Eur. Phys. J. C (2017) 77:709
DOI 10.1140/epjc/s10052-017-5297-4

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Exotic tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration

Si-Qiang Luo[1,2], **Kan Chen**[1,2], **Xiang Liu**[1,2,a], **Yan-Rui Liu**[3,b], **Shi-Lin Zhu**[4,5,6,c]

[1] School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China
[2] Research Center for Hadron and CSR Physics, Lanzhou University and Institute of Modern Physics of CAS, Lanzhou 730000, China
[3] School of Physics and Key Laboratory of Particle Physics and Particle Irradiation (MOE), Shandong University, Jinan 250100, China
[4] School of Physics and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China
[5] Collaborative Innovation Center of Quantum Matter, Beijing 100871, China
[6] Center of High Energy Physics, Peking University, Beijing 100871, China

Received: 31 August 2017 / Accepted: 10 October 2017 / Published online: 25 October 2017

Abstract In this work, we study systematically the mass splittings of the $qq\bar{Q}\bar{Q}$ ($q = u, d, s$ and $Q = c, b$) tetraquark states with the color-magnetic interaction by considering color mixing effects and estimate roughly their masses. We find that the color mixing effect is relatively important for the $J^P = 0^+$ states and possible stable tetraquarks exist in the $nn\bar{Q}\bar{Q}$ ($n = u, d$) and $ns\bar{Q}\bar{Q}$ systems either with $J = 0$ or with $J = 1$. Possible decay patterns of the tetraquarks are briefly discussed.

1 Introduction

Searching for exotic hadronic states is an interesting research topic full of opportunities and challenges. In the past decade, the reported charmonium-like states like $X(3872)$ [1], $Y(3940)$ [2], $Y(4140)$ [3,4], $Z^+(4430)$ [5], and $Z_c^+(4200)$ [6], the bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ [7], and the open-heavy flavor meson $X(5568)$ [8] have stimulated extensive discussions on their exotic assignments. The interested reader may refer to the recent literature [9–15] for the comprehensive review of progress.

Among the various exotic state assignments, the tetraquark configuration is most popular in explaining these observed novel phenomena. Thus, studying tetraquark states has become an important issue of exploring exotic hadronic matter. To find where and how to identify tetraquark states from these observed XYZ states is a major task. Usually, it is easy to identify a hadronic state as an exotic one if it has exotic quantum numbers like 0^{--}, 0^{+-}, 1^{-+}, 2^{+-}, and so on. In order to identify a tetraquark state, we need to pay attention to not only its quark component but also its special properties. If a hadronic state has valence quarks with four different flavors, we may conclude that this hadron is probably a tetraquark state when the molecule interpretation is not favored. The newly observed $X(5568)$ in the $B_s^0\pi^\pm$ channel [8] is a typical example since the $X(5568)$ contains valence quarks of four different flavors and the $\bar{B}K$ interaction in the isovector channel is not strong enough to form a molecule. Besides, if a hadron has the $qq\bar{Q}\bar{Q}$ configuration, where Q denotes b or c quark and q is a light quark, we may identify it to be a tetraquark state when it is far below relevant meson–meson thresholds.

Until now, the tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration have not been reported by experiments. On the other hand, the existence and stability of such states have been discussed by theorists for a long time. Different models [16–40] with various potentials as well as the associated interactions were introduced to describe the $qq\bar{Q}\bar{Q}$ system, suggesting that the $qq\bar{Q}\bar{Q}$ states are stable against breakup into the $q\bar{Q} - q\bar{Q}$ meson pair. In the framework of QCD sum rule, the mass spectrum of the $qq\bar{Q}\bar{Q}$ states has been studied in Refs. [41,42]. In addition, lattice QCD simulations gave us more hints on this issue [43–57]. An intuitive picture [55] in Lattice QCD is that in a tetraquark with $qq\bar{Q}\bar{Q}$ configuration, when the two heavy quarks are in a long separation, the gluon exchange force between them is screened by the two light quarks. This system is similar to a hydrogen molecule [41]. Thus, a $(q\bar{Q} - q\bar{Q})$ loosely bound state can be formed [58,59]. On the other hand, if the two heavy quarks have a small separation distance, the $\bar{Q}\bar{Q}$ component can form a color source. As a result, the exotic tetraquark state $(\bar{Q}\bar{Q} - qq)$ can be generated after pairing with the light component qq. Although many exotic $qq\bar{Q}\bar{Q}$ tetraquarks were also obtained in Lattice QCD simulation, the binding energy of such tetraquarks strongly depends on the details of

[a] e-mail: xiangliu@lzu.edu.cn
[b] e-mail: yrliu@sdu.edu.cn
[c] e-mail: zhusl@pku.edu.cn

the dynamical models. To understand further the properties of the tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration, both theorists and experimentalists still need to make more efforts to explore them.

In this work, we continue to pay attention to the tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration. We will again adopt the framework of the simple color-magnetic interaction, although it is not a dynamical model. Recently, we have systematically applied it to tetraquark and pentaquark states in order to understand the nature of the observed exotic hadrons and to predict exotic phenomena [60–63]. Here, we calculate the mass splittings of the tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration and estimate their mass spectrum, with which we further discuss their decay patterns. Hopefully, the information presented in this work will be helpful to further experimental search for them.

This paper is organized as follows. After the introduction section, we present the deduction of the elements of the interaction matrices in Sect. 2. In Sect. 3, the adopted parameters and numerical results are given in detail. In Sect. 4, we provide some discussion of these systems. Finally, the paper ends with a short summary in Sect. 5.

2 Formalism

The color magnetic interaction (CMI) in a hadron can be written as

$$H_{\rm CM} = -\sum_{i<j} C_{ij}\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j, \tag{1}$$

where the subscript i denotes the ith constituent quark in the hadron and λ_i and σ_i represent the Gell-Mann matrices and the Pauli matrices, respectively. For an antiquark, λ_i should be replaced by $-\lambda_i^*$. This Hamiltonian is deduced from part of the one gluon exchange interaction [64]. The C_{ij} describes the effective coupling constant between one quark and another quark or antiquark, which incorporates the effects from the spatial wave function and the quark mass. This constant will be estimated in the next section.

For the ground state of a tetraquark, the above CMI Hamiltonian leads to the mass formula

$$H = \sum_{i=1}^{4} m_i + \langle H_{\rm CM}\rangle \tag{2}$$

where m_i is the effective mass of the ith constituent quark.

We use the diquark–antidiquark bases to discuss the wave functions of ground tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration, where $Q = c$ or b and $q = n$ or s with $n = u$ or d. Since we consider the mixing between different color–spin structures, all the other bases will finally result in the same mass splittings. In order to obtain all the ground states

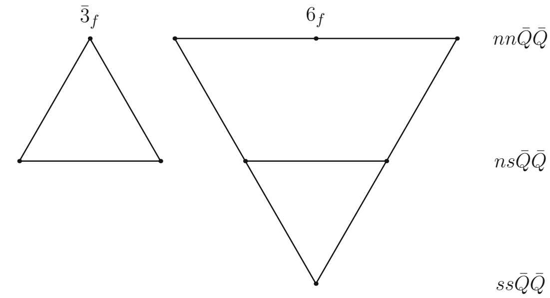

Fig. 1 Flavor representations for the $qq\bar{Q}\bar{Q}$ systems. Here $Q = c, b$ and $q = n, s$ with $n = u, d$

satisfying the flavor-color–spin symmetry constraint from Pauli principle, we need to exhaust all the possible spin and color wave functions of a diquark–antidiquark system and combine them appropriately with the flavor wave functions. With the obtained wave functions, one may calculate the matrix elements using the approach illustrated in Refs. [65,66].

The possible spin wave functions for the tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration are

$$\begin{aligned}
\chi_1 &= |(q_1q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_2, \quad \chi_2 = |(q_1q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_1,\\
\chi_3 &= |(q_1q_2)_1(\bar{Q}_3\bar{Q}_4)_1\rangle_0, \quad \chi_4 = |(q_1q_2)_1(\bar{Q}_3\bar{Q}_4)_0\rangle_1,\\
\chi_5 &= |(q_1q_2)_0(\bar{Q}_3\bar{Q}_4)_1\rangle_1, \quad \chi_6 = |(q_1q_2)_0(\bar{Q}_3\bar{Q}_4)_0\rangle_0,
\end{aligned} \tag{3}$$

where the subscripts denote the spins of the light diquark, the heavy antidiquark, and the tetraquark state. The color wave function can be analyzed by applying the SU(3) group theory, where the direct product of the diquark–antidiquark components reads

$$3_c \otimes 3_c \otimes \bar{3}_c \otimes \bar{3}_c = (6_c \oplus \bar{3}_c) \otimes (\bar{6}_c \oplus 3_c). \tag{4}$$

Obviously, we have two combinations to form a color singlet tetraquark wave function, i.e.,

$$\begin{aligned}
\phi_1 &= |(q_1q_2)^6(\bar{Q}_3\bar{Q}_4)^{\bar{6}}\rangle\\
&= \frac{1}{2\sqrt{6}}\Big[2(rr\bar{r}\bar{r} + gg\bar{g}\bar{g} + bb\bar{b}\bar{b}) + rb\bar{b}\bar{r} + br\bar{b}\bar{r}\\
&\quad + gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} + bg\bar{b}\bar{g} + gr\bar{r}\bar{g} + rg\bar{r}\bar{g}\\
&\quad + gb\bar{g}\bar{b} + bg\bar{g}\bar{b} + rb\bar{r}\bar{b} + br\bar{r}\bar{b}\Big]
\end{aligned} \tag{5}$$

and

$$\begin{aligned}
\phi_2 &= |(q_1q_2)^{\bar{3}}(\bar{Q}_3\bar{Q}_4)^{3}\rangle\\
&= \frac{1}{2\sqrt{3}}\Big(rb\bar{b}\bar{r} - br\bar{b}\bar{r} - gr\bar{g}\bar{r} + rg\bar{g}\bar{r} + gb\bar{b}\bar{g} - bg\bar{b}\bar{g}\\
&\quad + gr\bar{r}\bar{g} - rg\bar{r}\bar{g} - gb\bar{g}\bar{b} + bg\bar{g}\bar{b} - rb\bar{r}\bar{b} + br\bar{r}\bar{b}\Big).
\end{aligned} \tag{6}$$

In flavor space, the heavy quark is treated as an SU(3) singlet and we have states belonging to 6_f and $\bar{3}_f$ (Fig. 1).

For the $nn\bar{Q}\bar{Q}$ case, the isovector states and the isoscalar states do not mix since we do not consider isospin breaking effects. For the $ns\bar{Q}\bar{Q}$ case, the fact $m_n \neq m_s$ leads to SU(3) breaking and then the state mixing between 6_f and $\bar{3}_f$.

