

I in generalized supergravity

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Abstract We showed in previous work that for homogeneous Yang–Baxter (YB) deformations of $\text{AdS}_5 \times S^5$ the open string metric and coupling and as a result the closed string density $e^{-2\Phi} \sqrt{g}$ remain undeformed. In this work, in addition to extending these results to the deformation associated with the modified CYBE or η -deformation, we identify the Page forms as the open string counterpart for RR fields and demonstrate case by case that the non-zero Page forms remain invariant under YB deformations. We give a physical meaning to the Killing vector I of generalized supergravity and show for all YB deformations: (1) I appears as a current for the center of mass motion on the worldvolume of a D-brane probing the background, (2) I is equal to the divergence of the noncommutativity parameter, (3) I exhibits “holographic” behavior where the radial component of I vanishes at the AdS boundary and (4) in pure spinor formalism I is related to a certain state in the BRST cohomology.

1 Introduction

Over recent years we have witnessed an explosion in interest in integrability preserving deformations of $\text{AdS}_5 \times S^5$ superstring, collectively dubbed Yang–Baxter (YB) deformations [1–4] which include T-duality shift T-duality (TsT) transformations [5] as a special case. As a by-product of this breakthrough a large family of supergravity solutions, or geometries, of varying complexity can be defined. In [6, 7] it was observed that within the ensuing geometric madness was a simplifying principle: for homogeneous YB deformations it was shown that in open string frame the metric and string coupling were undeformed and all information about the deformation resided in a noncommutativity (NC) parameter or antisymmetric bivector Θ , thus unlocking a key to the AdS/CFT interpretation (see also [8, 9]). In this letter, noting

the key role played by pure gauge contributions to the NSNS two-form, we extend the results to η -deformations.

Concurrently, significant progress has been made in the Green–Schwarz (GS) formulation of superstring theory. From old work [10] the on-shell condition of type II supergravities ensures the invariance of the GS string action under kappa-symmetry (which is a local fermionic symmetry and necessary to get the right number of physical spacetime fermions). On the other hand, the reverse statement had not been clarified even after more than three decades. In groundbreaking work by Tseytlin and Wulff [11] it has been shown that the kappa-symmetry of the GS string action defined on an arbitrary background leads to *generalized* type II supergravities [12] including an extra Killing vector field I , rather than the usual well-known supergravities. The derivation of generalized supergravity is robust and the result is so striking that the fundamental aspects of superstring theory must be carefully reconsidered. Hence, the study of generalized type II supergravities is a fundamental issue concerned with the formulation of superstring theory. Here we confine our attention to generalized type IIB supergravity. We reproduce the equations of motion (EOM) of the NSNS sector in the appendix.

It is worth making a comment on the Weyl invariance of the string theory when the target space is given by a solution of the generalized supergravity. In the original paper [12] the worldsheet theory was described as non-Weyl invariant. Then in recent work [13], the modified Fradkin–Tseytlin (FT) term was proposed and the Weyl invariance was shown in the doubled formalism where the counter-term is non-local due to the dual coordinate dependence of the dilaton. We recall here that the interpretation of the FT term itself is different from that of the usual counter-term. In particular, it breaks the classical Weyl invariance. However, this issue can be overcome because the FT term should be regarded as a one-loop quantum correction rather than the classical one. With the same logic the non-locality of the modified FT

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term can also be avoided. In fact the extra vector field I is identified with a trace of non-geometric Q -flux as shown in [14]. Hence the solutions of the generalized supergravities are non-geometric. To account for the global property of the non-geometric background the non-locality of the FT term should be rather natural.

In generalized type IIB supergravity itself there are many open questions, including: *What is the physical interpretation of I ?* In order to tackle these issues it is useful to employ the YB deformation because this can be regarded as a solution generation technique in generalized supergravity. Although we appreciate the profound derivation of the generalized type IIB supergravity from the kappa-invariance nowadays it is remarkable that its discovery was a by-product of an enriched understanding of integrable deformations of the $\text{AdS}_5 \times \text{S}^5$ superstring.

The integrable deformation technique that concerns us was invented by Klimcik for the principal chiral models based on the modified classical Yang–Baxter equation (mCYBE) [1,2]. According to this procedure an integrable deformation is specified by taking a classical r -matrix satisfying the mCYBE. Hence this procedure is often called the Yang–Baxter deformation. Klimcik’s work was subsequently generalized to the symmetric coset case [3] and the homogeneous CYBE [4]. Since the classical action of the $\text{AdS}_5 \times \text{S}^5$ superstring [15] has a construction based on the supercoset

$$\frac{PSU(2, 2|4)}{SO(1, 4) \times SO(5)}, \quad (1)$$

the \mathbf{Z}_4 -grading of the superalgebra leads to classical integrability [16]. Thus, the YB deformation is also applicable to the $\text{AdS}_5 \times \text{S}^5$ superstring theory [17,18]. The pioneering work of Delduc, Magro and Vicedo [17], based on the mCYBE, is the standard q -deformation of $\mathfrak{su}(2, 2|4)$ with the classical r -matrix of Drinfeld–Jimbo type. This is often called the η -deformation. After performing a supercoset construction the full background, including the RR sector, was worked out [19], which led to the surprise that the resulting background was not a solution of the usual type IIB supergravity but a generalized variant [12].

To date, a considerable amount of community effort has focused on the largely technical exercise of mapping out the dictionary between r -matrices and geometries, or YB deformations, including connections to (non-Abelian) T-duality [20–32]. Following the latter thread further, the relation between Double Field Theory (DFT) [33–35] was elucidated in [13,14,36,37] and some implications for the AdS/CFT dual gauge theory, including whether planar integrability is preserved [38,39], have been teased out in [6–9,40,41]. For studies of integrability, including classical solutions on the η -deformed background, see [42–53]. In this letter, we return to the physical interpretation of the Killing vector I .

We recall for homogeneous YB deformations that the Killing vector I is related to the divergence of the noncommutativity (NC) parameter, or bivector Θ [6,7],

$$I^N = \nabla_M \Theta^{MN}, \quad (2)$$

where ∇ is the covariant derivative with respect to the open string metric. The validity of this relation has been checked for homogeneous YB deformations on a case by case basis and an outline of its derivation was given in [7]. In this letter we extend this result to the well-known η -deformations [17,19,54,55] and provide a derivation in terms of D3-branes probing the background, where I may be interpreted as a current for the center of mass motion on the D3-brane world-volume.

The conclusion that (2) holds for all YB deformations, including η -deformations, hinges on the observation that Page forms [56], special combinations of RR fluxes and NSNS two-form that appear in the D-brane action, facilitate a simple rewriting of the equations of generalized supergravity and that the Page five-form associated with the D3-brane is invariant, which we have confirmed on a case by case basis. Recalling that the corresponding quantized Page charges are mapped to the ranks of gauge groups in the dual field theory, one arrives at some intuitive understanding for this invariance through AdS/CFT. More generally, we conjecture that the non-zero Page forms are invariants of YB deformations. In addition, through complementary comments on the role of I as a certain state in the BRST cohomology in pure spinor formalism, our work provides I with a more physical flavor.

