Revisit assignments of the new excited Ω_c states with QCD sum rules

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Abstract In this article, we distinguish the contributions of the positive parity and negative parity Ω_c states, study the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ using the QCD sum rules in a consistent way, and we revisit the assignments of the new narrow excited Ω_c^0 states. The predictions support assigning $\Omega_c(3000)$ to the 1P Ω_c state with $J^P = \frac{1}{2}^-$, assigning $\Omega_c(3090)$ to the 1P Ω_c state with $J^P = \frac{3}{2}^-$ or the 2S Ω_c state with $J^P = \frac{1}{2}^+$, and assigning $\Omega_c(3119)$ to the 2S Ω_c state with $J^P = \frac{3}{2}^+$.

1 Introduction

Recently, the LHCb collaboration studied the $\Xi_c^+ K^-$ mass spectrum and observed five new narrow excited Ω_c states, $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$, $\Omega_c(3119)$ [1]. The measured masses and widths are

$$\begin{aligned} \Omega_c(3000) : M &= 3000.4 \pm 0.2 \pm 0.1 \text{ MeV}, \\ \Gamma &= 4.5 \pm 0.6 \pm 0.3 \text{ MeV}, \\ \Omega_c(3050) : M &= 3050.2 \pm 0.1 \pm 0.1 \text{ MeV}, \\ \Gamma &= 0.8 \pm 0.2 \pm 0.1 \text{ MeV}, \\ \Omega_c(3066) : M &= 3065.6 \pm 0.1 \pm 0.3 \text{ MeV}, \\ \Gamma &= 3.5 \pm 0.4 \pm 0.2 \text{ MeV}, \\ \Omega_c(3090) : M &= 3090.2 \pm 0.3 \pm 0.5 \text{ MeV}, \\ \Gamma &= 8.7 \pm 1.0 \pm 0.8 \text{ MeV}, \\ \Omega_c(3119) : M &= 3119.1 \pm 0.3 \pm 0.9 \text{ MeV}, \\ \Gamma &= 1.1 \pm 0.8 \pm 0.4 \text{ MeV}. \end{aligned} \quad (1)$$

There have been several assignments for those new Ω_c states, such as the 2S Ω_c states with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ [2–5], the P-wave Ω_c states with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$ [3–12], the

pentaquark states or molecular pentaquark states with $J^P = \frac{1}{2}^-, \frac{3}{2}^-$ or $\frac{5}{2}^-$ [13–16], or the D-wave Ω_c states [17].

In Refs. [2,5], Agaev, Azizi and Sundu construct the interpolating currents without introducing the relative P-wave to study the Ω_c states by taking into account the 1S, 1P, 2S states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions in the QCD sum rules. They use the 1S state plus continuum model to obtain the masses and pole residues of the 1S states firstly, then take them as input parameters and use the 1S state plus 1P state plus continuum model to obtain the masses and pole residues of the 1P states, and finally use the 1S state plus 1P state plus 2S state plus continuum model to obtain the masses and pole residues of the 2S states. In Ref. [12], Aliev, Bilmis and Savci use the same interpolating currents to study the Ω_c states by taking into account the 1S and 1P states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions in the QCD sum rules. The potential quark models predict that the 1P and 2S Ω_c states have the masses about 3.0–3.2 GeV [18,19]. If the 1P and 2S Ω_c states lie in the same energy region, it is difficult to distinguish their contributions in the QCD sum rules [2,5,12].

In Refs. [20–25], we construct the interpolating currents without introducing the relative P-wave to study the $J^P = \frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ heavy, doubly heavy and triply heavy baryon states with the QCD sum rules in a systematic way by subtracting the contributions from the corresponding $J^P = \frac{1}{2}^\mp$ and $\frac{3}{2}^\mp$ heavy, doubly heavy and triply heavy baryon states, and obtain satisfactory results. In Ref. [8], we study the new excited Ω_c states with the QCD sum rules by introducing an explicit P-wave involving the two s quarks, the predictions support assigning $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$ to the P-wave baryon states with $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively.

In this article, we distinguish the contributions of the S-wave (positive parity) and P-wave (negative parity) Ω_c states, study the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states, with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$, using the QCD sum

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rules, in detail, and we revisit the assignments of the new narrow excited Ω_c^0 states.

The article is arranged as follows: we derive the QCD sum rules for the masses and pole residues of the S-wave and P-wave $\frac{1}{2}$ and $\frac{3}{2}$ Ω_c states in Sect. 2; in Sect. 3, we present the numerical results and discussions; and Sect. 4 is reserved for our conclusion.

2 QCD sum rules for the $\frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ Ω_c states

Firstly, we write down the two-point correlation functions $\Pi(p)$ and $\Pi_{\alpha\beta}(p)$ in the QCD sum rules,

$$\begin{aligned}\Pi(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \} | 0 \rangle, \\ \Pi_{\alpha\beta}(p) &= i \int d^4x e^{ip \cdot x} \langle 0 | T \{ \eta_\alpha(x) \bar{\eta}_\beta(0) \} | 0 \rangle,\end{aligned}\quad (2)$$

where

$$\begin{aligned}\eta(x) &= \varepsilon^{ijk} s_i^T(x) C \gamma_\alpha s_j(x) \gamma_5 \gamma^\alpha c_k(x), \\ \eta_\alpha(x) &= \varepsilon^{ijk} s_i^T(x) C \gamma_\alpha s_j(x) c_k(x),\end{aligned}\quad (3)$$

i, j and k are color indices, and C is the charge conjugation matrix. In this article, we choose the simple Ioffe type interpolating currents.