Now we can combine the flavor, color, and spin wave functions together. In this procedure, one needs to include the constraint from the Pauli principle. In the diquark–antidiquark picture, the possible wave function bases are

$$\begin{aligned}
\phi_1\chi_1 &= |(q_1q_2)_1^6(\bar{Q}_3\bar{Q}_4)_1^{\bar{6}}\rangle_2\delta_{12}^S\delta_{34},\\
\phi_2\chi_1 &= |(q_1q_2)_1^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_1^{3}\rangle_2\delta_{12}^A,\\
\phi_1\chi_2 &= |(q_1q_2)_1^6(\bar{Q}_3\bar{Q}_4)_1^{\bar{6}}\rangle_1\delta_{12}^S\delta_{34},\\
\phi_2\chi_2 &= |(q_1q_2)_1^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_1^{3}\rangle_1\delta_{12}^A,\\
\phi_1\chi_3 &= |(q_1q_2)_1^6(\bar{Q}_3\bar{Q}_4)_1^{\bar{6}}\rangle_0\delta_{12}^S\delta_{34},\\
\phi_2\chi_3 &= |(q_1q_2)_1^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_1^{3}\rangle_0\delta_{12}^A,\\
\phi_1\chi_4 &= |(q_1q_2)_1^6(\bar{Q}_3\bar{Q}_4)_0^{\bar{6}}\rangle_1\delta_{12}^S,\\
\phi_2\chi_4 &= |(q_1q_2)_1^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_0^{3}\rangle_1\delta_{12}^A\delta_{34},\\
\phi_1\chi_5 &= |(q_1q_2)_0^6(\bar{Q}_3\bar{Q}_4)_1^{\bar{6}}\rangle_1\delta_{12}^A\delta_{34},\\
\phi_2\chi_5 &= |(q_1q_2)_0^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_1^{3}\rangle_1\delta_{12}^S,\\
\phi_1\chi_6 &= |(q_1q_2)_0^6(\bar{Q}_3\bar{Q}_4)_0^{\bar{6}}\rangle_0\delta_{12}^A,\\
\phi_2\chi_6 &= |(q_1q_2)_0^{\bar{3}}(\bar{Q}_3\bar{Q}_4)_0^{3}\rangle_0\delta_{12}^S\delta_{34}.
\end{aligned} \tag{7}$$

Here, we present the wave functions with the notation $|(q_1q_2)_{\rm spin}^{\rm color}(\bar{Q}_3\bar{Q}_4)_{\rm spin}^{\rm color}\rangle_{\rm total\ spin}$. Since not all the wave functions are allowed for a given set of quantum numbers, we have introduced three factors δ_{12}^S, δ_{12}^A, and δ_{34} to reflect this symmetry requirement. When the two light quarks in flavor space are symmetric (antisymmetric), we have $\delta_{12}^S = 0$ ($\delta_{12}^A = 0$). When the two heavy quarks are identical, we have $\delta_{34} = 0$. If the factor cannot be 0, its value is set to be 1. We may easily ignore irrelevant wave functions with these factors. Then the considered tetraquark states can be categorized into six classes:

1. The $(nn\bar{c}\bar{c})^{I=1}$, $(nn\bar{b}\bar{b})^{I=1}$, $ss\bar{c}\bar{c}$, and $ss\bar{b}\bar{b}$ states with $\delta_{12}^S = \delta_{34} = 0$;
2. The $(nn\bar{c}\bar{c})^{I=0}$, $(nn\bar{b}\bar{b})^{I=0}$ states with $\delta_{12}^A = \delta_{34} = 0$;
3. The $(nn\bar{c}\bar{b})^{I=1}$ and $ss\bar{c}\bar{b}$ states with $\delta_{12}^S = 0$ and $\delta_{34} = 1$;
4. The $(nn\bar{c}\bar{b})^{I=0}$ states with $\delta_{12}^A = 0$ and $\delta_{34} = 1$;
5. The $ns\bar{c}\bar{c}$ and $ns\bar{b}\bar{b}$ states with $\delta_{12}^S = \delta_{12}^A = 1$ and $\delta_{34} = 0$;
6. The $ns\bar{c}\bar{b}$ states with $\delta_{12}^S = \delta_{12}^A = \delta_{34} = 1$.

By selecting the corresponding tetraquark wave functions, we obtain the CMI matrices for different cases. The expressions for classes 1 and 2, 3 and 4, and 5 and 6 are shown in Tables 1, 2, and 3, respectively. Here, we have simplified the expressions by using the following definitions: $\alpha = C_{12} + C_{34}$, $\beta = C_{13} + C_{14} + C_{23} + C_{24}$, $\gamma = C_{13} + C_{14} - C_{23} - C_{24}$, $\delta = C_{13} - C_{14} + C_{23} - C_{24}$, $\mu = C_{13} - C_{14} - C_{23} + C_{24}$, $\eta = C_{12} - 3C_{34}$, and $\theta = 3C_{12} - C_{34}$.

Table 1 The quantum numbers, color–spin wave functions, and the corresponding CMI matrices for the $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, $ss\bar{c}\bar{c}$, and $ss\bar{b}\bar{b}$ systems. Here, $(qq)^S$ ($(qq)^A$) means that the flavor wave function of the two light quarks is symmetric (antisymmetric)

Symmetry	J^P	Wave functions	$\langle H_{CM}\rangle$
$(qq)^S$	0^+	$(\phi_2\chi_3, \phi_1\chi_6)^T$	$\begin{pmatrix} \frac{8}{3}(\alpha-\beta) & 2\sqrt{6}\beta \\ & 4\alpha \end{pmatrix}$
	1^+	$(\phi_2\chi_2)$	$\frac{4}{3}(2\alpha-\beta)$
	2^+	$(\phi_2\chi_1)$	$\frac{4}{3}(2\alpha+\beta)$
$(qq)^A$	1^+	$(\phi_2\chi_5, \phi_1\chi_4)^T$	$\begin{pmatrix} -\frac{8}{3}\theta & -2\sqrt{2}\beta \\ & -\frac{4}{3}\eta \end{pmatrix}$

3 Numerical results

3.1 Parameters

We need to determine the values of the relevant coefficients C_{qq}, $C_{\bar{Q}q}$, and C_{QQ} in estimating the mass splittings of the possible tetraquark states. Here, the subscripts $Q = c, b$ and $q = n, s$ with $n = u, d$. The parameters $C_{nn} = 18.3$ MeV and $C_{ns} = 12.3$ MeV can be determined from the CMI relation between $N-\Delta$ and $\Sigma-\Sigma^*$ systems. One further obtains $C_{ss} = 6.4$ MeV by using the relation $2M_\Omega + M_\Delta - (2M_{\Xi^*} + M_\Xi) = 8C_{ss} + 8C_{nn}$. The coupling constant $C_{\bar{c}n}$ ($C_{\bar{c}s}$) is estimated by considering the mass splitting between the pseudoscalar and vector charmed (charmed-strange) mesons. Similarly, $C_{\bar{b}n}$ ($C_{\bar{b}s}$) is determined with the bottom (bottom-strange) mesons. We extract the $C_{c\bar{c}}$ ($C_{b\bar{b}}$) from the associated charmonium (bottomium) mesons and use the mass of B_c^* estimated from GI model [67] to determine $C_{c\bar{b}}$. For the remaining constants C_{QQ} (C_{cc}, C_{bc}, C_{bb}), however, there are no observed or confirmed doubly heavy baryons in experiments at present. The approximation $C_{QQ} = C_{Q\bar{Q}}$, i.e., $C_{c\bar{c}} = C_{cc}$, $C_{c\bar{b}} = C_{cb}$, and $C_{b\bar{b}} = C_{bb}$, is used in our analysis of the $qq\bar{Q}\bar{Q}$ tetraquarks. We collect the determined parameters in Table 4. In getting these parameters and in the following evaluation, we use the following meson masses [68]: $M_D = 1867.21$ MeV, $M_{D^*} = 2008.56$ MeV, $M_{D_s} = 1968.27$ MeV, $M_{D_s^*} = 2112.1$ MeV, $M_B = 5279.31$ MeV, $M_{B^*} = 5324.65$ MeV, $M_{B_s} = 5366.82$ MeV, and $M_{B_s^*} = 5415.4$ MeV.

Since we only consider the color magnetic interaction between valence quarks in a $qq\bar{Q}\bar{Q}$ system and no dynamical effects are involved in this calculation, we will discuss the masses of tetraquarks with two schemes. In the first

Table 2 The quantum numbers, color–spin wave functions, and the corresponding CMI matrices for the $nn\bar{c}\bar{b}$ and $ss\bar{c}\bar{b}$ systems. Here, $(qq)^S$ ($(qq)^A$) means that the flavor wave function of the two light quarks is symmetric (antisymmetric)

Symmetry	J^P	Wave functions	$\langle H_{CM}\rangle$
$(qq)^S$	0^+	$(\phi_2\chi_3, \phi_1\chi_6)^T$	$\begin{pmatrix} \frac{8}{3}(\alpha-\beta) & 2\sqrt{6}\beta \\ & 4\alpha \end{pmatrix}$
	1^+	$(\phi_2\chi_4, \phi_2\chi_2, \phi_1\chi_5)^T$	$\begin{pmatrix} \frac{8}{3}\eta & \frac{4\sqrt{2}}{3}\delta & -2\sqrt{2}\beta \\ & \frac{4}{3}(2\alpha-\beta) & 4\delta \\ & & \frac{4}{3}\theta \end{pmatrix}$
	2^+	$(\phi_2\chi_1)$	$\frac{4}{3}(2\alpha+\beta)$
$(qq)^A$	0^+	$(\phi_2\chi_6, \phi_1\chi_3)^T$	$\begin{pmatrix} -8\alpha & 2\sqrt{6}\beta \\ & -\frac{4}{3}(\alpha+5\beta) \end{pmatrix}$
	1^+	$(\phi_2\chi_5, \phi_1\chi_4, \phi_1\chi_2)^T$	$\begin{pmatrix} -\frac{8}{3}\theta & -2\sqrt{2}\beta & 4\delta \\ & -\frac{4}{3}\eta & \frac{10\sqrt{2}}{3}\delta \\ & & -\frac{2}{3}(2\alpha+5\beta) \end{pmatrix}$
	2^+	$(\phi_1\chi_1)$	$\frac{2}{3}(-2\alpha+5\beta)$

Table 3 The quantum numbers, color–spin wave functions, and the corresponding CMI matrices for the $ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$, and $ns\bar{c}\bar{b}$ systems

System	J^P	Wave functions	$\langle H_{CM}\rangle$
$ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$	0^+	$(\phi_2\chi_3, \phi_1\chi_6)^T$	$\begin{pmatrix} \frac{8}{3}(\alpha-\beta) & 2\sqrt{6}\beta \\ & 4\alpha \end{pmatrix}$
	1^+	$(\phi_2\chi_5, \phi_1\chi_4, \phi_2\chi_2)^T$	$\begin{pmatrix} -\frac{8}{3}\theta & -2\sqrt{2}\beta & -\frac{4\sqrt{2}}{3}\gamma \\ & -\frac{4}{3}\eta & -4\gamma \\ & & \frac{4}{3}(2\alpha-\beta) \end{pmatrix}$
	2^+	$(\phi_2\chi_1)$	$\frac{4}{3}(2\alpha+\beta)$
$ns\bar{c}\bar{b}$	0^+	$(\phi_2\chi_6, \phi_1\chi_3, \phi_2\chi_3, \phi_1\chi_6)^T$	$\begin{pmatrix} -8\alpha & 2\sqrt{6}\beta & -\frac{4}{\sqrt{3}}\mu & 0 \\ & -\frac{4}{3}(\alpha+5\beta) & 4\sqrt{2}\mu & -\frac{10\mu}{\sqrt{3}} \\ & & \frac{8}{3}(\alpha-\beta) & 2\sqrt{6}\beta \\ & & & 4\alpha \end{pmatrix}$
	1^+	$(\phi_2\chi_5, \phi_1\chi_4, \phi_1\chi_2, \phi_2\chi_4, \phi_1\chi_5, \phi_2\chi_2)^T$	$\begin{pmatrix} -\frac{8}{3}\theta & -2\sqrt{2}\beta & 4\delta & \frac{4}{3}\mu & -\frac{4\sqrt{2}}{3}\gamma & 0 \\ & -\frac{4}{3}\eta & \frac{10\sqrt{2}}{3}\delta & 0 & -4\gamma & \frac{10}{3}\mu \\ & & -\frac{2}{3}(2\alpha+5\beta) & -4\gamma & 2\sqrt{2}\mu & -\frac{10\sqrt{2}}{3}\gamma \\ & & & \frac{8}{3}\eta & \frac{4\sqrt{2}}{3} & -2\sqrt{2}\beta \\ & & & & \frac{4}{3}(2\alpha-\beta) & 4\delta \\ & & & & & \frac{4}{3}\theta \end{pmatrix}$
	2^+	$(\phi_1\chi_1, \phi_2\chi_1)^T$	$\begin{pmatrix} \frac{2}{3}(-2\alpha+5\beta) & -2\sqrt{2}\mu \\ & \frac{4}{3}(2\alpha+\beta) \end{pmatrix}$

Table 4 The parameters obtained with mass splittings of conventional mesons and baryons

