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On generalized Melvin solution for the Lie algebra E_6

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Abstract A multidimensional generalization of Melvin's solution for an arbitrary simple Lie algebra $\mathcal{G}$ is considered. The gravitational model in D dimensions, $D \geq 4$, contains n 2-forms and $l \geq n$ scalar fields, where n is the rank of $\mathcal{G}$. The solution is governed by a set of n functions $H_s(z)$ obeying n ordinary differential equations with certain boundary conditions imposed. It was conjectured earlier that these functions should be polynomials (the so-called fluxbrane polynomials). The polynomials $H_s(z)$, $s = 1, \ldots, 6$, for the Lie algebra E_6 are obtained and a corresponding solution for $l = n = 6$ is presented. The polynomials depend upon integration constants Q_s, $s = 1, \ldots, 6$. They obey symmetry and duality identities. The latter ones are used in deriving asymptotic relations for solutions at large distances. The power-law asymptotic relations for E_6-polynomials at large z are governed by the integer-valued matrix $\nu = A^{-1}(I + P)$, where A^{-1} is the inverse Cartan matrix, I is the identity matrix and P is a permutation matrix, corresponding to a generator of the Z_2-group of symmetry of the Dynkin diagram. The 2-form fluxes Φ^s, $s = 1, \ldots, 6$, are calculated.

1 Introduction

In this paper we deal with a multidimensional generalization of the Melvin solution [1] which was considered earlier in Ref. [2]. This solution is governed by a simple finite-dimensional Lie algebra. It is a special case of the so-called generalized fluxbrane solutions from [3]. For generalizations of the Melvin solution, fluxbrane solutions and their applications, see Refs. [4–33] and the references therein.

We remind the reader that Melvin's original solution in 4d space-time describes the gravitational field of a magnetic flux tube. The multidimensional analog of such a flux tube, supported by a certain configuration of fields of forms, is referred to as a fluxbrane (a "thickened brane" of magnetic flux). The appearance of fluxbrane solutions was motivated by superstring/M-theory models. A physical interest in such solutions is that they supply an appropriate background geometry for studying various processes involving branes, instantons, Kaluza–Klein monopoles, pair production of magnetically charged black holes and other configurations which can be studied via a special kind of Kaluza–Klein reduction ("modding technique") of a certain multidimensional model in the presence of $U(1)$ isometry subgroup.

The Melvin solution is geodesically complete [34]. Its group of isometry is $U(1) \times P(1,1)$, where $P(1,1)$ is 3-dimensional isometry group of 2-dimensional Minkowski space. $P(1,1)$ is semi-direct product of $O(1,1)$ and $\mathbb{R}^2$.

In Ref. [2] the electro-vacuum Melvin solution was generalized for the D-dimensional model which contains metric g, n 2-form fields $F^s = dA^s$ and l scalar fields φ^α. The model also includes n dilatonic coupling vectors belonging to $\mathbb{R}^l$. The D-dimensional warped product solution from Ref. [2] comprises two factor spaces: 1-dimensional subspace M_1 and a $(D-2)$-dimensional Ricci-flat subspace M_2. Here M_1 is either $\mathbb{R}$ or S^1. For $M_1 = S^1$ we have a cylindrically symmetric solution with the isometry group $U(1) \times \mathrm{Isom}(M_2)$, where $\mathrm{Isom}(M_2)$ is the isometry group of M_2.

The generalized fluxbrane solutions from Ref. [2] are governed by functions $H_s(z) > 0$ defined on the interval $(0, +\infty)$ which obey the non-linear differential equations

$$\frac{d}{dz}\left(\frac{z}{H_s}\frac{dd}{dz}H_s\right) = P_s \prod_{s'=1}^{n} H_{s'}^{-A_{ss'}}, \qquad (1.1)$$

with the following boundary conditions:

$$H_s(+0) = 1, \qquad (1.2)$$

$s = 1, \ldots, n$, where $P_s > 0$ for all s. Parameters P_s are proportional to Q_s^2, where Q_s are integration constants and

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$z = \rho^2$, where ρ is a radial parameter. The boundary condition (1.2) guarantees the absence of a conic singularity (in the metric) for $\rho = +0$. The integration constants Q_s are coinciding up to a sign with values of magnetic fields on the axis of the symmetry.

In this paper we assume that $(A_{ss'})$ is a Cartan matrix for some simple finite-dimensional Lie algebra $\mathcal{G}$ of rank n ($A_{ss} = 2$ for all s).

According to a conjecture suggested in [3], the solutions to Eqs. (1.1), (1.2) governed by the Cartan matrix $(A_{ss'})$ are polynomials:

$$H_s(z) = 1 + \sum_{k=1}^{n_s} P_s^{(k)} z^k, \tag{1.3}$$

where $P_s^{(k)}$ are constants ($P_s^{(1)} = P_s$). Here $P_s^{(n_s)} \neq 0$ and

$$n_s = 2 \sum_{s'=1}^{n} A^{ss'} \tag{1.4}$$

where we denote $(A^{ss'}) = (A_{ss'})^{-1}$. Integers n_s are components of a twice dual Weyl vector in the basis of simple co-roots [35].

The set of fluxbrane polynomials H_s defines a special solution to open Toda chain equations [36,37] corresponding to a simple finite-dimensional Lie algebra $\mathcal{G}$; see Ref. [38]. In Refs. [2,39] a program (in Maple) for calculation of these polynomials for classical series of Lie algebras (of A-, B-, C- and D-series) was suggested.

It should be noted that the open Toda chain corresponding to the Lie algebra $\mathcal{G}$ has a hidden symmetry group $G_T = \exp(\mathcal{G})$. The solution from Ref. [2] corresponding to this group is a special case of solutions from [3]. It may be obtained by using an 1-dimensional sigma-model [40–42] with $(2+l+n)$-dimensional target space. The isometry group of this target space $G_{\rm sm}$ (related to the sigma model) was studied in detail in [43]. For another more general setup with non-diagonal metrics (which is valid for flat M_2) see also [9]. The group $G_{\rm sm}$ is another hidden symmetry group related to our model. Here the Toda Lagrangian $L_{\rm T}$ may be obtained from the sigma-model one after integrating the Maxwell-type equations corresponding to potentials $\Phi^s(u) = A^s_\phi(u)$, where u is a radial variable and ϕ is a coordinate on M_1 ($0 < \phi < 2\pi$ for $M_1 = S^1$), and obtaining integration constants Q_s. The Toda Lagrangian $L_{\rm T} = L_{\rm T}(x, \dot{x}, Q)$ ($\dot{x} = \frac{dx}{du}$) is responsible for the equations of motion for 2 scale factors and l scalar fields described by $x = (x^a)$ for fixed $Q = (Q_s)$.

We note also that there are several multidimensional aspects of generalized Melvin solution from Ref. [2]: (1) the space-time dimension D (for Melvin's solution $D = 4$), (2) the rank of the Toda group $G_{\rm T}$ which is equal to n (in Melvin's case $n = 1$) and (3) the dimension of the target space of the corresponding sigma-model which is equal to $N = n + l + 2$ (in Melvin's case $N = 3$).

Here we verify the conjecture from Ref. [3] for the Lie algebra E_6. In Sect. 2 the generalized Melvin solution for an arbitrary simple finite-dimensional Lie algebra $\mathcal{G}$ is considered. The exact solution for the Lie algebra E_6 is presented in Sect. 3, while the fluxbrane polynomials are listed in the appendix. Here duality relations for the polynomials $H_s(z)$ and asymptotic formulas for $z \to +\infty$ are presented, as well as the asymptotics for the solutions at large distances and a calculation of flux integrals. We find that any flux Φ^s depends upon the integration constant Q_s and does not depend upon the other constants $Q_{s'}$, $s' \neq s$. The flux Φ^s is proportional to $n_s Q_s^{-1}$, where n_s are integer numbers (1.4): $n_s = 16, 30, 42, 30, 16, 22$ for $s = 1, 2, 3, 4, 5, 6$, respectively.

2 The main solution

We consider a model governed by the action

$$S = \int d^D x \sqrt{|g|} \Big\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \frac{1}{2} \sum_{s=1}^{n} \exp[2\lambda_s(\varphi)](F^s)^2 \Big\}, \tag{2.1}$$

where $g = g_{MN}(x) dx^M \otimes dx^N$ is a metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $F^s = dA^s = \frac{1}{2} F^s_{MN} dx^M \wedge dx^N$ is a 2-form, λ_s is a 1-form on $\mathbb{R}^l$: $\lambda_s(\varphi) = \lambda_{s\alpha}\varphi^\alpha$, $s = 1, \ldots, n$; $\alpha = 1, \ldots, l$. In (2.1), we denote $|g| = |\det(g_{MN})|$, $(F^s)^2 = F^s_{M_1 M_2} F^s_{N_1 N_2} g^{M_1 N_1} g^{M_2 N_2}$, $s = 1, \ldots, n$.

Here we consider a family of exact solutions to the field equations corresponding to the action (2.1) and depending on one variable ρ. The solutions are defined on the manifold

$$M = (0, +\infty) \times M_1 \times M_2, \tag{2.2}$$

where M_1 is a one-dimensional manifold (say S^1 or $\mathbb{R}$) and M_2 is a (D-2)-dimensional Ricci-flat manifold. The solution reads [2]

$$g = \Big(\prod_{s=1}^{n} H_s^{2h_s/(D-2)} \Big) \Big\{ w d\rho \otimes d\rho + \Big(\prod_{s=1}^{n} H_s^{-2h_s} \Big) \rho^2 d\phi \otimes d\phi + g^2 \Big\}, \tag{2.3}$$

$$\exp(\varphi^\alpha) = \prod_{s=1}^{n} H_s^{h_s \lambda_s^\alpha}, \tag{2.4}$$

$$F^s = -Q_s \left(\prod_{s'=1}^{n} H_{s'}^{-A_{ss'}} \right) \rho d\rho \wedge d\phi, \tag{2.5}$$

$\alpha = 1, \ldots, l$ and $s = 1, \ldots, n$, where $w = \pm 1$, $g^1 = d\phi \otimes d\phi$ is a metric on M_1 and g^2 is a Ricci-flat metric on M_2.