At the hadron side, we insert a complete set of intermediate Ω_c states with the same quantum numbers as the current operators $\eta(x)$, $i\gamma_5\eta(x)$, $\eta_\alpha(x)$ and $i\gamma_5\eta_\alpha(x)$ into the correlation functions $\Pi(p)$ and $\Pi_{\alpha\beta}(p)$ to obtain the hadronic representation [26,27]. We isolate the pole terms of the lowest 1S, 1P, 2S and 2P Ω_c states (Ω_c and Ω'_c), and obtain the results

$$\begin{aligned}\Pi(p) &= \lambda_{\frac{1}{2}}^{+2} \frac{\not{p} + M_+}{M_+^2 - p^2} + \lambda_{\frac{1}{2}}^{-2} \frac{\not{p} - M_-}{M_-^2 - p^2} + \lambda_{\frac{1}{2}}^{' +2} \frac{\not{p} + M'_+}{M'^2_+ - p^2} \\ &\quad + \lambda_{\frac{1}{2}}^{' -2} \frac{\not{p} - M'_-}{M'^2_- - p^2} + \dots, \\ &= \Pi_{\frac{1}{2}}(p^2) + \dots, \\ \Pi_{\alpha\beta}(p) &= \left(\lambda_{\frac{3}{2}}^{+2} \frac{\not{p} + M_+}{M_+^2 - p^2} + \lambda_{\frac{3}{2}}^{-2} \frac{\not{p} - M_-}{M_-^2 - p^2} \right. \\ &\quad \left. + \lambda_{\frac{3}{2}}^{' +2} \frac{\not{p} + M'_+}{M'^2_+ - p^2} + \lambda_{\frac{3}{2}}^{' -2} \frac{\not{p} - M'_-}{M'^2_- - p^2} \right) \\ &\quad \times \left(-g_{\alpha\beta} + \frac{\gamma_\alpha \gamma_\beta}{3} + \frac{2p_\alpha p_\beta}{3p^2} - \frac{p_\alpha \gamma_\beta - p_\beta \gamma_\alpha}{3\sqrt{p^2}} \right) + \dots, \\ &= \Pi_{\frac{3}{2}}(p^2) (-g_{\alpha\beta}) + \dots.\end{aligned}\quad (4)$$

The currents $\eta(0)$ and $\eta_\alpha(0)$ couple potentially to the spin-parity $J^P = \frac{1}{2}^\pm$ and $\frac{3}{2}^\pm$ Ω_c states $\Omega_c^{(\prime)\pm}$ and $\Omega_{\frac{3}{2}}^{(\prime)\pm}$, respectively [23–25,28–31],

$$\begin{aligned}\langle 0 | \eta(0) | \Omega_{\frac{1}{2}}^{(\prime)+}(p) \rangle &= \lambda_{\frac{1}{2}}^{(\prime)+} U^+(p, s), \\ \langle 0 | \eta_\alpha(0) | \Omega_{\frac{3}{2}}^{(\prime)+}(p) \rangle &= \lambda_{\frac{3}{2}}^{(\prime)+} U_\alpha^+(p, s),\end{aligned}\quad (6)$$

$$\begin{aligned}\langle 0 | \eta(0) | \Omega_{\frac{1}{2}}^{(\prime)-}(p) \rangle &= \lambda_{\frac{1}{2}}^{(\prime)-} i \gamma_5 U^-(p, s), \\ \langle 0 | \eta_\alpha(0) | \Omega_{\frac{3}{2}}^{(\prime)-}(p) \rangle &= \lambda_{\frac{3}{2}}^{(\prime)-} i \gamma_5 U_\alpha^-(p, s),\end{aligned}\quad (7)$$

where $\lambda_{\frac{1}{2}}^{(\prime)\pm}$ and $\lambda_{\frac{3}{2}}^{(\prime)\pm}$ are the pole residues or the current–baryon couplings, the spinors $U^\pm(p, s)$ and $U_\alpha^\pm(p, s)$ satisfy the relations

$$\begin{aligned}\sum_s U(p, s) \bar{U}(p, s) &= \not{p} + M_\pm^{(\prime)}, \\ \sum_s U_\alpha(p, s) \bar{U}_\beta(p, s) \\ &= (\not{p} + M_\pm^{(\prime)}) \left(-g_{\alpha\beta} + \frac{\gamma_\alpha \gamma_\beta}{3} + \frac{2p_\alpha p_\beta}{3p^2} - \frac{p_\alpha \gamma_\beta - p_\beta \gamma_\alpha}{3\sqrt{p^2}} \right),\end{aligned}\quad (8)$$

and $p^2 = M_\pm^{(\prime)2}$ on mass-shell, the s are the polarizations or spin indices of the spinors, and they should be distinguished from the s quark or the energy s .

We obtain the hadronic spectral densities at hadron side through the dispersion relation

$$\begin{aligned}\frac{\text{Im} \Pi_j(s)}{\pi} &= \not{p} \left[\lambda_j^{+2} \delta(s - M_+^2) + \lambda_j^{-2} \delta(s - M_-^2) \right. \\ &\quad \left. + \lambda_j^{' +2} \delta(s - M'^2_+) + \lambda_j^{' -2} \delta(s - M'^2_-) \right] \\ &\quad + \left[M_+ \lambda_j^{+2} \delta(s - M_+^2) - M_- \lambda_j^{-2} \delta(s - M_-^2) \right. \\ &\quad \left. + M'_+ \lambda_j^{' +2} \delta(s - M'^2_+) - M'_- \lambda_j^{' -2} \delta(s - M'^2_-) \right] \\ &= \not{p} \rho_{j,H}^1(s) + \rho_{j,H}^0(s),\end{aligned}\quad (9)$$

where $j = \frac{1}{2}, \frac{3}{2}$, the subscript H denotes the hadron side, then we introduce the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules at the hadron side,

$$\begin{aligned}\int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,H}^1(s) + \rho_{j,H}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) \\ = 2M_+ \lambda_j^{+2} \exp\left(-\frac{M_+^2}{T^2}\right) \\ + 2M'_+ \lambda_j^{' +2} \exp\left(-\frac{M'^2_+}{T^2}\right),\end{aligned}\quad (10)$$

$$\begin{aligned}
& \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,H}^1(s) - \rho_{j,H}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) \\
&= 2M_- \lambda_j^{-2} \exp\left(-\frac{M_-^2}{T^2}\right) \\
&+ 2M'_- \lambda_j'^{-2} \exp\left(-\frac{M_-'^2}{T^2}\right), \quad (11)
\end{aligned}$$

where the s_0 are the continuum thresholds and the T^2 are the Borel parameters [31]. We distinguish the contributions of the positive parity and negative parity Ω_c states unambiguously according to Eqs. (9)–(11).

