorana ones. Depending on the nature of their mass, the exact freeze-out mechanism will be different. There are three possible dark photons A^0_μ, A^1_μ and the KK $U(1)$ coming from the 5d metric that could also be DM or their associated gaugini could also play a similar role, again depending on the compactification of the extra coordinate, on how supersymmetry breaking is implemented, as well as on the radion stabilisation mechanism. In general there could be a very rich phenomenology in the gravitational sector.

Regarding LHC or FCC phenomenology it is going to depend on how the SM fields are included in this setup, we will leave that to a forthcoming publication [20]. In general this theory will have KK massive resonances that could be strongly coupled to the SM in a similar fashion as in Randall–Sundrum [13] models.

Note added After the completion of this work, we received the paper [21] which contains very similar results.

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Appendix A: Conventions

Our convention for the five-dimensional Minkowski metric is

$$\eta_{mn} = \mathrm{diag}(-,+,+,+,+), \tag{A.1}$$

where $m, n, \ldots$ are inert indices and $m = 1, \ldots, 5$. For Γ-matrices we write

$$\Gamma_{mn} \equiv \Gamma_{[m}\Gamma_{n]} \equiv \frac{1}{2}(\Gamma_m\Gamma_n - \Gamma_n\Gamma_m). \tag{A.2}$$

We also have that

$$\Gamma^5 = \Gamma_5 = i\gamma^5 = i\gamma_5, \tag{A.3}$$

where γ^5 is the standard γ^5 in four-dimensions, such that in the Dirac representation

$$\Gamma^5 = i\gamma^5 = \begin{pmatrix} 0_{2\times 2} & i1_{2\times 2} \\ i1_{2\times 2} & 0_{2\times 2} \end{pmatrix}. \tag{A.4}$$

The five-dimensional bulk metric of the LST dual is given by

$$g_{MN} = \begin{pmatrix} e^{-\frac{2}{\sqrt{3}}Cy}\eta_{\mu\nu} & 0_{4\times 1} \\ 0_{1\times 4} & e^{-\frac{2}{\sqrt{3}}Cy} \end{pmatrix} = e^{-\frac{2}{\sqrt{3}}Cy}\eta_{MN}. \tag{A.5}$$

Appendix B: Einstein equation in 5D

In our conventions, the Einstein equation takes the form

$$G_{MN} = T_{MN}, \tag{B.1}$$

where G_{MN} and T_{MN} are the Einstein and the energy–momentum tensor respectively. Moreover, we have that

$$\begin{aligned} G_{MN} =& \frac{3}{2}\bigg[\frac{1}{2}\partial_M\Xi\partial_N\Xi + \partial_M\partial_N\Xi \\ &- \eta_{MN}\left(\partial_l\partial^l\Xi - \frac{1}{2}\partial_l\Xi\partial^l\Xi\right)\bigg], \end{aligned} \tag{B.2}$$

where $\Xi = \Xi(y) = \frac{2}{\sqrt{3}}Cy$ in our case. This gives

$$G_{55} = \frac{3}{2}\left(\frac{d\Xi}{dy}\right)^2 = 2C^2. \tag{B.3}$$

In addition,

$$T_{MN} = (\partial_M\Phi)(\partial_N\Phi) - g_{MN}\left(\frac{1}{2}(\partial_K\Phi)(\partial^K\Phi) + e^{\frac{2}{\sqrt{3}}\Phi}\Lambda\right), \tag{B.4}$$

so $T_{55} = \frac{1}{2}C^2 - \Lambda$. The Einstein equation $G_{55} = T_{55}$ then gives

$$C = \frac{gA}{\sqrt{2}}, \tag{B.5}$$

where we have used (26).