In the remainder of the letter we study η -deformations. Concretely, we demonstrate that the appropriate inclusion of pure gauge contributions to the NSNS two-form indeed results in an open string metric that is undeformed. For the explicit Arutyunov, Borsato and Frolov (ABF) solution [19,54] we check that the well-known T-duality invariant $e^{-2\Phi} \sqrt{g}$ is also an invariant of the deformation. This extends an earlier observation, originally only valid for homogeneous YB deformations [7], to YB deformations associated with η -deformations. We use this fact and (2) to determine the NSNS sector of η -deformations of $\text{AdS}_5 \times \text{S}^5$ [55],¹ in the process finding that the open string metric is undeformed, in line with our expectations. We check all expressions against the dilaton EOM which we show is satisfied in all cases.

¹ In [55] only the metric and NSNS two-form are computed. The remainder of the solution, including the dilaton and RR sector, can be determined by performing the supercoset construction directly, as explained in [19,28]. Alternatively, it can also be fixed by analytic continuations [29], starting from the original η -deformed background [19,54].

2 Open string frame

In the context of string theory and its related effective field theories, depending on the probes used and the associated degrees of freedom, one may conveniently describe the system in different frames. These frames are related by field redefinitions and carry the name of the associated probe. For example, closed strings probe string frame and particles probe Einstein frame. Within string theory, especially when we have D-brane probes, or when we have spacetimes with a causal boundary like AdS geometry, one may choose to probe spacetime with D-branes or “open strings”. It is well known, especially after the seminal work of Seiberg and Witten [57], that the open string and closed string frames in the NSNS sector are different if we have a Kalb–Ramond B -field in the background. In this section, we first review the open string frame in the NSNS sector and then introduce the open string frame fields for the RR sector.

2.1 Open string frame, NSNS sector

Given a triple (g_{MN}, B_{MN}, Φ) , comprising the closed string metric g , the closed string coupling $g_s e^\Phi$ and the NSNS two-form B , an open string metric G , an antisymmetric bivector Θ and open string coupling G_s can be defined [57],²

$$G_{MN} = \left(g - B g^{-1} B \right)_{MN}, \tag{3}$$

$$\Theta^{MN} = - \left((g + B)^{-1} B (g - B)^{-1} \right)^{MN}, \tag{4}$$

$$G_s = g_s e^\Phi \left(\frac{\det(g + B)}{\det g} \right)^{\frac{1}{2}}. \tag{5}$$

The above are called open string fields because they are the combinations naturally appearing in the DBI part of the brane action describing open strings. Also it can directly be shown that these are the fields appearing in the low-energy effective limit of open string scatterings [57,59].

For r -matrix solutions to the homogeneous CYBE it was noted in [6,7] that following the frame change, the open string metric is simply undeformed AdS₅ with constant open string coupling $G_s = g_s$ and that all information about the YB deformation resides in the bivector Θ . Invariance of the open string metric and coupling under $O(d, d)$ transformations had been observed earlier [60]. In addition the bivector exhibits holographic commutativity as Θ depends on the holographic direction z but all dependence drops out at the boundary $z = 0$. To be more precise, it was noted that $\Theta^{\mu z}|_{z=0} = 0$. At $z = 0$ one is expected to make contact with a NC deformation of $\mathcal{N} = 4$ super Yang–Mills: in support of this correspondence it has been shown case by case that the bivector, evaluated at $z = 0$, agrees with the

² See [58] for an earlier incidence of open string metric and coupling.

Moyal bracket arising from the Drinfeld twist [61] of the conformal algebra. At the level of algebra this establishes an equivalence between homogeneous YB deformations and conformal twists, thereby substantiating an earlier conjecture of van Tongeren [8].

A corollary of these results is the relation [7],

$$\sqrt{\det G} = e^{-2\Phi} \sqrt{\det g}. \tag{6}$$

G being an undeformed AdS₅ × S⁵ metric implies that the density $e^{-2\Phi} \sqrt{\det g}$, in addition to being a well-known invariant of Buscher’s T-duality, is more generally an invariant of YB deformations based on r -matrix solutions to the homogeneous CYBE. We will show later that this extends to η -deformations [17,54,55].

It was shown in [6,7] that for homogeneous YB deformations we always have Eq. (2), thus relating I to the open string bivector Θ . This relation was explicitly checked for an exhaustive set of backgrounds associated with conformal twists. Moreover, we gave a symmetry based argument for (2) but with an incomplete derivation. In this letter we complete the derivation and demonstrate below that (2) as well as the symmetry based argument leading to it remain valid for the mCYBE case associated with η -deformations. In addition we show that an expression for the dilaton can be read off from (6), once one uses the fact that G is undeformed.

2.2 Open string frame, RR sector

In closed string theory we have RR form fields too and one may wonder what are their corresponding fields in the open string frame. Similarly to the NSNS sector one may look again into the D-brane action but now the Chern–Simons or Wess–Zumino part of it which contains RR forms. Here we will conveniently denote this part of a p -brane action by S_{WZ} which, when the field strength of the gauge field on the brane is set to zero, takes the form

$$S_{WZ} = \int_{\Sigma_p} \sum_{n \leq p+1} C_n \wedge e^B, \tag{7}$$

where C_n are n -form RR fields pulled back to the Dp-brane worldvolume Σ_p .

The above suggests that combinations of RR forms and B -field are appropriate for open string frame RR forms. When B is a constant exterior derivative (“field strength”) the above sum is related to a $p + 2$ -form Page form [56]:

$$\begin{aligned} Q_1 &= F_1, & Q_3 &= F_3 + B F_1, & Q_5 &= F_5 + B F_3 + \frac{1}{2} B^2 F_1, \\ Q_7 &= - * F_3 + B F_5 + \frac{1}{2} B^2 F_3 + \frac{1}{3!} B^3 F_1, \\ Q_9 &= * F_1 - B * F_3 + \frac{1}{2} B^2 F_5 + \frac{1}{3!} B^3 F_3 + \frac{1}{4!} B^4 F_1, \end{aligned} \tag{8}$$

where we have omitted wedge products and employed $B^2 = B \wedge B$, etc. Closure of Q_n is guaranteed by the Bianchi/EOMs of the usual type IIB supergravity. We take Page forms (8) as the open string frame fields corresponding to RR forms.

One may view the pull back of the above Page forms on the brane as “Page charge densities” and define Page charges [56] as integrals of them over compact cycles (where the branes wrap). Page charges have been shown to be localized, conserved and most importantly quantized [62], so that they may be identified with the rank of gauge groups in the dual theory.

