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Quintessential inflation at low reheating temperatures

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Abstract We have tested some simple quintessential inflation models, imposing the requirement that they match with the recent observational data provided by the BICEP and Planck team and leading to a reheating temperature, which is obtained via gravitational particle production after inflation, supporting the nucleosynthesis success. Moreover, for the models coming from supergravity one needs to demand low temperatures in order to avoid problems such as the gravitino overproduction or the gravitational production of moduli fields, which are obtained only when the reheating temperature is due to the production of massless particles with a coupling constant very close to its conformal value.

1 Introduction

Quintessential inflation [1] is a good candidate to unify the early and late time acceleration of our universe. These models are a combination of an inflationary potential, used to explain the early acceleration of the universe, and a quintessential one – which could only be a cosmological constant –, which takes into account the current cosmic acceleration. At early times the inflationary acceleration is the one that dominates and it ceases to be dominant in a phase transition where the universe enters in a kinetic regime [2]. At this moment, particles coupled with gravity are produced to reheat the universe and match with the current hot one. Finally, at very late times, the quintessential potential dominates and the universe starts to accelerate again.

In order to ensure the viability of these models, they are required to fit well with the recent observational data provided by the BICEP and Planck's teams [3,4], but also the reheating temperatures have to be compatible with the nucleosynthesis and baryogenesis bounds, preventing the increase

of entropy due to the decays of gravitational relics such as moduli fields and gravitinos. It is well known that the success of nucleosynthesis constrains the reheating temperature to be between 1 MeV and 10^9 GeV [5–11], whereas in order to overcome the overproduction of very light gravitinos (particles which appear in supersymmetric gravitational theories), i.e., with masses in the range 10^{-3} MeV $\lesssim m_{3/2} \lesssim 10^{-1}$ MeV, it is stated that the reheating temperature has to be less than 10^2 GeV [12]. It is more complicated to avoid the increase of entropy due to the production of moduli fields, which also appears in supergravity models, during the inflationary period; in that case, as it was showed in [13], the reheating temperature has to be less than 1 GeV. However, this low temperature seems to be problematic with baryogenesis, according to which it is commonly assumed that the reheating temperature is at least of the order of the electroweak scale (10^2 GeV). But, fortunately, there are mechanisms that explain the baryon asymmetry at very low temperatures [14]. For all these reasons, we will consider that those supergravity models with a reheating temperature in the MeV regime, i.e., between 1 MeV and 1 GeV, succeed in solving all these problems.

Having this in mind, the main goal of the present work is to study the viability of some well-known inflationary potentials, some of them coming from supersymmetric theories, adapted to quintessence. To do it in a simple way, first of all we consider a universe with a cosmological constant, we choose positive inflationary potentials that vanish at some value of the scalar field and then we extend them to zero for the other values of the field. Hence, we obtain a potential with a phase transition that models an inflationary universe at early times, which reheats the universe via gravitational particle production after inflation and it is finally dominated by the cosmological constant. Once we have these potentials, we calculate their spectral parameters, the number of e-folds, which has to be in quintessential inflation between 63 and 73

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as we show, and its reheating temperature in three cases that will be analytically calculated: via the production of heavy massive particles conformally coupled with gravity, massless nearly conformally coupled particles and particles far from the conformal coupling. This last case has also been studied in [15] in the context of braneworld inflation for exponential and power-law potentials. Note that there is another way to reheat the universe, via the so-called "instant preheating" [13], which has been applied to quintessential inflation in [16–19] for exponential and power-law potentials. This kind of reheating deserves future investigation for different potentials such as the ones studied in this paper, which we will deal with in a future work.

The paper is organized as follows: in Sect. 2 we build a non-singular model based on a universe filled with a barotropic fluid with a non-linear equation of state where the universe starts and finishes in a de Sitter phase. Via the reconstruction method we find the potential that mimics these dynamics when the universe is filled with a scalar field, we find the analytic solution that mimics the fluid model, and we see that it provides spectral quantities such as the spectral index, its running and the tensor to scalar ratio that match with the current observational data. However, this solution appears to be unstable at early times, in the sense that any small perturbation leads to a solution which is singular at early times, in the same way as happens with the Starobinsky model [20,21]. Moreover, with our criterion, the model only supports a reheating via the creation of massless particles nearly conformally coupled with gravity. In Sect. 3, we consider a universe with a small cosmological constant and we adapt three simple inflationary potentials [exponential SUSY inflation potential (ESI), the Higgs inflation potential in Einstein frame (HI), and the power-law inflation potential (PLI)] to quintessence, showing that the HI potential leads to an unacceptable number of e-folds, the ESI potential, which comes from a supersymmetric potential and thus suffers the gravitino and moduli problems, only supports a reheating via the production of massless particles nearly conformally coupled with gravity, and the PLI, which is only viable in the quadratic case, supports the three kind of reheating studied in the work because, since it does not come from any supergravity potential, gravitinos or relic moduli fields do not appear in this theory. In the last section we adapt other well-known potentials [Witten–O'Raifeartaigh inflation (WRI), Kähler moduli inflation I (KMII), open string tachyonic inflation (OSTI), brane inflation (BI) and loop inflation (LI)] coming from inflation, to quintessence and we study its viability. Finally, in the appendix we review from a critical viewpoint the overproduction of gravitational waves in quintessential inflation.

The units used throughout the paper are $\hbar = c = 1$ and, with these units, $M_{\mathrm{pl}} = \frac{1}{\sqrt{8\pi G}}$ is the reduced Planck's mass.

2 A non-singular model

In this section we are going to study inflation coming from fluids [22,23] and to develop the idea of a non-singular universe proposed in [24], where the universe was filled with a barotropic fluid whose non-linear equation of state (EoS), namely $P = P(\rho)$ P being the pressure and ρ the energy density, satisfies $P + \rho = 0$ at two different scales, leading, at early and late times, to two de Sitter eras.

The simplest realization of this model, which could be generalized introducing viscosity [25], is to consider three parameters $H_i \gg H_E \gg H_f$, where H_i and H_f are two fixed points of the dynamical system and H_E is the value of the Hubble parameter when the transition from inflation to kinetic regime is produced. To simplify, we assume that $H_i = M_{\mathrm{pl}}$, i.e., the universe starts, in a de Sitter phase, at Planck scales, and in order to reproduce the current cosmic acceleration we have to take $H_f \sim H_0$. The simplest way to obtain the model is to consider the following differential equation:

$$\dot{H} = \begin{cases} -k(M_{\mathrm{pl}} - H)^2 & \text{for } H \geq H_E, \\ -3(H - H_f)^2 & \text{for } H \leq H_E, \end{cases} \tag{1}$$

where we have chosen $k = \frac{3(H_E - H_f)^2}{(M_{\mathrm{pl}} - H_E)^2} \cong \frac{3H_E^2}{M_{\mathrm{pl}}^2}$ to ensure the continuity of $\dot{H}$. Regarding the value of k, it will be determined in the next section so that our model matches with the current observational data. The dynamical system can be analytically solved leading to the following Hubble parameter:

$$H(t) = \begin{cases} \frac{H_E - kt M_{\mathrm{pl}}(M_{\mathrm{pl}} - H_E)}{1 - kt(M_{\mathrm{pl}} - H_E)} & \text{when } t \leq 0, \\ \frac{H_E + 3H_f t(H_E - H_f)}{1 + 3t(H_E - H_f)} & \text{when } t \geq 0, \end{cases} \tag{2}$$

and the corresponding scale factor is

$$a(t) = \begin{cases} a_E(1 - kt(M_{\mathrm{pl}} - H_E))^{1/k} e^{M_{\mathrm{pl}}t} & \text{when } t \leq 0, \\ a_E(1 + 3t(H_E - H_f))^{1/3} e^{H_f t} & \text{when } t \geq 0. \end{cases} \tag{3}$$

Moreover, the effective EoS parameter, namely w_{eff}, which is defined as $w_{\mathrm{eff}} \equiv \frac{P}{\rho} = -1 - \frac{2\dot{H}}{3H^2}$, for our model is given by

$$w_{\mathrm{eff}} = \begin{cases} -1 + \frac{2k}{3}\left(\frac{M_{\mathrm{pl}}}{H} - 1\right)^2 & \text{when } H \geq H_E, \\ -1 + 2\left(1 - \frac{H_f}{H}\right)^2 & \text{when } H \leq H_E, \end{cases} \tag{4}$$

which shows that for $H \sim M_{\mathrm{pl}}$ one has $w_{\mathrm{eff}}(H) \cong -1$, meaning that we have an early inflationary quasi-de Sitter period. At the phase transition, i.e. when $H \cong H_E$, the EoS parameter satisfies $w_{\mathrm{eff}}(H) \cong 1$ and the universe enters in a kinetic or deflationary period [26,27], and finally, for $H \cong H_f$ one also has $w_{\mathrm{eff}}(H) \cong -1$ depicting the current cosmic acceleration.

2.1 Cosmological perturbations

In order to study the cosmological perturbations, one needs to introduce the slow-roll parameters [28]

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \tag{5}$$

which allow us to calculate the associated inflationary parameters, such as the spectral index (n_s), its running (α_s) and the ratio of tensor to scalar perturbations (r) defined by

$$n_s - 1 = -6\epsilon_* + 2\eta_*, \quad \alpha_s = \frac{H_* \dot{n}_s}{H_*^2 + \dot{H}_*}, \quad r = 16\epsilon_*, \tag{6}$$

where the star ($*$) means that the quantities are evaluated when the pivot scale crosses the Hubble radius. In our case, these quantities become

$$\frac{n_s - 1}{2k} = \frac{1}{H_*^2}(M_{pl} - H_*)(2M_{pl} - H_*),$$
$$\alpha_s = \frac{-2k^2 M_{pl}(M_{pl} - H_*)^2(4M_{pl} - 3H_*)}{H_*^2(H^2 - k(M_{pl} - H)^2)},$$
$$r = 16k\left(\frac{M_{pl}}{H_*} - 1\right)^2. \tag{7}$$

From the theoretical [28] and the observational [29] value of the power spectrum

$$\mathcal{P} \cong \frac{H_*^2}{8\pi^2 \epsilon_* M_{pl}^2} \sim 2 \times 10^{-9}, \tag{8}$$

and the observational value of the spectral index obtained by Planck2015 data, $n_s = 0.968 \pm 0.006$, we can find the value of H_* and k that fits in our model with these data by inserting the value of $k = \frac{10^9 H_*^4}{16\pi^2(M_{pl} - H_*)^2 M_{pl}^2}$ (coming from Eq. (8), using $\epsilon_* = k\left(1 - \frac{M_{pl}}{H_*}\right)^2$ into the first equation in (7). This is equivalent to finding a root to $f(H) = \alpha(M_{pl} - H)M_{pl}^2 - H^2(2M_{pl} - H)$, where $\alpha = 8 \times 10^{-9}\pi^2(1 - n_s) > 0$. Since $f(0) > 0, \lim_{H \to -\infty} f(H) < 0, \lim_{H \to \infty} f(H) > 0$ and $f(M_{pl}) < 0$, from Bolzano's theorem, there is always a unique root H_* such that $0 < H_* < M_{pl}$.

Hence, for the corresponding range of values of n_s, we obtain that $3 \times 10^{-5} < \frac{H_*}{M_{pl}} < 4 \times 10^{-5}$. Thus, it is straightforward that $7 \times 10^{-12} < k < 10^{-11}$ and, by using the equations in (7), one gets $0.1040 < r < 0.1520$ and $-7 \times 10^{-4} < \alpha_s < -3 \times 10^{-4}$. Therefore, some values of n_s in the 1-dimensional marginalized 2σ C.L. ($0.970 < n_s < 0.974$) correspond to a ratio of tensor to scalar perturbations which agrees with the constraint $r < 0.12$, provided by the BICEP and Planck Collaborations [3]. It fulfills, as well, the limits obtained for α_s in Planck2015 [4] ($\alpha_s = -0.003 \pm 0.007$).