Appendix C: Spacetime calculations

In the following $M, N, \ldots$ are coordinate indices and $n, m, \ldots$ are (inert) frame indices of the five-dimensional spacetime. We have that

$$g_{MN} = e^m_M\eta_{mn}e^n_N. \tag{C.1}$$

The only non-vanishing components of the vielbein e^m are thus

$$e^a_\mu = e^{-\frac{1}{\sqrt{3}}Cy}\delta^a_\mu, \quad e^5_5 = e^{-\frac{1}{\sqrt{3}}Cy}, \tag{C.2}$$

where $\mu, \nu, \ldots$ are the coordinate and $a, b, \ldots$ the frame indices on the four-dimensional brane respectively. Moreover,

$$e^{a5} = g^{55}e^a_5 = 0 \quad , \quad e^{55} = g^{55}e^5_5 = e^{\frac{2}{\sqrt{3}}Cy}e^5_5 \tag{C.3}$$

and

$$e^{a\nu} = g^{\nu\kappa}e^a_\kappa = e^{\frac{2}{\sqrt{3}}Cy}\eta^{\nu\kappa}e^a_\kappa, \quad e_{\mu b} = \eta_{ab}e^a_\mu. \tag{C.4}$$

Consequently,

$$\not{\partial}\Phi = (\partial_M \Phi)\Gamma^M = (\partial_M \Phi)e_m^M \Gamma^m = (\partial_M \Phi)(e_M^m)^{-1}\Gamma^m$$
$$= C(e_5^5)^{-1}\Gamma^5 = Ce^{\frac{1}{\sqrt{3}}Cy}\Gamma^5. \tag{C.5}$$

Using the second of the Eq. (27), the second of the Eq. (12) then takes the form (in the vacuum)

$$\tilde{\delta}\lambda_i = -\frac{1}{2}e^{\frac{1}{\sqrt{3}}Cy}\left(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ij}\delta^{jk}\epsilon_k\right). \tag{C.6}$$

The components of the spacetime spin-connection are given by

$$\omega_M^{mn}(e) = 2e^{[mN}e^{n]}_{[N,M]} + e^{m\Lambda}e^{nP}e^l_{[\Lambda,P]}e_{Ml}. \tag{C.7}$$

Consequently,

$$\omega_\mu^{ab}(e) = \left(-e^{[a5}e^{b]}_{\mu,5} + \frac{1}{2}e^{a\Lambda}e^{b5}e^l_{\Lambda,5}e_{\mu l} - \frac{1}{2}e^{b\Lambda}e^{a5}e^l_{\Lambda,5}e_{\mu l}\right)$$
$$= 0, \tag{C.8}$$

since $e^{a5} = 0$. Moreover,

$$\omega_\mu^{a5}(e) = \left(-e^{[a5}e^{5]}_{\mu,5} + \frac{1}{2}e^{a\Lambda}e^{55}e^l_{\Lambda,5}e_{\mu l}\right)$$
$$= \left(\frac{1}{2}e^{55}e^a_{\mu,5} + \frac{1}{2}e^{a\nu}e^{55}e^b_{\nu,5}e_{\mu b}\right)$$
$$= e^{55}\left(\partial_5 e^{-\frac{C}{\sqrt{3}}y}\right)\left(\frac{1}{2}\delta^a_\mu + e^{\frac{1}{\sqrt{3}}Cy}\eta^{\nu\kappa}\delta^a_\kappa\delta^b_\nu\eta_{cb}e^c_\mu\right)$$
$$= -\frac{C}{\sqrt{3}}\delta^a_\mu\,. \tag{C.9}$$

Similarly, we find that

$$\omega_5^{ab} = \omega_5^{a5} = 0. \tag{C.10}$$

Since $\partial_\mu\epsilon_{i1} = 0$, we have that (in the vacuum) on the brane

$$D_\mu\epsilon_i = \frac{1}{4}\omega_\mu^{mn}\Gamma_{mn}\epsilon_i = -\frac{C}{2\sqrt{3}}\Gamma_\mu\Gamma_5\epsilon_i. \tag{C.11}$$

Then, using the first of the Eq. (27), the first of the Eq. (12) takes the following form on the brane

$$\tilde{\delta}\psi_{\mu i} = \frac{i}{2\sqrt{3}}\Gamma_\mu\left(iC\Gamma^5\epsilon_i + \frac{gA}{\sqrt{2}}\varepsilon_{ij}\delta^{jk}\epsilon_k\right), \tag{C.12}$$

while the fifth component of the first of the Eq. (12) takes the form

$$\tilde{\delta}\psi_{5i} = \partial_5\epsilon_i + \frac{igA}{2\sqrt{6}}\Gamma_5\varepsilon_{ij}\delta^{jk}\epsilon_k. \tag{C.13}$$

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