3 Page forms and generalized supergravity

It is well known that homogeneous YB deformations which are equivalent to (generalized) T-dualities [63–65] result in supergravity solutions, provided an unimodular condition holds [32]. For non-unimodular YB deformations one finds a solution to generalized supergravity [12] where, as explained in the appendix, the EOMs involve an extra vector field I which is a Killing vector of the original and the deformed background. The full set of field equations of generalized supergravity may be found in [11, 12].

Here, we make the observation that the Bianchi and EOMs of generalized supergravity simplify considerably once expressed in terms of Page forms (8). When $I \neq 0$, the equations of generalized supergravity read [12],

$$dF_1 = i_I F_3 + i_I B \wedge F_1, \quad (9)$$

$$dF_3 + H \wedge F_1 = i_I F_5 + i_I B \wedge F_3, \quad (10)$$

$$dF_5 + H \wedge F_3 = -i_I * F_3 + i_I B \wedge F_5, \quad (11)$$

$$d * F_3 - H \wedge F_5 = -i_I * F_1 + i_I B \wedge * F_3, \quad (12)$$

$$d * F_1 - H \wedge * F_3 = i_I B \wedge * F_1. \quad (13)$$

Recast in terms of the Page forms (8), the modified equations take a remarkably simple form

$$\begin{aligned} dQ_1 &= i_I Q_3, & dQ_3 &= i_I Q_5, & dQ_5 &= i_I Q_7, \\ dQ_7 &= i_I Q_9. \end{aligned} \quad (14)$$

We omit dQ_9 due to its length. It is now clear that the Page forms are no longer closed. Nevertheless, the above are a consistent set of equations if I is a Killing vector field.

To get a better feel for the rewriting, let us explicitly demonstrate how the first two equations in (14) come about. A similar logic applies to the later equations. From $i_I F_1 = 0$ which is a property of all generalized supergravity solutions [12] we see that (9) is simply the first equation in (14). Furthermore, combining (9) and (10), we find

$$\begin{aligned} d(F_3 + BF_1) &= i_I F_5 + i_I BF_3 + B(i_I F_3 + i_I BF_1), \\ &= i_I F_5 + i_I (BF_3) + \frac{1}{2} i_I (B^2 F_1) \\ &= i_I Q_5, \end{aligned} \quad (15)$$

which explains the second equation in (14).

It is easy to convince oneself through explicit calculation, either case by case or employing a general ansatz, that the *non-zero* Page forms and associated Page charges are invariants of TsT transformations. For more general YB deformations this can be checked case by case for explicit solutions. In particular, we have checked all examples presented in [31] where $I \neq 0$. This leads to the conjecture

Non-zero Page forms and associated Page charges are invariants of all Yang–Baxter deformations.

The above extends our earlier result for invariance of open string metric and coupling to the RR sector.

At an intuitive level this statement is obvious and expected: in the current setting we start with $\text{AdS}_5 \times S^5$, a geometry sourced exclusively by D3-branes. We perform a special transformation that introduces a continuous constant deformation parameter which is *a priori* not quantized. In this light it is reasonable to expect that the non-zero Page forms and charges are invariant.

It is instructive to illustrate our point in the simplest example of the Hashimoto–Itzhaki, Maldacena–Russo geometry [66, 67] (see also [68]),

$$\begin{aligned} ds^2 &= \frac{(-dt^2 + dx_3^2 + dz^2)}{z^2} + \frac{z^2(dx_1^2 + dx_2^2)}{(z^4 + \eta^2)} + ds^2(S^5), \\ B &= \frac{\eta}{z^4 + \eta^2} dx_1 \wedge dx_2, & e^{2\Phi} &= \frac{z^4}{(z^4 + \eta^2)}, \\ F_3 &= \frac{4\eta}{z^5} dt \wedge dx_3 \wedge dz, \\ F_5 &= (1 + *) \frac{4}{z(z^4 + \eta^2)} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge dz. \end{aligned} \quad (16)$$

A short calculation reveals that the only non-zero Page forms associated with this geometry are

$$Q_3 = \frac{4\eta}{z^5} dt \wedge dx_3 \wedge dz, \quad Q_5 = (1 + *) 4 \text{vol}(AdS_5). \quad (17)$$

The important point from this example is that Q_5 has not changed in the deformation and is simply the original undeformed five-form flux. While we defer the general proof to an upcoming publication, it may be checked by direct computation for an exhaustive list of examples that Q_5 is the undeformed five-form flux for all YB deformations, homogeneous or modified, of $\text{AdS}_5 \times S^5$ (for η -deformations associated with mCYBE this can be checked using the ABF solution [54]). Our conjecture applies equally to YB deformations of different geometries where different Page forms will be invariant.

4 Physical meaning of I

In this section, we arrive at the main point of this letter, namely to address the physical meaning of the Killing vector I of generalized supergravity. Two complementary perspectives will be given: I will be identified as a current on the D-brane worldvolume and its meaning will be elucidated in the pure spinor formalism of the string σ -model. Our treatment improves on earlier analysis [7] by specifying the D-brane, in this case a D3-brane, and illustrating how it couples to the background through a Wess–Zumino term.

4.1 Λ -symmetry and I on the brane

In [6, 7] we argued that (2) may be derived from a symmetry principle, the Λ -symmetry: (generalized) supergravity equations are invariant under the Λ -symmetry,

$$B \rightarrow B + d\Lambda, \tag{18}$$

where Λ is an arbitrary one-form. At the level of generalized gravity this is manifest because the EOMs depend only on the field strength $H = dB$ (see appendix B of [7]). In the presence of open strings or D-branes, however, we have a $U(1)$ gauge field on the brane, A , with field strength $F = dA$. If we have a stack of N coincident D-branes this $U(1)$ symmetry gets enhanced to $U(N)$ [69] and one may decompose it into $SU(N)$ fields and the “center of mass” $U(1)$ part. In the following part we set the $SU(N)$ part of the gauge fields to zero, since it does not mix with the closed string fields. We only focus on this center of mass $U(1)$ part.

In the presence of D-branes the Λ -symmetry (18) is replaced with [69, 70]

$$B \rightarrow B + d\Lambda, \quad A \rightarrow A - \Lambda. \tag{19}$$

Besides H there is another Λ -gauge invariant combination

$$\mathcal{F} = B + F = B + dA, \tag{20}$$

and it can be checked that the DBI part of the brane action only involves \mathcal{F} [71]:

$$S_{DBI} = \int_{\Sigma_p} \frac{e^{-\Phi}}{g_s} \sqrt{\det(g + \mathcal{F})}. \tag{21}$$

We comment in passing that \mathcal{F} and hence the DBI action are invariant under the $U(1)$ center of mass gauge symmetry, the λ -symmetry, under which B remains invariant and $A \rightarrow A + d\lambda$, where λ is an arbitrary scalar function.