The functions $H_s(z) > 0$, $z = \rho^2$, obey Eqs. (1.1) with the boundary conditions (1.2) and

$$P_s = \frac{1}{4} K_s Q_s^2. \tag{2.6}$$

The parameters h_s satisfy the relations

$$h_s = K_s^{-1}, \qquad K_s = B_{ss} > 0, \tag{2.7}$$

where

$$B_{ss'} \equiv 1 + \frac{1}{2-D} + \lambda_{s\alpha} \lambda_{s'\beta} h^{\alpha\beta}, \tag{2.8}$$

$s, s' = 1, \ldots, n$, with $(h^{\alpha\beta}) = (h_{\alpha\beta})^{-1}$. Here $\lambda_s^{\alpha} = h^{\alpha\beta} \lambda_{s\beta}$ and

$$(A_{ss'}) = (2B_{ss'}/B_{s's'}) \tag{2.9}$$

is the Cartan matrix for a simple Lie algebra $\mathcal{G}$ of rank n.

It may be shown that if the matrix $(h_{\alpha\beta})$ has an Euclidean signature and $l \geq n$, there exists a set of co-vectors $\lambda_1, \ldots, \lambda_n$ obeying (2.9). Thus the solution is valid at least when $l \geq n$ and the matrix $(h_{\alpha\beta})$ is positive-definite.

The solution under consideration is as a special case of the fluxbrane (for $w = +1$, $M_1 = S^1$) and S-brane ($w = -1$) solutions from [3] and [31], respectively.

If $w = +1$ and the (Ricci-flat) metric g_2 has a pseudo-Euclidean signature, we get a multidimensional generalization of Melvin's solution [1].

Melvin's solution (without scalar field) corresponds to $D = 4$, $n = 1$, $M_1 = S^1$ $(0 < \phi < 2\pi)$, $M_2 = \mathbb{R}^2$, $g^2 = -dt \otimes dt + d\xi \otimes d\xi$ and $\mathcal{G} = A_1$.

For $w = -1$ and g^2 of Euclidean signature we obtain a cosmological solution with a horizon (as $\rho = +0$) if $M_1 = \mathbb{R}$ $(-\infty < \phi < +\infty)$.

3 The solution for the Lie algebra E_6

Here we deal with the solution for $n = l = 6$, $w = +1$ and $M_1 = S^1$, which corresponds to the Lie algebra E_6. We put here $h_{\alpha\beta} = \delta_{\alpha\beta}$ and denote $(\lambda_{sa}) = (\lambda_s^a) = \vec{\lambda}_s$, $s = 1, \ldots, 6$.

The matrix $A = (A_{ss'})$ coincides with the Cartan matrix for the exceptional Lie algebra E_6

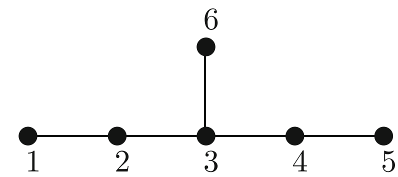

Fig. 1 The Dynkin diagram for the Lie algebra E_6

$$A = (A_{ss'}) = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & -1 \\ 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 \end{pmatrix}. \tag{3.1}$$

This matrix is graphically depicted at Fig. 1 by the Dynkin diagram.

3.1 Fluxbrane polynomials for Lie algebra E_6

The inverse Cartan matrix for E_6,

$$A^{-1} = (A^{ss'}) = \begin{pmatrix} \frac{4}{3} & \frac{5}{3} & 2 & \frac{4}{3} & \frac{2}{3} & 1 \\ \frac{5}{3} & \frac{10}{3} & 4 & \frac{8}{3} & \frac{4}{3} & 2 \\ 2 & 4 & 6 & 4 & 2 & 3 \\ \frac{4}{3} & \frac{8}{3} & 4 & \frac{10}{3} & \frac{5}{3} & 2 \\ \frac{2}{3} & \frac{4}{3} & 2 & \frac{5}{3} & \frac{4}{3} & 1 \\ 1 & 2 & 3 & 2 & 1 & 2 \end{pmatrix}, \tag{3.2}$$

implies due to (1.4)

$$(n_1, n_2, n_3, n_4, n_5, n_6) = (16, 30, 42, 30, 16, 22). \tag{3.3}$$

For the Lie algebra E_6 we find the set of six fluxbrane polynomials, which are listed in the appendix. Here as in [38] we parametrize the polynomials by using other parameters (here denoted B_s) instead of P_s:

$$P_s = n_s B_s, \tag{3.4}$$

$s = 1, \ldots, 6$. This is necessary to avoid huge denominators in monomials of H_s.

The polynomials have the following structure:

$$\begin{aligned} H_1 &= 1 + 16 B_1 z + 120 B_1 B_2 z^2 + \cdots \\ &\quad + 120 B_1^2 B_2^3 B_3^4 B_4^2 B_5 B_6^2 z^{14} \\ &\quad + 16 B_1^2 B_2^3 B_3^4 B_4^3 B_5 B_6^2 z^{15} + B_1^2 B_2^3 B_3^4 B_4^3 B_5^2 B_6^2 z^{16}, \\ H_2 &= 1 + 30 B_2 z + (120 B_1 B_2 + 315 B_3 B_2) z^2 \cdots \\ &\quad + (120 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^8 + 315 B_1^3 B_2^6 B_4^5 B_5^3 B_6^4 B_3^7) z^{28} \\ &\quad + 30 B_1^3 B_2^6 B_3^8 B_4^5 B_5^3 B_6^4 z^{29} + B_1^3 B_2^6 B_3^8 B_4^6 B_5^3 B_6^4 z^{30}, \end{aligned}$$

$$
\begin{aligned}
H_3 &= 1 + 42B_3z + (315B_2B_3 + 315B_4B_3 + 231B_6B_3)z^2 \cdots \\
&\quad + (315B_1^4B_2^7B_4^8B_5^4B_6^6B_3^{11} + 315B_1^4B_2^8B_4^7B_5^4B_6^6B_3^{11} \\
&\quad + 231B_1^4B_2^8B_4^8B_5^4B_6^5B_3^{11})z^{40} \\
&\quad + 42B_1^4B_2^8B_3^{11}B_4^8B_5^4B_6^6z^{41} + B_1^4B_2^8B_3^{12}B_4^8B_5^4B_6^6z^{42}, \\
H_4 &= 1 + 30B_4z + (315B_3B_4 + 120B_5B_4)z^2 \cdots \\
&\quad + (120B_1^2B_2^5B_4^6B_3^8B_5^3B_6^4 + 315B_1^3B_2^5B_4^6B_5^3B_6^4B_3^7)z^{28} \\
&\quad + 30B_1^3B_2^5B_3^8B_4^6B_5^3B_6^4z^{29} + B_1^3B_2^6B_3^8B_4^6B_5^3B_6^4z^{30}, \\
H_5 &= 1 + 16B_5z + 120B_4B_5z^2 + \cdots \\
&\quad + 120B_1B_2^2B_3^4B_4^3B_5^2B_6^2z^{14} \\
&\quad + 16B_1B_2^3B_3^4B_4^3B_5^2B_6^2z^{15} + B_1^2B_2^3B_3^4B_4^3B_5^2B_6^2z^{16}, \\
H_6 &= 1 + 22B_6z + 231B_3B_6z^2 + \cdots \\
&\quad + 231B_1^2B_2^4B_3^5B_4^4B_5^2B_6^3z^{20} \\
&\quad + 22B_1^2B_2^4B_3^6B_4^4B_5^2B_6^3z^{21} + B_1^2B_2^4B_3^6B_4^4B_5^2B_6^4z^{22}. \qquad (3.5)
\end{aligned}
$$

The powers of polynomials are in agreement with Eq. (3.3). In what follows we denote

$$H_s = H_s(z) = H_s(z, (B_i)), \qquad (3.6)$$

$s = 1, \ldots, 6$; where $(B_i) = (B_1, B_2, B_3, B_4, B_5, B_6)$.

Due to (3.5) the polynomials have the following asymptotical behavior:

$$H_s = H_s(z, (B_i)) \sim \left(\prod_{l=1}^{6} (B_l)^{\nu^{sl}}\right) z^{n_s} \equiv H_s^{as}(z, (B_i)), \qquad (3.7)$$

$s = 1, \ldots, 6$, as $z \to \infty$. Here

$$\nu = (\nu^{sl}) = \begin{pmatrix} 2 & 3 & 4 & 3 & 2 & 2 \\ 3 & 6 & 8 & 6 & 3 & 4 \\ 4 & 8 & 12 & 8 & 4 & 6 \\ 3 & 6 & 8 & 6 & 3 & 4 \\ 2 & 3 & 4 & 3 & 2 & 2 \\ 2 & 4 & 6 & 4 & 2 & 4 \end{pmatrix}. \qquad (3.8)$$

The matrix (3.8) is related to the inverse Cartan matrix as follows:

$$\nu = A^{-1}(I + P), \qquad (3.9)$$

where I is 6×6 identity matrix and

$$P = \begin{pmatrix} 0&0&0&0&1&0 \\ 0&0&0&1&0&0 \\ 0&0&1&0&0&0 \\ 0&1&0&0&0&0 \\ 1&0&0&0&0&0 \\ 0&0&0&0&0&1 \end{pmatrix}. \qquad (3.10)$$

is permutation matrix. This matrix corresponds to the permutation $\sigma \in S_6$ (S_6 is the symmetric group)

$$\sigma : (1, 2, 3, 4, 5, 6) \mapsto (5, 4, 3, 2, 1, 6), \qquad (3.11)$$

by the relation $P = (P^i_j) = (\delta^i_{\sigma(j)})$. Here σ is the generator of the group $G = \{\sigma, id\}$ which is the symmetry group of the Dynkin diagram. G is isomorphic to the group Z_2. σ is a composition of two transpositions: (1 5) and (2 4).