Finally, the number of e-folds is given by

$$N = \int_{t_*}^{t_{end}} H \, dt = -\int_{H_{end}}^{H_*} \frac{H}{\dot{H}} dH$$
$$= \frac{1}{k}\left[\ln\left(\frac{M_{pl} - H_*}{M_{pl} - H_{end}}\right) + M_{pl}\left(\frac{1}{M_{pl} - H_*} - \frac{1}{M_{pl} - H_{end}}\right)\right] \tag{9}$$

where (end) stands for the end of inflation, i.e., $\epsilon_{end} = 1$, that is, $H_{end} = \frac{M_{pl}\sqrt{k}}{1 + \sqrt{k}}$. Thus, by using the calculated values of k and H_*, we obtain a number of e-folds satisfying $52 < N < 77$. Now, we are going to compare this value with the one that we will obtain from the following equation [30] in an analogous way to [31]:

$$\frac{k_*}{a_0 H_0} = e^{-N} \frac{H_*}{H_0} \frac{a_{end}}{a_E} \frac{a_E}{a_R} \frac{a_R}{a_M} \frac{a_M}{a_0}$$
$$= e^{-N} \frac{H_*}{H_0} \frac{a_{end}}{a_E} \frac{\rho_R^{-1/12} \rho_M^{1/4}}{\rho_E^{1/6}} \frac{a_M}{a_0}, \tag{10}$$

where R and M symbolize the beginning of radiation era and the beginning of the matter domination era and we have used relations $(a_E/a_R)^6 = \rho_R/\rho_E$ and $(a_R/a_M)^4 = \rho_M/\rho_R$. We use that $H_0 \approx 2 \times 10^{-4} \text{Mpc}^{-1}$ and, as usual, we choose $a_0 = 1$ taking as a physical value of the pivot scale $k_{phys} = 0.02 \text{ Mpc}^{-1}$ (value used by Planck 2015 [4]). Then one sees that, in co-moving coordinates, the pivot scale will be $k_* \equiv a_0 k_{phys} = 0.02 \text{ Mpc}^{-1}$. Moreover, we know that the process after reheating is adiabatic, i.e. $T_0 = \frac{a_M}{a_0} T_M$, and that we have the relations $\rho_M \approx \frac{\pi^2}{15} g_M T_M^4$ and $\rho_R \approx \frac{\pi^2}{30} g_R T_R^4$ (where $\{g_i\}_{i=R,M}$ are the relativistic degrees of freedom [32]). Hence,

$$N = -4.61 + \ln\left(\frac{H_*}{H_0}\right) + \ln\left(\frac{a_{end}}{a_E}\right) + \frac{1}{4}\ln\left(\frac{2g_M}{g_R}\right)$$
$$+ \frac{1}{6}\ln\left(\frac{\rho_R}{\rho_E}\right) + \ln\left(\frac{T_0}{T_R}\right). \tag{11}$$

We use $H_0 \sim 6 \times 10^{-61} M_{pl}$ and, from Eq. (8), we infer that $H_* \sim 4 \times 10^4 \sqrt{\epsilon_*} M_{pl}$. We know as well that $T_0 \sim 2 \times 10^{-13} \text{GeV}$ and $g_M = 3.36$ [32]. Also, $g_R = 107, 90$ and 11 for $T_R \geq 135 \text{ GeV}, 175 \text{ GeV} \geq T_R \geq 200 \text{ MeV}$ and $200 \text{ MeV} \geq T_R \geq 1 \text{ MeV}$, respectively [32]. On the other hand,

$$\ln\left(\frac{a_{end}}{a_E}\right) = \int_{H_E}^{H_{end}} \frac{H}{\dot{H}} dH$$
$$\approx -\frac{1}{k}\left[\frac{1}{1 - \sqrt{k}} - \frac{1}{1 - \sqrt{\frac{k}{3}}} + \ln\left(\frac{1 - \sqrt{k}}{1 - \sqrt{\frac{k}{3}}}\right)\right]$$
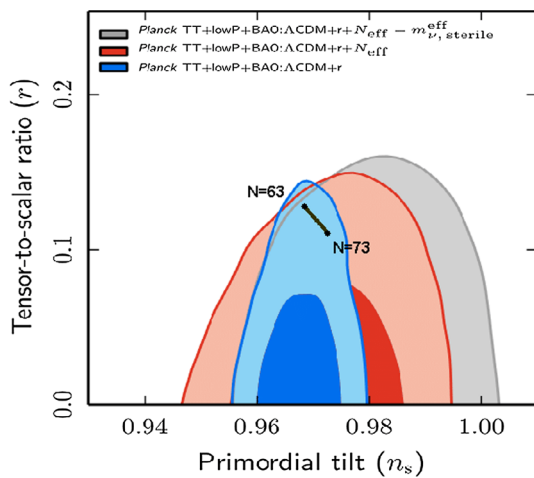$$\cong -0.3. \tag{12}$$

Fig. 1 Marginalized joint confidence contours for (n_s, r), at the 68 and 95% CL, without the presence of running of the spectral indices. We have drawn the curve from $N = 63$ to 73 e-folds. (Figure courtesy of the Planck2015 Collaboration)

Thus, we obtain

$$N \approx 54.5 + \frac{1}{2} \ln \epsilon_* - \frac{1}{3} \ln \left(\frac{g_R^{1/4} T_R H_E}{M_{\text{pl}}^2} \right). \tag{13}$$

Therefore, with the values in our model and with the range $1 \, \text{MeV} \leq T_R \leq 10^9 \, \text{GeV}$ required in order to have a successful nucleosynthesis [33], we find that $63 \lesssim N \lesssim 73$. With Eq. (9), this range is verified in our values for $0.969 < n_s < 0.973$.

In conclusion, with the intersection of the bounds obtained from the constraints $r < 0.12$ and $63 \lesssim N \lesssim 73$, in our model we have $0.970 < n_s < 0.973$ with the corresponding values of $k \sim 8 \times 10^{-12}$ and $H_* \sim 3 \times 10^{-5} M_{\text{pl}}$ (corresponding to $H_E \sim 2 \times 10^{-6} M_{\text{pl}}$), thereby having a number of e-folds of $66 \leq N \leq 73$. Hence, our model satisfies all the values obtained from recent observations. Moreover, one can see in Fig. 1 that our model also provides theoretical values that enter in the marginalized 2σ C.L. contour in the plane (n_s, r).

2.2 The scalar field

In this section we are going to mimic the perfect fluid that fills the FLRW universe with a scalar field. If we represent the energy density and the pressure by the notations ρ_φ, p_φ, respectively, then they assume the following simplest forms:

$$\rho_\varphi = \frac{\dot{\varphi}^2}{2} + V(\varphi), \quad p_\varphi = \frac{\dot{\varphi}^2}{2} - V(\varphi). \tag{14}$$

Now, using Eq. (14) and the Raychaudhuri equation $\dot{H} = -\frac{\dot{\varphi}^2}{2M_{\text{pl}}^2}$, we find

$$\varphi = -M_{\text{pl}} \int \sqrt{-2\dot{H}} \, dt = -M_{\text{pl}} \int \sqrt{-\frac{2}{\dot{H}}} \, dH, \tag{15}$$

which can be analytically solved,

$$\varphi = \begin{cases} M_{\text{pl}} \sqrt{\frac{2}{k}} \ln \left(\frac{M_{\text{pl}} - H}{M_{\text{pl}}} \right) & \text{for } H \geq H_E \\ -M_{\text{pl}} \sqrt{\frac{2}{3}} \ln \left(\frac{H - H_f}{H_E - H_f} \right) + \varphi_E & \text{for } H \leq H_E, \end{cases} \tag{16}$$

where $\varphi_E = M_{\text{pl}} \sqrt{\frac{2}{k}} \ln \left(\frac{M_{\text{pl}} - H_E}{M_{\text{pl}}} \right) \approx -\sqrt{\frac{2}{k}} H_E$.

The potential is given by $V(H) = 3H^2 M_{\text{pl}}^2 + \dot{H} M_{\text{pl}}^2$. Hence,

$$V(\varphi) = \begin{cases} M_{\text{pl}}^4 \left[3 \left(1 - e^{\frac{\varphi}{M_{\text{pl}}} \sqrt{\frac{k}{2}}} \right)^2 - k e^{\frac{\varphi}{M_{\text{pl}}} \sqrt{2k}} \right] & \text{for } \varphi \leq \varphi_E \\ 3 M_{\text{pl}}^2 H_f^2 \left[1 + 2 \left(\frac{H_E}{H_f} - 1 \right) e^{-\sqrt{\frac{3}{2}} \frac{\varphi - \varphi_E}{M_{\text{pl}}}} \right] & \text{for } \varphi \geq \varphi_E. \end{cases} \tag{17}$$

Now, we aim to analyze the stability of the solution to our model. Using the potential found in Eq. (17), we will study the dynamics of the equation:

$$\ddot{\varphi} + 3H(\varphi, \dot{\varphi})\dot{\varphi} + V_\varphi = 0, \tag{18}$$

where $H(\varphi, \dot{\varphi}) = \sqrt{\frac{1}{3} \left(\frac{\dot{\varphi}^2}{2} + V(\varphi) \right)}$.

The numerical results are presented in Fig. 2. For clarity in the understanding of the dynamical system, we have used a value of k which differs in various orders of magnitude from the one in our model. Yet the behavior is exactly the same as with the real values. Therefore we observe that the analytical orbit is the only one with two de Sitter points. All the other orbits start with an infinite energy at $t \to -\infty$, they experience a phase transition at $\varphi = \varphi_E$ and then asymptotically approach the analytical orbit for $t \to \infty$, reaching at $t \to \infty$ the de Sitter point $H = H_f$.

An important remark is in order: From the phase space portrait we can see that there is only one non-stable non-singular solution, i.e., if one takes initial conditions near the analytical solution one obtains a past singular orbit. This is exactly the same that happens in the Starobinsky model [20] (see Figure 3 of [21] for details).