The NC description for the DBI action with the open string field Θ as NC parameter, as considered in [57], is only a valid description in the specific Λ -gauge $\Lambda = A$. In this gauge $\mathcal{F} = B$. Hereafter, we will be working in this Λ -gauge. In fact, the closed string-open string frame map (3)–(5), expressed in this gauge [57], yields

$$\frac{1}{G_s} \sqrt{\det G} = \frac{e^{-\Phi}}{g_s} \sqrt{\det(g + B)}. \tag{22}$$

We will prove below that (2) is a consequence of Λ -symmetry, of course once we fix the $\mathcal{F} = B$ gauge. To this end, we first note that M, N indices are bulk AdS₅ indices, while the fields appearing in the brane action are pullbacks on the brane worldvolume. Thus, we may consider a brane that is AdS-filling, for example a D7-brane wrapping three dimensions on the S^5 piece. Alternatively, we may consider a D3-brane on the AdS₅ part and recall that the brane worldvolume is codimension-one on this 5D spacetime.

We shall choose the latter, where it is easy to show that if the D3-brane has no expansion along the radial direction in Poincaré patch of AdS₅, parameterized by z coordinate, we have

$$\begin{aligned} S_{DBI} &= 4 \int_{\text{AdS}_5} \frac{e^{-\Phi}}{g_s} \sqrt{\det(g + B)} \\ &= 4 \int_{\text{AdS}_5} \frac{1}{G_s} \sqrt{\det G}, \end{aligned} \tag{23}$$

where we used (22). In addition we used the facts that open string metric is simply the undeformed AdS₅ metric, G_s is a constant and the factor of 4 has appeared as a result of exterior differentiation of the D3-brane DBI which produces a 5D action from a 4D action. Our treatment here mirrors a similar analysis in [72].

Once again we point out that the action (23) is written in a specific Λ -gauge. A consistency condition of fixing this gauge, recalling (19), implies

$$\delta_\Lambda S_{DBI} = -\delta_A S_{DBI}, \tag{24}$$

and hence the EOM for the gauge field A is related to the Λ invariance of the Λ -gauge fixed action. We shall return to this point momentarily.

The full D3-brane action besides the DBI part also involves a Wess–Zumino part which for a D3-brane in $\mathcal{F} = B$ Λ -gauge may be written

$$S_{WZ} = - \int_{\text{AdS}_5} \left(F_5 + B \wedge F_3 + \frac{1}{2} B^2 \wedge F_1 \right). \tag{25}$$

As a side comment we note that with the factor 4 in the DBI part (23) we get the “no force” condition cancellation of DBI and the WZ parts for flat, undeformed D3-branes.

Now we vary the overall action $S_{DBI} + S_{WZ}$ with respect to Λ . From the variation of the DBI term we get [7]

$$\delta_\Lambda S_{DBI} = -4 \frac{\sqrt{\det G}}{G_s} \nabla_M \Theta^{MN} \delta \Lambda_N, \tag{26}$$

where we have used (22) and $\Theta^{MN} = (g + B)^{[MN]}$, whereas from the variation of S_{WZ} we get

$$\begin{aligned} \delta_\Lambda S_{WZ} &= -(F_3 + B \wedge F_1) \wedge d\delta\Lambda \\ &= -i_I Q_5 \wedge \delta\Lambda, \\ &= -\frac{4}{g_s} i_I \text{vol}(AdS_5) \wedge \delta\Lambda = \frac{4}{g_s} \sqrt{\det G} I^N \delta\Lambda_N. \end{aligned} \tag{27}$$

In the second line we have used (14) and in the third line we have employed

$$Q_5 = \frac{4}{g_s} [\text{vol}(AdS_5) + \text{vol}(S^5)], \tag{28}$$

where we have reinstated g_s . Recalling that $G_s = g_s$, combining the two contributions, we arrive at (2). In other words, (2) is nothing but the EOM for the gauge field A in the $\Lambda = A$ gauge or (2) is the condition for consistency of $\Lambda = A$ gauge fixing. We stress that the λ -symmetry is still unfixed and that our arguments above do not depend on λ -symmetry gauge fixing.³

Given the above result one may ask if it can be written in a different Λ -gauge. The answer is simply yes: if we fix the $B = 0$ gauge, using (24), it can readily be seen that the EOM for the gauge field A of the action

$$S = \int_{\Sigma_4} \frac{1}{G_s} \sqrt{G + F} - A_\mu I^\mu \tag{29}$$

yields the pullback of (2) on the brane worldvolume. The above action is interesting because it provides a new perspective on the vector field I : I is a current coupled to the center of mass $U(1)$.

Our derivation above is true for all YB deformations of $AdS_5 \times S^5$ as long as (28) holds. It can be checked that this is the case by the exhaustive set of examples associated with CYBE discussed in [7]. For the mCYBE case this is shown explicitly in the next section.

4.2 I and pure spinor formulation

In the previous section we gave two meanings to the I vector in the open string frame: we derived (2) and argued that I may be viewed as a current for the center of mass $U(1)$ gauge symmetry that we are left with after fixing the Λ -gauge. In [11] it was suggested that the vector field I may be related to some non-physical states that appear in the pure spinor analysis of supergravity [73–75]. In this section we give additional evidence for this relation.

Type IIB string worldsheet theory can be characterized by the following data [76]: (i) a conformally invariant action

³ We should note that expressions for the Page forms (8) are not Λ -gauge invariant as they are written in a specific Λ -gauge. This Λ -gauge is the same as the one in which we have written our closed string to open string map. Moreover, this is the same gauge in which the Z_M vector field of generalized supergravity (67) is written. We thank Ben Hoare for a clarifying discussion about this point.

S_{IIB} , (ii) a pair of nilpotent BRST operators Q_L and Q_R satisfying

$$Q_L^2 = Q_R^2 = \{Q_L, Q_R\} = 0 \tag{30}$$

and a grading, the *ghost number*, for which $\text{gh}(Q_L) = \text{gh}(Q_R) = +1$. Further (iii), a pair of b -ghosts b and \bar{b} satisfying $\{Q, b\} = T$ and $\{Q, \bar{b}\} = \bar{T}$, where $Q = Q_L + Q_R$. Let us now assume the existence of a theory satisfying these axioms.

Infinitesimal deformations parametrized by the *integrated vertex operators* $V_i^{(2)}$ are defined by

$$S = S_{IIB} + \eta \int V_1^{(2)} + \eta^2 \int V_2^{(2)} + \dots \tag{31}$$

$$Q = Q + \eta Q_1 + \eta^2 Q_2 + \dots$$

For sigma models on $\mathfrak{g} = \mathfrak{psu}(2, 2|4)$ the β -deformations [5] are described by vertex operators

$$V_1^{(2)} = \frac{1}{2} r^{ab} j_a \wedge j_b, \tag{32}$$

where j_a are conserved currents of global symmetries, r^{ab} are components of the tensor $r \in (\mathfrak{g} \wedge \mathfrak{g})_0 / \mathfrak{g}$ and $(\mathfrak{g} \wedge \mathfrak{g})_0$ stands for the subspace of $(\mathfrak{g} \wedge \mathfrak{g})$ generated by objects of the form $a \wedge b$ with $[a, b] = 0$. An important feature in the present analysis is that the second order expansion in η implies that r satisfies the homogeneous CYBE. The simplest example is given by the case when $r \in \mathfrak{su}(4)$. As we know this deformation gives the Lunin–Maldacena background [5] that can be obtained by a TsT transformation. See [73] for further details.