At the QCD side, we calculate the light quark parts of the correlation functions $\Pi(p)$ and $\Pi_{\alpha\beta}(p)$ with the full light quark propagators $S_{ij}(x)$ in the coordinate space [32],

$$\begin{aligned}
S_{ij}(x) = & \frac{i\delta_{ij}\not{x}}{2\pi^2 x^4} - \frac{\delta_{ij}m_s}{4\pi^2 x^2} - \frac{\delta_{ij}\langle\bar{s}s\rangle}{12} + \frac{i\delta_{ij}\not{x}m_s\langle\bar{s}s\rangle}{48} \\
& - \frac{\delta_{ij}x^2\langle\bar{s}g_s\sigma Gs\rangle}{192} \\
& + \frac{i\delta_{ij}x^2\not{x}m_s\langle\bar{s}g_s\sigma Gs\rangle}{1152} \\
& - \frac{ig_s G_{\alpha\beta}^a t_{ij}^a (\not{x}\sigma^{\alpha\beta} + \sigma^{\alpha\beta}\not{x})}{32\pi^2 x^2} \\
& - \frac{1}{8} \langle\bar{s}_j\sigma^{\mu\nu}s_i\rangle\sigma_{\mu\nu} + \dots, \quad (12)
\end{aligned}$$

and we take the full c -quark propagator $C_{ij}(x)$ in the momentum space [27],

$$\begin{aligned}
C_{ij}(x) = & \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot x} \left\{ \frac{\delta_{ij}}{\not{k} - m_c} \right. \\
& - \frac{g_s G_{\alpha\beta}^n t_{ij}^n \sigma^{\alpha\beta} (\not{k} + m_c) + (\not{k} + m_c) \sigma^{\alpha\beta}}{4(k^2 - m_c^2)^2} \\
& - \frac{g_s^2 (t^a t^b)_{ij} G_{\alpha\beta}^a G_{\mu\nu}^b (f^{\alpha\beta\mu\nu} + f^{\alpha\mu\beta\nu} + f^{\alpha\nu\mu\beta})}{4(k^2 - m_c^2)^5} \\
& \left. + \dots \right\}, \quad (13)
\end{aligned}$$

$$\begin{aligned}
f^{\alpha\beta\mu\nu} = & (\not{k} + m_c) \gamma^\alpha (\not{k} + m_c) \gamma^\beta (\not{k} + m_c) \gamma^\mu (\not{k} + m_c) \gamma^\nu \\
& \times (\not{k} + m_c), \quad (14)
\end{aligned}$$

$t^n = \frac{\lambda^n}{2}$, λ^n is the Gell-Mann matrix. In Eq. (12), we add the term $\langle\bar{s}_j\sigma_{\mu\nu}s_i\rangle$ originating from the Fierz re-ordering of $\langle s_i\bar{s}_j\rangle$ to absorb the gluons emitted from other quark lines to form $\langle\bar{s}_j g_s G_{\alpha\beta}^a t_{mn}^a \sigma_{\mu\nu} s_i\rangle$ to extract the mixed condensate $\langle\bar{s}g_s\sigma Gs\rangle$. The term $-\frac{1}{8}\langle\bar{s}_j\sigma^{\mu\nu}s_i\rangle\sigma_{\mu\nu}$ was introduced in Refs. [33–35]. We compute the integrals both in the coordinate space and momentum space to obtain the correlation functions $\Pi_j(p^2)$; then we obtain the QCD spectral densities through the dispersion relation

$$\frac{\text{Im}\Pi_j(s)}{\pi} = \not{p} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s), \quad (15)$$

where $j = \frac{1}{2}, \frac{3}{2}$, the explicit expressions of the QCD spectral densities $\rho_{j,QCD}^1(s)$ and $\rho_{j,QCD}^0(s)$ can be rewritten in a concise form after multiplying the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the integrals $\int_{m_c^2}^{\infty} ds \sqrt{s} \rho_{j,QCD}^1(s) \exp\left(-\frac{s}{T^2}\right)$ and $\int_{m_c^2}^{\infty} ds \rho_{j,QCD}^0(s) \exp\left(-\frac{s}{T^2}\right)$.

We take the quark–hadron duality, introduce the continuum thresholds s_0 and the weight function $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules:

$$\begin{aligned}
& \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,H}^1(s) + \rho_{j,H}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) \\
&= \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right), \quad (16)
\end{aligned}$$

$$\begin{aligned}
& \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,H}^1(s) - \rho_{j,H}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) \\
&= \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right), \quad (17)
\end{aligned}$$

where $j = \frac{1}{2}, \frac{3}{2}$,

$$\begin{aligned}
\rho_{j,QCD}^0(s) &= m_c \rho_j^0(s), \\
\rho_{j,QCD}^1(s) &= \rho_j^1(s), \quad (18)
\end{aligned}$$

$$\begin{aligned}
\rho_{\frac{1}{2}}^0(s) = & \frac{3}{32\pi^4} \int_{x_i}^1 dx (1-x)^2 (s - \tilde{m}_c^2)^2 - \frac{3m_s\langle\bar{s}s\rangle}{2\pi^2} \int_{x_i}^1 dx \\
& + \frac{1}{32\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \frac{(1-x)^2}{x^2} \left[1 - \frac{s}{3} \delta(s - \tilde{m}_c^2) \right] \\
& + \frac{1}{32\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \frac{2-3x}{x} \\
& - \frac{m_s\langle\bar{s}g_s\sigma Gs\rangle}{4\pi^2} \int_{x_i}^1 dx \frac{1}{x} \delta(s - \tilde{m}_c^2) \\
& + \frac{5m_s\langle\bar{s}g_s\sigma Gs\rangle}{12\pi^2} \delta(s - m_c^2) + \frac{4\langle\bar{s}s\rangle^2}{3} \delta(s - m_c^2) \\
& + \frac{\langle\bar{s}s\rangle\langle\bar{s}g_s\sigma Gs\rangle}{3T^2} \left(1 - \frac{2m_c^2}{T^2} \right) \delta(s - m_c^2) \\
& - \frac{\langle\bar{s}g_s\sigma Gs\rangle^2}{12T^4} \left(1 + \frac{3m_c^2}{T^2} - \frac{m_c^4}{T^4} \right) \delta(s - m_c^2), \quad (19)
\end{aligned}$$

$$\begin{aligned}
\rho_{\frac{1}{2}}^1(s) = & \frac{1}{16\pi^4} \int_{x_i}^1 dx x(1-x)^3 (5s - 3\tilde{m}_c^2)(s - \tilde{m}_c^2) \\
& - \frac{m_s\langle\bar{s}s\rangle}{\pi^2} \int_{x_i}^1 dx x \\
& + \frac{m_s\langle\bar{s}s\rangle}{\pi^2} \int_{x_i}^1 dx x(1-x) [3 + s \delta(s - \tilde{m}_c^2)] \\
& + \frac{1}{48\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \{ (1-x) [3 + s \delta(s - \tilde{m}_c^2)] \}
\end{aligned}$$