2.3 Reheating constraints

Before studying the reheating constraints in our model, we are going to carry out some useful approximations that will help us in our further calculations. Since $k \cong \frac{10^9 H_*^4}{16\pi^2 M_{\text{pl}}^4}$, we easily obtain $H_E \cong \sqrt{\frac{k}{3}} M_{\text{pl}} \cong \frac{10^4}{4\pi} \sqrt{\frac{10}{3}} \frac{H_*^2}{M_{\text{pl}}^2} M_{\text{pl}}$. On the other
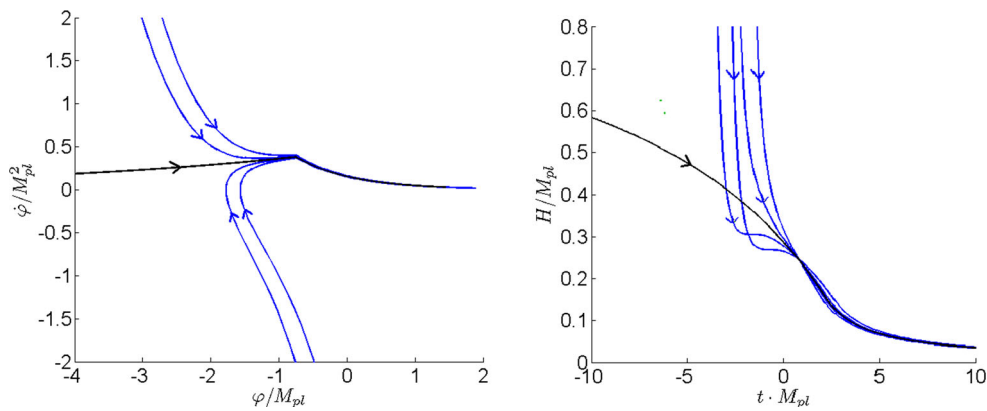
Fig. 2 Phase portrait in the plane $(\varphi/M_{\rm pl}, \dot{\varphi}/M_{\rm pl}^2)$ (left) for some orbits, with the analytical one represented in black. Evolution of $H/M_{\rm pl}$ as a function of the time $t \times M_{\rm pl}$ (right) for the same orbits represented in the phase portrait
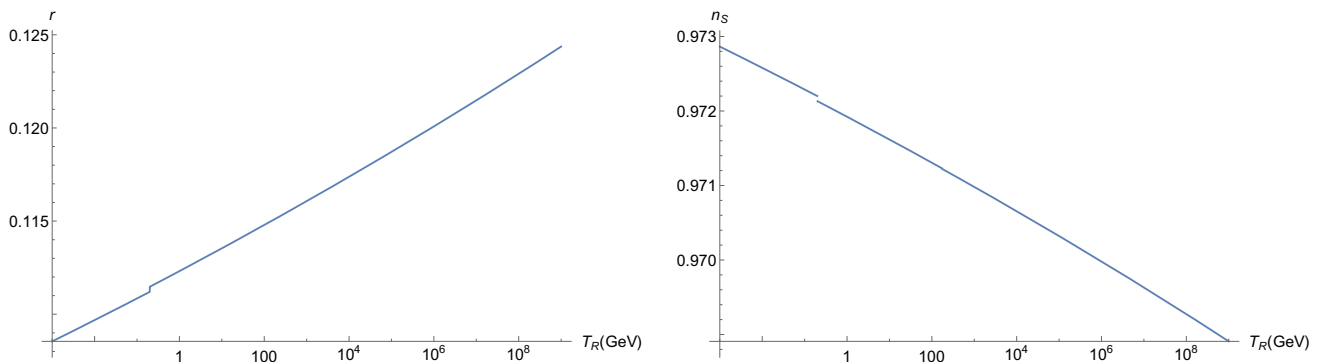

Fig. 3 Evolution of the tensor/scalar ratio r (left) and the spectral index n_s (right) versus the reheating temperature T_R

hand, it is also fulfilled that $0 \cong \alpha M_{\rm pl}^2 - 2H_*^2$ and, thus, we obtain all the following approximative expressions:

$$k \cong 10^{-9}\pi^2(1-n_s)^2, \qquad \alpha \cong 8 \times 10^{-9}\pi^2(1-n_s),$$
$$H_* \cong 6 \times 10^{-5}\pi\sqrt{1-n_s}M_{\rm pl},$$
$$H_{\rm end} \cong 3 \times 10^{-5}\pi(1-n_s)M_{\rm pl}, \qquad (19)$$
$$H_E \cong 2 \times 10^{-5}\pi(1-n_s)M_{\rm pl},$$

and, hence, $r = 16\epsilon_* \cong 16k\frac{M_{\rm pl}^2}{H_*^2} \cong 4(1-n_s)$.

Moreover, the number of e-folds is

$$N \cong \frac{1}{k}\left(\ln\left(\frac{1-\sqrt{\alpha/2}}{1-\sqrt{k}}\right) + \frac{1}{1-\sqrt{\alpha/2}} - \frac{1}{1-\sqrt{k}}\right)$$
$$\cong \frac{\alpha}{4k} - \frac{1}{2} \cong \frac{2}{1-n_s} - \frac{1}{2}, \qquad (20)$$

but using Eq. (13) we also obtain

$$N \cong 71.1 + \frac{1}{6}\ln(1-n_s) - \frac{1}{3}\ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\rm GeV}\right). \qquad (21)$$

Equating both quantities one obtains the equation

$$Y + \frac{1}{2}\ln Y = 217.5 - \ln\left(\frac{g_R^{\frac{1}{4}}T_R}{\rm GeV}\right), \qquad (22)$$

where we have introduced the notation $Y = \frac{6}{1-n_s}$.

Hence, from this equation we obtain the following relation between the cosmological parameters (n_s, r) and T_R, showing that the suitable constraints will be verified for $1\,{\rm MeV} \le T_R \le 10^5\,{\rm GeV}$, as we can see in Fig. 3.

Now, let us consider the gravitational production of χ-particles. Firstly, we treat the case when the produced particles are very massive and conformally coupled with gravity. Since a classical picture of the universe is only possible at energy densities less than the Planck one, it seems natural to choose that at this scale the quantum field χ is in the vacuum. In fact, we will choose as a vacuum state the adiabatic one, defined by the modes [34]

$$\chi_k = \frac{1}{\sqrt{2\Omega_k}}e^{-i\int \Omega_k{\rm d}\eta}, \qquad (23)$$

where Ω_k satisfies the equation

$$\Omega_k^2 = (k^2 + a^2 m^2) + \frac{3\Omega_k'^2}{4\Omega_k^2} - \frac{\Omega_k''}{2\Omega_k}. \tag{24}$$

Then, to obtain an approximate expression of Ω_k, one can use the WKB approximation, which holds when $m \geq H$. For this reason, since at the Planck epoch $H \sim M_{\rm pl}$, in order to obtain the WKB solution one has to assume that the mass of the field, namely m, satisfies $m \geq M_{\rm pl}$. However, since for $m > \sqrt{4\pi} M_{\rm pl}$ the produced particles are micro black holes [35], whose thermodynamical description is unknown [36], one has to choose $m \sim M_{\rm pl}$ so as to avoid this black hole production.

The reheating temperature caused by the decay of these particles into lighter ones, with a thermalization rate – the same used in [5,37] – equal to $\Gamma = \frac{\beta^2}{m} \rho_\chi^{1/2}$ with $\beta^2 \sim 10^{-3}$ [26], is of the order (see for details [31,38])

$$T_R \sim 10^{-1} \left(\frac{H_E}{M_{\rm pl}}\right)^2 \left(\frac{H_E}{m}\right) M_{\rm pl}$$
$$\sim 2 \times 10^{-13} (1 - n_s)^3 \frac{M_{\rm pl}^2}{m}$$
$$\sim 2 \times 10^{-13} (1 - n_s)^3 M_{\rm pl} \tag{25}$$

and, inserting this expression in (22), one obtains

$$Y - \frac{5}{2} \ln Y \cong 197.9, \tag{26}$$

which leads to a spectral index $n_s \cong 0.9716$, a tensor/scalar ratio $r \cong 0.1136$ and a reheating temperature $T_R \sim 11$ GeV. Since this model does not come from supergravity, we do not need to take into account the gravitino and moduli fields problems. Hence, these results lead to a viable model.

On the other hand, when considering massless particles nearly conformally coupled with gravity, the reheating temperature becomes [37,39]

$$T_R \sim \mathcal{N}^{3/4} \left|\xi - \frac{1}{6}\right|^{3/2} \frac{H_E^2}{M_{\rm pl}^2} M_{\rm pl}$$
$$\sim 3 \times 10^{-9} \left|\xi - \frac{1}{6}\right|^{3/2} (1 - n_s)^2 M_{\rm pl} \tag{27}$$

with ξ the coupling constant, $\mathcal{N} = \frac{1}{8\pi^2} \int_0^\infty s |g(s)|^2 ds$, with $g(s) = \frac{1}{H_E^2 a_E^2} \int_{-\infty}^\infty e^{-2is\tau} a^2 \left(\frac{\tau}{H_E}\right) R \left(\frac{\tau}{H_E}\right) \frac{d\tau}{H_E}$, and where R is the scalar curvature. It has been numerically computed that $\mathcal{N} \sim 1$. So as to verify the bounds $1 \, {\rm MeV} \leq T_R \leq 10^5$ GeV (coming from the restriction $r \leq 0.12$ in our model), the coupling constant ξ must satisfy

$$3 \times 10^{-7} \lesssim \left|\xi - \frac{1}{6}\right| \lesssim 6 \times 10^{-2}. \tag{28}$$

If we consider now massless particles far from the conformal coupling with gravity, using the results of [40,41] which

we will discuss in the appendix, the reheating temperature was calculated in [38], leading to

$$T_R \sim 3 \times 10^{-2} \frac{H_E^2}{M_{\rm pl}^2} M_{\rm pl} \sim 10^{-10} (1 - n_s)^2 M_{\rm pl}$$
$$\sim 2 \times 10^5 \, {\rm GeV}. \tag{29}$$

In this case combining Eqs. (22) and (29), one obtains

$$Y - \frac{3}{2} \ln Y \cong 193.5, \tag{30}$$

whose solution is $n_s = 0.970$, corresponding to $r = 0.119$, coinciding thus with the limit where our model starts being valid. Hence, our model supports the presence of these particles.

3 Simple quintessential inflation models

In this section, we are going to proceed in an inverse way from the one we have just done. We are going to take some simple and well-known potentials, some of them with a similar form from the one already studied, and we are going to verify whether we can adjust the parameters so as to fulfill all the corresponding constraints for the cosmological parameters. To simplify, we will consider some well-known positive inflationary potentials that vanish at some value of the scalar field, namely φ_E, and we will extend it to zero for the other values of the field. Moreover, we introduce a cosmological constant $\Lambda \sim 3H_0^2$ (H_0 being the current Hubble cosmological constant) to ensure the current cosmic acceleration.

3.1 Exponential SUSY inflation (ESI)

The first potential we are going to study is an exponential SUSY inflation (ESI) style potential [42,43],

$$V(\varphi) = \begin{cases} \lambda M_{\rm pl}^4 (1 - e^{\frac{\varphi}{M_{\rm pl}}}) & \varphi < 0 \\ 0 & \varphi \geq 0, \end{cases} \tag{31}$$

λ being a dimensionless positive parameter. By using the following approximate expressions of the slow-roll parameters as a function of the potential:

$$\epsilon \approx \frac{M_{\rm pl}^2}{2} \left(\frac{V_\varphi}{V}\right)^2 \quad \eta \approx M_{\rm pl}^2 \frac{V_{\varphi\varphi}}{V}, \tag{32}$$

we obtain $\epsilon_* = \frac{s_*^2}{2}$ and $\eta_* = -s_*$, where $s_* = \frac{e^{\frac{\varphi_*}{M_{\rm pl}}}}{1 - e^{\frac{\varphi_*}{M_{\rm pl}}}} \cong e^{\frac{\varphi_*}{M_{\rm pl}}}$. Hence, we can compute the associated inflationary parameters, analogously as we did in Sect. 3:

$$n_s - 1 = -s_* \left(\frac{3}{2} s_* + 2 \right)$$

$$\alpha_s = -\frac{\sqrt{2\epsilon_*}}{1 - \epsilon_*} \frac{3s_* + 2}{4 \sinh^2 \left(\frac{\varphi_*}{2M_{pl}} \right)} \quad r = 16\epsilon_*, \tag{33}$$

where we have used $\dot{H} = -\frac{\dot{\varphi}^2}{2M_{pl}^2}$ for the calculation of α_s. It is also straightforward to calculate the power spectrum:

$$P \approx \frac{H_*^2}{8\pi^2 \epsilon_* M_{pl}^2} \approx \frac{V(\varphi_*)}{24\pi^2 M_{pl}^4 \epsilon_*} = \frac{\lambda (1 - e^{\frac{\varphi_*}{M_{pl}}})}{12\pi^2 s_*^2}, \tag{34}$$

where we have used $V(\varphi_*) \approx 3M_{pl}^2 H_*^2$. We will guarantee that the value $P \sim 2 \times 10^{-9}$ is verified in our model by taking $\lambda = \frac{24 \times 10^{-9} \pi^2 s_*^2}{1 - e^{\frac{\varphi_*}{M_{pl}}}}$. Finally, regarding the number of e-folds,

$$N = \int_{t_*}^{t_{end}} H \, dt = \frac{1}{M_{pl}} \int_{\varphi_*}^{\varphi_{end}} \frac{1}{\sqrt{2\epsilon}} \, d\varphi$$

$$= e^{-\frac{\varphi_*}{M_{pl}}} - \frac{1 + \sqrt{2}}{\sqrt{2}} + \frac{\varphi_*}{M_{pl}} - \ln \left(\frac{\sqrt{2}}{1 + \sqrt{2}} \right). \tag{35}$$

So as to fit our model with the recent observational values, we will initially take, as before, the spectral index obtained by Planck2015 data, $n_s = 0.968 \pm 0.006$ and we will find the corresponding value φ_* from the first equation in (33), which is $-4.37 \leq \frac{\varphi_*}{M_{pl}} \leq -3.99$. Therefore, the other cosmological parameters are $0.0013 \leq r \leq 0.0028$ and $-7 \times 10^{-4} \leq \alpha_s \leq -3 \times 10^{-4}$. Finally, the corresponding number of e-folds is $49 \lesssim N \lesssim 73$.