It is easy to write a perturbative expansion similar to the deformations by an integrated vertex (31) for the YB deformed σ -model action. Using the projector $P_-^{\alpha\beta} = \frac{1}{2}(\gamma^{\alpha\beta} - \epsilon^{\alpha\beta})$ where $\gamma_{\alpha\beta}$ is the worldsheet metric, the YB deformations [17, 18] of $AdS_5 \times S^5$ are defined by

$$\mathcal{L} = \frac{(1 + c\eta^2)^2}{2(1 - c\eta^2)} P_-^{\alpha\beta} \text{Str} \left(A_\alpha \circ d \circ \frac{1}{1 - \eta R \circ d} \circ A_\beta \right), \tag{33}$$

where

$$d \equiv P_1 + \frac{2}{1 - c\eta^2} P_2 - P_3, \tag{34}$$

and P_i are projectors onto the \mathbb{Z}_4 grading of the algebra $\mathfrak{psu}(2, 2|4)$, with $c = 0$ for deformations based on the homogeneous CYBE [18] and $c = 1$ for the mCYBE considered in [17].

The perturbative expansion for the case $c = 0$ is obvious: since the perturbation by integrated vertex in the pure spinor formalism (31) is related to the homogeneous CYBE at $\mathcal{O}(\eta^2)$ it is clear that both pictures must be physically equivalent.

In what follows the analysis of the I field refers to deformations corresponding to homogeneous CYBE solutions.

We briefly discuss what is to be expected from the analysis of deformations built from mCYBE solutions. In this case, given an infinitesimal parameter η , we have the expansion

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2} P_-^{\alpha\beta} \text{Str} (A_\alpha \circ d_0 \circ A_\beta) \\ & - \frac{1}{2} \eta P_-^{\alpha\beta} \text{Str} [A_\alpha \circ (d_0 \circ R \circ d_0) \circ A_\beta] \\ & \times \frac{1}{2} \eta^2 P_-^{\alpha\beta} \text{Str} [A_\alpha \circ (d_0 \circ R \circ d_0 \circ R \circ d_0 \\ & + 2P_2 + 3d_0) \circ A_\beta] + \mathcal{O}(\eta^3) \end{aligned} \tag{35}$$

where $d_0 \equiv d|_{\eta=0}$ and the first term is the Metsaev–Tseytlin action [15]. Therefore, based on our previous experience, we may conjecture that there exists a perturbation by a vertex in the pure spinor formalism of the form

$$\tilde{V}_1^{(2)} = \frac{1}{2} \tilde{r}^{ab} j_a \wedge j_b, \tag{36}$$

where \tilde{r} satisfies the mCYBE. All in all we see that these two deformations pictures are consistent with each other, provided this mild conjecture is satisfied.

Let us return to the homogeneous CYBE picture. Consistency with supergravity imposes the additional condition on the vertex operator (32)

$$r^{ab} f_{ab}{}^c = 0, \tag{37}$$

where $f_{ab}{}^c$ are the structure constants of the Lie algebra \mathfrak{g} . The authors of [73] have shown that when this condition is not satisfied there are vertex operators associated with states that we call η -states,⁴ which are not present in the type IIB spectrum.

In [74] Mikhailov considers the flat space limit of the deformation (31) and performs an analysis of the η -states. But since his analysis is in flat space and at first order in the η -expansion there are many problems to be solved for finite transformations and curved backgrounds. There is one particular open question that can be solved using the connection between vertex operators and YB deformations: *Do the η -states survive at all perturbative orders in η in curved backgrounds?*

From the YB deformation viewpoint the condition (37), called *unimodularity condition*, appears in [32] and implies that deformed backgrounds satisfy the type II supergravity equations of motion, if and only if (37) is satisfied. Failure of the unimodularity condition (37) means that these YB deformed backgrounds satisfy the so-called *generalized supergravity equations of motion* that take into account a covector X with components

$$X_M \equiv \partial_M \Phi + (g_{NM} + B_{NM}) I^N, \tag{38}$$

⁴ *Non-physical states* in their terminology.

where I^M is a Killing vector in the generalized type II background. Furthermore, in the special case $I = 0$ we get $X = \partial\Phi$ which can be interpreted as the gradient of the dilaton field Φ . In this case we recover the usual type II supergravity equations of motion.

Therefore, just one of the η -states in [73] survives at all perturbative orders in η . It should correspond to the vector field I in the YB deformation context. In order to make this correspondence more explicit we follow a simple strategy: if we know that type IIB supergravity does not have any vector field in the physical spectrum we should keep track of states that give origin to vectors in the BRST cohomology of [75]. Just one of these vector-like states may be realized as a descendant of a physical type IIB state, namely the gradient of the dilaton $V = \partial\Phi$.

As usual kappa-symmetry plays the crucial role restricting the extra degrees of freedom in a supergravity theory, including the generalized supergravity in the absence of Weyl invariance of the worldsheet theory. In pure spinor formalism the correct number of degrees of freedom is carried and imposed through BRST charges. Hence, one would expect to find a counterpart of I through BRST cohomology analysis.

The BRST cohomology in the flat space limit⁵ and at order η has been performed in [75]. In our analysis the essential states in this cohomology are the covectors A^+ and A^- which satisfy the conditions

$$\begin{aligned} \partial_{[M} A_{N]}^+ &= 0, \\ \partial_{(M} A_{N)}^- &= 0. \end{aligned} \tag{39}$$

The vector field A^+ has been identified as the gradient of the dilaton

$$A_M^+ := \partial_M \Phi. \tag{40}$$

The state A^- is clearly an η -state, as there is no additional vector field in the type IIB string spectrum. Furthermore, through some mild assumptions, it is reasonable to conjecture that this state is linearly related to the vector I , that is $I \sim A^-$.