$$\begin{aligned}
& + (1 - 2x) \Big\} \\
& - \frac{m_c^2}{72\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \frac{(1-x)^3}{x^2} \left(1 + \frac{s}{2T^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{3\pi^2} \int_{x_i}^1 dx x \left(1 + \frac{s}{2T^2} \right) \delta(s - \tilde{m}_c^2) \\
& - \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{8\pi^2} \int_{x_i}^1 dx \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{4\pi^2} \delta(s - m_c^2) + \frac{2 \langle \bar{s} s \rangle^2}{3} \delta(s - m_c^2) \\
& - \frac{\langle \bar{s} s \rangle \langle \bar{s} g_s \sigma G s \rangle}{6T^2} \left(1 + \frac{2m_c^2}{T^2} \right) \delta(s - m_c^2) \\
& - \frac{m_c^2 \langle \bar{s} g_s \sigma G s \rangle^2}{24T^6} \left(1 - \frac{m_c^2}{T^2} \right) \delta(s - m_c^2), \quad (20)
\end{aligned}$$

$$\begin{aligned}
\rho_{\frac{3}{2}}^0(s) &= \frac{1}{64\pi^4} \int_{x_i}^1 dx (x+2)(1-x)^2 (s - \tilde{m}_c^2)^2 \\
& - \frac{m_s \langle \bar{s} s \rangle}{4\pi^2} \int_{x_i}^1 dx (2-x) \\
& - \frac{m_c^2}{576\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \frac{(x+2)(1-x)^2}{x^3} \delta(s - \tilde{m}_c^2) \\
& + \frac{1}{192\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \left[\frac{(x+2)(1-x)^2}{x^2} - (2-x) \right] \\
& + \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{24\pi^2} \int_{x_i}^1 dx \delta(s - \tilde{m}_c^2) \\
& + \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{12\pi^2} \delta(s - m_c^2) \\
& + \frac{\langle \bar{s} s \rangle^2}{3} \delta(s - m_c^2) - \frac{m_c^2 \langle \bar{s} s \rangle \langle \bar{s} g_s \sigma G s \rangle}{6T^4} \delta(s - m_c^2) \\
& - \frac{m_c^2 \langle \bar{s} g_s \sigma G s \rangle^2}{24T^6} \left(1 - \frac{m_c^2}{2T^2} \right) \delta(s - m_c^2), \quad (21)
\end{aligned}$$

$$\begin{aligned}
\rho_{\frac{1}{2}}^1(s) &= \frac{1}{64\pi^4} \int_{x_i}^1 dx x(x+2)(1-x)^2 (s - \tilde{m}_c^2)^2 \\
& - \frac{m_s \langle \bar{s} s \rangle}{4\pi^2} \int_{x_i}^1 dx x(2-x) \\
& - \frac{m_c^2}{576\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx \frac{(x+2)(1-x)^2}{x^2} \delta(s - \tilde{m}_c^2) \\
& - \frac{1}{192\pi^2} \left\langle \frac{\alpha_s GG}{\pi} \right\rangle \int_{x_i}^1 dx x(2-x) \\
& + \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{24\pi^2} \int_{x_i}^1 dx x \delta(s - \tilde{m}_c^2)
\end{aligned}$$

$$\begin{aligned}
& + \frac{m_s \langle \bar{s} g_s \sigma G s \rangle}{12\pi^2} \delta(s - m_c^2) \\
& + \frac{\langle \bar{s} s \rangle^2}{3} \delta(s - m_c^2) - \frac{\langle \bar{s} s \rangle \langle \bar{s} g_s \sigma G s \rangle}{6T^2} \left(1 + \frac{m_c^2}{T^2} \right) \delta(s - m_c^2) \\
& + \frac{m_c^4 \langle \bar{s} g_s \sigma G s \rangle^2}{48T^8} \delta(s - m_c^2), \quad (22)
\end{aligned}$$

$$\tilde{m}_c^2 = \frac{m_c^2}{x}, x_i = \frac{m_c^2}{s}.$$

The QCD sum rules can be written more explicitly,

$$\begin{aligned}
2M_{+\lambda_j^+} \exp\left(-\frac{M_{+}^2}{T^2}\right) + 2M'_{+\lambda_j'^+} \exp\left(-\frac{M_{+}'^2}{T^2}\right) \\
= \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right), \quad (23)
\end{aligned}$$

$$\begin{aligned}
2M_{-\lambda_j^-} \exp\left(-\frac{M_{-}^2}{T^2}\right) + 2M'_{-\lambda_j'^-} \exp\left(-\frac{M_{-}'^2}{T^2}\right) \\
= \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right). \quad (24)
\end{aligned}$$

The contributions of the positive parity and negative parity Ω_c states are separated explicitly.

Firstly, we choose low continuum threshold parameters s_0 so as not to include the contributions of the 2S and 2P Ω_c states (Ω_c'), and obtain the QCD sum rules for the masses of the 1S and 1P Ω_c states,

$$M_{+}^2 = \frac{-\frac{d}{d(1/T^2)} \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)}{\int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)}, \quad (25)$$

$$M_{-}^2 = \frac{-\frac{d}{d(1/T^2)} \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)}{\int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right)}, \quad (26)$$

then obtain the pole residues λ_j^+ and λ_j^- .

Now we take the masses and pole residues of the 1S and 1P Ω_c states as input parameters, and postpone the continuum threshold parameters s_0 to larger values to include the contributions of the 2S and 2P Ω_c states, and obtain the QCD sum rules for the masses of the 2S and 2P Ω_c states,

$$M_+^{\prime 2} = \frac{-\frac{d}{d(1/T^2)} \left\{ \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) - 2M_+ \lambda_j^{+2} \exp\left(-\frac{M_+^2}{T^2}\right) \right\}}{\int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) - 2M_+ \lambda_j^{+2} \exp\left(-\frac{M_+^2}{T^2}\right)}, \quad (27)$$

$$M_-^{\prime 2} = \frac{-\frac{d}{d(1/T^2)} \left\{ \int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) - 2M_- \lambda_j^{-2} \exp\left(-\frac{M_-^2}{T^2}\right) \right\}}{\int_{m_c^2}^{s_0} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right) - 2M_- \lambda_j^{-2} \exp\left(-\frac{M_-^2}{T^2}\right)}, \quad (28)$$

then obtain the pole residues $\lambda_j^{\prime +}$ and $\lambda_j^{\prime -}$.