We note that the matrix ν is symmetric and

$$\sum_{s=1}^{6} \nu^{sl} = n_l, \qquad (3.12)$$

$l = 1, \ldots, 6$.

Let us denote $\hat{B}_i = B_{\sigma(i)}$, $i = 1, \ldots, 6$. We call the ordered set $(\hat{B}_i)$ a dual one to the ordered set (B_i). By using the relations for polynomials from the appendix we are led to the following two identities which are verified with the aid of Mathematica.

Proposition 1 *For all B_i and z*

$$H_{\sigma(s)}(z, (B_i)) = H_s(z, (\hat{B}_i)), \qquad (3.13)$$

$s = 1, \ldots, 6$.

Proposition 2 *For all $B_i \neq 0$ and $z \neq 0$*

$$H_s(z, (B_i)) = H_s^{as}(z, (B_i)) H_s(z^{-1}, (\hat{B}_i^{-1})), \qquad (3.14)$$

$s = 1, \ldots, 6$.

We call (3.13) symmetry relations, and (3.14) duality ones.

3.2 Exact solution for E_6, fluxes and asymptotics

The solution (2.3)–(2.5) in our case reads

$$
\begin{aligned}
g &= \Big(\prod_{s=1}^{6} H_s^{2h/(D-2)}\Big)\Big\{d\rho \otimes d\rho \\
&\quad + \Big(\prod_{s=1}^{6} H_s^{-2h}\Big)\rho^2 d\phi \otimes d\phi + g^2\Big\}, \qquad (3.15)
\end{aligned}
$$

$$\exp(\varphi^a) = \prod_{s=1}^{6} H_s^{h\lambda_s^a}, \qquad (3.16)$$

$$F^s = \mathcal{B}^s \rho d\rho \wedge d\phi, \qquad (3.17)$$

$a, s = 1, \ldots, 6$, where $g^1 = d\phi \otimes d\phi$ is a metric on $M_1 = S^1$ ($0 < \phi < 2\pi$), g^2 is a Ricci-flat metric on M_2 of signature $(-, +, \ldots, +)$. Here

$$\mathcal{B}^s = -Q_s \left(\prod_{l=1}^{6} H_l^{-A_{sl}}\right), \qquad (3.18)$$

and due to (2.7)–(2.9)

$$K = K_s = \frac{D-3}{D-2} + \vec{\lambda}_s^2, \qquad (3.19)$$

$h_s = h = K^{-1}$,

$$\vec{\lambda}_s \vec{\lambda}_{s'} = \frac{1}{2} K A_{ss'} - \frac{D-3}{D-2} \equiv \Gamma_{ss'}, \tag{3.20}$$

$s, s' = 1, \ldots, 6$. For large enough K there exist vectors $\vec{\lambda}_s$ of equal length which obey relations (3.20). Indeed, the matrix $(\Gamma_{ss'})$ is positive-definite for $K > K_0$, where K_0 is some positive number. Hence there exists a matrix Λ, such that $\Lambda^T \Lambda = \Gamma$. We put $(\Lambda_{as}) = (\lambda_s^a)$ and get the set of vectors obeying (3.20).

Remark Let us put $h_{\alpha\beta} = -\delta_{\alpha\beta}$. It may be shown (along a line as was done for $h_{\alpha\beta} = \delta_{\alpha\beta}$) that, for $K < K_0$, where K_0 is some negative number, there exist vectors $\vec{\lambda}_s$ of equal length which obey relations

$$-\vec{\lambda}_s \vec{\lambda}_{s'} = \frac{1}{2} K A_{ss'} - \frac{D-3}{D-2}, \tag{3.21}$$

following from (2.8) and (2.9). Thus, for both choices of signatures $h_{\alpha\beta} = \pm\delta_{\alpha\beta}$ we get the same algebra (in our case E_6) and the same hidden group $G_{\rm T}$. So, the properties of the matrix $(h_{\alpha\beta})$ are not a priori known from the properties of the group $G_{\rm T}$. In the case of phantom scalar fields, when $h_{\alpha\beta} = -\delta_{\alpha\beta}$, we get solutions which are defined for $\rho < \rho_0$, where $\rho_0 > 0$. The cosmological analogs of such solutions with phantom scalar fields were considered for Lie algebras of rank 2 and 3 in Refs. [44] and [45], respectively. We note that another (sigma model) hidden group $G_{\rm sm}$ (see Introduction) depends upon the choice of the matrix $(h_{\alpha\beta})$ [43].

Now let us consider oriented 2-dimensional manifold $M_* = (0, +\infty) \times S^1$. The flux integrals

$$\Phi^s = \int_{M_*} F^s = 2\pi \int_0^{+\infty} d\rho \rho \mathcal{B}^s, \tag{3.22}$$

are convergent since due to

$$H_s \sim C_s \rho^{2n_s}, \qquad C_s = \prod_{l=1}^{6} B_l^{\nu^{sl}}, \tag{3.23}$$

for $\rho \to +\infty$, and the equality $\sum_1^6 A_{sl} n_l = 2$ (following from (1.4)), we get

$$\mathcal{B}^s \sim -Q_s C^s \rho^{-4}, \tag{3.24}$$

as $\rho \to +\infty$, where

$$C^s = \prod_{l=1}^{6} C_l^{-A_{sl}} = \prod_{k=1}^{6} \prod_{l=1}^{6} B_l^{-A_{sk} \nu^{ks}}, \tag{3.25}$$

$s = 1, \ldots, 6$. Due to (3.9) we get $A\nu = I + P$

$$C^s = \prod_{l=1}^{6} B_l^{-(I+P)_{sl}} = \prod_{l=1}^{6} B_l^{-\delta_s^l - \delta_{\sigma(s)}^l} = B_s^{-1} B_{\sigma(s)}^{-1}, \tag{3.26}$$

$s = 1, \ldots, 6$.

By using Eq. (1.1) we obtain

$$\begin{aligned} \int_0^{+\infty} d\rho \rho \mathcal{B}^s &= -Q_s P_s^{-1} \frac{1}{2} \int_0^{+\infty} dz \frac{d}{dz} \left(\frac{z}{H_s} \frac{d}{dz} H_s \right) \\ &= -\frac{1}{2} Q_s P_s^{-1} \lim_{z \to +\infty} \left(\frac{z}{H_s} \frac{d}{dz} H_s \right) \\ &= -\frac{1}{2} n_s Q_s P_s^{-1}, \end{aligned} \tag{3.27}$$

which implies (see (2.6))

$$\Phi^s = -4\pi n_s Q_s^{-1} h, \qquad h = K^{-1}, \tag{3.28}$$

$s = 1, \ldots, 6$.

It is remarkable that any flux Φ^s depends only upon n_s and the integration constant Q_s, which for $D = 4$ and $g^2 = -dt \otimes dt + dx \otimes dx$ is coinciding up to a sign with the value of the x-component of the magnetic field on the axis of symmetry.

Analogous relations were found recently in Ref. [46] for solutions corresponding to Lie algebras of rank 2; see also Ref. [47].

The asymptotic relations for the solution under consideration for $\rho \to +\infty$ read

$$\begin{aligned} g_{as} &= \left(\prod_{l=1}^{6} B_l^{n_l} \right)^{2h/(D-2)} \rho^{2A} \Big\{ d\rho \otimes d\rho \\ &\quad + \left(\prod_{s=1}^{6} B_s^{n_s} \right)^{-2h} \rho^{2-2A(D-2)} d\phi \otimes d\phi + g^2 \Big\}, \end{aligned} \tag{3.29}$$

$$\varphi_{as}^a = h \sum_{s=1}^{6} \lambda_s^a \Big(\sum_{l=1}^{6} \nu^{sl} \ln B_l + 2n_s \ln \rho \Big), \tag{3.30}$$

$$F_{as}^s = -Q_s B_s^{-1} B_{\sigma(s)}^{-1} \rho^{-3} d\rho \wedge d\phi, \tag{3.31}$$

$a, s = 1, \ldots, 6$, where

$$A = (2h/(D-2)) \sum_{s=1}^{6} n_s = (312h/(D-2)). \tag{3.32}$$

In derivation of asymptotic relations Eqs. (3.12), (3.23), (3.24) and (3.26) were used.

4 Conclusions

Here we have obtained a multidimensional generalization of Melvin's solution for the Lie algebra E_6. The solution is governed by a set of six fluxbrane polynomials $H_s(z)$, $s = 1, \ldots, 6$, which are presented in the appendix. These polynomials define special solutions to open Toda chain equations corresponding to the Lie algebra E_6.

The polynomials $H_s(z)$ depend also upon parameters Q_s, which are coinciding for $D = 4$ (up to a sign) with the values of colored magnetic fields on the axis of symmetry. The symmetry and duality identities for polynomials were verified. The duality identities may be used in deriving $(1/\rho)$-expansion for solutions at large distances ρ, e.g. for asymptotic relations, which are presented in the paper. The power-law asymptotic relations for E_6-polynomials at large z are governed by integer-valued matrix ν. This matrix is related to the inverse Cartan matrix A^{-1} by the formula $\nu = A^{-1}(I + P)$, where I is identity matrix and P is permutation matrix. The matrix P corresponds to a permutation $\sigma \in S_6$, which is the generator of the Z_2-group of symmetry of the Dynkin diagram.