Hadron	CMI	Hadron	CMI	Parameter (MeV)
N	$-8C_{nn}$	Δ	$8C_{nn}$	$C_{nn}=18.3$
Σ	$\frac{8}{3}C_{nn}-\frac{32}{3}C_{ns}$	Σ^*	$\frac{8}{3}C_{nn}+\frac{16}{3}C_{ns}$	$C_{ns}=12.3$
D	$-16C_{\bar{c}n}$	D^*	$\frac{16}{3}C_{\bar{c}n}$	$C_{\bar{c}n}=6.6$
D_s	$-16C_{\bar{c}s}$	D_s^*	$\frac{16}{3}C_{\bar{c}s}$	$C_{\bar{c}s}=6.7$
B	$-16C_{\bar{b}n}$	B^*	$\frac{16}{3}C_{\bar{b}n}$	$C_{\bar{b}n}=2.1$
B_s	$-16C_{\bar{b}s}$	B_s^*	$\frac{16}{3}C_{\bar{b}s}$	$C_{\bar{b}s}=2.3$
η_c	$-16C_{\bar{c}c}$	J/ψ	$\frac{16}{3}C_{\bar{c}c}$	$C_{\bar{c}c}=5.3$
η_b	$-16C_{\bar{b}b}$	Υ	$\frac{16}{3}C_{\bar{b}b}$	$C_{\bar{b}b}=2.9$
B_c	$-16C_{\bar{c}b}$	B_c^* [67]	$\frac{16}{3}C_{\bar{c}b}$	$C_{\bar{c}b}=3.3$

Table 5 The effective constituent quark masses extracted from conventional baryons

Mass formula	Quark mass (MeV)
$M_N = 3m_n - 8C_{nn}$	$m_n = 361.8$
$M_\Omega = 3m_s + 8C_{ss}$	$m_s = 540.4$
$M_{\Sigma_c} = \frac{8}{3}C_{nn} - \frac{32}{3}C_{nc} + 2m_q + m_c$	$m_c = 1724.8$
$M_{\Sigma_c^*} = \frac{8}{3}C_{nn} + \frac{16}{3}C_{nc} + 2m_n + m_c$	
$M_{\Sigma_b} = 2m_q + m_b + \frac{8}{3}C_{nn} - \frac{32}{3}C_{bn}$	$m_b = 5052.9$
$M_{\Sigma_b^*} = 2m_q + m_b + \frac{8}{3}C_{nn} + \frac{16}{3}C_{bn}$	

scheme, we use the experimental data to obtain the effective constituent quark masses and estimate the values of the tetraquark masses by introducing the mass shifts due to CMI. The corresponding formula is $M = \sum_i m_i + \langle H_{\rm CM}\rangle$. In Table 5, we show the procedure of determination. In the second scheme, we relate the tetraquark system to a reference meson–meson system and compare the masses to the threshold of the meson–meson state by the equation $M = M_{\rm ref} - \langle H_{\rm CM}\rangle_{\rm ref} + \langle H_{\rm CM}\rangle$. In the first scheme, we find that the predicted masses are generally overestimated. The main reason is probably from the fact that the dynamical effects cannot be simply absorbed into the effective quark masses. This scheme can give an upper limit for the masses of the tetraquark states. In the following discussions, we mainly focus on the results obtained from the threshold scheme. By substituting the C_{ij} parameters into the corresponding CMI matrices given in the previous section, one gets the values of the mass shifts from the color magnetic interaction after we diagonalize the numerical CMI matrices.

Since the corresponding antiparticle $\bar{q}\bar{q}QQ$ has identical mass to the $qq\bar{Q}\bar{Q}$, we only present the results for the $qq\bar{Q}\bar{Q}$ systems. In the previous section, we have divided such systems into six classes. In the following, we discuss the systems according to their SU(3) classification in the flavor space.

3.2 Systems with strangeness = 0

The quark content of these systems may be $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, or $nn\bar{c}\bar{b}$. We present the calculated CMI matrices, the eigenvectors, the eigenvalues, and the estimated masses in both schemes in Table 6. As mentioned earlier, the constitute quark scheme gives higher mass predictions. In the following discussions, we will use the threshold scheme to discuss their properties. In Fig. 2, the rough positions of the studied tetraquark states, various thresholds, and relevant rearrangement decay patterns are shown.

Table 6 Numerical results for the $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, and $nn\bar{c}\bar{b}$ systems in units of MeV, where $n = u$ or d. The masses in the sixth column are estimated with the masses of effective quarks and those in the last column with the thresholds of $DD/BB/BD$

System	J^P	$\langle H_{CM}\rangle$	Eigenvalues	Eigenvectors	Mass	$DD/BB/BD$
$(nn\bar{c}\bar{c})^{I=1}$	0^+	$\begin{pmatrix} -7.5 & 129.3 \\ 129.3 & 94.4 \end{pmatrix}$	$\begin{bmatrix} 182.5 \\ -95.5 \end{bmatrix}$	$\begin{bmatrix} (0.56, 0.83) \\ (-0.83, 0.56) \end{bmatrix}$	$\begin{bmatrix} 4356 \\ 4078 \end{bmatrix}$	$\begin{bmatrix} 4128 \\ 3850 \end{bmatrix}$
	1^+	27.7	27.7	1.00	4201	3973
	2^+	98.1	98.1	1.00	4271	4044
$(nn\bar{c}\bar{c})^{I=0}$	1^+	$\begin{pmatrix} -132.3 & -74.7 \\ -74.7 & -3.2 \end{pmatrix}$	$\begin{bmatrix} -166.4 \\ 31.0 \end{bmatrix}$	$\begin{bmatrix} (0.91, 0.42) \\ (-0.42, 0.91) \end{bmatrix}$	$\begin{bmatrix} 4007 \\ 4204 \end{bmatrix}$	$\begin{bmatrix} 3779 \\ 3977 \end{bmatrix}$
$(nn\bar{b}\bar{b})^{I=1}$	0^+	$\begin{pmatrix} 34.1 & 41.2 \\ 41.2 & 84.8 \end{pmatrix}$	$\begin{bmatrix} 107.8 \\ 11.1 \end{bmatrix}$	$\begin{bmatrix} (0.49, 0.87) \\ (-0.87, 0.49) \end{bmatrix}$	$\begin{bmatrix} 10937 \\ 10841 \end{bmatrix}$	$\begin{bmatrix} 10734 \\ 10637 \end{bmatrix}$
	1^+	45.3	45.3	1.00	10875	10671
	2^+	67.7	67.7	1.00	10897	10694
$(nn\bar{b}\bar{b})^{I=0}$	1^+	$\begin{pmatrix} -138.7 & -23.8 \\ -23.8 & -12.8 \end{pmatrix}$	$\begin{bmatrix} -143.0 \\ -8.5 \end{bmatrix}$	$\begin{bmatrix} (0.98, 0.18) \\ (-0.18, 0.98) \end{bmatrix}$	$\begin{bmatrix} 10686 \\ 10821 \end{bmatrix}$	$\begin{bmatrix} 10483 \\ 10617 \end{bmatrix}$
$(nn\bar{c}\bar{b})^{I=1}$	0^+	$\begin{pmatrix} 11.2 & 85.2 \\ 85.2 & 86.4 \end{pmatrix}$	$\begin{bmatrix} 142.0 \\ -44.4 \end{bmatrix}$	$\begin{bmatrix} (0.55, 0.84) \\ (0.84, -0.55) \end{bmatrix}$	$\begin{bmatrix} 7643 \\ 7457 \end{bmatrix}$	$\begin{bmatrix} 7428 \\ 7241 \end{bmatrix}$
	1^+	$\begin{pmatrix} 22.4 & 17.0 & -49.2 \\ 17.0 & 34.4 & 36.0 \\ -49.2 & 36.0 & 68.8 \end{pmatrix}$	$\begin{bmatrix} 46.4 \\ -28.0 \\ 107.2 \end{bmatrix}$	$\begin{bmatrix} (0.59, 0.81, -0.01) \\ (0.69, -0.49, 0.53) \\ (-0.43, 0.32, 0.85) \end{bmatrix}$	$\begin{bmatrix} 7548 \\ 7473 \\ 7609 \end{bmatrix}$	$\begin{bmatrix} 7332 \\ 7258 \\ 7393 \end{bmatrix}$
	2^+	80.8	80.8	1.00	7582	7367
$(nn\bar{c}\bar{b})^{I=0}$	0^+	$\begin{pmatrix} -172.8 & 85.2 \\ 85.2 & -144.8 \end{pmatrix}$	$\begin{bmatrix} -245.2 \\ -72.4 \end{bmatrix}$	$\begin{bmatrix} (-0.76, 0.65) \\ (0.65, 0.76) \end{bmatrix}$	$\begin{bmatrix} 7256 \\ 7429 \end{bmatrix}$	$\begin{bmatrix} 7041 \\ 7213 \end{bmatrix}$
	1^+	$\begin{pmatrix} -137.6 & -49.2 & 36.0 \\ -49.2 & -11.2 & 42.4 \\ 36.0 & 42.4 & -86.8 \end{pmatrix}$	$\begin{bmatrix} -180.2 \\ 14.9 \\ -70.3 \end{bmatrix}$	$\begin{bmatrix} (-0.81, -0.35, 0.47) \\ (0.23, -0.92, -0.31) \\ (0.54, -0.14, 0.83) \end{bmatrix}$	$\begin{bmatrix} 7321 \\ 7516 \\ 7431 \end{bmatrix}$	$\begin{bmatrix} 7106 \\ 7301 \\ 7215 \end{bmatrix}$
	2^+	29.2	29.2	1.00	7530	7315

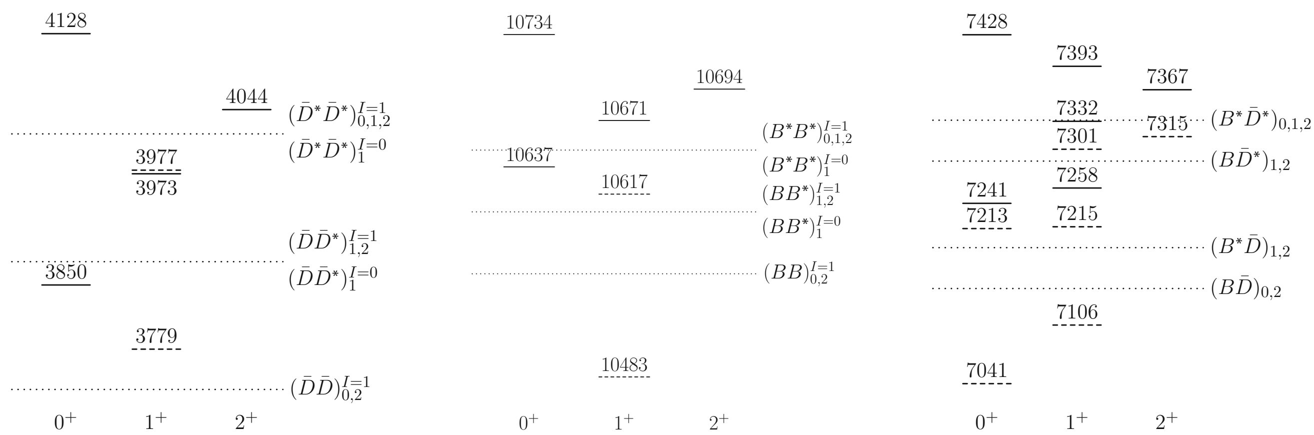

Fig. 2 The estimated masses (units: MeV) of the $nn\bar{c}\bar{c}$ (left), $nn\bar{b}\bar{b}$ (middle), and $nn\bar{c}\bar{b}$ (right) systems in the threshold scheme. The solid (dashed) lines correspond to the $I = 1$ ($I = 0$) case. The dotted lines are thresholds of the possible decay channels. When the isospin conservation is satisfied and the total spin of a tetraquark is equal to a subscript of the symbol for a meson–meson channel, the corresponding S- or D-wave decay is allowed

3.2.1 $nn\bar{c}\bar{c}$

In literature, a compact tetraquark state with the quark content $nn\bar{c}\bar{c}$ is usually called T_{cc}. Because of the highly symmetric constraint from the Pauli principle, the total number of the T_{cc} states is restricted to be six: four isovector states and two isoscalar states. In the isovector case, only one scalar state has probably attractive color-magnetic interaction. The mixing with the other scalar state makes the gap between the two states larger, from 100 to 280 MeV (see Table 6), which results in the heaviest T_{cc} and the lightest 0^+ T_{cc}. In the isoscalar case, the two CMI matrix elements for the axial-vector states can be both negative, depending on the parameters. The state mixing finally leads to one tetraquark with attractive CMI and one with repulsive CMI. This lower tetraquark corresponds to the lowest T_{cc}, which was widely discussed in the literature. In Ref. [39], we argued that the tetraquark mixing is suppressed because the transition needs a spin-flip for the cc state. Now, one sees that the mass shift for the $I = 0$ tetraquarks due to the state mixing is about 30 MeV and it is not a big number. On the other hand, the mass shift for the isovector 0^+ states due to the state mixing is about 90 MeV and the state mixing may have effects on the decay widths.