3 Numerical results and discussions

The input parameters are taken to be the standard values $\langle \bar{q}q \rangle = -(0.24 \pm 0.01 \text{ GeV})^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1) \langle \bar{q}q \rangle$, $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $m_0^2 = (0.8 \pm 0.1) \text{ GeV}^2$, $\left\langle \frac{\alpha_s GG}{\pi} \right\rangle = (0.33 \text{ GeV})^4$ at the energy scale $\mu = 1 \text{ GeV}$ [26,27,36,37], $m_c(m_c) = (1.275 \pm 0.025) \text{ GeV}$ and $m_s(\mu = 2 \text{ GeV}) = (0.095 \pm 0.005) \text{ GeV}$ from the Particle Data Group [38]. The updated values from the Particle Data Group in version 2016 [38] are slightly different from the corresponding ones in version 2014, we take the old values to make consistent predictions with the same parameters and criteria chosen in previous work. If we choose the updated values $m_c(m_c) = (1.28 \pm 0.03) \text{ GeV}$ and $m_s(\mu = 2 \text{ GeV}) = 0.096_{-0.004}^{+0.008} \text{ GeV}$ [38], the central value of the predicted mass of $\Omega_c(1S)$ is 2.6991 GeV rather than 2.6983 GeV, the predicted mass presented in Table 2 survives, so the old values are OK. The values of m_0^2 , $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$ and $\langle \bar{s}g_s \sigma Gs \rangle / \langle \bar{q}g_s \sigma Gq \rangle$ vary in rather large ranges from different theoretical determinations, for example, in Ref. [39], $\langle \bar{s}g_s \sigma Gs \rangle / \langle \bar{q}g_s \sigma Gq \rangle = 0.95 \pm 0.15$, which differs from the standard value $\langle \bar{s}g_s \sigma Gs \rangle / \langle \bar{q}g_s \sigma Gq \rangle = \langle \bar{s}s \rangle / \langle \bar{q}q \rangle = 0.8 \pm 0.1$ remarkably [36]. In this article, we take the standard values or the old values still accepted now [36,37].

We take into account the energy-scale dependence of the input parameters from the renormalization group equation,

$$\begin{aligned} \langle \bar{s}s \rangle(\mu) &= \langle \bar{s}s \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{4}{9}}, \\ \langle \bar{s}g_s \sigma Gs \rangle(\mu) &= \langle \bar{s}g_s \sigma Gs \rangle(Q) \left[\frac{\alpha_s(Q)}{\alpha_s(\mu)} \right]^{\frac{2}{27}}, \\ m_s(\mu) &= m_s(2 \text{ GeV}) \left[\frac{\alpha_s(\mu)}{\alpha_s(2 \text{ GeV})} \right]^{\frac{4}{9}}, \\ m_c(\mu) &= m_c(m_c) \left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{25}}, \end{aligned}$$

$$\begin{aligned} \alpha_s(\mu) &= \frac{1}{b_0 t} \left[1 - \frac{b_1 \log t}{b_0^2 t} \right. \\ &\quad \left. + \frac{b_1^2 (\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2} \right], \end{aligned} \quad (29)$$

where $t = \log \frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857 - \frac{5033}{9}n_f + \frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213 \text{ MeV}$, 296 MeV and 339 MeV for the flavors $n_f = 5, 4$ and 3, respectively [38], and evolve all the input parameters to the optimal energy scales μ to extract the masses of the Ω_c states. The energy-scale dependence of the quark masses and quark condensates is known beyond the leading order, the energy-scale dependence of the mixed quark condensates is only known in the leading order [40,41]. In this article, we take the leading order approximation in a consistent way, and take the energy-scale dependence of the mixed condensates presented in Refs. [40,41], while a quite different energy-scale dependence of the mixed condensates is presented in Refs. [39,42]. It is interesting to take the energy-scale dependence presented in Refs. [39,42], this may be our next work. For the heavy degrees of freedom, we take the favors $n_f = 4$, the power in $m_c(\mu)$ is $\frac{12}{25}$. For the light degrees of freedom, we take the flavors $n_f = 3$, the powers in $\langle \bar{s}s \rangle(\mu)$, $\langle \bar{s}g_s \sigma Gs \rangle(\mu)$ and $m_s(\mu)$ are $\frac{4}{9}$ (or $\frac{12}{27}$), $\frac{2}{27}$ and $\frac{4}{9}$, respectively. If we take the favors $n_f = 4$, the powers in $\langle \bar{s}s \rangle(\mu)$, $\langle \bar{s}g_s \sigma Gs \rangle(\mu)$ and $m_s(\mu)$ are $\frac{12}{25}$, $\frac{2}{25}$ and $\frac{12}{25}$, respectively, in fact, the induced tiny difference in numerical calculations can be neglected. As far as the fine constant $\alpha_s(\mu)$ is concerned, we choose the next-to-next-to-leading order approximation, which is consistent with the values determined experimentally [38].

In Fig. 1, we plot the correlation functions $\Pi_{j,+}$ and $\Pi_{j,-}$ with variations of the energy scales μ and the Borel parameters T^2 ,

$$\Pi_{j,+} = \int_{m_c^2}^{\infty} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) + \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right), \quad (30)$$

Fig. 1 The correlation functions with variations of the energy scales μ and Borel parameters T^2 , where A, B, C and D correspond to $\Pi_{\frac{1}{2},+}$, $\Pi_{\frac{3}{2},+}$, $\Pi_{\frac{1}{2},-}$ and $\Pi_{\frac{3}{2},-}$, respectively

$$\Pi_{j,-} = \int_{m_c^2}^{\infty} ds \left[\sqrt{s} \rho_{j,QCD}^1(s) - \rho_{j,QCD}^0(s) \right] \exp\left(-\frac{s}{T^2}\right). \quad (31)$$

From the figure, we can see that $\Pi_{j,+}$ and $\Pi_{j,-}$ increase remarkably with increase of the energy scale μ at the region $T^2 > 4.0 \text{ GeV}^2$, while at the region $T^2 < 3.0 \text{ GeV}^2$, $\Pi_{j,+}$ and $\Pi_{j,-}$ increase slowly with increase of the energy scale μ . All in all, we cannot obtain energy-scale independent QCD sum rules, some constraints are needed to determine the energy scales of the QCD spectral densities in a consistent way.