Hence, all the values of r and α_s fit to our restrictions and, regarding the number of e-folds, in order to have $63 \lesssim N \lesssim 73$ we need to take the range of $0.970 \leq n_s \leq 0.974$ for the spectral index, obtaining thus $0.0013 < r < 0.0017$ and $-5 \times 10^{-4} < \alpha_s < -3 \times 10^{-4}$. Moreover, the theoretical values provided by our model enters in the marginalized 1σ C.L. contour in the plane (n_s, r) as one can see in Fig. 4.

Since the approximative expression of the cosmological parameters is only valid during the inflation process, we cannot directly calculate the Hubble constant at the transition, namely H_E. As we can verify in the former model where we had the analytical form of the dynamical system, it is of the same order as at the end of the inflation (H_{end}). Since $\epsilon_{end} = 1$ and $\varphi_{end} = \ln \left(\frac{\sqrt{2}}{1 + \sqrt{2}} \right) M_{pl}$ and using $\epsilon_{end} = 1 \implies \dot{H}_{end} = -H_{end}^2$ and $V = 3H^2 M_{pl}^2 + \dot{H} M_{pl}^2$, one can conclude that $H_{end} = \sqrt{\frac{V(\varphi_{end})}{2M_{pl}^2}} = M_{pl} \sqrt{\frac{\lambda}{2(1 + \sqrt{2})}}$, meaning that for our model one has $H_{end} \sim 3 \times 10^{-6} M_{pl}$.

So as to study the reheating constraints for this model, we are going to proceed as in the former case, i.e., with the approximative expressions $r \approx 2(1 - n_s)^2$, $H_E \sim H_{end} \approx \frac{\sqrt{\lambda}}{2} M_{pl} \approx \frac{\pi(1 - n_s)\sqrt{3P}}{2} \approx 10^{-4}(1 - n_s) M_{pl}$ and
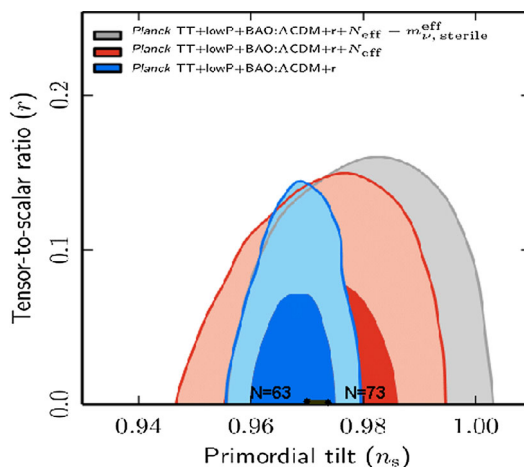

Fig. 4 Marginalized joint confidence contours for (n_s, r), at the 68 and 95% CL, without the presence of running of the spectral indices. We have drawn the curve from $N = 63$ to 73 e-folds. (Figure courtesy of the Planck2015 Collaboration)

$N \approx \frac{2}{1 - n_s} + \ln \left(\frac{1 - n_s}{2} \right)$. Now we cannot reproduce the same calculation as we did for the former case so as to obtain Eq. (13), because we do not have an analytic expression to perform the calculations given in (12), but since this value is negligible we can continue using it.

Therefore, by combining Eq. (13) and the just obtained approximate value of N, we have

$$Y - \ln Y = 212.0 - \ln \left(\frac{g_R^{1/4} T_R}{\text{GeV}} \right), \tag{36}$$

again using $Y = \frac{6}{1 - n_s}$. Hence, from solving this equation, one can see in Fig. 5 that the constraints on r and n_s are fulfilled, as we had already pointed out.

Now, with regards to the case when the produced particles are very massive and conformally coupled with gravity (again choosing $m \sim M_{pl}$ for the same argument as in previous section), we have to proceed as follows:

The square modulus of the β-Bogoliubov coefficient is given by the expression [31]

$$|\beta_k|^2 \cong \frac{m^4 a_E^{10} (\ddot{H}_E^+ - \ddot{H}_E^-)^2}{256(k^2 + m^2 a_E^2)^5}, \tag{37}$$

where a_E is the value of the scale factor at the transition phase, and $\ddot{H}_E^\pm$ are the value of the second derivative of the Hubble parameter after and before the phase transition. Since we do not have an analytic expression of the Hubble parameter, to obtain its second derivative we use the equation $\ddot{H} = -\frac{\dot{\varphi}\ddot{\varphi}}{M_{pl}^2}$. From the conservation equation we deduce that $|\dddot{\varphi}_E^+ - \dddot{\varphi}_E^-| = |V_\varphi(0^-)| = \lambda M_{pl}^3$. On the other hand, at the transition time all the energy density is kinetic, which means

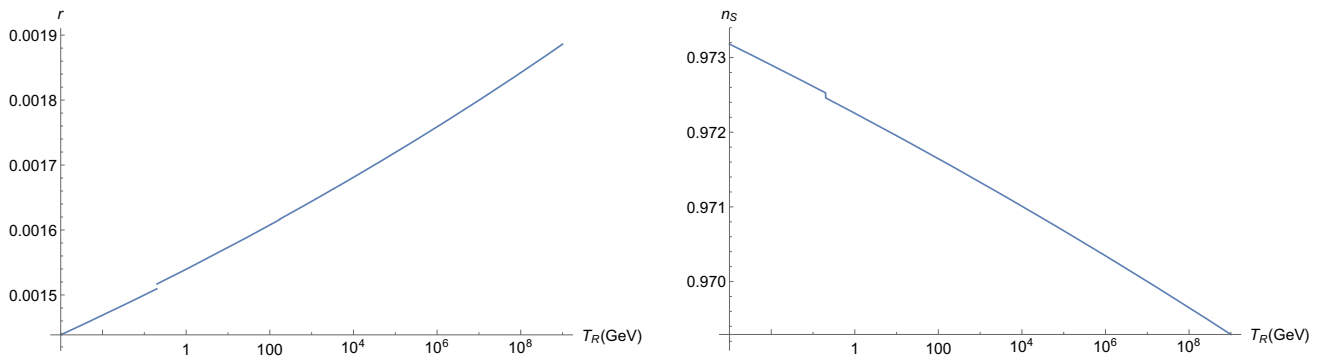**Fig. 5** Evolution of the tensor/scalar ratio r (left) and the spectral index n_s (right) versus the reheating temperature T_R

that $\dot{\varphi}_E = \sqrt{6}H_E M_{\rm pl} \cong \sqrt{6}H_{\rm end} M_{\rm pl} = \sqrt{\frac{3\lambda}{2}} M_{\rm pl}^2$, and thus, $(\ddot{H}_E^+ - \ddot{H}_E^-)^2 \cong \frac{3\lambda^3}{2} M_{\rm pl}^6 \cong 32 H_{\rm end}^6$.

A simple integration leads to an energy density for the produced particles of $\rho_\chi \cong \frac{5 H_{\rm end}^6}{16^3 \pi m^2}$, and following the calculations made in [31] one gets the following reheating temperature:

$$T_R \sim \frac{1}{\sqrt{3}} \left(\frac{5}{16\pi}\right)^{5/8} \left(\frac{H_{\rm end}}{M_{\rm pl}}\right)^2 \frac{H_{\rm end}}{m} M_{\rm pl}$$
$$\sim 10^{-1} \left(\frac{H_{\rm end}}{M_{\rm pl}}\right)^2 \left(\frac{H_{\rm end}}{m}\right) M_{\rm pl}$$
$$\sim 10^{-13}(1 - n_s)^3 M_{\rm pl}, \tag{38}$$

which leads, by combination with (36), to

$$Y - 4 \ln Y \cong 193.1, \tag{39}$$

obtaining that $n_s \cong 0.9720$, $r \cong 1.56 \times 10^{-3}$ and $T_R \sim 5\,{\rm GeV}$. This result again satisfies the gravitino-overproduction problem but not the moduli fields one.

In order to study massless particles nearly conformally coupled with gravity, we will use the same formula as used in (27) and, since we do not have the analytical expression of the Hubble constant throughout all the time, we will assume that $\mathcal{N} \sim 1$ as in the other case. Hence, the reheating temperature is

$$T_R \sim 10^{-8} \left|\xi - \frac{1}{6}\right|^{3/2} (1 - n_s)^2 M_{\rm pl} \tag{40}$$

obtaining, thus, the result that for the bounds of T_R coming from the nucleosynthesis it should be verified that $10^{-7} \lesssim \left|\xi - \frac{1}{6}\right| \lesssim 16$. This only gives us a restriction for the lower bound, since given that we have considered the particles to be nearly conformally coupled with gravity $\left|\xi - \frac{1}{6}\right| \lesssim 10^{-1}$, which adds the constraint that $T_R \leq 7 \times 10^5\,{\rm GeV}$. By considering a reheating temperature in the MeV scale, one would obtain $10^{-7} \lesssim \left|\xi - \frac{1}{6}\right| \lesssim 10^{-5}$. Finally, regarding massless particles far from the conformal coupling with gravity, the reheating temperature becomes

$$T_R \sim 3 \times 10^{-2} \frac{H_E^2}{M_{\rm pl}^2} M_{\rm pl} \sim 3 \times 10^{-10}(1 - n_s)^2 M_{\rm pl}$$
$$\sim 6 \times 10^5\,{\rm GeV}, \tag{41}$$

which leads, using also Eq. (36), to

$$Y - 3 \ln Y \cong 186.9, \tag{42}$$

whose solution is $n_s = 0.970$, corresponding to $r = 0.0018$, which now clearly falls within the bounds imposed in our model so as to satisfy the observational data, but again fails to fulfill the bound $T_R \leq 1\,{\rm GeV}$ suggested in the introduction.

Finally we are going to briefly study the dynamics of the equation $\ddot{\varphi} + 3H(\varphi, \dot{\varphi})\dot{\varphi} + V_\varphi = 0$. In Fig. 6 we have represented in blue some orbits which are solution of these dynamical system, observing that, as in the former case, they start with infinite energy at $t \to -\infty$, they experience a phase transition at $\varphi = 0$ and then evolve towards a de Sitter phase for $t \to \infty$. Moreover, we have drawn in black the slow-roll solution $\dot{\varphi} = -\frac{V_\varphi}{3H} = -\frac{V_\varphi M_{\rm pl}}{\sqrt{3V(\varphi)}} = \sqrt{\frac{2\epsilon(\varphi)V(\varphi)}{3}}$.