From Fig. 2, the lowest T_{cc} should be stable if our estimation is reasonable. It is below the $\bar{D}\bar{D}^*$ threshold and its strong decay into $\bar{D}\bar{D}$ is forbidden by both isospin and angular momentum conservations. Whether it can be observed would be crucial information in distinguishing models of genuine multiquark states, although there is still no experimental signal for this state. If the other isoscalar 1^+ state is narrow [39], the isovector 1^+ T_{cc} might also be narrow since they have similar masses and rearrangement decay patterns. Another interesting state is the tensor T_{cc} with $I = 1$. Its decay into $\bar{D}\bar{D}$ or $\bar{D}\bar{D}^*$ is through D wave and relevant partial widths should not be large. Although this tetraquark can decay into $\bar{D}^*\bar{D}^*$ through S wave, the width may not be so large because of the suppression in phase space. This feature of narrow width for high-spin state is similar to the $J = \frac{5}{2}$ pentaquark case which was studied in Ref. [63]. The lower 0^+ T_{cc} has only one S-wave decay channel $\bar{D}\bar{D}$ while the higher one can also decay into $\bar{D}^*\bar{D}^*$ through S or D wave. The decay into the channel $\bar{D}\bar{D}^*$ is forbidden either by kinematics or by angular momentum conservation. Probably the higher scalar T_{cc} could not be observed even if it really exists because of its broad width. From our estimations, probably a search in the $I = 1$ $\bar{D}^{(*)}\bar{D}^{(*)}$ channel may show us some exotic signals.

3.2.2 $nn\bar{b}\bar{b}$

Similarly, such a system is called T_{bb}. From Table 1, the basic features of spectrum should be similar to the T_{cc} case. Because the concrete interaction strengths are different, however, the relevant positions of T_{bb}'s are changed compared with their charmed partners. Accordingly, the decay properties are different. We now take a look at the tetraquark properties from Fig. 2. Below the BB^* threshold, there is only one state, the stable 1^+ T_{bb} with $I = 0$. The second heavier state is the other 1^+ T_{bb} with $I = 0$, which is just above the BB^* threshold and should be narrower than its charmed analog. The remaining T_{bb} states are all isovector mesons and are all above these two states. Although the lowest 0^+ T_{bb} is above the BB^* threshold, its decay into this channel is forbidden and the width may be comparable to the lowest 0^+ T_{cc}. Now, the $I(J^P) = 1(1^+)$ T_{bb} is above the B^*B^* threshold. Because the decay into this channel through S wave is forbidden, its dominant decays should be S-wave

BB^*, D-wave BB^*, and D-wave B^*B^* and thus the width is larger than the $I(J^P) = 0(1^+)$ T_{bb}. The features of the tensor T_{bb} and the heaviest T_{bb} are similar to their charmed analogs, respectively.

3.2.3 $nn\bar{c}\bar{b}$

Now we focus on the systems with two identical light quarks but different heavy quarks. We use T_{cb} to denote such tetraquarks. Obviously, the wave functions of the heavy antidiquarks are not constrained by the Pauli principle and the number of states is bigger than that in the $nn\bar{c}\bar{c}$ systems. From Tables 1 and 2, by comparing T_{cb} with the T_{cc} in the $I = 1$ case, we have the same number of 0^+ and 2^+ tetraquark states and two more 1^+ states. In the isoscalar case, two 0^+ states, one 2^+ state, and one more 1^+ state are allowed.

The heaviest and the lightest states in the $I = 1$ case are the two 0^+ T_{cb}'s, whose mass difference is about 180 MeV. The masses of the 1^+ and 2^+ states lie between these two tetraquarks. The $I = 0$ T_{cb} masses are generally lower than those $I = 1$ masses. Different from the T_{cc} and T_{bb} mesons, now the lowest state is a scalar tetraquark.

From Fig. 2, there are four possible rearrangement decay channels for the T_{cb} states, $B\bar{D}$, $B^*\bar{D}$, $B\bar{D}^*$, and $B^*\bar{D}^*$, two of which are pseudoscalr+vector (PV) type channels. Since no symmetry constraint is required among isospin, spin, and orbital spaces for these meson–meson states, each T_{cb} can decay into these channels once the angular momentum conservation is satisfied and the mass is high enough. We do not need to consider the isospin conservation in considering the meson–meson decay properties.

Two states, one scalar meson and one axial-vector meson, are below the threshold of $B\bar{D}$ and they should be narrow and stable tetraquarks. Two higher 0^+ states can only decay into $B\bar{D}$ through S wave although they are above the threshold of $B^*\bar{D}$. The highest 0^+ state may be broad and might not be observed since it can decay into $B^*\bar{D}^*$ through S and D wave and into $B\bar{D}$ through S wave. Each excited 1^+ T_{cb} has the S-wave decay channel $B^*\bar{D}$. The S-wave channel $B\bar{D}^*$ is opened for three of them and the S-wave $B^*\bar{D}^*$ is also opened for the highest one. For the two tensor states, the lower one can only decay through D wave and the higher one can also decay into $B^*\bar{D}^*$ through S wave. Needless to say, these arguments strongly depend on the estimation of the T_{cb} masses, which is the dominant uncertainty of the present method. If our estimation is reasonable, probably exotic states could be observed in the $B^{(*)}\bar{D}^{(*)}$ channels. If our estimation is underestimated around 100 MeV, the lowest two states may be above respective S-wave decay channels and their signals may also be observed through two-body strong decays. Anyway, interesting phenomena may exist in the $B^{(*)}\bar{D}^{(*)}$ channels.

3.3 Systems with strangeness $= -1$

The system $ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$, or $ns\bar{c}\bar{b}$ contains a strange quark and the isospin of each state is $I = \frac{1}{2}$. We list the numerical results in Table 7 and show their rough positions in Fig. 3. In the table, one finds that some nondiagonal elements of $\langle H_{CM}\rangle$ are close to zero. They are from $\gamma = (C_{13} - C_{23}) + (C_{14} - C_{24})$ and $\mu = (C_{13} - C_{23}) - (C_{14} - C_{24})$ and their nonvanishing values reflect the effects of the flavor symmetry breaking. Comparing with the previous $nn\bar{Q}\bar{Q}$ systems, the total number of states does not change. Although the constraint from isospin conservation is removed, we find that $ns\bar{Q}\bar{Q}$ and $nn\bar{Q}\bar{Q}$ have some similar properties of rearrangement decays.

First, we concentrate on the $ns\bar{c}\bar{c}$ systems. From Fig. 3, it is clear that the heaviest state is still a 0^+ tetraquark and the lightest one is still a 1^+ meson. This lowest 1^+ state does not decay into $\bar{D}D_s^-$ and this highest 0^+ state can decay into $\bar{D}D_s^-$ and $\bar{D}^*D_s^{*-}$ through S wave, which is similar to corresponding states in Fig. 2. Similar features of decay properties for other mesons also exist: the lower 0^+ state decays into $\bar{D}D_s^-$ through S wave, the two higher 1^+ states have PV type S-wave decay channels, and the 2^+ state might be a narrow tetraquark.

Next, we move on to the $ns\bar{b}\bar{b}$ systems. The positions of masses for the $ns\bar{b}\bar{b}$ states and those of various thresholds are similar to those in the $nn\bar{b}\bar{b}$ case. The only difference is that the positions for the lower 0^+ state and the second 1^+ state are exchanged. The similarities in mass spectrum would result in those in widths.

Finally, we take a look at the systems composed of four different flavors, $ns\bar{c}\bar{b}$. The lowest state is a stable scalar tetraquark and the lowest 1^+ is also below the corresponding open-charm decay channel, which is a similar feature to the $nn\bar{c}\bar{b}$ case. The decay properties of other states in the $ns\bar{c}\bar{b}$ case and in the $nn\bar{c}\bar{b}$ case should have similar features, too. A slightly different feature is that almost degenerate states exist in the 1^+ $ns\bar{c}\bar{b}$ case. There are two states around 7330 MeV and two states around 7405 MeV. By inspecting the contributions in the wave functions, one finds that the dominant wave functions in one state belong to $I = 1$ $nn\bar{c}\bar{b}$ and those in the other state belong to $I = 0$ $nn\bar{c}\bar{b}$. It is the state mixing that results in the occasional degeneracy.

When we estimate the $ns\bar{c}\bar{b}$ masses by using the reference threshold BD_s or DB_s, the mass difference is around 10 MeV. This value is from the SU(3) flavor symmetry breaking. From $M = M_{\text{ref}} - \langle H_{\text{CM}}\rangle_{\text{ref}} + \langle H_{\text{CM}}\rangle$ and Table 4, the difference resulting from the two thresholds is $[(M_{D_s} - M_D) - (M_{B_s} - M_B)] + 16[(C_{\bar{c}n} - C_{\bar{c}s}) - (C_{\bar{b}n} - C_{\bar{b}s})]$. If the $SU(3)$ flavor symmetry is strict, $M_{D_s} = M_D$, $M_{B_s} = M_B$, $C_{\bar{c}n} = C_{\bar{c}s}$, $C_{\bar{b}n} = C_{\bar{b}s}$, and the difference vanishes. One may also estimate the $ns\bar{Q}\bar{Q}$ masses with the PV type thresholds and similar uncertainty occurs because of the SU(3) sym-

Table 7 Numerical results for the $ns\bar{c}\bar{c}$, $ns\bar{b}b$, and $ns\bar{c}\bar{b}$ systems in units of MeV, where $n = u$ or d. The masses in the sixth column are estimated with the masses of effective quarks and those in the last column with the thresholds of $DD_s/BB_s/DB_s$. For the $ns\bar{c}\bar{b}$ system, the result estimated with the threshold of BD_s is about 10 MeV higher than that with DB_s