Now we take a short digression to discuss how to choose the optimal energy scales. In the heavy quark limit, the heavy quark Q serves as a static well potential and combines with a light quark q to form a heavy diquark in color antitriplet, or combines with a light diquark in color antitriplet to form a heavy baryon in color singlet. The heavy antiquark $\bar{Q}$ serves as another static well potential and combines with a light antiquark $\bar{q}'$ to form a heavy antidiquark in color triplet, or combines with a light antidiquark in color triplet to form a heavy antibaryon in color singlet. Then the heavy diquark and heavy antidiquark combine together to form a hidden-charm or hidden-bottom tetraquark state. The heavy baryons B and tetraquark states $X/Y/Z$ are characterized by the effective

heavy quark masses $\mathbb{M}_Q$ (or constituent quark masses) and the virtuality $V = \sqrt{M_B^2 - \mathbb{M}_Q^2}$, $\sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$ (or bound energy not as robust). The diquark–quark type baryon states and diquark–antidiquark type tetraquark states are expected to have the same effective Q -quark masses $\mathbb{M}_Q$, which embody the net effects of the complex dynamics [33–35, 43]. In Refs. [33–35, 44, 45], we study the acceptable energy scales of the QCD spectral densities for the hidden-charm (hidden-bottom) tetraquark states and molecular states in the QCD sum rules in detail for the first time, and we suggest an energy-scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2\mathbb{M}_Q)^2}$ by setting $\mu = V$ to determine the optimal energy scales with the effective heavy quark masses $\mathbb{M}_Q$.

We fit the effective c -quark mass $\mathbb{M}_c$ to reproduce the experimental value of the mass of $Z_c^\pm(3900)$ in the scenario of tetraquark state [33–35]. In this article, we use the empirical energy-scale formula $\mu = \sqrt{M_{\Omega_c}^2 - \mathbb{M}_c^2}$ to determine the optimal energy scales of the QCD spectral densities, and we take the updated value of the effective c -quark mass $\mathbb{M}_c = 1.82 \text{ GeV}$ [46]. For detailed discussions as regards the energy-scale formula $\mu = \sqrt{M_{\Omega_c}^2 - \mathbb{M}_c^2}$, one can consult Ref. [43]. According to the energy-scale formula $\mu = \sqrt{M_{\Omega_c}^2 - \mathbb{M}_c^2}$, we extract the masses of the ground states

Fig. 2 The masses and pole residues of the Ω_c states with variations of the energy scale μ for the central values of the Borel parameters and threshold parameters shown in Table 1, where A, B, C and D correspond to $\Omega_c(1S, \frac{1}{2})$, $\Omega_c(1S, \frac{3}{2})$, $\Omega_c(1P, \frac{1}{2})$ and $\Omega_c(1P, \frac{3}{2})$, respectively

Table 1 The optimal energy scales μ , Borel parameters T^2 , continuum threshold parameters s_0 , pole contributions (pole) and perturbative contributions (perturbative) for Ω_c states

	J^P	μ (GeV)	T^2 (GeV ²)	$\sqrt{s_0}$ (GeV)	Pole	Perturbative
$\Omega_c(1S)$	$\frac{1}{2}^+$	2.0	2.3 – 2.9	3.30 ± 0.10	(41 – 69)%	(86 – 90)%
$\Omega_c(1S)$	$\frac{3}{2}^+$	2.1	2.4 – 3.0	3.40 ± 0.10	(46 – 72)%	(87 – 91)%
$\Omega_c(1P)$	$\frac{1}{2}^-$	2.4	2.2 – 2.8	3.40 ± 0.10	(40 – 68)%	(117 – 130)%
$\Omega_c(1P)$	$\frac{3}{2}^-$	2.5	2.2 – 2.8	3.50 ± 0.10	(39 – 67)%	(106 – 114)%
$\Omega_c(2S)$	$\frac{1}{2}^+$	2.5	2.6 – 3.2	3.45 ± 0.10	(43 – 68)%	(90 – 93)%
$\Omega_c(2S)$	$\frac{3}{2}^+$	2.5	2.7 – 3.3	3.50 ± 0.10	(45 – 69)%	(90 – 93)%
$\Omega_c(2P)$	$\frac{1}{2}^-$	2.9	2.4 – 3.0	3.70 ± 0.10	(53 – 78)%	(111 – 118)%
$\Omega_c(2P)$	$\frac{3}{2}^-$	2.9	2.4 – 3.0	3.75 ± 0.10	(49 – 75)%	(104 – 108)%

(see Eqs. (25), (26)) and the first radial excited states (see Eqs. (27), (28)) at different energy scales.

In Fig. 2, we plot the masses and pole residues of $\Omega_c(1S, \frac{1}{2})$, $\Omega_c(1S, \frac{3}{2})$, $\Omega_c(1P, \frac{1}{2})$ and $\Omega_c(1P, \frac{3}{2})$ with variations of the energy scale μ for the central values of the Borel parameters and threshold parameters shown in Table 1. From the figure, we can see that the predicted masses decrease monotonously but mildly with increase of the energy scale μ , the constraint $\mu = \sqrt{M_{\Omega_c}^2 - \mathbb{M}_c^2}$ is not difficult to satisfy. On the other hand, the pole residues increase monotonously and mildly with increase of the energy scale μ , which is consistent with Fig. 1, as the Borel parameters are chosen as $T^2 < 3.0 \text{ GeV}^2$. In the vicinities of the energy scales presented in Table 1, the uncertainties induced by the uncertainties of the energy scales are tiny.

For $Z_c(3900)$, the uncertainty of the energy scale of the QCD spectral density is about $\delta\mu = 0.1 \text{ GeV}$, the uncertainty of the effective c -quark mass $\mathbb{M}_c$ can be estimated as $\delta\mathbb{M}_c = \frac{\mu_0}{4\mathbb{M}_c} \delta\mu = 0.02 \text{ GeV}$ from the equation

$$\begin{aligned} \mu &= \sqrt{M_{X/Y/Z}^2 - 4(\mathbb{M}_c \pm \delta\mathbb{M}_c)^2} \\ &= \sqrt{M_{X/Y/Z}^2 - 4\mathbb{M}_c^2} \sqrt{1 \mp \frac{8\mathbb{M}_c\delta\mathbb{M}_c}{M_{X/Y/Z}^2 - 4\mathbb{M}_c^2}} \\ &= \mu_0 \left(1 \mp \frac{4\mathbb{M}_c\delta\mathbb{M}_c}{\mu_0^2} \right) = \mu_0 \mp \delta\mu, \end{aligned} \quad (32)$$

where μ_0 is the central value. The uncertainties $\delta\mu$ in this article can be estimated as $\delta\mu = \frac{\mathbb{M}_c}{\mu_0} \delta\mathbb{M}_c < 0.02 \text{ GeV}$ from the equation,

$$\mu = \sqrt{M_{\Omega_c}^2 - (\mathbb{M}_c \pm \delta\mathbb{M}_c)^2} = \mu_0 \mp \frac{\mathbb{M}_c}{\mu_0} \delta\mathbb{M}_c. \quad (33)$$

The predicted masses and pole residues are not sensitive to variations of the energy scales, the small uncertainty $\delta\mathbb{M}_c = 0.02 \text{ GeV}$ or $\delta\mu < 0.02 \text{ GeV}$ can be neglected safely.