We have also calculated $2d$ flux integrals Φ^s, $s = 1, \ldots, 6$. Any flux Φ^s depends only upon one parameter Q_s, while the integrand F^s depends upon all parameters $Q_1, \ldots, Q_6$. An open question is how to apply the approach of this paper to other finite-dimensional simple Lie algebras.

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Appendix

In this appendix we present polynomials corresponding to the Lie algebra E_6. The polynomials were calculated by using a certain program in Mathematica. We denote the variable z in bold and capital inside the polynomials for better readability:

$$
\boxed{H_1} = B_1^2 B_2^3 B_3^4 B_4^3 B_5^2 B_6^2 \mathbf{Z}^{16} + 16 B_1^2 B_2^3 B_3^4 B_4^3 B_5 B_6^2 \mathbf{Z}^{15}
+ 120 B_1^2 B_2^3 B_3^4 B_4^2 B_5 B_6^2 \mathbf{Z}^{14} + 560 B_1^2 B_2^3 B_3^3 B_4^2 B_5 B_6^2 \mathbf{Z}^{13}
+ (1050 B_1^2 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 770 B_1^2 B_2^3 B_4^2 B_5 B_6 B_3^3) \mathbf{Z}^{12}
+ (672 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^3 + 3696 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^3) \mathbf{Z}^{11}
+ (3696 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3 + 4312 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^2) \mathbf{Z}^{10}
+ (8800 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2 + 2640 B_1^2 B_2^2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^9
+ (660 B_1^2 B_2^2 B_4 B_6 B_3^2 + 4125 B_1 B_2 B_4^2 B_5 B_6 B_3^2
+ 8085 B_1 B_2^2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^8 + (2640 B_1 B_2^2 B_4 B_6 B_3^2
+ 8800 B_1 B_2 B_4 B_5 B_6 B_3^2) \mathbf{Z}^7 + (4312 B_1 B_2 B_4 B_6 B_3^2
+ 3696 B_1 B_2 B_4 B_5 B_6 B_3) \mathbf{Z}^6 + (672 B_1 B_2 B_3 B_4 B_5
+ 3696 B_1 B_2 B_3 B_4 B_6) \mathbf{Z}^5 + (1050 B_1 B_2 B_3 B_4
+ 770 B_1 B_2 B_3 B_6) \mathbf{Z}^4 + 560 B_1 B_2 B_3 \mathbf{Z}^3
+ 120 B_1 B_2 \mathbf{Z}^2 + 16 B_1 \mathbf{Z} + 1,
$$

$$
\boxed{H_2} = B_1^3 B_2^6 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{30} + 30 B_1^3 B_2^6 B_3^8 B_4^5 B_5^3 B_6^4 \mathbf{Z}^{29}
+ (120 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^8 + 315 B_1^3 B_2^6 B_4^5 B_5^3 B_6^4 B_3^7) \mathbf{Z}^{28}
+ (1050 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 2240 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^7
+ 770 B_1^3 B_2^6 B_4^5 B_5^3 B_6^3 B_3^7) \mathbf{Z}^{27} + (1050 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7
+ 9450 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 4200 B_1^3 B_2^6 B_4^4 B_5^2 B_6^4 B_3^7
+ 5775 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 6930 B_1^3 B_2^6 B_4^5 B_5^2 B_6^3 B_3^7) \mathbf{Z}^{26}
+ (10752 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 31500 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7
+ 8316 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 59136 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7
+ 23100 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^7 + 9702 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6) \mathbf{Z}^{25}
+ (45360 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7 + 92400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7
+ 249480 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 + 36750 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6
+ 26950 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6 + 8085 B_1^3 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6
+ 107800 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 26950 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^6) \mathbf{Z}^{24}
+ (443520 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7 + 94080 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6
+ 16500 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6 + 32340 B_1^2 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6
+ 316800 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 1132560 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6) \mathbf{Z}^{23}
+ (44100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 44550 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6
+ 202125 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 3256110 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6
+ 242550 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 495000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6
+ 32340 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 177870 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6
+ 1358280 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5) \mathbf{Z}^{22} + (2674100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6
+ 2182950 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6 + 168960 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^6
+ 178200 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^6 + 711480 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6
+ 23100 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 4928000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5
+ 1131900 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 + 1478400 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^5
+ 830060 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^5) \mathbf{Z}^{21} + (155232 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6
+ 3234000 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^6 + 349272 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^6
+ 970200 B_1^2 B_2^5 B_4^3 B_5 B_6^3 B_3^6 + 853776 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6
+ 9315306 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 + 7074375 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^5
+ 1559250 B_1^3 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5 + 577500 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^5
+ 5082 B_1^2 B_2^4 B_4^4 B_5^3 B_6^2 B_3^5 + 3811500 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^5
+ 996072 B_1^3 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5 + 1143450 B_1^3 B_2^5 B_4^3 B_5^2 B_6^2 B_3^5) \mathbf{Z}^{20}
+ (2069760 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^6 + 1478400 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5
+ 4331250 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5 + 21801780 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5
+ 369600 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^5 + 3326400 B_1^2 B_2^5 B_4^3 B_5 B_6^3 B_3^5
+ 693000 B_1^3 B_2^4 B_4^3 B_5 B_6^3 B_3^5 + 11384100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5
+ 6225450 B_1^2 B_2^5 B_4^3 B_5^2 B_6^2 B_3^5 + 2439360 B_1^3 B_2^4 B_4^3 B_5^2 B_6^2 B_3^5
+ 508200 B_1^3 B_2^5 B_4^3 B_5 B_6^2 B_3^5) \mathbf{Z}^{19} + (1559250 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5
$$