System	J^P	$\langle H_{CM}\rangle$	Eigenvalues	Eigenvectors	Mass	$DD_s/BB_s/DB_s$
$ns\bar{c}\bar{c}$	0^+	$\begin{pmatrix} -24.0 & 130.3 \\ 130.3 & 70.4 \end{pmatrix}$	$\begin{bmatrix} 161.8 \\ -115.4 \end{bmatrix}$	$\begin{bmatrix} (0.57, 0.82) \\ (-0.82, 0.57) \end{bmatrix}$	$\begin{bmatrix} 4514 \\ 4236 \end{bmatrix}$	$\begin{bmatrix} 4210 \\ 3933 \end{bmatrix}$
	1^+	$\begin{pmatrix} -84.3 & -75.2 & 0.4 \\ -75.2 & 4.8 & 0.8 \\ 0.4 & 0.8 & 11.5 \end{pmatrix}$	$\begin{bmatrix} -127.2 \\ 47.7 \\ 11.5 \end{bmatrix}$	$\begin{bmatrix} (-0.87, -0.50, 0.01) \\ (0.50, -0.87, -0.01) \\ (0.01, -0.01, 1.0) \end{bmatrix}$	$\begin{bmatrix} 4225 \\ 4400 \\ 4363 \end{bmatrix}$	$\begin{bmatrix} 3921 \\ 4096 \\ 4060 \end{bmatrix}$
	2^+	82.4	82.4	1.00	4434	4131
$ns\bar{b}\bar{b}$	0^+	$\begin{pmatrix} 17.1 & 43.1 \\ 43.1 & 60.8 \end{pmatrix}$	$\begin{bmatrix} 87.3 \\ -9.4 \end{bmatrix}$	$\begin{bmatrix} (0.52, 0.85) \\ (-0.85, 0.52) \end{bmatrix}$	$\begin{bmatrix} 11095 \\ 10999 \end{bmatrix}$	$\begin{bmatrix} 10804 \\ 10707 \end{bmatrix}$
	1^+	$\begin{pmatrix} -90.7 & -24.9 & 0.8 \\ -24.9 & -4.8 & 1.6 \\ 0.8 & 1.6 & 28.8 \end{pmatrix}$	$\begin{bmatrix} -97.4 \\ 28.9 \\ 1.8 \end{bmatrix}$	$\begin{bmatrix} (0.97, 0.26, -0.01) \\ (0.00, 0.05, 1.0) \\ (0.26, -0.96, 0.05) \end{bmatrix}$	$\begin{bmatrix} 10911 \\ 11037 \\ 11010 \end{bmatrix}$	$\begin{bmatrix} 10619 \\ 10745 \\ 10718 \end{bmatrix}$
	2^+	52.3	52.3	1.00	11060	10769
$ns\bar{c}\bar{b}$	0^+	$\begin{pmatrix} -124.8 & 86.7 & -0.2 & 0.0 \\ 86.7 & -138.8 & 0.6 & -0.6 \\ -0.2 & 0.6 & -5.6 & 86.7 \\ 0.0 & -0.6 & 86.7 & 62.4 \end{pmatrix}$	$\begin{bmatrix} -218.8 \\ 121.5 \\ -64.7 \\ -44.8 \end{bmatrix}$	$\begin{bmatrix} (0.68, -0.74, 0.00, 0.00) \\ (0.00, 0.00, 0.56, 0.83) \\ (0.02, 0.01, -0.83, 0.56) \\ (0.73, 0.68, 0.02, -0.01) \end{bmatrix}$	$\begin{bmatrix} 7461 \\ 7801 \\ 7615 \\ 7635 \end{bmatrix}$	$\begin{bmatrix} 7158 \\ 7498 \\ 7312 \\ 7332 \end{bmatrix}$
	1^+	$\begin{pmatrix} -89.6 & -50.1 & 35.6 & 0.1 & 0.6 & 0.0 \\ -50.1 & -3.2 & 42.0 & 0.0 & 1.2 & 0.3 \\ 35.6 & 42.0 & -79.8 & 1.2 & 0.3 & 1.4 \\ 0.1 & 0.0 & 1.2 & 6.4 & 16.8 & -50.1 \\ 0.6 & 1.2 & 0.3 & 16.8 & 18.0 & 35.6 \\ 0.0 & 0.3 & 1.4 & -50.1 & 35.6 & 44.8 \end{pmatrix}$	$\begin{bmatrix} -149.8 \\ 86.0 \\ -49.2 \\ -46.4 \\ 30.2 \\ 25.7 \end{bmatrix}$	$\begin{bmatrix} (0.69, 0.41, -0.60, 0.01, -0.01, 0.01) \\ (0.00, 0.01, 0.01, -0.45, 0.32, 0.83) \\ (-0.60, -0.01, -0.70, 0.26, -0.17, 0.21) \\ (0.24, 0.00, 0.29, 0.62, -0.45, 0.51) \\ (0.06, -0.20, -0.07, -0.57, -0.79, 0.00) \\ (0.31, -0.89, -0.25, 0.12, 0.18, 0.01) \end{bmatrix}$	$\begin{bmatrix} 7530 \\ 7766 \\ 7631 \\ 7634 \\ 7710 \\ 7706 \end{bmatrix}$	$\begin{bmatrix} 7227 \\ 7462 \\ 7327 \\ 7330 \\ 7407 \\ 7402 \end{bmatrix}$
	2^+	$\begin{pmatrix} 38.2 & -0.3 \\ -0.3 & 65.2 \end{pmatrix}$	$\begin{bmatrix} 65.2 \\ 38.2 \end{bmatrix}$	$\begin{bmatrix} (-0.01, 1.0) \\ (-1.0, -0.01) \end{bmatrix}$	$\begin{bmatrix} 7745 \\ 7718 \end{bmatrix}$	$\begin{bmatrix} 7442 \\ 7415 \end{bmatrix}$

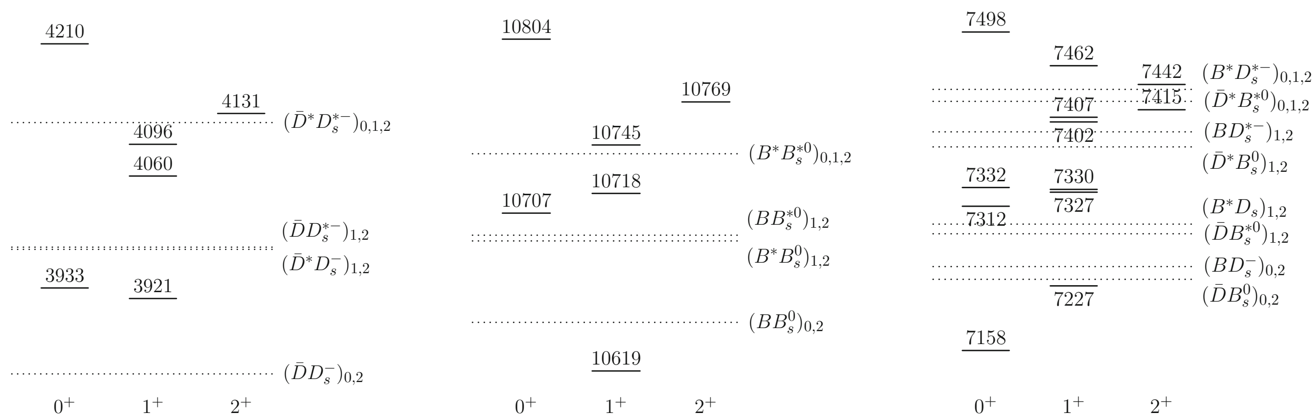

Fig. 3 The estimated masses (units: MeV) of the $ns\bar{c}\bar{c}$ (left), $ns\bar{b}\bar{b}$ (middle), and $ns\bar{c}\bar{b}$ (right) systems in the threshold scheme. The dotted lines are thresholds of the possible decay channels. When the total spin of a tetraquark is equal to a subscript of the symbol for a meson–meson channel, the corresponding S- or D-wave decay is allowed

Table 8 Numerical results for the $ss\bar{c}\bar{c}$, $ss\bar{b}\bar{b}$, and $ss\bar{c}\bar{b}$ systems in units of MeV. The masses in the sixth column are estimated with the masses of effective quarks and those in the last column with the thresholds of $D_sD_s/B_sB_s/D_sB_s$

System	J^P	$\langle H_{CM}\rangle$	Eigenvalues	Eigenvectors	Mass	$D_sD_s/B_sB_s/D_sB_s$
$ss\bar{c}\bar{c}$	0^+	$\begin{pmatrix} -40.3 & 131.3 \\ 131.3 & 46.8 \end{pmatrix}$	$\begin{bmatrix} 141.6 \\ -135.1 \end{bmatrix}$	$\begin{bmatrix} (0.59, 0.81) \\ (-0.81, 0.59) \end{bmatrix}$	$\begin{bmatrix} 4672 \\ 4395 \end{bmatrix}$	$\begin{bmatrix} 4293 \\ 4016 \end{bmatrix}$
	1^+	-4.5	-4.5	1.00	4526	4146
	2^+	66.9	66.9	1.00	4597	4218
$ss\bar{b}\bar{b}$	0^+	$\begin{pmatrix} 0.3 & 45.1 \\ 45.1 & 37.2 \end{pmatrix}$	$\begin{bmatrix} 67.4 \\ -30.0 \end{bmatrix}$	$\begin{bmatrix} (0.56, 0.83) \\ (-0.83, 0.56) \end{bmatrix}$	$\begin{bmatrix} 11254 \\ 11157 \end{bmatrix}$	$\begin{bmatrix} 10875 \\ 10777 \end{bmatrix}$
	1^+	12.5	12.5	1.00	11199	10820
	2^+	37.1	37.1	1.00	11224	10844
$ss\bar{c}\bar{b}$	0^+	$\begin{pmatrix} -22.1 & 88.2 \\ 88.2 & 38.8 \end{pmatrix}$	$\begin{bmatrix} 101.6 \\ -85.0 \end{bmatrix}$	$\begin{bmatrix} (0.58, 0.81) \\ (-0.81, 0.58) \end{bmatrix}$	$\begin{bmatrix} 7960 \\ 7774 \end{bmatrix}$	$\begin{bmatrix} 7581 \\ 7394 \end{bmatrix}$
	1^+	$\begin{pmatrix} -9.3 & 16.6 & -50.9 \\ 16.6 & 1.9 & 35.2 \\ -50.9 & 35.2 & 21.2 \end{pmatrix}$	$\begin{bmatrix} -65.5 \\ 65.4 \\ 13.8 \end{bmatrix}$	$\begin{bmatrix} (0.67, -0.47, 0.58) \\ (-0.48, 0.32, 0.81) \\ (0.57, 0.82, 0.01) \end{bmatrix}$	$\begin{bmatrix} 7793 \\ 7924 \\ 7872 \end{bmatrix}$	$\begin{bmatrix} 7414 \\ 7545 \\ 7493 \end{bmatrix}$
	2^+	49.9	49.9	1.00	7908	7529

metry breaking. For example, the difference resulting from the thresholds BB_s^{*0} and $B^*B_s^0$ is $[(M_{B_s^*}-M_{B_s})-(M_{B_s}-M_B)+\frac{64}{3}(C_{\bar{b}n}-C_{\bar{b}s})]$, which vanishes in the limit $m_n=m_s$. In fact, what the above mentioned similarity reflects is also the underlying SU(3) flavor symmetry. More results will be discussed later.

3.4 Systems with strangeness $= -2$

The expressions of the CMI matrices for the systems $ss\bar{c}\bar{c}$ and $ss\bar{b}\bar{b}$ ($ss\bar{c}\bar{b}$) are the same as those for the system $nn\bar{c}\bar{c}$ ($nn\bar{c}\bar{b}$) with $I=1$, which has been shown in Table 1 (2). As a result, the spectrum for $ss\bar{Q}\bar{Q}$ has similar features to the isovector $nn\bar{Q}\bar{Q}$ case. One may find the estimated masses and their rough positions in Table 8 and Fig. 4, respectively. All the obtained tetraquarks except the tensor one have S-wave decay channels and probably are broad states. The 2^+ $ss\bar{c}\bar{c}$ ($ss\bar{c}\bar{b}$) is around the threshold of $D_s^{*-}D_s^{*-}$ ($D_s^{*-}B_s^{*0}$). Its decay is mainly through D wave and may be a narrow state. The 2^+ $ss\bar{b}\bar{b}$ is slightly above the threshold of $B_s^{*0}B_s^{*0}$ and may have narrower width compared with its 0^+ and 1^+ partner states.