We search for the ideal Borel parameters T^2 and continuum threshold parameters s_0 according to the four criteria:

Table 2 The masses and pole residues of the Ω_c states, the masses are compared with the experimental data, the values of $\Omega_c(1P)$ with $J^P = \frac{5}{2}^-$ are taken from Ref. [8]

	J^P	M (GeV)	λ (GeV ³)	(expt) (MeV)
$\Omega_c(1S)$	$\frac{1}{2}^+$	$2.70^{+0.11}_{-0.13}$	$1.09^{+0.17}_{-0.15} \times 10^{-1}$	2695.2
$\Omega_c(1S)$	$\frac{3}{2}^+$	$2.76^{+0.11}_{-0.12}$	$0.64^{+0.09}_{-0.08} \times 10^{-1}$	2765.9
$\Omega_c(1P)$	$\frac{1}{2}^-$	$3.02^{+0.12}_{-0.07}$	$0.90^{+0.13}_{-0.10} \times 10^{-1}$	? 3000.4
$\Omega_c(1P)$	$\frac{3}{2}^-$	$3.09^{+0.08}_{-0.06}$	$0.29^{+0.04}_{-0.04} \times 10^{-1}$	? 3090.2
$\Omega_c(2S)$	$\frac{1}{2}^+$	$3.09^{+0.11}_{-0.12}$	$0.82^{+0.09}_{-0.09} \times 10^{-1}$	? 3090.2
$\Omega_c(2S)$	$\frac{3}{2}^+$	$3.12^{+0.12}_{-0.12}$	$0.37^{+0.03}_{-0.04} \times 10^{-1}$	? 3119.1
$\Omega_c(2P)$	$\frac{1}{2}^-$	$3.40^{+0.10}_{-0.10}$	$0.91^{+0.09}_{-0.09} \times 10^{-1}$	
$\Omega_c(2P)$	$\frac{3}{2}^-$	$3.46^{+0.10}_{-0.11}$	$0.27^{+0.04}_{-0.03} \times 10^{-1}$	
$\Omega_c(1P)$	$\frac{5}{2}^-$	$3.11^{+0.10}_{-0.10}$	$1.07^{+0.17}_{-0.17} \times 10^{-1}$ GeV	? 3119.1

? denote possible assignments

1. pole dominance at the hadron side, the pole contributions are about (40–70)%;
2. convergence of the operator product expansion, the dominant contributions come from the perturbative terms;
3. appearance of the Borel platforms;
4. satisfying the energy-scale formula $\mu = \sqrt{M_{\Omega_c}^2 - \mathbb{M}_c^2}$, by try and error, and present the optimal energy scales μ , ideal Borel parameters T^2 , continuum threshold parameters s_0 , pole contributions and perturbative contributions in Table 1. From Table 1, we can see that the criteria 1 and 2 can be satisfied, the two basic criteria of the QCD sum rules can be satisfied, and we expect to make reliable predictions.

We take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states, which are shown explicitly in Table 2. From Table 2, we can see that the criterion 4 can be satisfied. In Figs. 3 and 4, we plot the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states with variations of the Borel parameters T^2 at much larger intervals than the Borel windows shown in Table 1. In the Borel windows, the uncertainties originate from the Borel parameters T^2 are very small, the Borel platforms exist, the criterion 3 can be satisfied. Now the four criteria are all satisfied, and we expect to make reliable predictions. In the Borel windows, the uncertainties of the predicted masses are about (3–5)%, as we obtain the masses from a ratio, see Eqs. (25)–(28), the uncertainties originate from a special parameter in the numerator and denominator cancel out with each other, so the net uncertainties are very small. On the other hand, the uncertainties of the pole residues are about (10–16)%, which are much larger. The uncertainties $\delta\lambda_{\Omega_c}$ are compatible with the uncertainties of the decay constants $f_\pi = 127 \pm 15$ MeV and $f_\rho = 213 \pm 20$ MeV from the QCD sum rules [37].

In Table 2, we also present the experimental values [1, 38]. The present predictions support assigning $\Omega_c(3000)$ to the

1P Ω_c state with $J^P = \frac{1}{2}^-$, assigning $\Omega_c(3090)$ to the 1P Ω_c state with $J^P = \frac{3}{2}^-$ or the 2S Ω_c state with $J^P = \frac{1}{2}^+$, and assigning $\Omega_c(3119)$ to the 2S Ω_c state with $J^P = \frac{3}{2}^+$ (or the 1P Ω_c state with $J^P = \frac{5}{2}^-$ [8]). The present predictions indicate that the 1P Ω_c state with $J^P = \frac{3}{2}^-$ and the 2S Ω_c state with $J^P = \frac{1}{2}^+$ have degenerate masses, it is difficult to distinguish them by the masses alone, we have to study their strong decays. Other predictions can be confronted with the experimental data in the future.

In Refs. [2, 5], Agaev, Azizi and Sundu study the Ω_c states by taking into account the 1S, 1P, 2S states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions, and assign $\Omega_c(3000)$, $\Omega_c(3050)$, $\Omega_c(3066)$ and $\Omega_c(3119)$ to the $(1P, \frac{1}{2}^-)$, $(1P, \frac{3}{2}^-)$, $(2S, \frac{1}{2}^+)$ and $(2S, \frac{3}{2}^+)$ states, respectively. In Ref. [12], Aliev, Bilimis and Savci use the same interpolating currents to study the Ω_c states by taking into account the 1S and 1P states with $J = \frac{1}{2}$ and $\frac{3}{2}$ in the pole contributions, and assign $\Omega_c(3000)$ and $\Omega_c(3066)$ to the $(1P, \frac{1}{2}^-)$ and $(1P, \frac{3}{2}^-)$ states, respectively. In Refs. [2, 5, 12], the contributions of the Ω_c states with positive parity and negative parity are not separated explicitly, there are some contaminations from the 2S or 1P states. In Ref. [8], we separate the contributions of the positive parity and negative parity Ω_c states explicitly, and study the new excited Ω_c states with the QCD sum rules by introducing an explicit P-wave involving the two s quarks. The predictions support assigning $\Omega_c(3050)$, $\Omega_c(3066)$, $\Omega_c(3090)$ and $\Omega_c(3119)$ to the P-wave Ω_c states with $J^P = \frac{1}{2}^-, \frac{3}{2}^-, \frac{3}{2}^-$ and $\frac{5}{2}^-$, respectively. Compared with Refs. [2, 5, 12], the methods used in the present work and Ref. [8] have the advantage that the contributions of the Ω_c states with positive parity and negative parity are separated explicitly, there are no contaminations from the 2S or 1P states.