$$+3056130B_1B_2^4B_4^3B_5^2B_6^3B_3^5+14437500B_1^2B_2^3B_4^3B_5^2B_6^3B_3^5$$
$$+14314300B_1^2B_2^4B_4^3B_5B_6^3B_3^5+2032800B_1B_2^4B_4^4B_5^2B_6^2B_3^5$$
$$+8575875B_1^2B_2^3B_4^4B_5^2B_6^2B_3^5+28420210B_1^2B_2^4B_4^3B_5^2B_6^2B_3^5$$
$$+127050B_1^2B_2^4B_4^4B_5B_6^2B_3^5+3176250B_1^2B_2^5B_4^3B_5B_6^2B_3^5$$
$$+1372140B_1^3B_2^4B_4^3B_5B_6^2B_3^5+6338640B_1^2B_2^4B_4^3B_5^2B_6^3B_3^4$$
$$+2371600B_1^2B_2^4B_4^4B_5^2B_6^2B_3^4+711480B_1^3B_2^4B_4^3B_5^2B_6^2B_3^4)\mathbf{Z}^{18}$$
$$+(5913600B_1B_2^3B_4^3B_5^2B_6^3B_3^5+1774080B_1B_2^4B_4^3B_5B_6^3B_3^5$$
$$+10187100B_1^2B_2^3B_4^3B_5B_6^3B_3^5+577500B_1^2B_2^4B_4^2B_5B_6^3B_3^5$$
$$+3811500B_1B_2^3B_4^4B_5^2B_6^2B_3^5+7470540B_1B_2^4B_4^3B_5^2B_6^2B_3^5$$
$$+32524800B_1^2B_2^3B_4^3B_5^2B_6^2B_3^5+18705960B_1^2B_2^4B_4^3B_5B_6^2B_3^5$$
$$+8731800B_1^2B_2^3B_4^3B_5^2B_6^3B_3^4+8279040B_1^2B_2^4B_4^3B_5B_6^3B_3^4$$
$$+4446750B_1^2B_2^3B_4^4B_5^2B_6^2B_3^4+16625700B_1^2B_2^4B_4^3B_5^2B_6^2B_3^4$$
$$+711480B_1^3B_2^4B_4^3B_5B_6^2B_3^4)\mathbf{Z}^{17}+(4527600B_1B_2^3B_4^3B_5B_6^3B_3^5$$
$$+18295200B_1B_2^3B_4^3B_5^2B_6^2B_3^5+5488560B_1B_2^4B_4^3B_5B_6^2B_3^5$$
$$+24901800B_1^2B_2^3B_4^3B_5B_6^2B_3^5+508200B_1^2B_2^4B_4^2B_5B_6^2B_3^5$$
$$+3880800B_1B_2^3B_4^3B_5^2B_6^3B_3^4+11884950B_1^2B_2^3B_4^3B_5B_6^3B_3^4$$
$$+2182950B_1^2B_2^4B_4^2B_5B_6^3B_3^4+2268750B_1B_2^3B_4^4B_5^2B_6^2B_3^4$$
$$+4446750B_1B_2^4B_4^3B_5^2B_6^2B_3^4+45530550B_1^2B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+1334025B_1^2B_2^4B_4^2B_5^2B_6^2B_3^4+18478980B_1^2B_2^4B_4^3B_5B_6^2B_3^4$$
$$+108900B_1^3B_2^4B_4^2B_5B_6^2B_3^4+1584660B_1^2B_2^4B_4^3B_5^2B_6B_3^4)\mathbf{Z}^{16}$$
$$+(15937152B_1B_2^3B_4^3B_5B_6^2B_3^5+5588352B_1B_2^3B_4^3B_5B_6^3B_3^4$$
$$+3234000B_1^2B_2^3B_4^2B_5B_6^3B_3^4+34036496B_1B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+5808000B_1^2B_2^3B_4^2B_5^2B_6^2B_3^4+5808000B_1B_2^4B_4^3B_5B_6^2B_3^4$$
$$+53742416B_1^2B_2^3B_4^3B_5B_6^2B_3^4+6203600B_1^2B_2^4B_4^2B_5B_6^2B_3^4$$
$$+5588352B_1^2B_2^3B_4^3B_5^2B_6B_3^4+3234000B_1^2B_2^4B_4^3B_5B_6B_3^4$$
$$+15937152B_1^2B_2^3B_4^3B_5^2B_6^2B_3^3)\mathbf{Z}^{15}+(108900B_1^2B_3^4B_4^2B_6^2B_2^4$$
$$+1334025B_1B_3^4B_4^2B_5B_6^2B_2^4+508200B_1^2B_3^3B_4^2B_5B_6^2B_2^4$$
$$+2182950B_1^2B_3^4B_4^2B_5B_6B_2^4+1584660B_1B_3^4B_4^2B_5B_6^3B_2^3$$
$$+18295200B_1B_3^3B_4^3B_5^2B_6^2B_2^3+4446750B_1B_3^4B_4^2B_5^2B_6^2B_2^3$$
$$+5488560B_1^2B_3^3B_4^2B_5^2B_6^2B_2^3+45530550B_1B_3^4B_4^3B_5B_6^2B_2^3$$
$$+24901800B_1^2B_3^3B_4^3B_5B_6^2B_2^3+18478980B_1^2B_3^4B_4^2B_5B_6^2B_2^3$$
$$+3880800B_1B_3^4B_4^3B_5^2B_6B_2^3+4527600B_1^2B_3^3B_4^3B_5^2B_6B_2^3$$
$$+11884950B_1^2B_3^4B_4^3B_5B_6B_2^3+2268750B_1B_3^4B_4^3B_5^2B_6^2B_2^2)\mathbf{Z}^{14}$$
$$+(577500B_1^2B_3^3B_4^2B_5B_6B_2^4+711480B_1^2B_3^4B_4^2B_6^2B_2^3$$
$$+7470540B_1B_3^3B_4^2B_5^2B_6^2B_2^3+32524800B_1B_3^3B_4^3B_5B_6^2B_2^3$$
$$+16625700B_1B_3^4B_4^2B_5B_6^2B_2^3+18705960B_1^2B_3^3B_4^2B_5B_6^2B_2^3$$
$$+5913600B_1B_3^3B_4^3B_5^2B_6B_2^3+1774080B_1^2B_3^3B_4^2B_5^2B_6B_2^3$$
$$+8731800B_1B_3^4B_4^3B_5B_6B_2^3+10187100B_1^2B_3^3B_4^3B_5B_6B_2^3$$
$$+8279040B_1^2B_3^4B_4^2B_5B_6B_2^3+3811500B_1B_3^3B_4^3B_5^2B_6^2B_2^2$$
$$+4446750B_1B_3^4B_4^3B_5B_6^2B_2^2)\mathbf{Z}^{13}+(711480B_1B_2^3B_4^2B_6^2B_3^4$$
$$+2371600B_1B_2^2B_4^2B_5B_6^2B_3^4+6338640B_1B_2^3B_4^2B_5B_6B_3^4$$
$$+1372140B_1^2B_2^3B_4^2B_6^2B_3^3+2032800B_1B_2^2B_4^2B_5^2B_6^2B_3^3$$
$$+8575875B_1B_2^2B_4^3B_5B_6^2B_3^3+28420210B_1B_2^3B_4^2B_5B_6^2B_3^3$$
$$+127050B_1^2B_2^2B_4^2B_5B_6^2B_3^3+3176250B_1^2B_2^3B_4B_5B_6^2B_3^3$$
$$+1559250B_1B_2^2B_4^3B_5^2B_6B_3^3+3056130B_1B_2^3B_4^2B_5^2B_6B_3^3$$
$$+14437500B_1B_2^3B_4^3B_5B_6B_3^3+14314300B_1^2B_2^3B_4^2B_5B_6B_3^3)\mathbf{Z}^{12}$$
$$+(2439360B_1B_3^3B_4^2B_6^2B_2^3+508200B_1^2B_3^3B_4B_6^2B_2^3$$
$$+6225450B_1B_3^3B_4B_5B_6^2B_2^3+693000B_1^2B_3^3B_4^2B_6B_2^3$$
$$+21801780B_1B_3^3B_4^2B_5B_6B_2^3+2069760B_1^2B_3^2B_4^2B_5B_6B_2^3$$
$$+3326400B_1^2B_3^3B_4B_5B_6B_2^3+11384100B_1B_3^3B_4^2B_5B_6^2B_2^2$$
$$+1478400B_1B_3^3B_4^2B_5^2B_6B_2^2+4331250B_1B_3^3B_4^3B_5B_6B_2^2$$
$$+369600B_1^2B_3^3B_4^2B_5B_6B_2^2)\mathbf{Z}^{11}+(1143450B_1B_3^3B_4B_6^2B_2^3$$
$$+1559250B_1B_3^3B_4^2B_6B_2^3+577500B_1^2B_3^3B_4B_6B_2^3$$
$$+3234000B_1B_3^2B_4^2B_5B_6B_2^3+7074375B_1B_3^3B_4B_5B_6B_2^3$$
$$+970200B_1^2B_3^2B_4B_5B_6B_2^3+996072B_1B_3^3B_4^2B_6^2B_2^2$$
$$+5082B_3^3B_4^2B_5B_6^2B_2^2+853776B_1B_3^2B_4^2B_5B_6^2B_2^2$$
$$+3811500B_1B_3^3B_4B_5B_6^2B_2^2+155232B_1B_3^2B_4^2B_5^2B_6B_2^2$$
$$+9315306B_1B_3^3B_4^2B_5B_6B_2^2+349272B_1^2B_3^2B_4^2B_5B_6B_2^2)\mathbf{Z}^{10}$$
$$+(1478400B_1B_3^3B_4B_6B_2^3+178200B_1^2B_3^2B_4B_6B_2^3$$
$$+2182950B_1B_3^2B_4B_5B_6B_2^3+830060B_1B_3^3B_4B_6^2B_2^2$$
$$+711480B_1B_3^2B_4B_5B_6^2B_2^2+1131900B_1B_3^3B_4^2B_6B_2^2$$
$$+23100B_3^3B_4^2B_5B_6B_2^2+2674100B_1B_3^2B_4^2B_5B_6B_2^2$$
$$+4928000B_1B_3^3B_4B_5B_6B_2^2+168960B_1^2B_3^2B_4B_5B_6B_2^2)\mathbf{Z}^9$$
$$+(495000B_1B_3^2B_4B_6B_2^3+177870B_1B_3^2B_4B_6^2B_2^2$$
$$+44100B_1B_3^2B_4^2B_5B_2^2+242550B_1B_3^2B_4^2B_6B_2^2$$
$$+1358280B_1B_3^3B_4B_6B_2^2+32340B_1^2B_3^2B_4B_6B_2^2$$
$$+44550B_3^2B_4^2B_5B_6B_2^2+3256110B_1B_3^2B_4B_5B_6B_2^2$$
$$+202125B_1B_3^2B_4^2B_5B_6B_2)\mathbf{Z}^8$$
$$+(94080B_1B_3^2B_4B_5B_2^2+1132560B_1B_3^2B_4B_6B_2^2$$
$$+32340B_3^2B_4B_5B_6B_2^2+443520B_1B_3B_4B_5B_6B_2^2$$
$$+16500B_3^2B_4^2B_5B_6B_2+316800B_1B_3^2B_4B_5B_6B_2)\mathbf{Z}^7$$
$$+(36750B_1B_3^2B_4B_2^2+45360B_1B_3B_4B_5B_2^2$$
$$+26950B_1B_3^2B_6B_2^2+8085B_3^2B_4B_6B_2^2+249480B_1B_3B_4B_6B_2^2$$
$$+107800B_1B_3^2B_4B_6B_2+26950B_3^2B_4B_5B_6B_2$$
$$+92400B_1B_3B_4B_5B_6B_2)\mathbf{Z}^6+(31500B_1B_3B_4B_2^2$$
$$+23100B_1B_3B_6B_2^2+10752B_1B_3B_4B_5B_2+9702B_3^2B_4B_6B_2$$
$$+59136B_1B_3B_4B_6B_2+8316B_3B_4B_5B_6B_2)\mathbf{Z}^5$$
$$+(4200B_1B_3B_2^2+9450B_1B_3B_4B_2+1050B_3B_4B_5B_2$$
$$+6930B_1B_3B_6B_2+5775B_3B_4B_6B_2)\mathbf{Z}^4$$
$$+(2240B_1B_2B_3+1050B_2B_4B_3+770B_2B_6B_3)\mathbf{Z}^3$$
$$+(120B_1B_2+315B_3B_2)\mathbf{Z}^2+30B_2\mathbf{Z}+1,$$

$$\boxed{H_3}=B_1^4B_2^8B_3^{12}B_4^8B_5^4B_6^6\mathbf{Z}^{42}+42B_1^4B_2^8B_3^{11}B_4^8B_5^4B_6^6\mathbf{Z}^{41}$$
$$+(315B_1^4B_2^7B_4^8B_5^4B_6^6B_3^{11}+315B_1^4B_2^8B_4^7B_5^4B_6^6B_3^{11}$$
$$+231B_1^4B_2^8B_4^8B_5^4B_6^5B_3^{11})\mathbf{Z}^{40}$$
$$+(560B_1^3B_2^7B_4^8B_5^4B_6^6B_3^{11}+4200B_1^4B_2^7B_4^7B_5^4B_6^6B_3^{11}$$
$$+560B_1^4B_2^8B_4^7B_5^3B_6^6B_3^{11}+3080B_1^4B_2^7B_4^8B_5^4B_6^5B_3^{11}$$
$$+3080B_1^4B_2^8B_4^7B_5^4B_6^5B_3^{11})\mathbf{Z}^{39}$$
$$+(9450B_1^3B_2^7B_4^7B_5^4B_6^6B_3^{11}+9450B_1^4B_2^7B_4^7B_5^3B_6^6B_3^{11}$$
$$+6930B_1^3B_2^7B_4^8B_5^4B_6^5B_3^{11}+51975B_1^4B_2^7B_4^7B_5^4B_6^5B_3^{11}$$
$$+6930B_1^4B_2^8B_4^7B_5^3B_6^5B_3^{11}+11025B_1^4B_2^7B_4^7B_5^4B_6^6B_3^{10}$$
$$+8085B_1^4B_2^7B_4^8B_5^4B_6^5B_3^{10}+8085B_1^4B_2^8B_4^7B_5^4B_6^5B_3^{10})\mathbf{Z}^{38}$$
$$+(24192B_1^3B_2^7B_4^7B_5^3B_6^6B_3^{11}+133056B_1^3B_2^7B_4^7B_5^4B_6^5B_3^{11}$$
$$+133056B_1^4B_2^7B_4^7B_5^3B_6^5B_3^{11}+44100B_1^3B_2^7B_4^7B_5^4B_6^6B_3^{10}$$
$$+44100B_1^4B_2^7B_4^7B_5^3B_6^6B_3^{10}+32340B_1^3B_2^7B_4^8B_5^4B_6^5B_3^{10}$$