4 Discussions

4.1 The spectrum

We have explored various tetraquark configurations in a model with the color-magnetic interaction, $QQ\bar{Q}\bar{Q}$ in [60], $QQ\bar{Q}\bar{q}$ in [61], and $cs\bar{c}\bar{s}$ in [62]. A general feature for the mass spectra of these systems is that both the lightest and the heaviest states are those with $J^P=0^+$. This observation is

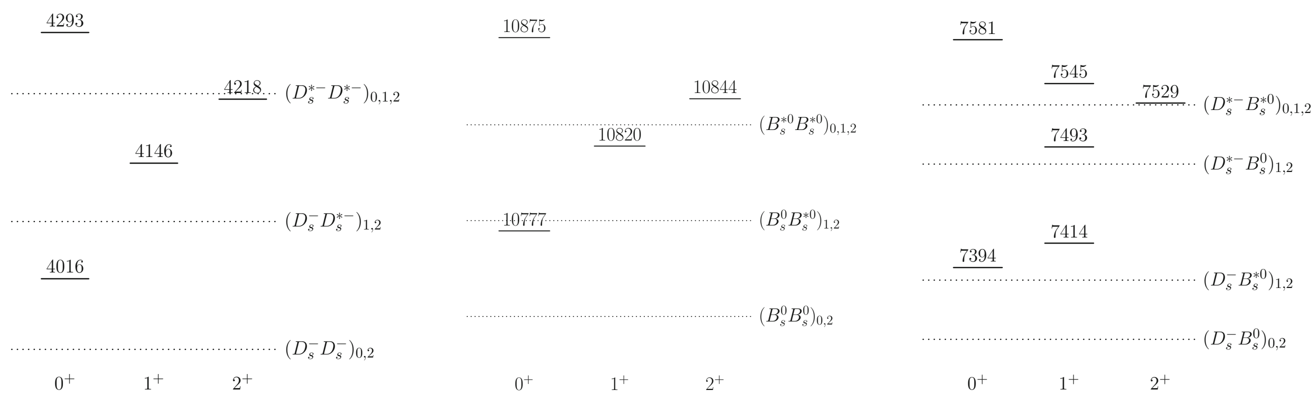

Fig. 4 The estimated masses (units: MeV) of the $ss\bar{c}\bar{c}$ (left), $ss\bar{b}\bar{b}$ (middle), and $ss\bar{c}\bar{b}$ (right) systems in the threshold scheme. The dotted lines are thresholds of the possible decay channels. When the total spin of a tetraquark is equal to a subscript of the symbol for a meson–meson channel, the corresponding S- or D-wave decay is allowed

also valid for the $nn\bar{c}\bar{b}$, $ns\bar{c}\bar{b}$, $ss\bar{c}\bar{c}$, $ss\bar{b}\bar{b}$, and $ss\bar{c}\bar{b}$ systems. In the $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, $ns\bar{c}\bar{c}$, and $ns\bar{b}\bar{b}$ cases, now the quantum numbers of all the lightest states are $J^P = 1^+$. Since the meson–meson channels with the lowest threshold have the quantum numbers $J^P = 0^+$, the stable scalar tetraquarks must be below such thresholds. Because of the angular momentum conservation, the axial-vector tetraquarks above the lowest thresholds may also be stable. This feature is unique for the $qq\bar{Q}\bar{Q}$ systems.

By comparing the diagonal elements of $\langle H_{CM}\rangle$ with the eigenvalues, from Tables 6, 7, and 8, we may understand the color mixing effects for various tetraquark states. The mass shifts due to the color mixing range from several MeVs to a value around 100 MeV. The effects for the $(nn\bar{c}\bar{c})^{I=1}$, $ns\bar{c}\bar{c}$, and $ss\bar{c}\bar{c}$ states with $J^P = 0^+$ are large while those for the $(nn\bar{b}\bar{b})^{I=0}$ and $ns\bar{b}\bar{b}$ states with $J^P = 1^+$ are small. In a given system, the effects for the 0^+ states are usually larger than the effects for the 1^+ states.

For multiquark masses, the obtained values are just estimations which rely on the parameters extracted from the conventional hadrons and the adopted reference masses. As noted in Ref. [69], some dynamical contributions are not appropriately included in the present method and the tetraquark masses need to be improved in a more elaborate model. One may guess that not all the studied tetraquarks exist. However, if one state could be observed, its partner states may be searched for with the relative positions shown in Figs. 2, 3, and 4. Such a study can be used to test the present model.

4.2 Mass relations

In principle, one may use the Gell-Mann–Okubo mass formula to discuss the mass splittings in the same representation of flavor SU(3). In the present studied systems, the mixing between two flavor representations occurs and it is difficult to find a general mass formula for all the states. However, it is interesting that the equal mass splitting relation $m_{nn\bar{Q}\bar{Q}}+m_{ss\bar{Q}\bar{Q}} = 2m_{ns\bar{Q}\bar{Q}}$ in 6_f still holds for some states. In the $qq\bar{c}\bar{c}$ case, the sets $[m_{nn\bar{c}\bar{c}}, m_{ns\bar{c}\bar{c}}, m_{ss\bar{c}\bar{c}}] = [4128, 4210, 4293]$ MeV and [3850, 3933, 4016] MeV for the 0^+ states and the set $[m_{nn\bar{c}\bar{c}}, m_{ns\bar{c}\bar{c}}, m_{ss\bar{c}\bar{c}}] = [4044, 4131, 4218]$ MeV for the 2^+ states satisfy this relation. The set $[m_{nn\bar{c}\bar{c}}, m_{ns\bar{c}\bar{c}}, m_{ss\bar{c}\bar{c}}] = [3973, 4060, 4146]$ MeV for the 1^+ states also satisfies this relation. In the $qq\bar{b}\bar{b}$ case, the sets satisfying this relation are: $[m_{nn\bar{b}\bar{b}}, m_{ns\bar{b}\bar{b}}, m_{ss\bar{b}\bar{b}}] = [10734, 10804, 10875]$ MeV and [10637, 10707, 10777] MeV for the 0^+ states, [10694, 10769, 10844] for the 2^+ states, and [10617, 10718, 10820] MeV and [10671, 10745, 10820] MeV for the 1^+ states. In the $qq\bar{c}\bar{b}$ case, the relation is roughly satisfied for the sets: $[m_{nn\bar{c}\bar{b}}, m_{ns\bar{c}\bar{b}}, m_{ss\bar{c}\bar{b}}] = [7428, 7498, 7581]$ MeV and [7241, 7312, 7394] MeV for the 0^+ states, [7367, 7442, 7529] MeV for the 2^+ states, and [7393, 7462, 7545] MeV, [7332, 7407, 7493] MeV, and [7258, 7330, 7414] MeV for the 1^+ states.

If one checks the mass formula in the estimation procedure, one finds that the above mass relations actually reflect the flavor SU(3) symmetry and its breaking. For example, for the 2^+ $qq\bar{Q}\bar{Q}$ systems, the relation is the occasional result of $C_{nn} + C_{ss} = 2C_{ns}$ from which one may recover the relations $2M_N + 2M_\Xi = 3M_\Lambda + M_\Sigma$ and $M_{\Sigma^*} - M_\Delta = M_{\Xi^*} - M_{\Sigma^*} = M_{\Omega_c} - M_{\Xi^*}$. Conversely, it is easy to understand the relation $C_{nn} + C_{ss} = 2C_{ns}$ in the CMI model from the flavor symmetry. In principle, the equal mass relations in the mixing case should also be from the relation $C_{nn} + C_{ss} = 2C_{ns}$ and thus from the flavor symmetry. As a byproduct of this relation, we get $\Sigma^* - \Delta = \Xi - \Sigma$.

4.3 Comparison with other work

As mentioned in Sect. 1, there is also other work as regards $qq\bar{Q}\bar{Q}$ tetraquark states. Various approaches such as the nonrelativistic quark model [24,25], the relativistic quark model [32], and the QCD sum rule [41] have been used to explore the spectra. For comparison, we briefly list the obtained results in the present model and those in other theoretical methods in Table 9. One may consult Table V of Ref. [32] for more comparison. According to these two tables, most masses are below the upper limits of the present estimation (Scheme I) and are in the reasonable range. The upper limits of masses for the $ss\bar{Q}\bar{Q}$ systems in QCD sum rule seem to be high. Experimental investigations on tetraquark states may be used to test theoretical methods according to their predictions.

The isoscalar T_{cc} is especially interesting. It is the most promising tetraquark to be found first. Most work gives a mass around 3900 MeV. In the present work, we consider the configuration mixing and get a lower mass around 3780 MeV (Scheme II). A comparable value 3764 MeV was obtained in a chiral constituent quark model in Ref. [70]. Whether this T_{cc} is stable enough needs experimental judgment. Because of the limitations of the present model (it is not a dynamical model, it has problems of parameters, and so on), probably the mass is underestimated.

If all the predicted masses are 100 MeV underestimated, many "stable" states would be unstable. However, the lowest $I(J^P) = 0(0^+)$ T_{cb} and the lowest $I(J^P) = 0(1^+)$ T_{bb} are still below their respective thresholds of open-charm/bottom decay channels, $B\bar{D}$ (7146 MeV) and BB^* (10604 MeV), and seem to be still stable. To search for these exotic tetraquarks in addition to the T_{cc} is also called for. Although the present model is oversimplified, the basic features of spectra should be roughly reasonable.

4.4 Production and decay

The production of the $qq\bar{Q}\bar{Q}$ tetraquark states needs high energy processes. For example, the C.M. energy for the production of a $ss\bar{b}\bar{b}$ ($ss\bar{c}\bar{b}$) at an electron-positron collider should be $\sqrt{s} > 22$ ($\sqrt{s} > 15$) GeV. Of these exotic states, the production of the lowest T_{cc} at various facilities (Tevatron, RHIC, LHC, KEK) has been considered [38–40,71–74]. Because of its clean background, the electron-positron collision experiment has its advantage in searching for T_{cc}. At Belle/BelleII, the $ns\bar{c}\bar{c}$ tetraquarks can also be searched for.

From Figs. 2, 3, and 4, it is easy to get a feature about the rearrangement decays and the stability of the possible tetraquarks $qq\bar{Q}\bar{Q}$. For the lowest stable states, the rearrangement decay channels are not opened. One has to adopt weak or electromagnetic decay modes to search for them: (1) for the lowest $T_{cc}^{I=0,J=1}$, one may use $D^{*-}K^+\pi^-$ or $D^-\bar{D}^0\gamma$ [40,75]; (2) for the lowest $T_{bb}^{I=0,J=1}$, one may use $\bar{D}\pi B^*$, $\bar{D}^0D_s^+B^{*0}$, or $D^{*-}D_s^{*+}B^{*+}$; (3) for the lowest $T_{cb}^{I=0,J=0}$ ($T_{cb}^{I=0,J=1}$), one may use $\bar{D}\bar{D}\pi$ or $BK\pi$ ($\bar{D}\bar{D}^*\pi$ or $B^*K\pi$); (4) for the lowest $(ns\bar{c}\bar{c})^{J=1}$, one may use $D_s^{*-}K\pi$, $\bar{D}^*K\bar{K}$, or $\bar{D}D_s^-\gamma$; (5) for the lowest $(ns\bar{b}\bar{b})^{J=1}$, one may use $\bar{D}\pi B_s^{*0}$, $B^*D_s^-\pi^+$, or $B^*D_s^{(*)+}D_s^{(*)-}$; (6) for the lowest $(ns\bar{c}\bar{b})^{J=0}$, one may use $K\pi B_s^0$, $BK\bar{K}$, $\bar{D}D_s^-\pi^+$, or $\bar{D}D_s^{(*)+}D_s^{(*)-}$; and (7) for the lowest $(ns\bar{c}\bar{b})^{J=1}$, one may use $K\pi B_s^{*0}$, $B^*K\bar{K}$, $\bar{D}\pi D_s^{*-}$, $\bar{D}^*D_s^-\pi^+$, or $\bar{D}^*D_s^{(*)+}D_s^{(*)-}$. For higher states, the two-body meson–meson channels are the dominant decay modes. Searching for them in meson–meson channels can probably give interesting exotic signals. In addition, their transition to lower tetraquarks is also allowed, e.g. an $I(J^P) = 1(1^+)$ $nn\bar{c}\bar{b}$ state may decay to an $I(J^P) = 0(0^+)$ $nn\bar{c}\bar{b}$ state by emitting a pion. However, the decay into OZI-allowed (light-baryon)+(heavy-antibaryon) final state is forbidden by kinematics even if the doubly heavy baryons are observed.