In the diquark–quark models for the heavy baryon states, the angular momentum between the two light quarks is denoted by L_ρ , while the angular momentum between the

Fig. 3 The masses of the Ω_c states with variations of the Borel parameters T^2 , where A, B, C, D, E, F, G and H correspond to the Ω_c states with the quantum numbers $(1S, \frac{1}{2}^+)$, $(1S, \frac{3}{2}^+)$, $(1P, \frac{1}{2}^-)$, $(1P, \frac{3}{2}^-)$, $(2S, \frac{1}{2}^+)$, $(2S, \frac{3}{2}^+)$, $(2P, \frac{1}{2}^-)$ and $(2P, \frac{3}{2}^-)$, respectively

Fig. 4 The pole residues of the Ω_c states with variations of the Borel parameters T^2 , where A, B, C, D, E, F, G and H correspond to the Ω_c states with the quantum numbers $(1S, \frac{1}{2}^+)$, $(1S, \frac{3}{2}^+)$, $(1P, \frac{1}{2}^-)$, $(1P, \frac{3}{2}^-)$, $(2S, \frac{1}{2}^+)$, $(2S, \frac{3}{2}^+)$, $(2P, \frac{1}{2}^-)$ and $(2P, \frac{3}{2}^-)$, respectively

Table 3 The possible assignments of the new Ω_c states based on the QCD sum rules

	J^P	nL	References
$\Omega_c(3000)$	$\frac{1}{2}^-$	1P	[2, 5, 8, 12, 22] and this work
$\Omega_c(3050)$	$\frac{3}{2}^-$	1P	[2, 5]
$\Omega_c(3050)$	$\frac{1}{2}^-$	1P	[8]
$\Omega_c(3066)$	$\frac{3}{2}^-$	1P	[8, 12]
$\Omega_c(3066)$	$\frac{1}{2}^+$	2S	[2, 5]
$\Omega_c(3090)$	$\frac{3}{2}^-$	1P	[8] and this work
$\Omega_c(3090)$	$\frac{1}{2}^+$	2S	This work
$\Omega_c(3119)$	$\frac{3}{2}^+$	2S	[2, 5] and this work
$\Omega_c(3119)$	$\frac{5}{2}^-$	1P	[8]

light diquark and the heavy quark is denoted by L_λ . In Refs. [2, 5, 12] and present work, the currents with $L_\rho = L_\lambda = 0$ are chosen to explore the P-wave Ω_c states; although the currents couple potentially to the P-wave Ω_c states, we are unable to know the substructures of the P-wave Ω_c states, and we cannot distinguish whether they have $L_\lambda = 1$ or $L_\rho = 1$. In Ref. [8], we choose the currents with $L_\lambda = 1$ to interpolate the Ω_c states, and we obtain the predicted masses (3.06 ± 0.11) GeV and (3.06 ± 0.10) GeV for the $J^P = \frac{3}{2}^-$ Ω_c states with slightly different substructures, which support assigning the $\Omega_c(3066)$ and $\Omega_c(3090)$ to the P-wave Ω_c states with $J^P = \frac{3}{2}^-$ and $L_\lambda = 1$. In the present work, we obtain the mass $3.09^{+0.08}_{-0.06}$ GeV for the $J^P = \frac{3}{2}^-$ Ω_c state. If we take the central values of the predicted masses as references, $\Omega_c(3066)$ and $\Omega_c(3090)$ can be tentatively assigned to the $\frac{3}{2}^-$ Ω_c states with $L_\lambda = 1$ and $L_\rho = 1$, respectively. However, the assignment $\Omega_c(3090) = (2S, \frac{1}{2}^+)$ is also possible according to the predicted mass $3.09^{+0.11}_{-0.12}$ GeV for the $(2S, \frac{1}{2}^+)$ state.

Now we summarize the assignments based on the QCD sum rules in Table 3. From Table 3, we can see that all the calculations based on the QCD sum rules support assigning $\Omega_c(3000)$ to the $1P \frac{1}{2}^-$ state, while the assignments of the other Ω_c states are under debate. We have to study the decay widths to make the assignments on more solid foundation. In Ref. [5], Agaev, Azizi and Sundu study the decays of the Ω_c states to $\Xi_c^+ K^-$ by calculating the hadronic coupling constants $g_{\Omega_c \Xi_c K}$ with the light-cone QCD sum rules, however, they use an oversimplified hadronic representation and neglect the contributions of the excited Ξ_c states.

Experimentally, we can search for those new excited Ω_c states through strong decays and electromagnetic decays to the final states $\Xi_c^+ K^-$, $\Xi_c^0 \bar{K}^0$, $\Xi_c'^+ K^-$, $\Xi_c'^0 \bar{K}^0$, $\Xi_c^{*+} K^-$, $\Xi_c^{*0} \bar{K}^0$, $\Xi^- D^+$, $\Xi^0 D^0$, $\Omega_c(2695)\gamma$, $\Omega_c(2770)\gamma$, and measure the branching fractions precisely, which can shed light

on the nature of those Ω_c states. More theoretical work on the partial decay widths based on the QCD sum rules is still needed.

4 Conclusion

In this article, we distinguish the contributions of the S-wave and P-wave Ω_c states unambiguously, study the masses and pole residues of the 1S, 1P, 2S and 2P Ω_c states with the spin $J = \frac{1}{2}$ and $\frac{3}{2}$ using the QCD sum rules in a consistent way, and we revisit the assignments of the new narrow excited Ω_c states. The present predictions support assigning $\Omega_c(3000)$ to the 1P Ω_c state with $J^P = \frac{1}{2}^-$, assigning $\Omega_c(3090)$ to the 1P Ω_c state with $J^P = \frac{3}{2}^-$ or the 2S Ω_c state with $J^P = \frac{1}{2}^+$, and assigning $\Omega_c(3119)$ to the 2S Ω_c state with $J^P = \frac{3}{2}^+$. The present predictions indicate that the 1P Ω_c state with $J^P = \frac{3}{2}^-$ and the 2S Ω_c state with $J^P = \frac{1}{2}^+$ have degenerate masses, it is difficult to distinguish them by the masses alone, we have to study their strong decays. Other predictions can be confronted with the experimental data in the future.

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