$$+407484B_1^4B_2^7B_4^7B_5^4B_6^5B_3^{10}+32340B_1^4B_2^8B_4^7B_5^3B_6^5B_3^{10})\mathbf{Z}^{37}$$
$$+(369600B_1^3B_2^7B_4^7B_5^3B_6^5B_3^{11}+36750B_1^3B_2^6B_4^7B_5^4B_6^6B_3^{10}$$
$$+200704B_1^3B_2^7B_4^7B_5^3B_6^6B_3^{10}+36750B_1^4B_2^7B_4^6B_5^3B_6^6B_3^{10}$$
$$+26950B_1^3B_2^6B_4^8B_5^4B_6^5B_3^{10}+1539384B_1^3B_2^7B_4^7B_5^4B_6^5B_3^{10}$$
$$+202125B_1^4B_2^6B_4^7B_5^4B_6^5B_3^{10}+202125B_1^4B_2^7B_4^6B_5^4B_6^5B_3^{10}$$
$$+1539384B_1^4B_2^7B_4^7B_5^3B_6^5B_3^{10}+26950B_1^4B_2^8B_4^6B_5^3B_6^5B_3^{10}$$
$$+148225B_1^4B_2^7B_4^7B_5^4B_6^4B_3^{10}+916839B_1^4B_2^7B_4^7B_5^4B_6^5B_3^9)\mathbf{Z}^{36}$$
$$+(211680B_1^3B_2^6B_4^7B_5^3B_6^6B_3^{10}+211680B_1^3B_2^7B_4^6B_5^3B_6^6B_3^{10}$$
$$+1853280B_1^3B_2^6B_4^7B_5^4B_6^5B_3^{10}+1164240B_1^3B_2^7B_4^6B_5^4B_6^5B_3^{10}$$
$$+6044544B_1^3B_2^7B_4^7B_5^3B_6^5B_3^{10}+1164240B_1^4B_2^6B_4^7B_5^3B_6^5B_3^{10}$$
$$+1853280B_1^4B_2^7B_4^6B_5^3B_6^5B_3^{10}+853776B_1^3B_2^7B_4^7B_5^4B_6^4B_3^{10}$$
$$+853776B_1^4B_2^7B_4^7B_5^3B_6^4B_3^{10}+4527600B_1^3B_2^7B_4^7B_5^4B_6^5B_3^9$$
$$+1358280B_1^4B_2^6B_4^7B_5^4B_6^5B_3^9+1358280B_1^4B_2^7B_4^6B_5^4B_6^5B_3^9$$
$$+4527600B_1^4B_2^7B_4^7B_5^3B_6^5B_3^9+996072B_1^4B_2^7B_4^7B_5^4B_6^4B_3^9)\mathbf{Z}^{35}$$
$$+(396900B_1^3B_2^6B_4^6B_5^3B_6^6B_3^{10}+291060B_1^2B_2^6B_4^7B_5^4B_6^5B_3^{10}$$
$$+2182950B_1^3B_2^6B_4^6B_5^4B_6^5B_3^{10}+9168390B_1^3B_2^6B_4^7B_5^3B_6^5B_3^{10}$$
$$+9168390B_1^3B_2^7B_4^6B_5^3B_6^5B_3^{10}+2182950B_1^4B_2^6B_4^6B_5^3B_6^5B_3^{10}$$
$$+291060B_1^4B_2^7B_4^6B_5^2B_6^5B_3^{10}+1600830B_1^3B_2^6B_4^7B_5^4B_6^4B_3^{10}$$
$$+5336100B_1^3B_2^7B_4^7B_5^3B_6^4B_3^{10}+1600830B_1^4B_2^7B_4^6B_5^3B_6^4B_3^{10}$$
$$+13222440B_1^3B_2^6B_4^7B_5^4B_6^5B_3^9+8489250B_1^3B_2^7B_4^6B_5^4B_6^5B_3^9$$
$$+2546775B_1^4B_2^6B_4^6B_5^4B_6^5B_3^9+23654400B_1^3B_2^7B_4^7B_5^3B_6^5B_3^9$$
$$+8489250B_1^4B_2^6B_4^7B_5^3B_6^5B_3^9+13222440B_1^4B_2^7B_4^6B_5^3B_6^5B_3^9$$
$$+6225450B_1^3B_2^7B_4^7B_5^4B_6^4B_3^9+1867635B_1^4B_2^6B_4^7B_5^4B_6^4B_3^9$$
$$+1867635B_1^4B_2^7B_4^6B_5^4B_6^4B_3^9+6225450B_1^4B_2^7B_4^7B_5^3B_6^4B_3^9)\mathbf{Z}^{34}$$
$$+(2069760B_1^2B_2^6B_4^7B_5^3B_6^5B_3^{10}+24147200B_1^3B_2^6B_4^6B_5^3B_6^5B_3^{10}$$
$$+2069760B_1^3B_2^7B_4^6B_5^2B_6^5B_3^{10}+11383680B_1^3B_2^6B_4^7B_5^3B_6^4B_3^{10}$$
$$+11383680B_1^3B_2^7B_4^6B_5^3B_6^4B_3^{10}+205800B_1^3B_2^6B_4^6B_5^3B_6^6B_3^9$$
$$+3773000B_1^2B_2^6B_4^7B_5^4B_6^5B_3^9+9240000B_1^3B_2^5B_4^7B_5^4B_6^5B_3^9$$
$$+37560600B_1^3B_2^6B_4^6B_5^4B_6^5B_3^9+82222140B_1^3B_2^6B_4^7B_5^3B_6^5B_3^9$$
$$+82222140B_1^3B_2^7B_4^6B_5^3B_6^5B_3^9+37560600B_1^4B_2^6B_4^6B_5^3B_6^5B_3^9$$
$$+9240000B_1^4B_2^7B_4^5B_5^3B_6^5B_3^9+3773000B_1^4B_2^7B_4^6B_5^2B_6^5B_3^9$$
$$+27544440B_1^3B_2^6B_4^7B_5^4B_6^4B_3^9+13280960B_1^3B_2^7B_4^6B_5^4B_6^4B_3^9$$
$$+6225450B_1^4B_2^6B_4^6B_5^4B_6^4B_3^9+41164200B_1^3B_2^7B_4^7B_5^3B_6^4B_3^9$$
$$+13280960B_1^4B_2^6B_4^7B_5^3B_6^4B_3^9+27544440B_1^4B_2^7B_4^6B_5^3B_6^4B_3^9)\mathbf{Z}^{33}$$
$$+(5588352B_1^2B_2^6B_4^6B_5^3B_6^5B_3^{10}+5588352B_1^3B_2^6B_4^6B_5^2B_6^5B_3^{10}$$
$$+30735936B_1^3B_2^6B_4^6B_5^3B_6^4B_3^{10}+5197500B_1^2B_2^5B_4^7B_5^4B_6^5B_3^9$$
$$+10187100B_1^2B_2^6B_4^6B_5^4B_6^5B_3^9+38981250B_1^3B_2^5B_4^6B_5^4B_6^5B_3^9$$
$$+28385280B_1^2B_2^6B_4^7B_5^3B_6^5B_3^9+63669375B_1^3B_2^5B_4^7B_5^3B_6^5B_3^9$$
$$+440527626B_1^3B_2^6B_4^6B_5^3B_6^5B_3^9+63669375B_1^3B_2^7B_4^5B_5^3B_6^5B_3^9$$
$$+38981250B_1^4B_2^6B_4^5B_5^3B_6^5B_3^9+28385280B_1^3B_2^7B_4^6B_5^2B_6^5B_3^9$$
$$+10187100B_1^4B_2^6B_4^6B_5^2B_6^5B_3^9+5197500B_1^4B_2^7B_4^5B_5^2B_6^5B_3^9$$
$$+7470540B_1^2B_2^6B_4^7B_5^4B_6^4B_3^9+28586250B_1^3B_2^5B_4^7B_5^4B_6^4B_3^9$$
$$+87268104B_1^3B_2^6B_4^6B_5^4B_6^4B_3^9+202848030B_1^3B_2^6B_4^7B_5^3B_6^4B_3^9$$
$$+202848030B_1^3B_2^7B_4^6B_5^3B_6^4B_3^9+87268104B_1^4B_2^6B_4^6B_5^3B_6^4B_3^9$$
$$+28586250B_1^4B_2^7B_4^5B_5^3B_6^4B_3^9+7470540B_1^4B_2^7B_4^6B_5^2B_6^4B_3^9$$
$$+11884950B_1^3B_2^6B_4^6B_5^4B_6^5B_3^8+11884950B_1^4B_2^6B_4^6B_5^3B_6^5B_3^8$$
$$+8715630B_1^3B_2^6B_4^7B_5^4B_6^4B_3^8+2614689B_1^4B_2^6B_4^6B_5^4B_6^4B_3^8$$