4.5 "Good" diquark and stable tetraquarks

The studies on various multiquark states can be found in the literature. Usually, the multi-body problem is simplified by assuming the existence of possible substructures, e.g. diquark or triquark. In the conventional baryons, the color-antitriplet diquark with spin = 0 is called "good" diquark, while that with spin = 1 is called "bad" one because the color–spin interaction in the former (latter) case is attractive (repulsive). An example, because of their difference, is the mass splitting between Σ_c and Λ_c. In the multiquark study, one expects that states containing the scalar diquark have a lower mass and are easy to search for experimentally while those with the axial-vector diquark should be broad resonances even if they exist and are difficult to search for.

Because the interaction related with heavy quarks is suppressed, the properties of the tetraquarks $qq\bar{Q}\bar{Q}$ are determined mainly by the light diquark. In the present work, the number of states is significantly reduced if we consider tetraquarks containing the light "good" diquark only. Now the retained color–spin bases in Eq. (7) are $\phi_2\chi_5$ (total spin = 1) and $\phi_2\chi_6$ (total spin = 0). From Tables 1, 2 and 3, there is no "good" light diquark in the $ss\bar{Q}\bar{Q}$ tetraquark states and one needs to discuss only eight lowest states: 1^+ $nn\bar{c}\bar{c}$, 1^+ $nn\bar{b}\bar{b}$, 0^+ and 1^+ $nn\bar{c}\bar{b}$, 1^+ $ns\bar{c}\bar{c}$, 1^+ $ns\bar{b}\bar{b}$, and 0^+ and 1^+ $ns\bar{c}\bar{b}$. Without channel coupling, their masses are higher than the lowest states in Figs. 2 and 3. We list their values in Table 10 with the threshold scheme. Now, the 1^+ $nn\bar{c}\bar{b}$ state and the two $ns\bar{c}\bar{b}$ states are slightly above their corresponding thresholds of rearrangement decay channels. Stable states with the configurations $nn\bar{c}\bar{c}$, $nn\bar{b}\bar{b}$, $nn\bar{c}\bar{b}$, $ns\bar{c}\bar{c}$, and $ns\bar{b}\bar{b}$ are still possible.

However, as shown in Ref. [39], the color-sextet diquark with spin = 1 also has weak attraction but it does not exist in the conventional baryons. The contributions from such diquark and other diquarks other than the "good" diquark

Table 9 Comparison of results in various methods for tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration

System	J	Previous work				Our results	
		[24]	[25]	[41,42]	[32]	Scheme 1	Scheme 2
$(nn\bar{c}\bar{c})^{I=1}$	0	–	–	–	4056	4078/4356	3850/4128
	1	–	–	–	4079	4201	3973
	2	–	–	–	4118	4271	4044
$(nn\bar{c}\bar{c})^{I=0}$	1	3931	3892–3916	–	3935	4007/4204	3779/3977
$(nn\bar{b}\bar{b})^{I=1}$	0	–	–	9900–10600	10648	10841/10937	10637/10734
	1	10712	–	9900–10500	10657	10875	10671
	2	10735	10710–10740	–	10673	10897	10694
$(nn\bar{b}\bar{b})^{I=0}$	1	10525	10482–10514	9900–10500	10502	10686/10821	10483/10617
$(nn\bar{c}\bar{b})^{I=1}$	0	–	–	7040–7380	7383	7457/7643	7241/7428
	1	–	–	7030–7390	7396/7403	7473/7548/7609	7258/7332/7393
	2	–	–	–	7422	7582	7367
$(nn\bar{c}\bar{b})^{I=0}$	0	7206	7153–7183	7040–7380	7239	7256/7429	7041/7213
	1	7244	7204–7231	7030–7390	7246	7321/7431/7516	7106/7215/7301
	2	7422	–	–	–	7530	7315
$ns\bar{c}\bar{c}$	0	–	–	3840–4300	4221	4236/4514	3933/4210
	1	–	–	3950–5070	4143/4239	4225/4363/4400	3921/4060/4096
	2	–	–	–	4271	4434	4131
$ns\bar{b}\bar{b}$	0	–	–	9900–10600	10802	10999/11095	10707/10804
	1	10680	10631–10665	9900–11100	10706/10809	10911/11010/11037	10619/10718/10745
	2	10816	–	–	10823	11060	10769
$ns\bar{c}\bar{b}$	0	–	–	–	7444/7540	7461/7615/7635/7801	7158/7312/7332/7498
	1	–	–	–	7451/7552/7555	7530/7631/7634/ 7706/7710/7766	7227/7327/7330/ 7402/7407/7462
	2	7496	–	–	7572	7718/7745	7415/7442
$ss\bar{c}\bar{c}$	0	–	–	4160–5200	4359	4395/4672	4016/4293
	1	–	–	4010–5160	4375	4526	4146
	2	–	–	–	4402	4597	4218
$ss\bar{b}\bar{b}$	0	–	–	9900–11200	10932	11157/11254	10777/10875
	1	–	–	10100–11300	10939	11199	10820
	2	–	–	–	10950	11224	10844
$ss\bar{c}\bar{b}$	0	–	–	7120–7450	7673	7774/7960	7394/7581
	1	–	–	7210–7490	7683/7684	7793/7872/7924	7414/7493/7545
	2	–	–	–	7701	7908	7529

lead to more complex mass spectra of possible multiquark states. In the present study, we use "diquark" to denote the two-quark cluster which may have various quantum numbers. Our analysis shows that the channel coupling between the $6_c \otimes \bar{6}_c$ and $\bar{3}_c \otimes 3_c$ color configurations further lowers the $qq\bar{Q}\bar{Q}$ state where the qq pair forms a scalar diquark.

5 Summary

In this work, we systematically study the mass spectra of tetraquark states with the $qq\bar{Q}\bar{Q}$ configuration by using a simple color-magnetic model. All possible quantum numbers without orbital excitation are considered. We find that the color mixing effects are relatively important for the 0^+ states. The effect for the lowest T_{cc} (T_{bb}) is about 30 (7) MeV and is not large. If the results shown in Figs. 2, 3, and 4 are all reasonable, two types of $qq\bar{Q}\bar{Q}$ tetraquarks are probably stable: (1) $T_{cc}^{I=0}$, $T_{bb}^{I=0}$, $T_{cb}^{I=0}$, $ns\bar{c}\bar{c}$, $ns\bar{b}\bar{b}$, and $ns\bar{c}\bar{b}$ with $J^P = 1^+$; and (2) $T_{cb}^{I=0}$ and $ns\bar{c}\bar{b}$ with $J^P = 0^+$. The feature that the stable states could have $J = 1$ is unique compared with other tetraquark structures $QQ\bar{Q}\bar{Q}$, $QQ\bar{Q}\bar{q}$, and $cs\bar{c}\bar{s}$. Based on our estimations, possible strong decay patterns are briefly discussed. Up to now, none of the exotic

Table 10 Numerical results (units: MeV) for the systems containing light "good" diquark only. The tetraquark masses in the last column are estimated with the thresholds of $DD/BB/BD/DD_s/BB_s/DB_s$

System	J^P	$\langle H_{CM}\rangle$	$DD/BB/BD$ $/DD_s/BB_s/DB_s$
$(nn\bar{c}\bar{c})^{I=0}$	1^+	−132.3	3813
$(nn\bar{b}\bar{b})^{I=0}$	1^+	−138.7	10487
$(nn\bar{c}\bar{b})^{I=0}$	0^+	−172.8	7113
$(nn\bar{c}\bar{b})^{I=0}$	1^+	−137.6	7149
$(ns\bar{c}\bar{c})$	1^+	−84.3	3964
$(ns\bar{b}\bar{b})$	1^+	−84.3	10626
$(ns\bar{c}\bar{b})$	0^+	−124.8	7252
$(ns\bar{c}\bar{b})$	1^+	−89.6	7287

$qq\bar{Q}\bar{Q}$ states has been observed and more detailed investigations on their properties are still needed. Hopefully, our study on these interesting tetraquark states may be helpful to the future experimental searches.

Acknowledgements This paper is supported by the National Natural Science Foundation of China under Grant no. 11175073, no. 11275115, no. 11222547, no. 11261130311 and 973 program. Xiang Liu is also supported by the National Program for Support of Top-notch Young Professionals and the Fundamental Research Funds for the Central Universities.

References

1. S.K. Choi et al. [Belle Collaboration], Observation of a narrow charmonium—like state in exclusive $B^\pm \to K^\pm\pi^+\pi^- J/\psi$ decays. Phys. Rev. Lett. **91**, 262001 (2003). arXiv:hep-ex/0309032
2. K. Abe et al. [Belle Collaboration], Observation of a near-threshold $\omega J/\psi$ mass enhancement in exclusive $B \to K\omega J/\psi$ decays. Phys. Rev. Lett. **94**, 182002 (2005). arXiv:hep-ex/0408126
3. T. Aaltonen et al. [CDF Collaboration], Evidence for a narrow near-threshold structure in the $J/\psi\phi$ mass spectrum in $B^+ \to J/\psi\phi K^+$ decays. Phys. Rev. Lett. **102**, 242002 (2009). arXiv:0903.2229 [hep-ex]
4. T. Aaltonen et al. [CDF Collaboration], Observation of the $Y(4140)$ structure in the $J/\psi\,\phi$ Mass Spectrum in $B^\pm \to J/\psi\,\phi K$ decays. arXiv:1101.6058 [hep-ex]
5. S.K. Choi et al. [Belle Collaboration], Observation of a resonance-like structure in the $\pi^\pm\psi'$ mass distribution in exclusive $B \to K\pi^\pm\psi'$ decays. Phys. Rev. Lett. **100**, 142001 (2008). arXiv:0708.1790 [hep-ex]
6. K. Chilikin et al. [Belle Collaboration], Observation of a new charged charmoniumlike state in $\bar{B}^0 \to J/\psi K^-\pi^+$ decays. Phys. Rev. D **90**(11), 112009 (2014). arXiv:1408.6457 [hep-ex]
7. A. Bondar et al. [Belle Collaboration], Observation of two charged bottomonium-like resonances in $\Upsilon(5S)$ decays. Phys. Rev. Lett. **108**, 122001 (2012). arXiv:1110.2251 [hep-ex]
8. V.M. Abazov et al. [D0 Collaboration], Evidence for a $B_s^0\pi^\pm$ state. Phys. Rev. Lett. **117**(2), 022003 (2016). arXiv:1602.07588 [hep-ex]
9. X. Liu, An overview of XYZ new particles. Chin. Sci. Bull. **59**, 3815 (2014). arXiv:1312.7408 [hep-ph]
10. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, The hidden-charm pentaquark and tetraquark states. Phys. Rep. **639**, 1 (2016). arXiv:1601.02092 [hep-ph]
11. H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu, A review of the open charm and open bottom systems. Rep. Prog. Phys. **80**(7), 076201 (2017). arXiv:1609.08928 [hep-ph]
12. R.F. Lebed, R.E. Mitchell, E.S. Swanson, Heavy-quark QCD exotica. Prog. Part. Nucl. Phys. **93**, 143 (2017). arXiv:1610.04528 [hep-ph]
13. A. Esposito, A. Pilloni, A.D. Polosa, Multiquark resonances. Phys. Rep. **668**, 1 (2016). arXiv:1611.07920 [hep-ph]
14. A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, S. Yasui, Exotic hadrons with heavy flavors: X, Y, Z, and related states. PTEP **2016**(6), 062C01 (2016). arXiv:1603.09229 [hep-ph]
15. J.M. Richard, Exotic hadrons: review and perspectives. Few Body Syst. **57**(12), 1185 (2016). arXiv:1606.08593 [hep-ph]
16. J.P. Ader, J.M. Richard, P. Taxil, Do narrow heavy multi—quark states exist? Phys. Rev. D **25**, 2370 (1982)
17. J.l. Ballot, J.M. Richard, Four quark states in additive potentials. Phys. Lett. **123B**, 449 (1983)
18. H.J. Lipkin, A model independent approach to multi-quark bound states. Phys. Lett. B **172**, 242 (1986)
19. S. Zouzou, B. Silvestre-Brac, C. Gignoux, J.M. Richard, Four quark bound states. Z. Phys. C **30**, 457 (1986)
20. L. Heller, J.A. Tjon, On the existence of stable dimesons. Phys. Rev. D **35**, 969 (1987)
21. J. Carlson, L. Heller, J.A. Tjon, Stability of dimesons. Phys. Rev. D **37**, 744 (1988)
22. A.V. Manohar, M.B. Wise, Exotic $QQ\bar{q}\bar{q}$ states in QCD. Nucl. Phys. B **399**, 17 (1993). arXiv:hep-ph/9212236
23. B. Silvestre-Brac, C. Semay, Spectrum and decay properties of diquonia. Z. Phys. C **59**, 457 (1993)
24. B. Silvestre-Brac, C. Semay, Systematics of L = 0 $q^2\bar{q}^2$ systems. Z. Phys. C **57**, 273 (1993)
25. C. Semay, B. Silvestre-Brac, Diquonia and potential models. Z. Phys. C **61**, 271 (1994)
26. S. Pepin, F. Stancu, M. Genovese, J.M. Richard, Tetraquarks with color blind forces in chiral quark models. Phys. Lett. B **393**, 119 (1997). arXiv:hep-ph/9609348
27. D.M. Brink, F. Stancu, Tetraquarks with heavy flavors. Phys. Rev. D **57**, 6778 (1998)
28. J. Schaffner-Bielich, A.P. Vischer, Charmlets. Phys. Rev. D **57**, 4142 (1998). arXiv:nucl-th/9710064
29. J. Vijande, A. Valcarce, K. Tsushima, Dynamical study of bf $QQ\bar{u}\bar{d}$ mesons. Phys. Rev. D **74**, 054018 (2006). arXiv:hep-ph/0608316
30. Y. Cui, X.L. Chen, W.Z. Deng, S.L. Zhu, The possible heavy tetraquarks $qQ\bar{q}\bar{Q}$, $qq\bar{Q}\bar{Q}$ and $qQ\bar{Q}\bar{Q}$. HEPNP **31**, 7 (2007). arXiv:hep-ph/0607226
31. M. Zhang, H.X. Zhang, Z.Y. Zhang, $QQ\bar{q}\bar{q}$ four-quark bound states in chiral SU(3) quark model. Commun. Theor. Phys **50**, 437 (2008). arXiv:0711.1029 [nucl-th]
32. D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, Masses of tetraquarks with two heavy quarks in the relativistic quark model. Phys. Rev. D **76**, 114015 (2007). arXiv:0706.3853 [hep-ph]
33. F.S. Navarra, M. Nielsen, S.H. Lee, QCD sum rules study of $QQ\bar{u}\bar{d}$ mesons. Phys. Lett. B **649**, 166 (2007). arXiv:hep-ph/0703071