We observe that there is a long period in inflation where orbits coincide with the slow-roll one. Furthermore, as happened with the analytical solution in the former case we studied, the slow-roll solution is a limit orbit that starts with a de Sitter phase at $t \to -\infty$, since all the other orbits start with infinite energy.

3.2 Higgs inflation (HI)

Another potential that could work would be the following Higgs inflation (HI) style potential in the Einstein frame [44]:

$$V(\varphi) = \begin{cases} \lambda M_{\rm pl}^4 \left(1 - e^{\frac{\varphi}{M_{\rm pl}}}\right)^2, & \varphi < 0 \\ 0, & \varphi \geq 0, \end{cases} \tag{43}$$

λ being a dimensionless positive parameter. In this case, the slow-roll parameters are $\epsilon_* = 2s_*^2$ and $\eta_* = -s_*(1 - s_*)$, s_* being the same as above. In this case, the running, power spectrum and number of e-folds are
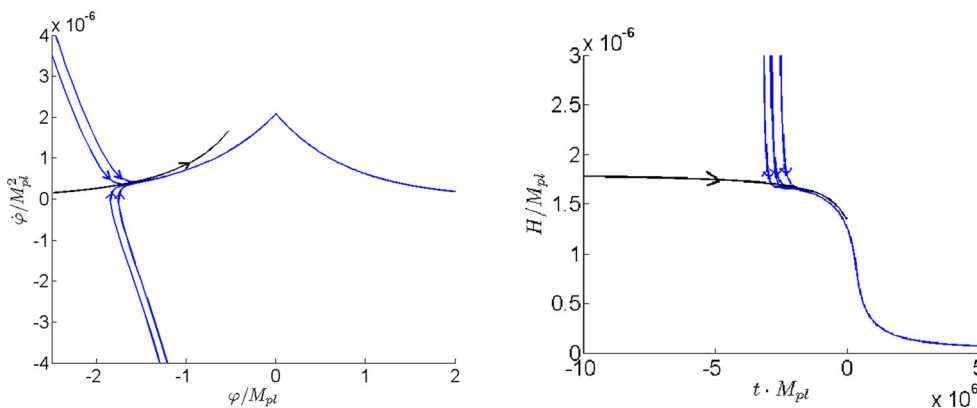
Fig. 6 Phase portrait in the plane $(\varphi/M_{\text{pl}}, \dot{\varphi}/M_{\text{pl}}^2)$ (left) for some orbits, with the slow-roll one represented in black. Evolution of H/M_{pl} in fuction of the time $t \times M_{\text{pl}}$ (right) for the same orbits represented in the phase portrait

$$\alpha_s = -\frac{\sqrt{2\epsilon_*}}{1-\epsilon_*} 2e^{\frac{\varphi_*}{M_{\text{pl}}}} \left(1 + 10(1+s_*)^2\right)$$

$$P \approx \frac{\lambda \left(1 - e^{\frac{\varphi_*}{M_{\text{pl}}}}\right)^2}{12\pi^2 s_*^2}$$

$$N = \int_{t_*}^{t_{\text{end}}} H \, dt = \frac{1}{M_{\text{pl}}} \int_{\varphi_*}^{\varphi_{\text{end}}} \frac{1}{\sqrt{2\epsilon}} d\varphi$$

$$= \frac{e^{-\frac{\varphi_*}{M_{\text{pl}}}} - (1+\sqrt{2})}{2} + \frac{\varphi_*}{2M_{\text{pl}}} - \frac{1}{2} \ln\left(\frac{1}{1+\sqrt{2}}\right). \tag{44}$$

However, in this model now we have $N \sim \frac{1}{s_*} \cong \frac{1}{1-n_s}$, which is bounded by 50 for the allowed values of the spectral index. Thus, the potential is not a viable quintessential inflation model because it leads to an insufficient number of e-folds.

3.3 Power-law inflation (PLI)

Now, we are going to study a power-law inflation (PLI) [45] potential, adapted to quintessence

$$V(\varphi) = \begin{cases} \lambda M_{\text{pl}}^4 \left(\frac{\varphi}{M_{\text{pl}}}\right)^{2n}, & \varphi < 0 \\ 0, & \varphi \geq 0. \end{cases} \tag{45}$$

With the same procedure as in the former cases, we obtain $\epsilon_* = 2n^2 \left(\frac{M_{\text{pl}}}{\varphi_*}\right)^2$ and $\eta = 2n(2n-1)\left(\frac{M_{\text{pl}}}{\varphi}\right)^2$. So, $\varphi_* = -M_{\text{pl}}\sqrt{\frac{4n(n+1)}{1-n_s}}$ and $N = \frac{n+1}{1-n_s} - \frac{n}{2}$. Hence, we can easily verify that, so as to obtain a number of e-folds $63 \leq N \leq 73$ with a spectral index $n_s = 0.968 \pm 0.006$, we need $0.65 < n < 1.35$.

Therefore, taking $n = 1$ is a good choice in order to match our model with the observational results. Thus, if we consider

the range of spectral index $0.9685 \leq n_s \leq 0.973$, we obtain the desired number of e-folds. Regarding the ratio of tensor to scalar perturbations $r = 16\epsilon_* = 4(1 - n_s)$, the constraint $r \leq 0.12$ is verified for $n_s \geq 0.97$. So does the running, which becomes $\alpha_s = -\frac{8\sqrt{2\epsilon_*}}{1-\epsilon_*}\left(\frac{M_{\text{pl}}}{\varphi_*}\right)^2 \approx -16\left|\frac{M_{\text{pl}}}{\varphi_*}\right|^3 = -\frac{(1-n_s)^{3/2}}{\sqrt{2}}$, resulting that $-3.7 \times 10^{-3} \leq \alpha_s \leq -3.1 \times 10^{-3}$ for $0.97 \leq n_s \leq 0.973$. Finally, the power spectrum obeys the following expression:

$$P \approx \frac{\lambda}{48\pi^2}\left(\frac{\varphi_*}{M_{\text{pl}}}\right)^4 = \frac{4\lambda}{3\pi^2(1-n_s)^2} \tag{46}$$

and, thus, since $P \sim 2 \times 10^{-9}$, we can determine λ, namely $\lambda \sim 10^{-11}$.

Remark We can see that the power-law potential for $n = 2$ is equivalent during the inflation, i.e. for $\varphi \ll -|\varphi_E|$ to the double well inflation (DWI) [46] potential, namely

$$V(\varphi) = \begin{cases} \lambda(\varphi^2 - \varphi_E^2)^2, & \varphi < \varphi_E \\ 0, & \varphi \geq \varphi_E. \end{cases} \tag{47}$$

Therefore, we have seen that the DWI is not a valid model that matches with the corresponding observational results.

Now, we are going to proceed analogously as for the potential previously studied. Therefore, we approximate the Hubble constant at the transition point by $H_E \sim H_{\text{end}} = M_{\text{pl}}\sqrt{\lambda} \approx 10^{-4}(1 - n_s)M_{\text{pl}}$. Now, combining Eq. (13) and the number of e-folds for this particular potential, we obtain

$$Y + \frac{1}{2}\ln Y = 213.5 - \ln\left(\frac{g_R^{1/4} T_R}{\text{GeV}}\right), \tag{48}$$

where, as before, $Y = \frac{6}{1-n_s}$. Thus, as we can see in Fig. 7, the bounds coming from the observational values used throughout all this paper are satisfied for $1 \text{ MeV} \leq T_R \leq 10^4 \text{ GeV}$ (corresponding to $0.970 \leq n_s \leq 0.973$).
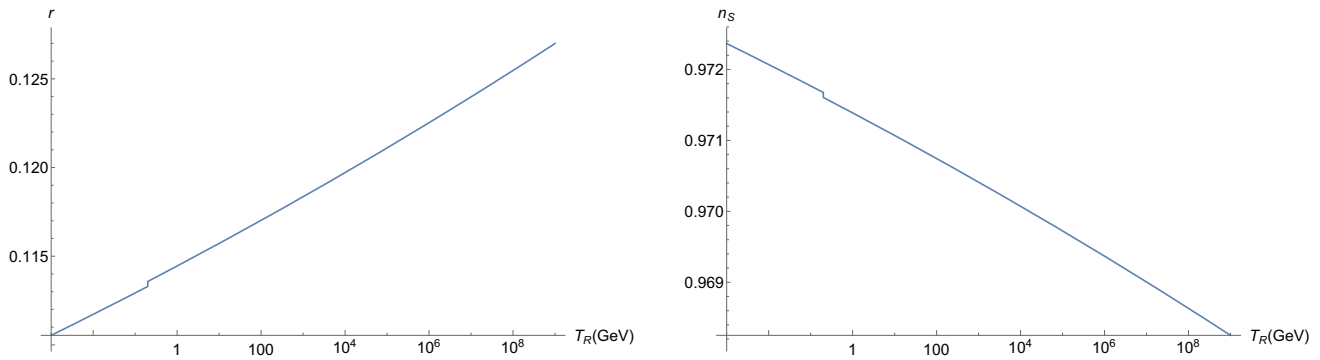
Fig. 7 Evolution of the tensor/scalar ratio r (left) and the spectral index n_s (right) versus the reheating temperature T_R

If we start considering the production of massless particles nearly conformally coupled to gravity, then the reheating temperature becomes

$$T_R \sim \left|\xi - \frac{1}{6}\right|^{3/2} \mathcal{N}^{3/4} \frac{H_E^2}{M_{\mathrm{pl}}}$$
$$\sim 10^{-8} \left|\xi - \frac{1}{6}\right|^{3/2} (1-n_s)^2 M_{\mathrm{pl}}. \tag{49}$$

Hence, for $1\,\mathrm{MeV} \leq T_R \leq 10^4\,\mathrm{GeV}$, we obtain $10^{-7} \lesssim \left|\xi - \frac{1}{6}\right| \lesssim 5 \times 10^{-3}$. On the other hand, in the case of heavy massive particles ($m \sim M_{\mathrm{pl}}$), since the first derivative of the potential is continuous at the transition phase, one has to use, for the β-Bogoliubov coefficient the expression given in [39],

$$|\beta_k|^2 \cong \frac{m^4 a_E^{12} (\dddot{H}_E^+ - \dddot{H}_E^-)^2}{1024(k^2 + m^2 a_E^2)^6}. \tag{50}$$

To obtain the third derivative of the Hubble parameter first of all we use the formula $\ddot{H} = -\frac{\dot{\varphi}^2 + \dot{\varphi}\ddot{\varphi}}{M_{\mathrm{pl}}^2}$. Using the conservation equation one gets $|\dddot{\varphi}_E^+ - \dddot{\varphi}_E^-| = |V_{\varphi\varphi}(0^-)\dot{\varphi}_E| \cong 2\sqrt{6}\lambda H_{\mathrm{end}} M_{\mathrm{pl}}^3$, where we have used the fact that at the transition time all energy density is kinetic. Then we have $(\dddot{H}_E^+ - \dddot{H}_E^-)^2 \cong 144\lambda^2 H_{\mathrm{end}}^4 M_{\mathrm{pl}}^4 = 144 H_{\mathrm{end}}^8$, where we have used $H_{\mathrm{end}} = \sqrt{\lambda} M_{\mathrm{pl}}$. Therefore, the energy density of produced particles is equal to $\rho_\chi \cong \frac{10^{-3}}{\pi} \frac{H_{\mathrm{end}}^8}{m^4}$, thus, following step by step the calculations made in [31] and, using the thermalization rate introduced in Sect. 2.3, one gets the following reheating temperature:

$$T_R \sim 5 \times 10^{-3} \left(\frac{H_{\mathrm{end}}}{M_{\mathrm{pl}}}\right)^2 \left(\frac{H_{\mathrm{end}}}{m}\right)^{9/4} M_{\mathrm{pl}}$$
$$\sim 5 \times 10^{-16} (1-n_s)^{13/4} M_{\mathrm{pl}}. \tag{51}$$

Remark 3.1 Note that the formula of the reheating temperature is a little bit different from the one obtained in [39], since there another thermalization rate has been used.