In the undeformed limit $\eta = 0$ we recover a type IIB supergravity solution where $I = 0$. Therefore, the perturbative expansion of this field must be of the form $I = \eta I^{(1)} + \eta^2 I^{(2)} + \mathcal{O}(\eta^3)$. In addition we already know that the vector field I is a Killing vector in the generalized supergravity solution and that the flat space limit of the Killing vector equation is simply

$$\begin{aligned} \nabla_M I_N + \nabla_N I_M = 0 & \xrightarrow{\text{flat space}} \partial_M I_N \\ + \partial_N I_M = 0, & \end{aligned} \tag{41}$$

⁵ This corresponds to a Taylor expansion of the $\text{AdS}_5 \times S_5$ solution around a point x_* [74].

which is, obviously, the second equation in (39). Therefore it is easy to see that the η -state A^- corresponds to the first order component of I in its perturbative expansion

$$A_M^- = I_M^{(1)}. \tag{42}$$

Using this perturbative analysis it can be conjectured that in a complete BRST analysis all states of η -states combine to give the Killing vector I in the Yang–Baxter deformation context. Additionally all other η -states in the BRST cohomology are obstructed by higher orders corrections and are, using the terminology of [73], truly *non-physical states*. Finally, Eqs. (41) and (42) realize explicitly the observation of [75] that η -states are in correspondence with global symmetries of the background. Evidently, the unarguable proof for this identification requires methods to calculate the complete BRST cohomology for finite deformations in the pure spinor formalism.

5 η -deformations and mCYBE

In previous work [6,7] we have largely discussed YB deformations based on r -matrix solutions to the homogeneous CYBE. For the analysis of the bulk D3-brane action we were ambivalent to whether the r -matrix solved the homogeneous CYBE or mCYBE. In principle we could check Q_5 for all the known η -deformations [55] but apart from the ABF solution [19,54], where Q_5 coincides with the undeformed five-form flux, the explicit form of the RR fluxes are not known. Thus, here we will adopt a different tack and demonstrate that Eq.(2) is valid for all known η -deformations. We will see that pure gauge contributions to the NSNS two-form play a special role.

To be clear, our approach is to take g and B which can be read off from the string σ -model [54,55], determine Θ and then use Eq. (2) to work out I , which we will directly compare with the Killing vector from the generalized supergravity solution. For the YB deformations identified by Delduc et al. [55] we will determine both I and Φ and check that the dilaton EOM [12],

$$R - \frac{1}{12} H_{MNP} H^{MNP} + 4\nabla_M X^M - 4X_M X^M = 0, \tag{43}$$

with X defined in (38), is satisfied. This provides a powerful consistency check on our results.

We recall the η -deformed geometry for $AdS_5 \times S^5$ [54],

$$\begin{aligned} ds^2 = & -\frac{(1 + \rho^2) dt^2}{1 - \kappa^2 \rho^2} + \frac{d\rho^2}{(1 + \rho^2)(1 - \kappa^2 \rho^2)} \\ & + \frac{\rho^2 d\zeta^2}{1 + \kappa^2 \rho^4 \sin^2 \zeta} + \frac{\rho^2 \cos^2 \zeta d\psi_1^2}{1 + \kappa^2 \rho^4 \sin^2 \zeta} \\ & + \rho^2 \sin^2 \zeta d\psi_2^2 + \frac{(1 - r^2) d\phi^2}{1 + \kappa^2 r^2} \end{aligned}$$

$$\begin{aligned} & + \frac{dr^2}{(1 - r^2)(1 + \kappa^2 r^2)} + \frac{r^2 d\xi^2}{1 + \kappa^2 r^4 \sin^2 \xi} \\ & + \frac{r^2 \cos^2 \xi d\phi_1^2}{1 + \kappa^2 r^4 \sin^2 \xi} + r^2 \sin^2 \xi d\phi_2^2, \end{aligned}$$

$$\begin{aligned} B = & -\frac{\kappa \rho^4 \sin 2\zeta}{2(1 + \kappa^2 \rho^4 \sin^2 \zeta)} d\zeta \wedge d\psi_1 + \frac{\kappa \rho}{1 - \kappa^2 \rho^2} dt \wedge d\rho \\ & - \frac{\kappa r^4 \sin 2\xi}{2(1 + \kappa^2 r^4 \sin^2 \xi)} d\phi_1 \wedge d\xi \\ & + \frac{\kappa r}{1 + \kappa^2 r^2} d\phi \wedge dr. \end{aligned} \tag{44}$$

Relative to its original incarnation we have retained the pure gauge terms in the B -field (see also [55]). It turns out that these terms are important for the open string metric to be undeformed. It is worth noting that there is a simple analytic continuation that takes the AdS_5 deformation to the S^5 deformation. Interestingly, just as for λ -deformations [77,78], it can be shown that the dilaton equation forces the same deformation parameter in AdS_5 and S^5 . Since η -deformations correspond to scaling limits of λ -deformations [79] this is not too surprising.

From (4), it is an easy exercise to determine Θ :

$$\begin{aligned} \Theta^\zeta \psi_1 &= \kappa \tan \zeta, & \Theta^{t\rho} &= \kappa \rho, \\ \Theta^{\xi \phi_1} &= -\kappa \tan \xi, & \Theta^{\phi r} &= -\kappa r, \end{aligned} \tag{45}$$

and I using (2),

$$I = \kappa(-4\partial_t + 2\partial_{\psi_1} + 4\partial_\phi - 2\partial_{\phi_1}), \tag{46}$$

which precisely agrees with the ABF result [19]. In contrast to ABF, the pure gauge terms in the B -field mean the dilaton is different, though X of course is the same.

To determine the dilaton we will now introduce a simple trick that has been largely overlooked. It is well-known that $e^{-2\Phi} \sqrt{g}$ is an invariant of the Buscher rules of T-duality. Existing results extend this to homogeneous YB deformations [7] and, having verified that the open string metric is undeformed for the ABF solution, it follows that this is also an invariant of η -deformations. This allows us to easily determine the dilaton for the ABF solution (44),

$$\begin{aligned} e^{-2\Phi} = & (1 - \kappa^2 \rho^2)(1 + \kappa^2 \rho^4 \sin^2 \zeta) \\ & \times (1 + \kappa^2 r^2)(1 + \kappa^2 r^4 \sin^2 \xi). \end{aligned}$$

For consistency we have checked the dilaton Eq. (43).

Emboldened by this success we proceed to analyze the η -deformations identified in [55]. Note that for these geometries the corresponding I , Φ and RR sector are not known. We proceed as before: we check that open string metric is undeformed in the presence of pure gauge contributions,⁶ determine the dilaton from the invariant and deduce the Killing

⁶ This allows us to identify and correct typos in [55].

vector from (2). In each case, we will show that the picture is complete by checking the dilaton EOM. One special feature of the η -deformations [55] is that the r -matrix is a permutation of the ABF r -matrix and only the AdS₅ part of the deformation differs from the ABF solution.