34. S.H. Lee, S. Yasui, W. Liu, C.M. Ko, Charmed exotics in heavy ion collisions. Eur. Phys. J. C **54**, 259 (2008). arXiv:0707.1747 [hep-ph]
35. Y. Yang, C. Deng, J. Ping, T. Goldman, S-wave $QQ\bar{q}\bar{q}$ state in the constituent quark model. Phys. Rev. D **80**, 114023 (2009)
36. J. Vijande, A. Valcarce, N. Barnea, Exotic meson–meson molecules and compact four-quark states. Phys. Rev. D **79**, 074010 (2009). arXiv:0903.2949 [hep-ph]
37. T.F. Carames, A. Valcarce, J. Vijande, Doubly charmed exotic mesons: a gift of nature? Phys. Lett. B **699**, 291 (2011)
38. J. Vijande, A. Valcarce, J.M. Richard, Adiabaticity and color mixing in tetraquark spectroscopy. Phys. Rev. D **87**(3), 034040 (2013). arXiv:1301.6212 [hep-ph]
39. T. Hyodo, Y.R. Liu, M. Oka, K. Sudoh, S. Yasui, Production of doubly charmed tetraquarks with exotic color configurations in electron-positron collisions. Phys. Lett. B **721**, 56 (2013). arXiv:1209.6207 [hep-ph]
40. T. Hyodo, Y.R. Liu, M. Oka, S. Yasui, arXiv:1708.05169 [hep-ph]
41. M.L. Du, W. Chen, X.L. Chen, S.L. Zhu, Exotic $QQ\bar{q}\bar{q}$, $QQ\bar{q}\bar{s}$ and $QQ\bar{s}\bar{s}$ states. Phys. Rev. D **87**(1), 014003 (2013). arXiv:1209.5134 [hep-ph]
42. W. Chen, T.G. Steele, S.L. Zhu, Exotic open-flavor $bc\bar{q}\bar{q}$, $bc\bar{s}\bar{s}$ and $qc\bar{q}\bar{b}$, $sc\bar{s}\bar{b}$ tetraquark states. Phys. Rev. D **89**(5), 054037 (2014). arXiv:1310.8337 [hep-ph]
43. C. Stewart, R. Koniuk, Hadronic molecules in lattice QCD. Phys. Rev. D **57**, 5581 (1998). arXiv:hep-lat/9803003
44. C. Michael et al. [UKQCD Collaboration], Two heavy-light mesons on a lattice. Phys. Rev. D **60**, 054012 (1999). arXiv:hep-lat/9901007
45. M.S. Cook, H.R. Fiebig, A Lattice study of interaction mechanisms in a heavy light meson meson system. arXiv:hep-lat/0210054
46. T. Doi, T.T. Takahashi, H. Suganuma, Meson–meson and meson-baryon interactions in lattice QCD. AIP Conf. Proc. **842**, 246 (2006). arXiv:hep-lat/0601008
47. W. Detmold, K. Orginos, M.J. Savage, BB Potentials in Quenched Lattice QCD. Phys. Rev. D **76**, 114503 (2007). arXiv:hep-lat/0703009
48. M. Wagner [ETM and Y Collaborations], Forces between static-light mesons. PoS LATTICE **2010**, 162 (2010). arXiv:1008.1538 [hep-lat]
49. G. Bali et al. [QCDSF Collaboration], Static-light meson–meson potentials. PoS LATTICE **2010**, 142 (2010). arXiv:1011.0571 [hep-lat]
50. M. Wagner [ETM Collaboration], Static-static-light-light tetraquarks in lattice QCD. Acta Phys. Polon. Supp. **4**, 747 (2011). arXiv:1103.5147 [hep-lat]
51. Z.S. Brown, K. Orginos, Tetraquark bound states in the heavy-light heavy-light system. Phys. Rev. D **86**, 114506 (2012). arXiv:1210.1953 [hep-lat]
52. Y. Ikeda et al., Charmed tetraquarks T_{cc} and T_{cs} from dynamical lattice QCD simulations. Phys. Lett. B **729**, 85 (2014). arXiv:1311.6214 [hep-lat]
53. B. Wagenbach, P. Bicudo, M. Wagner, Lattice investigation of heavy meson interactions. J. Phys. Conf. Ser. **599**(1), 012006 (2015). arXiv:1411.2453 [hep-lat]
54. A. Peters, P. Bicudo, K. Cichy, B. Wagenbach, M. Wagner, Exploring possibly existing $qq\bar{b}\bar{b}$ tetraquark states with $qq = ud, ss, cc$. PoS LATTICE **2015**, 095 (2016). arXiv:1508.00343 [hep-lat]
55. A. Peters, P. Bicudo, K. Cichy, M. Wagner, Investigation of $B\bar{B}$ four-quark systems using lattice QCD. J. Phys. Conf. Ser. **742**(1), 012006 (2016). arXiv:1602.07621 [hep-lat]
56. A. Peters, P. Bicudo, L. Leskovec, S. Meinel, M. Wagner, Lattice QCD study of heavy-heavy-light-light tetraquark candidates. PoS LATTICE **2016**, 104 (2016). arXiv:1609.00181 [hep-lat]
57. A. Francis, R.J. Hudspith, R. Lewis, K. Maltman, Doubly bottom strong-interaction stable tetraquarks from lattice QCD. Phys. Rev. Lett. **118**(14), 142001 (2017). arXiv:1607.05214 [hep-lat]
58. S. Ohkoda, Y. Yamaguchi, S. Yasui, K. Sudoh, A. Hosaka, Exotic mesons with double charm and bottom flavor. Phys. Rev. D **86**, 034019 (2012). arXiv:1202.0760 [hep-ph]
59. S. Sakai, L. Roca, E. Oset, Charm-beauty meson bound states from $B(B^*)D(D^*)$ and $B(B^*)\bar{D}(\bar{D}^*)$ interaction. arXiv:1704.02196 [hep-ph]
60. J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Heavy-flavored tetraquark states with the $QQ\bar{Q}\bar{Q}$ configuration. arXiv:1605.01134 [hep-ph]
61. K. Chen, X. Liu, J. Wu, Y.R. Liu, S.L. Zhu, Triply heavy tetraquark states with the $QQ\bar{Q}\bar{q}$ configuration. Eur. Phys. J. A **53**(1), 5 (2017). arXiv:1609.06117 [hep-ph]
62. J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, $X(4140)$, $X(4270)$, $X(4500)$ and $X(4700)$ and their $cs\bar{c}\bar{s}$ tetraquark partners. Phys. Rev. D **94**(9), 094031 (2016). arXiv:1608.07900 [hep-ph]
63. J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Hidden-charm pentaquarks and their hidden-bottom and B_c-like partner states. Phys. Rev. D **95**(3), 034002 (2017). arXiv:1701.03873 [hep-ph]
64. A. De Rujula, H. Georgi, S.L. Glashow, Hadron masses in a gauge theory. Phys. Rev. D **12**, 147 (1975)
65. H. Hogaasen, P. Sorba, The colour triplet qq anti-q cluster and pentaquark models. Mod. Phys. Lett. A **19**, 2403 (2004). arXiv:hep-ph/0406078
66. F. Buccella, H. Hogaasen, J.M. Richard, P. Sorba, Chromomagnetism, flavour symmetry breaking and S-wave tetraquarks. Eur. Phys. J. C **49**, 743 (2007). arXiv:hep-ph/0608001
67. S. Godfrey, N. Isgur, Mesons in a relativized quark model with chromodynamics. Phys. Rev. D **32**, 189 (1985)
68. C. Patrignani et al. [Particle Data Group], Review of particle physics. Chin. Phys. C **40**(10), 100001 (2016)
69. W. Park, A. Park, S.H. Lee, Dibaryons in a constituent quark model. Phys. Rev. D **92**(1), 014037 (2015). arXiv:1506.01123 [nucl-th]
70. J. Vijande, F. Fernandez, A. Valcarce, B. Silvestre-Brac, Tetraquarks in a chiral constituent quark model. Eur. Phys. J. A **19**, 383 (2004). arXiv:hep-ph/0310007
71. A. Del Fabbro, D. Janc, M. Rosina, D. Treleani, Production and detection of doubly charmed tetraquarks. Phys. Rev. D **71**, 014008 (2005). arXiv:hep-ph/0408258
72. S. Cho et al. [ExHIC Collaboration], Multi-quark hadrons from heavy ion collisions. Phys. Rev. Lett. **106**, 212001 (2011). arXiv:1011.0852 [nucl-th]
73. S. Cho et al. [ExHIC Collaboration], Studying exotic hadrons in heavy ion collisions. Phys. Rev. C **84**, 064910 (2011). arXiv:1107.1302 [nucl-th]
74. A. Esposito, M. Papinutto, A. Pilloni, A.D. Polosa, N. Tantalo, Doubly charmed tetraquarks in B_c and Ξ_{bc} decays. Phys. Rev. D **88**(5), 054029 (2013). arXiv:1307.2873 [hep-ph]
75. Y. Jin, S.Y. Li, Y.R. Liu, Z.G. Si, T. Yao, Search for a doubly charmed hadron at B factories. Phys. Rev. D **89**(9), 094006 (2014). arXiv:1401.6652 [hep-ph]