$$+8715630B_1^4B_2^7B_4^6B_5^3B_6^4B_3^8)\mathbf{Z}^{32}+(24948000B_1^2B_2^5B_4^6B_5^4B_6^5B_3^9$$
$$+40748400B_1^2B_2^5B_4^7B_5^3B_6^5B_3^9+177031008B_1^2B_2^6B_4^6B_5^3B_6^5B_3^9$$
$$+467082000B_1^3B_2^5B_4^6B_5^3B_6^5B_3^9+467082000B_1^3B_2^6B_4^5B_5^3B_6^5B_3^9$$
$$+177031008B_1^3B_2^6B_4^6B_5^2B_6^5B_3^9+40748400B_1^3B_2^7B_4^5B_5^2B_6^5B_3^9$$
$$+24948000B_1^4B_2^6B_4^5B_5^2B_6^5B_3^9+18295200B_1^2B_2^5B_4^7B_5^4B_6^4B_3^9$$
$$+35858592B_1^2B_2^6B_4^6B_5^4B_6^4B_3^9+137214000B_1^3B_2^5B_4^6B_5^4B_6^4B_3^9$$
$$+60984000B_1^2B_2^6B_4^7B_5^3B_6^4B_3^9+224116200B_1^3B_2^5B_4^7B_5^3B_6^4B_3^9$$
$$+1339753968B_1^3B_2^6B_4^6B_5^3B_6^4B_3^9+224116200B_1^3B_2^7B_4^5B_5^3B_6^4B_3^9$$
$$+137214000B_1^4B_2^6B_4^5B_5^3B_6^4B_3^9+60984000B_1^3B_2^7B_4^6B_5^2B_6^4B_3^9$$
$$+35858592B_1^4B_2^6B_4^6B_5^2B_6^4B_3^9+18295200B_1^4B_2^7B_4^5B_5^2B_6^4B_3^9$$
$$+29106000B_1^3B_2^5B_4^6B_5^4B_6^5B_3^8+191866752B_1^3B_2^6B_4^6B_5^3B_6^5B_3^8$$
$$+29106000B_1^4B_2^6B_4^5B_5^3B_6^5B_3^8+21344400B_1^3B_2^5B_4^7B_5^4B_6^4B_3^8$$
$$+66594528B_1^3B_2^6B_4^6B_5^4B_6^4B_3^8+71148000B_1^3B_2^6B_4^7B_5^3B_6^4B_3^8$$
$$+71148000B_1^3B_2^7B_4^6B_5^3B_6^4B_3^8+66594528B_1^4B_2^6B_4^6B_5^3B_6^4B_3^8$$
$$+21344400B_1^4B_2^7B_4^5B_5^3B_6^4B_3^8)\mathbf{Z}^{31}+(353089660B_1^2B_2^5B_4^6B_5^3B_6^5B_3^9$$
$$+247546530B_1^2B_2^6B_4^5B_5^3B_6^5B_3^9+707437500B_1^3B_2^5B_4^5B_5^3B_6^5B_3^9$$
$$+60555264B_1^2B_2^6B_4^6B_5^2B_6^5B_3^9+247546530B_1^3B_2^5B_4^6B_5^2B_6^5B_3^9$$
$$+353089660B_1^3B_2^6B_4^5B_5^2B_6^5B_3^9+111143340B_1^2B_2^5B_4^6B_5^4B_6^4B_3^9$$
$$+155636250B_1^2B_2^5B_4^7B_5^3B_6^4B_3^9+542666124B_1^2B_2^6B_4^6B_5^3B_6^4B_3^9$$
$$+1941993130B_1^3B_2^5B_4^6B_5^3B_6^4B_3^9+1941993130B_1^3B_2^6B_4^5B_5^3B_6^4B_3^9$$
$$+542666124B_1^3B_2^6B_4^6B_5^2B_6^4B_3^9+155636250B_1^3B_2^7B_4^5B_5^2B_6^4B_3^9$$
$$+111143340B_1^4B_2^6B_4^5B_5^2B_6^4B_3^9+5478396B_1^3B_2^6B_4^6B_5^3B_6^3B_3^9$$
$$+20212500B_1^2B_2^5B_4^6B_5^4B_6^5B_3^8+110387200B_1^2B_2^6B_4^6B_5^3B_6^5B_3^8$$
$$+388031490B_1^3B_2^5B_4^6B_5^3B_6^5B_3^8+388031490B_1^3B_2^6B_4^5B_5^3B_6^5B_3^8$$
$$+110387200B_1^3B_2^6B_4^6B_5^2B_6^5B_3^8+20212500B_1^4B_2^6B_4^5B_5^2B_6^5B_3^8$$
$$+14822500B_1^2B_2^5B_4^7B_5^4B_6^4B_3^8+29052100B_1^2B_2^6B_4^6B_5^4B_6^4B_3^8$$
$$+253998360B_1^3B_2^5B_4^6B_5^4B_6^4B_3^8+8715630B_1^3B_2^6B_4^5B_5^4B_6^4B_3^8$$
$$+181575625B_1^3B_2^5B_4^7B_5^3B_6^4B_3^8+1554121926B_1^3B_2^6B_4^6B_5^3B_6^4B_3^8$$
$$+8715630B_1^4B_2^5B_4^6B_5^3B_6^4B_3^8+181575625B_1^3B_2^7B_4^5B_5^3B_6^4B_3^8$$
$$+253998360B_1^4B_2^6B_4^5B_5^3B_6^4B_3^8+29052100B_1^4B_2^6B_4^6B_5^2B_6^4B_3^8$$
$$+14822500B_1^4B_2^7B_4^5B_5^2B_6^4B_3^8+6391462B_1^3B_2^6B_4^6B_5^4B_6^3B_3^8$$
$$+6391462B_1^4B_2^6B_4^6B_5^3B_6^3B_3^8)\mathbf{Z}^{30}+(651974400B_1^2B_2^5B_4^5B_5^3B_6^5B_3^9$$
$$+195592320B_1^2B_2^5B_4^6B_5^2B_6^5B_3^9+195592320B_1^2B_2^6B_4^5B_5^2B_6^5B_3^9$$
$$+651974400B_1^3B_2^5B_4^5B_5^2B_6^5B_3^9+1644128640B_1^2B_2^5B_4^6B_5^3B_6^4B_3^9$$
$$+1075757760B_1^2B_2^6B_4^5B_5^3B_6^4B_3^9+3585859200B_1^3B_2^5B_4^5B_5^3B_6^4B_3^9$$
$$+292723200B_1^2B_2^6B_4^6B_5^2B_6^4B_3^9+1075757760B_1^3B_2^5B_4^6B_5^2B_6^4B_3^9$$
$$+1644128640B_1^3B_2^6B_4^5B_5^2B_6^4B_3^9+413887320B_1^2B_2^5B_4^6B_5^3B_6^5B_3^8$$
$$+356548500B_1^2B_2^6B_4^5B_5^3B_6^5B_3^8+1189465200B_1^3B_2^5B_4^5B_5^3B_6^5B_3^8$$
$$+356548500B_1^3B_2^5B_4^6B_5^2B_6^5B_3^8+413887320B_1^3B_2^6B_4^5B_5^2B_6^5B_3^8$$
$$+268939440B_1^2B_2^5B_4^6B_5^4B_6^4B_3^8+48024900B_1^3B_2^5B_4^5B_5^4B_6^4B_3^8$$
$$+133402500B_1^2B_2^5B_4^7B_5^3B_6^4B_3^8+672348600B_1^2B_2^6B_4^6B_5^3B_6^4B_3^8$$
$$+4320547560B_1^3B_2^5B_4^6B_5^3B_6^4B_3^8+4320547560B_1^3B_2^6B_4^5B_5^3B_6^4B_3^8$$
$$+48024900B_1^4B_2^5B_4^5B_5^3B_6^4B_3^8+672348600B_1^3B_2^6B_4^6B_5^2B_6^4B_3^8$$
$$+133402500B_1^3B_2^7B_4^5B_5^2B_6^4B_3^8+268939440B_1^4B_2^6B_4^5B_5^2B_6^4B_3^8$$
$$+35218260B_1^3B_2^5B_4^6B_5^4B_6^3B_3^8+180457200B_1^3B_2^6B_4^6B_5^3B_6^3B_3^8$$
$$+35218260B_1^4B_2^6B_4^5B_5^3B_6^3B_3^8+119528640B_1^3B_2^5B_4^6B_5^4B_6^4B_3^7$$
$$+398428800B_1^3B_2^6B_4^6B_5^3B_6^4B_3^7+119528640B_1^4B_2^6B_4^5B_5^3B_6^4B_3^7)\mathbf{Z}^{29}$$