Focusing on the AdS₅ deformation, which is new relative to ABF, the first η -deformation may be written [55]

$$\begin{aligned}
 ds^2 = & -\frac{(1 + \rho^2)}{s} dt^2 \\
 & + \frac{1 + \kappa^2 \sin^2 \zeta - \kappa^2 \rho^2 (1 + \rho^2) \cos^2 \zeta \sin^2 \zeta}{(1 + \rho^2) f s} d\rho^2 \\
 & + \frac{\rho^2 (1 + \kappa^2 (1 + \rho^2) \cos^2 \zeta - \kappa^2 \rho^2 \sin^4 \zeta)}{f s} d\zeta^2 \\
 & + \frac{2\kappa^2 \rho (1 + \rho^2 \sin^2 \zeta) \cos \zeta \sin \zeta}{f s} d\rho d\zeta \\
 & + \frac{\rho^2 \cos^2 \zeta}{f} d\psi_1^2 + \rho^2 \sin^2 \zeta d\psi_2^2, \\
 B = & -\frac{1}{2\kappa} d \log f \wedge d\psi_1 - \frac{\kappa}{s} \left(\rho \sin^2 \zeta d\rho + \rho^2 \right. \\
 & \left. \times (1 + \rho^2) \sin \zeta \cos \zeta d\zeta \right) \wedge dt, \tag{47}
 \end{aligned}$$

where we have introduced the functions

$$\begin{aligned}
 f(\rho, \zeta) &= 1 + \kappa^2 + \kappa^2 \rho^2 \cos^2 \zeta, \\
 s(\rho, \zeta) &= 1 - \kappa^2 \rho^2 (1 + \rho^2 \cos^2 \zeta) \sin^2 \zeta, \\
 h(\rho, \zeta) &= 1 + \kappa^2 (1 + \rho^2) + \kappa^2 \rho^2 (1 + \rho^2) \cos^2 \zeta. \tag{48}
 \end{aligned}$$

The open string metric is undeformed as expected and Θ takes the form

$$\begin{aligned}
 \Theta^{t\rho} &= \kappa \rho \sin^2 \zeta, \quad \Theta^{t\zeta} = \kappa \cos \zeta \sin \zeta, \\
 \Theta^{\rho\psi_1} &= \kappa \left(\rho^{-1} + \rho \right), \quad \Theta^{\zeta\psi_1} = -\kappa \rho^{-2} \tan \zeta. \tag{49}
 \end{aligned}$$

Including the contribution from the internal space, the Killing vector may be determined from (2),

$$I = \kappa (-2\partial_t + 4\partial_{\psi_1} + 4\partial_\phi - 2\partial_{\phi_1}). \tag{50}$$

Finally, the dilaton can be calculated from the invariant $e^{-2\Phi} \sqrt{g}$:

$$e^{-2\Phi} = f s \left(1 + \kappa^2 r^2 \right) \left(1 + \kappa^2 r^4 \sin^2 \xi \right). \tag{51}$$

To confirm that our treatment is correct, we have checked that the dilaton EOM (43) is satisfied.

Finally, we consider the remaining η -deformation [55]:

$$\begin{aligned}
 ds^2 = & -\left(1 + \rho^2 \right) dt^2 + \frac{1 + \kappa^2 + \kappa^2 \rho^2 (2 + \rho^2) \cos^2 \zeta}{(1 + \rho^2) f h} d\rho^2 \\
 & + \frac{\rho^2 (1 + \kappa^2 (1 + \rho^2))}{f h} d\zeta^2 - \frac{\kappa^2 \rho^3 \sin(2\zeta)}{f h} d\rho d\zeta \\
 & + \frac{\rho^2 \cos^2 \zeta}{f} d\psi_1^2 + \frac{\rho^2 \sin^2 \zeta}{h} d\psi_2^2,
 \end{aligned}$$

$$\begin{aligned}
 B = & -\frac{1}{2\kappa} d \log f \wedge d\psi_1 - \frac{\kappa}{h} \left(\rho \sin^2 \zeta d\rho \right. \\
 & \left. + \rho^2 (1 + \rho^2) \sin \zeta \cos \zeta d\zeta \right) \wedge d\psi_2. \tag{52}
 \end{aligned}$$

It can be checked that the open string metric is again undeformed and the external bivector takes the form

$$\begin{aligned}
 \Theta^{\rho\psi_1} &= \kappa \left(\rho^{-1} + \rho \right), \quad \Theta^{\zeta\psi_1} = -\kappa \rho^{-2} \tan \zeta, \\
 \Theta^{\zeta\psi_2} &= \kappa \rho^{-2} (1 + \rho^2) \cot \zeta. \tag{53}
 \end{aligned}$$

Including the internal directions, the Killing vector is

$$I = \kappa (4\partial_{\psi_1} + 2\partial_{\psi_2} + 4\partial_\phi - 2\partial_{\phi_1}), \tag{54}$$

and the dilaton is

$$e^{-2\Phi} = f h \left(1 + \kappa^2 r^2 \right) \left(1 + \kappa^2 r^4 \sin^2 \xi \right). \tag{55}$$

Again, we have checked the dilaton equation.

We end this section with some comments. It is easy to check that the Jacobi identity for the bivector, or more concretely

$$\Theta^{[LP} \partial_P \Theta^{MN]} = 0, \tag{56}$$

is satisfied for the three examples of η -deformations we have considered. We observe that this is a necessary condition for the bivector Θ to correspond to a NC parameter.

Furthermore, in [7] it was noted that the bivector Θ exhibits holographic noncommutativity: although it may depend on the holographic direction z , working in Poincaré patch with the boundary at $z = 0$, the dependence on the holographic direction drops out consistently at the boundary, i.e. at the boundary both legs of Θ are along the boundary, $\Theta^{\mu z}|_{z=0} = 0$. To see that the same picture applies equally well to the ABF solution (44), the following map from (unit radius) global AdS₅ to the Poincaré metric can be employed:

$$\begin{aligned}
 \rho &= \frac{1}{2z} \sqrt{4\mathbf{x}^2 + (z^2 - 1 + \mathbf{x}^2 - t^2)^2}, \\
 \cos \tau &= \frac{z^2 + 1 + \mathbf{x}^2 - t^2}{\sqrt{4t^2 + (z^2 + 1 + \mathbf{x}^2 - t^2)^2}}, \\
 \cos \zeta &= \frac{2\sqrt{(x_1^2 + x_2^2)}}{\sqrt{4(x_1^2 + x_2^2 + x_3^2) + (z^2 - 1 + \mathbf{x}^2 - t^2)^2}}, \\
 \cos \psi_1 &= \frac{x_1}{\sqrt{x_1^2 + x_2^2}}, \\
 \cos \psi_2 &= \frac{2x_3}{\sqrt{4x_3^2 + (z^2 - 1 + \mathbf{x}^2 - t^2)^2}}, \tag{57}
 \end{aligned}$$

where $\mathbf{x} \equiv (x_1, x_2, x_3)$. Evaluating the ABF bivector (45) in Poincaré coordinates at the boundary, $\Theta^{\mu\nu} = \Theta^{MN}|_{z=0}$ we have confirmed that all z -dependence disappears leaving the following non-zero contributions to the bivector in Poincaré patch:

$$\begin{aligned}
\Theta^{tx_1} &= -\kappa \frac{tx_2}{2} (x_\mu x^\mu - 1), & \Theta^{tx_2} &= \kappa \frac{tx_1}{2} (x_\mu x^\mu - 1), \\
\Theta^{x_1 x_2} &= -\kappa \left(x_3^2 + \frac{1}{4} (x_\mu x^\mu - 1) (x_\mu x^\mu - x_1^2 - x_2^2 - 1) \right), \\
\Theta^{x_1 x_3} &= \kappa \frac{x_2 x_3}{2} (x_\mu x^\mu + 1), \\
\Theta^{x_2 x_3} &= -\kappa \frac{x_1 x_3}{2} (x_\mu x^\mu + 1).
\end{aligned} \tag{58}$$