By combining it with Eq. (48), namely

$$Y - \frac{11}{4} \ln Y \cong 200.0, \tag{52}$$

one obtains that $n_s \cong 0.9721$, $r \cong 0.1117$ and $T_R \sim 11\,\mathrm{MeV}$, which means this potential supports the production of heavy massive particles. If we consider massless particles far from the conformal coupling, then the reheating temperature becomes

$$T_R \sim 3 \times 10^{-2} \frac{H_E^2}{M_{\mathrm{pl}}^2} M_{\mathrm{pl}} \sim 3 \times 10^{-10} (1-n_s)^2 M_{\mathrm{pl}}$$
$$\sim 5 \times 10^5\,\mathrm{GeV}. \tag{53}$$

Therefore, it falls out of the bounds that we have previously found ($1\,\mathrm{MeV} \leq T_R \leq 10^4\,\mathrm{GeV}$) and, thus, our model does not support the production of massless particles far from the conformal coupling.

Regarding the dynamics of the equation $\ddot{\varphi} + 3H(\varphi,\dot{\varphi})\dot{\varphi} + V_\varphi = 0$, we have also proceeded analogously as with the other potential. In Fig. 8 we have represented some blue orbits solution of the dynamical system which start with infinite energy at $t \to -\infty$ and then evolve to the de Sitter phase for $t \to \infty$ after having gone through a phase transition at $\varphi = 0$. In this case, the slow-roll solution (in black) is $\dot{\varphi} = -\frac{V_\varphi}{3H} = M_{\mathrm{pl}}^2 \sqrt{\frac{4\lambda}{3}}$. We note that in this case the slow-roll solution does not start with a de Sitter phase.

We can compare this power-law potential with the one studied in [38], namely

$$V(\varphi) = \begin{cases} \lambda M_{\mathrm{pl}}^2 (\varphi^2 - \varphi_E^2), & \varphi < \varphi_E \neq 0 \\ 0, & \varphi \geq \varphi_E \neq 0. \end{cases} \tag{54}$$

Firstly we note that, for $\varphi \ll \varphi_E$, i.e. during all the inflation, the behavior will be analogous. Regarding the reheating temperature, the upper bound $T_R \leq 10^4\,\mathrm{GeV}$ coincides. It only differs the fact that for heavy massive particles nearly conformally coupled with gravity the reheating temperature is in the GeV regime, instead of the MeV one, for the model coming from the potential in Eq. (54).
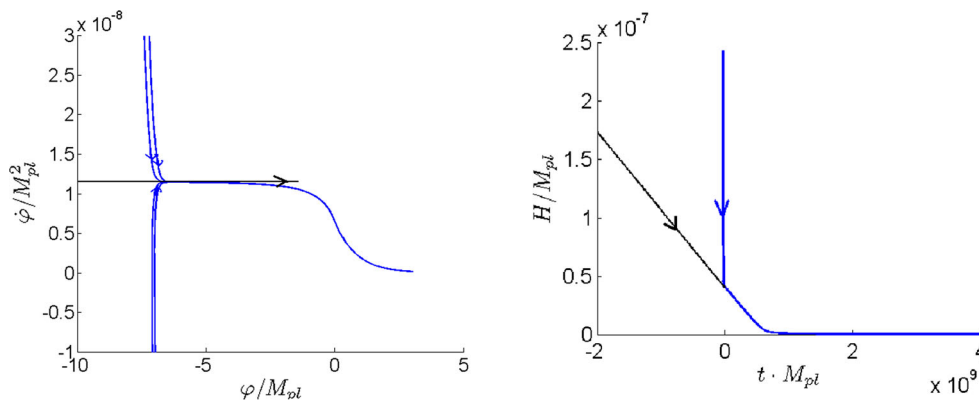
Fig. 8 Phase portrait in the plane $(\varphi/M_{\rm pl}, \dot{\varphi}/M_{\rm pl}^2)$ (left) for some orbits, with the slow-roll one represented in black. Evolution of $H/M_{\rm pl}$ in fuction of the time $t \times M_{\rm pl}$ (right) for the same orbits represented in the phase portrait

4 Other quintessential inflation potentials

In this last section we are going to study some other potentials that appear in [44], studying whether they can be implemented in the quintessential inflation.

4.1 Open string tachyonic inflation (OSTI)

We consider the following adapted form of the OSTI potential [47]:

$$
V(\varphi) = \begin{cases} -\lambda M_{\rm pl}^2 \varphi^2 \ln\left[\left(\frac{\varphi}{\varphi_0}\right)^2\right], & \varphi < 0, \\ 0, & \varphi \geq 0, \end{cases} \tag{55}
$$

where $|\varphi_0| \gg M_{\rm pl}$. We note that this potential is only non-negative for $\varphi > -|\varphi_0|$. Therefore, we will always move through this domain. Now, let us compute the cosmological parameters.

$$
\epsilon_* = 2\left(\frac{M_{\rm pl}}{\varphi_*}\right)^2 \left(\frac{1 + \ln\left[\left(\frac{\varphi_*}{\varphi_0}\right)^2\right]}{\ln\left[\left(\frac{\varphi_*}{\varphi_0}\right)^2\right]}\right)^2
$$

$$
\eta_* = 2\left(\frac{M_{\rm pl}}{\varphi_*}\right)^2 \frac{3 + \ln\left[\left(\frac{\varphi_*}{\varphi_0}\right)^2\right]}{\ln\left[\left(\frac{\varphi_*}{\varphi_0}\right)^2\right]}. \tag{56}
$$

Since ϵ cancels at $\varphi = -e^{-1/2}|\varphi_0|$, the inflation period will happen between this value and $\varphi_{\rm end}$ (corresponding to $\epsilon = 1$). Using that $|\varphi_0| \gg M_{\rm pl}$, we obtain $\varphi_{\rm end} \approx \sqrt{2}M_{\rm pl}$, $\varphi_* \approx \sqrt{\frac{8}{1-n_s}}M_{\rm pl}$ and, hence, $N \approx \frac{2}{1-n_s} - \frac{1}{2}$, $r \approx 4(1-n_s)$ and $\alpha_s = -\frac{(1-n_s)^2}{2}$. Thus, a number of e-folds comprised between 63 and 73 corresponds to a spectral index $0.9685 < n_s < 0.9728$ and a ratio of tensor to scalar perturbations $0.109 < r < 0.126$. Therefore, the constraint

$r \leq 0.12$ restricts this range to $66 \lesssim N \lesssim 73$, corresponding to $0.97 \lesssim n_s \lesssim 0.973$, $0.11 < r < 0.12$ and $-4.5 \times 10^{-4} \lesssim \alpha_s \lesssim -3.6 \times 10^{-4}$. As usual, the power spectrum $P \approx -\frac{4\lambda}{3\pi^2(1-n_s)^2} \ln\left[\left(\frac{M_{\rm pl}}{\varphi_0}\right)^2 \frac{8}{1-n_s}\right]$ is imposed to be $P \approx 2 \times 10^{-9}$ by choosing the suitable value of λ.

Now, we approximate the Hubble constant at the transition point by $H_E \sim H_{\rm end} = \sqrt{\frac{V(\varphi_{\rm end})}{2M_{\rm pl}^2}} \approx 10^{-4}\sqrt{1-n_s}M_{\rm pl}$. The combination of Eq. (13) and the number of e-folds for this potential leads to

$$
Y + \ln Y = 216.2 - \ln\left(\frac{g_R^{1/4}T_R}{\rm GeV}\right), \tag{57}
$$

showing, as in Fig. 9, that the valid bounds of the reheating temperature for this potential are $1\,{\rm MeV} \leq T_R \leq 10^5\,{\rm GeV}$.

As in the case of the power-law potential with $n = 1$, this potential has a continuous derivative at the transition phase. Thus, when considering the production of massless particles nearly conformally coupled to gravity, the reheating temperature is

$$
T_R \sim \left|\xi - \frac{1}{6}\right|^{3/2} \mathcal{N}^{3/4} \frac{H_E^2}{M_{\rm pl}}
$$

$$
\approx 10^{-8} \left|\xi - \frac{1}{6}\right|^{3/2} (1-n_s) M_{\rm pl}. \tag{58}
$$

Thus, for $1\,{\rm MeV} \leq T_R \leq 10^5\,{\rm GeV}$, we obtain $10^{-8} \lesssim \left|\xi - \frac{1}{6}\right| \lesssim 3 \times 10^{-3}$. However, dealing with heavy massive particles $(m \sim M_{\rm pl})$, one can see that the second derivative of the potential, and thus the third derivative of the Hubble parameter, diverges at the transition phase, which means that we cannot use the WKB solution to approximate the modes. Therefore, we are not able to compute, in this case, the reheating temperature. Regarding the case of massless particles far from the conformal coupling, the reheating temperature becomes
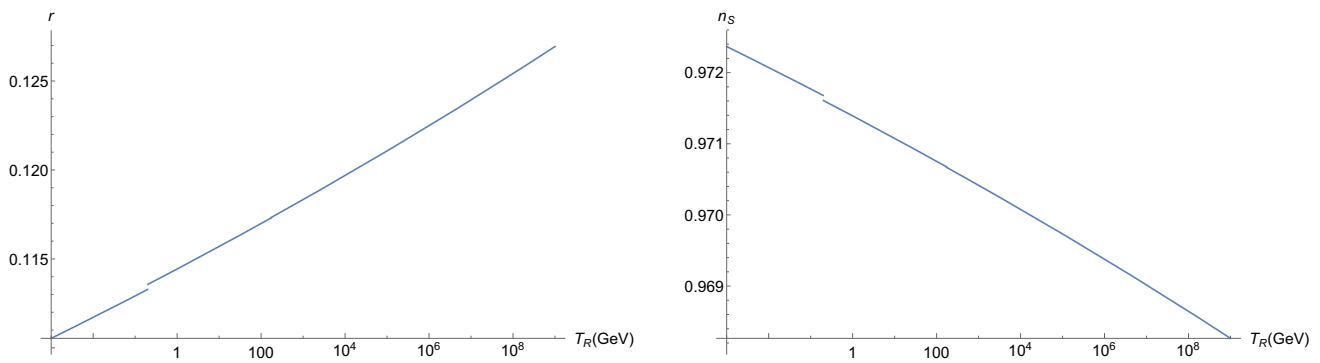
Fig. 9 Evolution of the tensor/scalar ratio r (left) and the spectral index n_s (right) versus the reheating temperature T_R
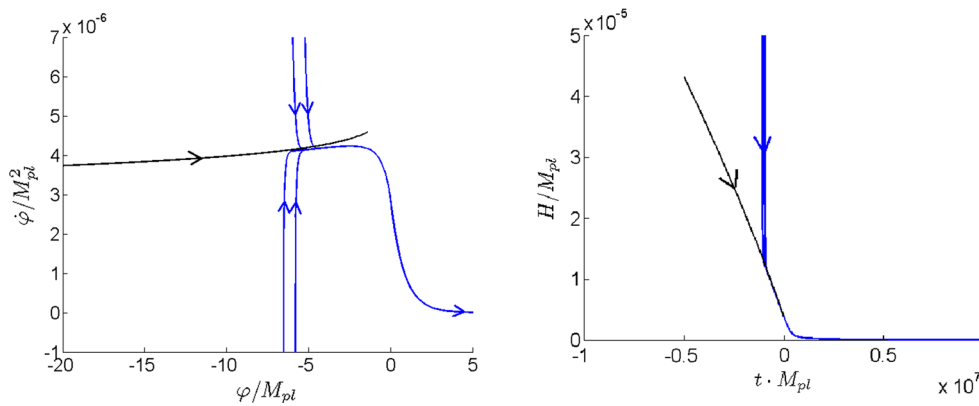

Fig. 10 Phase portrait in the plane $(\varphi/M_{\rm pl}, \dot{\varphi}/M_{\rm pl}^2)$ (left) for some orbits, with the slow-roll one represented in black. Evolution of $H/M_{\rm pl}$ as a function of the time $t \times M_{\rm pl}$ (right) for the same orbits represented in the phase portrait

$$T_R \sim 3 \times 10^{-2} \frac{H_E^2}{M_{\rm pl}^2} M_{\rm pl} \sim 3 \times 10^{-10}(1 - n_s)M_{\rm pl}$$

$$\sim 2 \times 10^7 \text{ GeV}, \tag{59}$$

which shows that this model neither supports the production of these particles.

Finally, as done with the other potentials, we study the dynamics of the equation $\ddot{\varphi} + 3H(\varphi, \dot{\varphi})\dot{\varphi} + V_\varphi = 0$. In Fig. 10 we see again that the blue orbits (solution of the dynamics equation) start with infinite energy at $t \to -\infty$ and then evolve to the de Sitter phase after a phase transition at $\varphi = 0$. As happened with the PLI potential, the slow-roll inflation, namely $\dot{\varphi} = 2M_{\rm pl}^2 \sqrt{\frac{\lambda}{3}} \frac{\left|1 + \ln\left[\left(\frac{\varphi}{\varphi_0}\right)^2\right]\right|}{\sqrt{\left|\ln\left[\left(\frac{\varphi}{\varphi_0}\right)^2\right]\right|}}$, does not start with a de Sitter phase.

4.2 Witten–O'Raifeartaigh inflation (WRI)

In this case, the version of the WRI [48,49] potential is

$$V(\varphi) = \begin{cases} \lambda M_{\rm pl}^4 \ln^2\left(\frac{-\varphi}{|\varphi_E|}\right), & \varphi < -|\varphi_E| \\ 0, & \varphi \geq -|\varphi_E| \end{cases}, \tag{60}$$

where $|\varphi_E| \gg M_{\rm pl}$. The cosmological parameters are

$$\epsilon_* = 2\left(\frac{M_{\rm pl}}{\varphi_* \ln\left(\frac{-\varphi_*}{|\varphi_E|}\right)}\right)^2$$

$$\eta_* = 2\left(\frac{M_{\rm pl}}{\varphi_*}\right)^2 \frac{\ln\left(\frac{-\varphi_*}{|\varphi_E|}\right) - 1}{\ln^2\left(\frac{-\varphi_*}{|\varphi_E|}\right)}. \tag{61}$$

From this equation we have $\varphi_* = -|\varphi_E|e^{\frac{M_{\rm pl}}{\varphi_*}\sqrt{\frac{2}{\epsilon_*}}}$. Since $|\varphi_*| \gg |\varphi_E| \gg M_{\rm pl}$ and $\epsilon_* \sim 10^{-2}$ we have $\left|\frac{M_{\rm pl}}{\varphi_*}\sqrt{\frac{2}{\epsilon_*}}\right| \ll 1$, and thus, $\varphi_* \cong -|\varphi_E|\left(1 + \frac{M_{\rm pl}}{\varphi_*}\sqrt{\frac{2}{\epsilon_*}}\right) \cong -|\varphi_E|$. Therefore, the approximation $\epsilon_* \approx 2\left(\frac{M_{\rm pl}}{|\varphi|_E \ln\left(\frac{-\varphi_*}{|\varphi_E|}\right)}\right)^2 \approx -\eta_*$ is valid and idem for $\epsilon_{\rm end}$. Hence, we get

$$\varphi_* \approx -|\varphi_E|e^{\frac{4M_{\rm pl}}{\sqrt{1-n_s}|\varphi_E|}} \quad \varphi_{\rm end} \approx -|\varphi_E|e^{\frac{\sqrt{2}M_{\rm pl}}{|\varphi_E|}} \tag{62}$$

With regards to the number of e-folds, it can be exactly integrated as $N = \frac{|\varphi_E|^2}{8M_{\rm pl}^2}\left(x^2(2\ln(x) - 1)\right)\big|_{x_*}^{x_{\rm end}}$ (where $x = \frac{-\varphi}{|\varphi_E|}$), whose expression up to order 2 in $\frac{M_{\rm pl}}{|\varphi_E|}$ is $N \approx \frac{4}{1-n_s} - \frac{1}{2}$,

which results in a too high number of e-folds, namely $N \gtrsim 100$.

4.3 Kähler moduli inflation I (KMII)

The expression of the Kähler moduli inflation I (KMII) potential [44] is

$$V(\varphi) = \begin{cases} \lambda M_{\text{pl}}^4 \left(1 - \alpha \frac{\varphi}{M_{\text{pl}}} e^{-\varphi/M_{\text{pl}}} \right), & \varphi > \varphi_E \\ 0, & \varphi \leq \varphi_E \end{cases}, \quad (63)$$

where α is a positive dimensionless constant such that $\alpha \geq e$ and φ_E is the value of φ where $\frac{\varphi}{M_{\text{pl}}} e^{-\varphi/M_{\text{pl}}} = 1/\alpha$ such that $\varphi_E \geq M_{\text{pl}}$. We note that, in contrast to the former potentials considered, in this one we are going to assume that $\dot{\varphi} < 0$. The cosmological parameters are

$$\epsilon = \frac{1}{2} \frac{\alpha^2 (x-1)^2 e^{-2x}}{(1 - \alpha x e^{-x})^2} \quad \eta = \frac{\alpha(2-x) e^{-x}}{1 - \alpha x e^x} \quad (64)$$

where $x = \varphi/M_{\text{pl}}$. By considering that $x_* \gg 1$, we obtain that $n_s - 1 \approx 2\alpha x_* e^{-x_*}$. Regarding the number of e-folds, they can be exactly integrated, namely

$$N = x_{\text{end}} - x_* + \ln \left(\frac{x_{\text{end}} - 1}{x_* - 1} \right) + \frac{e}{\alpha} \left(Ei(x_* - 1) - Ei(x_{\text{end}} - 1) \right), \quad (65)$$

where x_{end} is the value of x where $\epsilon = 1$ and E_i is the exponential integral function, which verifies that, for $x_* \gg 1$, $E_i(x_* - 1) \approx \frac{e^{x_*}}{x_*}$, this being the dominant term in the previous equation. Hence, by using $x_* e^{-x_*} \approx \frac{n_s - 1}{2\alpha}$, one obtains

$$N \approx \frac{2e}{1 - n_s}, \quad (66)$$

which does not fulfill our bounds for the number of e-folds and the spectral index. Therefore, we have proved that this potential is not a viable quintessential inflation model.

4.4 Brane inflation (BI)

The brane inflation (BI) potential behaves as [50]

$$V(\varphi) = \begin{cases} \lambda M_{\text{pl}}^4 \left[1 - \left(\frac{-\varphi}{\mu M_{\text{pl}}} \right)^{-p} \right], & \varphi < \varphi_E \\ 0, & \varphi \geq \varphi_E \end{cases}, \quad (67)$$

where μ and p are positive dimensionless parameters and $\varphi_E \equiv -\mu M_{\text{pl}}$. We will consider for simplicity only $p \in \mathbb{N}$. Thus, the cosmological parameters are

$$\epsilon = \frac{1}{\mu^2} \frac{p^2}{2x^2(1 - x^p)^2} \quad \eta = \frac{p(p+1)}{\mu^2 x^2 (1 - x^p)}, \quad (68)$$

where $x = \frac{-\varphi}{\mu M_{\text{pl}}}$. We are going to differ two cases:

(a) $\mu \ll 1$:

In this case one will have $x_* \gg 1$, and thus $\epsilon_* \cong \frac{p^2}{2\mu^2 x_*^{2(p+1)}}$ and $\eta_* \cong \frac{p(p+1)}{\mu^2 x_*^{p+2}}$, meaning that $n_s - 1 \cong 2\eta_* \cong -\frac{2p(p+1)}{\mu^2 x_*^{p+2}}$. On the other hand, one has

$$N = \frac{\mu^2}{p} \left[\frac{x_*^{p+2}}{p+2} - \frac{x_{\text{end}}^{p+2}}{p+2} - \frac{x_*^2}{2} + \frac{x_{\text{end}}^2}{2} \right].$$

Taking into account that $x_{\text{end}} \cong \left(\frac{p^2}{2\mu^2} \right)^{\frac{1}{2(p+1)}} \ll x_*$, one has $N \cong \frac{\mu^2}{p} \frac{x_*^{p+2}}{p+2}$, meaning that $N \approx \frac{2(p+1)}{(1-n_s)(p+2)}$, which enters in our range for values of p greater than 17. For the tensor/scalar ratio one has $r \cong \frac{8p^2}{\mu^2 x_*^{2(p+1)}} \ll \frac{8p^2}{\mu^2 x_*^{p+2}}$ for $p \geq 1$, namely $r \ll \frac{4p(1-n_s)}{p+1} \leq 4(1 - n_s)$. Hence, one can conclude that for all the values of 2σ CL of the spectral index it is verified that $r < 0.12$. We find as well that $\alpha_s \approx -\frac{(1-n_s)^2}{2}$, which also fulfills the constraints for the running. Finally, by adjusting λ so that $P \sim 2 \times 10^{-9}$, we can build a successful quintessential model.

Effectively, regarding the reheating constraints we see that for all the restricted values of the parameter p, $H_E \sim 10^{-4} \sqrt{1 - n_s} M_{\text{pl}}$. So, as usual, one obtains the result that the reheating temperature bounds from nucleosynthesis give the constraint $0.968 \leq n_s \leq 0.972$. For massive particles we have $T_R \sim 10^3$ GeV. In the case of massless particles nearly conformally coupled with gravity, we obtain $\left| \xi - \frac{1}{6} \right| \gtrsim 10^{-8}$ and the fact that $\left| \xi - \frac{1}{6} \right| \lesssim 1$ should be satisfied constrains our reheating temperature to less than 10^7 GeV. Finally, considering massless particles far from conformal coupling, one finds that $T_R \sim 10^7$ GeV.

(b) $\mu \gg 1$:

In this case by taking for example $\mu \geq 100p$, since ϵ_* or η_* have to be of the order of 10^{-2}, it is verified that $x_* \cong 1$ and, thus,

$$\epsilon_* \cong \frac{1}{\mu^2} \frac{p^2}{2(1 - x_*^p)^2}, \quad \eta_* \cong \frac{p(p+1)}{\mu^2 (1 - x_*^p)}. \quad (69)$$

So, given that $\epsilon_* \gg \eta_*$, $1 - n_s \cong 6\epsilon_* \cong \frac{1}{\mu^2} \frac{3p^2}{(1-x_*^p)^2}$. This means that $x_* \cong 1 - \sqrt{\frac{3}{1-n_s}} \frac{1}{\mu}$, as well as $x_{\text{end}} \cong 1 - \sqrt{\frac{1}{2}} \frac{1}{\mu}$. Consequently, $N \sim \frac{1}{2} \left(\frac{3}{1-n_s} - 1 \right) \sim 50$, which does not enter in our range.