We expect a similar outcome for other η -deformations. The corresponding expression for the Killing vector I can be written in terms of the Killing vectors for the Poincaré metric of AdS₅ as

$$I = 2\kappa(P_0 + K_0 + M_{12}), \tag{59}$$

where P_0 and K_0 denote translations and special conformal transformations in the temporal direction, respectively, and M_{12} is a rotation in the (x_1, x_2) -plane. We refer the reader to (A.2) of [7] for details of the notation. Note that the holographic behavior of I is dictated by (2).

6 Conclusions

One of the results of this letter is completing the derivation of Eq. (2) from the requirement that the action of D3-branes is invariant under Λ -symmetry. An interesting feature of the calculation is that the equations of generalized supergravity are incorporated through a bulk AdS₅ Wess–Zumino term which couples the D3-brane to the background. This provides an explanation for the relation which was observed to hold for all homogeneous Yang–Baxter deformations on a case by case basis. In this letter we show that η -deformations have an open string metric that is undeformed and satisfy the same equation. This allows us to interpret I as a current for the center of mass motion on the worldvolume of the D3-brane.

As a secondary result we noted that the Bianchi identities and EOMs of generalized supergravity take a strikingly simple form when expressed in terms of Page forms. We conjectured that the non-zero Page forms as well as associated Page charges are invariants of YB deformations. We showed that there always exist suitably chosen pure gauge contributions to the NSNS B -field, such that open string metric and consequently $e^{-2\Phi} \sqrt{g}$ remains undeformed for all (known) YB deformations. This may be used as a handy way to determine the dilaton for all YB deformations, including η -deformations.

Finally, using the deformation of the pure spinor action through integrated vertex operators, we have identified the states in the BRST cohomology that give origin to the vector field I in the η -deformation context. As a by-product we have shown that other non-physical states in the BRST cohomology of [75] are suppressed by higher order corrections in the curvature and/or deformation parameter η .

It is a straightforward exercise to extend the analysis presented here to η -deformations of AdS₃ × S³ and AdS₂ × S² geometries [80, 81]. Once pure gauge contributions to the NSNS two-form are introduced we expect the open string metric to be undeformed and we expect for Eq. (2) to hold with the slight story twist that it will arise from variations of the actions of D1-branes, D5-branes, intersecting D3-branes, etc.

One problem that has yet to be unraveled is the interpretation of the η -deformed geometries [17, 54] in the context of AdS/CFT. This problem is thorny, since in contrast to simpler homogeneous YB deformations one has to consider both AdS₅ and S⁵ deformations. As a result there is less residual global symmetry but the consensus is that the dual gauge theory, if it exists, is expected to be some NC deformation of $\mathcal{N} = 4$ super Yang–Mills. In this letter we have extracted a candidate NC parameter for the ABF solution [54], shown that it is associative in the sense that the Jacobi (56) is satisfied which means that it is consistent with a non-commuting open string description. Moreover, we have checked in Poincaré patch that the holographic direction decouples at the boundary, where the putative dual gauge theory is expected to reside.

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7 Review of generalized supergravity

The equations of motion for the metric, NSNS two-form and dilaton in the generalized type IIB supergravity are given by

$$\begin{aligned}
R_{MN} - \frac{1}{4} H_{MPQ} H_N{}^{PQ} + D_M X_N + D_N X_M &= T_{MN}, \\
\frac{1}{2} D^K H_{KMN} + \frac{1}{2} \mathcal{F}^K \mathcal{F}_{KMN} + \frac{1}{12} \mathcal{F}_{MNKLP} \mathcal{F}^{KLP} &= 0
\end{aligned}$$

$$\begin{aligned}
 &= X^K H_{KMN} + D_M X_N - D_N X_M, \\
 R - \frac{1}{12} H_{MNP} H^{MNP} + 4 D_M X^M - 4 X^M X_M &= 0,
 \end{aligned}
 \tag{60}$$

where the spacetime indices M, N, \dots run from 0 to 9, H is the field strength of the NSNS two-form. The appearance of a vector field X indicates the generalization. Note here that the dilaton Φ is implicitly included in X with a spacetime derivative and an extra vector field I as explained later. The flux contribution T_{MN} is defined as

$$\begin{aligned}
 T_{MN} \equiv & \frac{1}{2} \mathcal{F}_M \mathcal{F}_N + \frac{1}{4} \mathcal{F}_{MKL} \mathcal{F}_N^{KL} \\
 & + \frac{1}{4 \times 4!} \mathcal{F}_{MPQRS} \mathcal{F}_N^{PQRS} \\
 & - \frac{1}{4} G_{MN} \left(\mathcal{F}_K \mathcal{F}^K + \frac{1}{6} \mathcal{F}_{PQR} \mathcal{F}^{PQR} \right).
 \end{aligned}
 \tag{61}$$

Here $\mathcal{F}_M, \mathcal{F}_{MNK}, \mathcal{F}_{MNK PQ}$ are the rescaled RR field strengths, i.e.

$$\mathcal{F}_{n_1 n_2 \dots} = e^\Phi F_{n_1 n_2 \dots}
 \tag{62}$$

The vector X is decomposed as

$$X_M \equiv I_M + Z_M,
 \tag{63}$$

where I and Z satisfy relations

$$D_M I_N + D_N I_M = 0,
 \tag{64}$$

$$I^K H_{KMN} + D_M Z_N - D_N Z_M = 0,
 \tag{65}$$

$$I^M Z_M = 0,
 \tag{66}$$

where D is the closed string covariant derivative. Assuming that the Lie derivative of the NSNS two-form along the I -direction vanishes, one obtains the following expression:

$$Z_M = \partial_M \Phi - B_{MN} I^N.
 \tag{67}$$

We note that while the X and hence the Z fields are invariant under Λ -gauge transformations, the specific solution (67) is not. This is written in a specific gauge. This gauge is the same as the one we used in our analysis of Λ -invariance of the DBI action. Thus, in total, X can be regarded as a composite field comprising the dilaton derivative, the NSNS two-form and the extra vector field I .

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