4.5 Loop inflation (LI)

In this case the potential behaves as [51]

$$V(\varphi) = \begin{cases} 0, & \varphi \leq \varphi_E \equiv M_{\rm pl} e^{-\frac{1}{\alpha}} \\ \lambda M_{\rm pl}^4 \left(1 + \alpha \ln \left(\frac{\varphi}{M_{\rm pl}} \right) \right), & \varphi \geq \varphi_E \equiv M_{\rm pl} e^{-\frac{1}{\alpha}}, \end{cases}$$
(70)

where λ and α are positive dimensionless constants. The slow-roll parameters are

$$\epsilon = \frac{1}{2x^2} \frac{\alpha^2}{(1 + \alpha \ln x)^2}, \quad \eta = -\frac{1}{x^2} \frac{\alpha}{(1 + \alpha \ln x)},$$
(71)

where we have introduced the parameter $x \equiv \frac{\varphi}{M_{\rm pl}}$.

We consider to different asymptotic cases:

1. $0 < \alpha \ll 1$:

   In this case one has

   $$\epsilon_* \cong \frac{\alpha^2}{2x_*^2}, \quad \eta_* \cong -\frac{M_{\rm pl}^2 \alpha}{\varphi_*^2},$$
   (72)

   and thus, $n_s - 1 \cong -2\eta_* \cong \frac{2\alpha}{x_*^2}$. For the number of e-folds one has

   $$N \cong \frac{x_*^2}{2\alpha} \cong \frac{1}{1 - n_s},$$
   (73)

   which leads, as in the case of HI, to a not high enough number of e-folds.

2. $\alpha \gg 1$:

   Now the slow-roll parameters become

   $$\epsilon_* = \frac{1}{2x_*^2 \ln^2 x_*}, \quad \eta_* = -\frac{1}{x_*^2 \ln x_*},$$
   (74)

   and the spectral index and the tensor/scalar ratio will be as a function of x_*

   $$1 - n_s = \frac{1}{x_*^2 \ln^2 x_*}(3 + 2 \ln x_*), \quad r = \frac{8}{x_*^2 \ln^2 x_*}.$$
   (75)

   Then, at 2σ C.L., for the allowed values of the spectral index, we can see, after some numerics, that x_* ranges in the domain $6.94 \leq x_* \leq 7.98$. On the other hand, the number of e-folds is

   $$N = \frac{x_*^2}{2} \left(\ln x_* - \frac{1}{2} \right) - \frac{x_{\rm end}^2}{2} \left(\ln x_{\rm end} - \frac{1}{2} \right).$$
   (76)

   Using the range of values for x_* one finds that $34 \lesssim N \lesssim 50$, which results from the nucleosynthesis bounds.

5 Conclusions

We have adapted some inflationary potentials to quintessence inflation, extending them to zero after they vanish and adding a small cosmological constant. Once we have done it, we have tested the models imposing the requirements that: (1) they fit well with the current observational data provided by BICEP and Planck teams. (2) The number of e-folds must range between 63 and 73, this number is larger than the usual one used for potentials with a deep well, due to the kination phase after inflation. (3) The reheating temperature due to the gravitation particle production during the phase transition from inflation to kination has to be compatible with the nucleosynthesis success, i.e., it has to range between 1 MeV and 10^9 GeV, although if one wants to remove the gravitino and moduli problems, which appear in supergravity, this temperature has to be less than 1 GeV.

Our study shows that the potentials WRI and KMII lead to a too high number of e-folds, while for HI and LI potentials this number is too small. Other potentials such as ESI, PLI (only when the potential is quadratic), OSTI and BI satisfy the prescriptions 1 and 2. Dealing with the reheating temperature, all the four models, namely ESI, PLI, OSTI and BI, lead to a temperature compatible with the nucleosynthesis bounds. However, since the ESI potential comes from a supersymmetric gravitational one, the problems related with the gravitino and moduli overproduction will appear, which can only be removed when the reheating temperature is in the MeV regime. Then, for this potential, when reheating is due to the production of massless nearly conformally coupled particles (the coupling constant is very close to 1/6), these problems are removed because the reheating temperature is very low (in the MeV regime). On the contrary, reheating due to the production of very heavy massive particles conformally coupled with gravity or massless particles far from the conformal coupling leads to a too high reheating temperature, which does not avoid the gravitino and moduli problems for the ESI potential.

Fortunately, the other three potentials, namely PLI, OSTI and BI, do not come from supersymmetric potentials and, thus, do not suffer these problems because gravitino and moduli fields only appear in supergravity, meaning that the reheating via very massive particles conformally coupled with gravity or via massless particles far from the conformal coupling is also viable for these three potentials.

Appendix: A critical review of the gravitational wave particle production

Gravitons satisfy the same equation as scalar fields minimally coupled with gravity, so its energy density at the end of inflation is twice that of a single scalar field due to the two graviton polarization states. Experimental observations show that, at the end of inflation, the ratio of the energy density of the relic gravitational waves to the one of the produced particles is less than 10^{-2} [16]. In [37], using the results of [40] and [41], the authors take as the energy density of the gravitons $\rho_g = 2 \times 10^{-2} H_E^4$, where H_E is the value of the Hubble parameter at the transition time, so assuming that there are $\mathcal{N}_s$ different light scalar fields with a coupling constant near zero, one deduces that $\frac{\rho_g}{\rho_\chi} = \frac{2}{\mathcal{N}_s}$, where ρ_χ denotes the energy density of the produced particles. Then, according to the observational bound, this means that the number of different scalar fields must be greater than 100, which only happens in some supersymmetric theories [37].

Note that in this situation the reheating temperature is of the order $10^3 \mathcal{N}_s^{3/4}$ GeV $\sim 3 \times 10^4$ GeV, allowing the overproduction of gravitinos and moduli fields. The problem is even worse when one considers reheating via the production of heavy massive particles because this mechanism is less efficient than the one due to the creation of massless particles. The situation could be addressed when one considers models whose potential does not come from supergravity, which avoids the gravitino and moduli problems, and reheating is produced via oscillations of an auxiliary field in hybrid quintessential models [52] or via instant preheating [15–19] because these kinds of reheating are more efficient than the one due to gravitational particle production.

However, a more detailed investigation shows that the results used in [37] about the production of gravitons may not be correct and the conclusions could be wrong. Here, we summarize the problems that appear in this calculation

1. Gravitational particle production of massless particles far from the conformal coupling is only considered in toy models, where there is a transition from the exact de Sitter phase to a kination or radiation one. This happens because the vacuum modes are represented by Hankel functions and one can perform analytic calculation, but, as is well known, the inflationary period is not exactly de Sitter.

2. In the calculations, only the energy density of the particles is considered and vacuum polarization effects (see [53] for a detailed calculation) are disregarded, but as has been showed in [13] they could be very important in some inflationary scenarios leading to undesirable effects.

3. Performing the calculations, an UV divergence appears but it is removed by disregarding the modes inside the Hubble horizon, because they "do not feel" gravity and, thus, do not produce particles, but in the case of gravitons we obtain an IR divergence which is ignored and produces important consequences. Let us explain it in detail:

When one considers a transition from the de Sitter phase to radiation, in [40] it was showed that the density energy of the produced massless particles coupled with gravity, in the long wave-length limit, i.e., for modes that leave the Hubble radius after the phase transition ($k \leq a_E H_E$, H_E being the Hubble parameter during the de Sitter phase and a_E the scale factor at the transition time), satisfies

$$d\rho_\chi = \frac{\Gamma^2(\nu)}{32\pi^3} \left(\frac{a_E}{a(t)}\right)^4 \left(\nu - \frac{1}{2}\right)^2 \left(\frac{2H_E}{k}\right)^{2\nu+1} k^3 dk, \quad (77)$$

where $\nu^2 = \frac{9}{4} - 12\xi$, ξ being the coupling constant. This quantity is IR divergent for $\nu \geq 3/2$, that is, for $\xi \leq 0$, and in this case, to obtain the energy density of the produced particles, one cannot simply integrate in the domain $[0, a_E H_E]$. One can only do that when $\xi > 0$, obtaining the well-known result $\rho \sim 10^{-2} H_E^4 \left(\frac{a_E}{a(t)}\right)^4$. Thus, the case of gravitons has to be considered in another way, because for them $\xi = 0$. This was done in [54], where the IR singularity is removed assuming an early radiation phase before inflation (see [55]). Then, let a_i be the scale factor when the universe passes from the early radiation phase to inflation. In this situation, if one is only interested in the gravitons produced by the modes that leave the Hubble radius during the de Sitter phase, i.e., in modes satisfying $a_i H_E \leq k \leq a_E H_E$, one obtains [54]

$$\rho_g = \frac{H_E^4}{4\pi^2} \left(\frac{a_E}{a(t)}\right)^4 \ln\left(\frac{a_E}{a_i}\right) = \frac{H_E^4 \mathcal{N}}{4\pi^2} \left(\frac{a_E}{a(t)}\right)^4, \quad (78)$$

where $\mathcal{N} \equiv \ln\left(\frac{a_E}{a_i}\right)$ is the number of e-folds that the de Sitter phase lasts. On the other hand, dealing with massless particles with a small negative coupling constant, one has

$$\rho_\chi = \frac{4^\nu}{16\pi^3(2\nu-3)} \Gamma^2(\nu) \left(\nu - \frac{1}{2}\right)^2 H_E^{2\nu+1} \left(\frac{a_E}{a(t)}\right)^4$$
$$\times \left(e^{(2\nu-3)\mathcal{N}} - 1\right) \cong \frac{e^{(2\nu-3)\mathcal{N}} - 1}{8\pi^2(2\nu-3)} H_E^4 \left(\frac{a_E}{a(t)}\right)^4, \quad (79)$$

and thus

$$\frac{\rho_g}{\rho_\chi} \cong \frac{2(2\nu-3)\mathcal{N}}{e^{(2\nu-3)\mathcal{N}} - 1} \cong \frac{16|\xi|\mathcal{N}}{e^{8|\xi|\mathcal{N}} - 1}, \quad (80)$$

where we have used the fact that for negative small values of the coupling constant $\nu = \frac{3}{2} + 4|\xi|$. Since $\mathcal{N}$ has to be greater than N (the number of e-folds from the leaving of the pivot scale to the end of inflation), one can safely take $\mathcal{N} \geq 100$, then choosing for example $\mathcal{N} = 100$, and $\xi = -0.012$ the bound is reached. So, a unique massless quantum field with a small negative coupling constant satisfies the bound $\frac{\rho_g}{\rho_\chi} \leq 10^{-2}$, meaning that gravitational supersymmetric potentials are not needed and the gravitino and moduli problems do not appear, allowing a greater reheating temperature. In this case, the reheating temperature, taking as usual to obey $H_E \sim 10^{-6} M_{\text{pl}}$, is given by $T_R \sim 2 \times 10^4 \left(\frac{e^{8|\xi|\mathcal{N}}-1}{|\xi|}\right)^{3/4}$ GeV, leading for the values $\mathcal{N} = 100$ and $\xi = -0.012$ to the reheating temperature $T_R \sim 7 \times 10^8$ GeV, which enters in the viable range.

Summing up, the problem of the production of massless particles far from the conformal coupling and in particular the overproduction of gravitational waves in quintessential inflation is a problem that deserves future investigation for many reasons, but basically because nowadays we do not have any analytic method which allows us to calculate, in realistic models, the backreaction of a massless quantum field not conformally coupled with gravity, and only results that are valid for toy models which may not apply to viable models are used to perform analytic calculations.

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