$$+ (651974400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^5 B_3^9 + 3585859200 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^9$$
$$+ 1075757760 B_1^2 B_2^5 B_4^6 B_5^2 B_6^4 B_3^9 + 1075757760 B_1^2 B_2^6 B_4^5 B_5^2 B_6^4 B_3^9$$
$$+ 3585859200 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^9 + 54573750 B_1^2 B_2^4 B_4^6 B_5^3 B_6^5 B_3^8$$
$$+ 1671169500 B_1^2 B_2^5 B_4^5 B_5^3 B_6^5 B_3^8 + 298045440 B_1^2 B_2^5 B_4^6 B_5^2 B_6^5 B_3^8$$
$$+ 298045440 B_1^2 B_2^6 B_4^5 B_5^2 B_6^5 B_3^8 + 1671169500 B_1^3 B_2^5 B_4^5 B_5^2 B_6^5 B_3^8$$
$$+ 54573750 B_1^3 B_2^6 B_4^4 B_5^2 B_6^5 B_3^8 + 24502500 B_1^2 B_2^4 B_4^6 B_5^4 B_6^4 B_3^8$$
$$+ 48024900 B_1^2 B_2^5 B_4^5 B_5^4 B_6^4 B_3^8 + 4609356210 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^8$$
$$+ 300155625 B_1^3 B_2^4 B_4^6 B_5^3 B_6^4 B_3^8 + 3054383640 B_1^2 B_2^6 B_4^5 B_5^3 B_6^4 B_3^8$$
$$+ 14283282150 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^8 + 300155625 B_1^3 B_2^6 B_4^4 B_5^3 B_6^4 B_3^8$$
$$+ 446054400 B_1^2 B_2^6 B_4^6 B_5^2 B_6^4 B_3^8 + 3054383640 B_1^3 B_2^5 B_4^6 B_5^2 B_6^4 B_3^8$$
$$+ 4609356210 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^8 + 48024900 B_1^4 B_2^5 B_4^5 B_5^2 B_6^4 B_3^8$$
$$+ 24502500 B_1^4 B_2^6 B_4^4 B_5^2 B_6^4 B_3^8 + 35218260 B_1^2 B_2^5 B_4^6 B_5^4 B_6^3 B_3^8$$
$$+ 117394200 B_1^2 B_2^6 B_4^6 B_5^3 B_6^3 B_3^8 + 607296690 B_1^3 B_2^5 B_4^6 B_5^3 B_6^3 B_3^8$$
$$+ 607296690 B_1^3 B_2^6 B_4^5 B_5^3 B_6^3 B_3^8 + 117394200 B_1^3 B_2^6 B_4^6 B_5^2 B_6^3 B_3^8$$
$$+ 35218260 B_1^4 B_2^6 B_4^5 B_5^2 B_6^3 B_3^8 + 499167900 B_1^3 B_2^5 B_4^5 B_5^3 B_6^5 B_3^7$$
$$+ 186763500 B_1^2 B_2^5 B_4^6 B_5^4 B_6^4 B_3^7 + 56029050 B_1^3 B_2^5 B_4^5 B_5^4 B_6^4 B_3^7$$
$$+ 2655776970 B_1^3 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7 + 2655776970 B_1^3 B_2^6 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 56029050 B_1^4 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7 + 186763500 B_1^4 B_2^6 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 41087970 B_1^3 B_2^5 B_4^6 B_5^4 B_6^3 B_3^7 + 136959900 B_1^3 B_2^6 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+ 41087970 B_1^4 B_2^6 B_4^5 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{28} + (4079910912 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^9$$
$$+ 323400000 B_1^2 B_2^4 B_4^5 B_5^3 B_6^5 B_3^8 + 2253071744 B_1^2 B_2^5 B_4^5 B_5^2 B_6^5 B_3^8$$
$$+ 323400000 B_1^3 B_2^5 B_4^4 B_5^2 B_6^5 B_3^8 + 11383680 B_1 B_2^5 B_4^6 B_5^3 B_6^4 B_3^8$$
$$+ 830060000 B_1^2 B_2^4 B_4^6 B_5^3 B_6^4 B_3^8 + 18476731056 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^8$$
$$+ 1778700000 B_1^3 B_2^4 B_4^5 B_5^3 B_6^4 B_3^8 + 1778700000 B_1^3 B_2^5 B_4^4 B_5^3 B_6^4 B_3^8$$
$$+ 4063973760 B_1^2 B_2^5 B_4^6 B_5^2 B_6^4 B_3^8 + 4063973760 B_1^2 B_2^6 B_4^5 B_5^2 B_6^4 B_3^8$$
$$+ 18476731056 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^8 + 830060000 B_1^3 B_2^6 B_4^4 B_5^2 B_6^4 B_3^8$$
$$+ 11383680 B_1^3 B_2^6 B_4^5 B_5 B_6^4 B_3^8 + 871627680 B_1^2 B_2^5 B_4^6 B_5^3 B_6^3 B_3^8$$
$$+ 766975440 B_1^2 B_2^6 B_4^5 B_5^3 B_6^3 B_3^8 + 2414513024 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^8$$
$$+ 766975440 B_1^3 B_2^5 B_4^6 B_5^2 B_6^3 B_3^8 + 871627680 B_1^3 B_2^6 B_4^5 B_5^2 B_6^3 B_3^8$$
$$+ 887409600 B_1^2 B_2^5 B_4^5 B_5^3 B_6^5 B_3^7 + 887409600 B_1^3 B_2^5 B_4^5 B_5^2 B_6^5 B_3^7$$
$$+ 50820000 B_1^2 B_2^4 B_4^6 B_5^4 B_6^4 B_3^7 + 99607200 B_1^2 B_2^5 B_4^5 B_5^4 B_6^4 B_3^7$$
$$+ 3252073440 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7 + 622545000 B_1^3 B_2^4 B_4^6 B_5^3 B_6^4 B_3^7$$
$$+ 2075150000 B_1^2 B_2^6 B_4^5 B_5^3 B_6^4 B_3^7 + 19480302576 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 622545000 B_1^3 B_2^6 B_4^4 B_5^3 B_6^4 B_3^7 + 2075150000 B_1^3 B_2^5 B_4^6 B_5^2 B_6^4 B_3^7$$
$$+ 3252073440 B_1^3 B_2^6 B_4^5 B_5^2 B_6^4 B_3^7 + 99607200 B_1^4 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 50820000 B_1^4 B_2^6 B_4^4 B_5^2 B_6^4 B_3^7 + 73045280 B_1^2 B_2^5 B_4^6 B_5^4 B_6^3 B_3^7$$
$$+ 21913584 B_1^3 B_2^5 B_4^5 B_5^4 B_6^3 B_3^7 + 1016898960 B_1^3 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+ 1016898960 B_1^3 B_2^6 B_4^5 B_5^3 B_6^3 B_3^7 + 21913584 B_1^4 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 73045280 B_1^4 B_2^6 B_4^5 B_5^2 B_6^3 B_3^7)\mathbf{Z}^{27} + (356548500 B_1^2 B_2^4 B_4^5 B_5^2 B_6^5 B_3^8$$
$$+ 356548500 B_1^2 B_2^5 B_4^4 B_5^2 B_6^5 B_3^8 + 48024900 B_1 B_2^4 B_4^6 B_5^3 B_6^4 B_3^8$$
$$+ 94128804 B_1 B_2^5 B_4^5 B_5^3 B_6^4 B_3^8 + 5042614500 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^8$$
$$+ 1961016750 B_1^2 B_2^5 B_4^4 B_5^3 B_6^4 B_3^8 + 588305025 B_1^2 B_2^4 B_4^6 B_5^2 B_6^4 B_3^8$$
$$+ 27835512516 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^8 + 1961016750 B_1^3 B_2^4 B_4^5 B_5^2 B_6^4 B_3^8$$
$$+ 588305025 B_1^2 B_2^6 B_4^4 B_5^2 B_6^4 B_3^8 + 5042614500 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^8$$
$$+ 94128804 B_1^3 B_2^5 B_4^5 B_5 B_6^4 B_3^8 + 48024900 B_1^3 B_2^6 B_4^4 B_5 B_6^4 B_3^8$$
$$+ 264136950 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^8 + 4790284884 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^8$$
$$+ 880456500 B_1^2 B_2^5 B_4^6 B_5^2 B_6^3 B_3^8 + 880456500 B_1^2 B_2^6 B_4^5 B_5^2 B_6^3 B_3^8$$
$$+ 4790284884 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^8 + 264136950 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^8$$
$$+ 305613000 B_1^2 B_2^4 B_4^5 B_5^3 B_6^5 B_3^7 + 1669054464 B_1^2 B_2^5 B_4^5 B_5^2 B_6^5 B_3^7$$
$$+ 305613000 B_1^3 B_2^5 B_4^4 B_5^2 B_6^5 B_3^7 + 34303500 B_1^2 B_2^4 B_4^5 B_5^4 B_6^4 B_3^7$$
$$+ 1413304200 B_1^2 B_2^4 B_4^6 B_5^3 B_6^4 B_3^7 + 30824064054 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 6565308750 B_1^3 B_2^4 B_4^5 B_5^3 B_6^4 B_3^7 + 6565308750 B_1^3 B_2^5 B_4^4 B_5^3 B_6^4 B_3^7$$
$$+ 3902976000 B_1^2 B_2^5 B_4^6 B_5^2 B_6^4 B_3^7 + 3902976000 B_1^2 B_2^6 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 30824064054 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 1413304200 B_1^3 B_2^6 B_4^4 B_5^2 B_6^4 B_3^7$$
$$+ 34303500 B_1^4 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7 + 25155900 B_1^2 B_2^4 B_4^6 B_5^4 B_6^3 B_3^7$$
$$+ 49305564 B_1^2 B_2^5 B_4^5 B_5^4 B_6^3 B_3^7 + 1445944500 B_1^2 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+ 308159775 B_1^3 B_2^4 B_4^6 B_5^3 B_6^3 B_3^7 + 1027199250 B_1^2 B_2^6 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 8951787306 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 308159775 B_1^3 B_2^6 B_4^4 B_5^3 B_6^3 B_3^7$$
$$+ 1027199250 B_1^3 B_2^5 B_4^6 B_5^2 B_6^3 B_3^7 + 1445944500 B_1^3 B_2^6 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+ 49305564 B_1^4 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 25155900 B_1^4 B_2^6 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 8199664704 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^6)\mathbf{Z}^{26}$$
$$+ (409812480 B_1 B_2^4 B_4^5 B_5^3 B_6^4 B_3^8$$
$$+ 122943744 B_1 B_2^5 B_4^5 B_5^2 B_6^4 B_3^8 + 7991343360 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^8$$
$$+ 7991343360 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^8 + 122943744 B_1^2 B_2^5 B_4^5 B_5 B_6^4 B_3^8$$
$$+ 409812480 B_1^3 B_2^5 B_4^4 B_5 B_6^4 B_3^8 + 2253968640 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^8$$
$$+ 8611029504 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^8 + 2253968640 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^8$$
$$+ 599001480 B_1^2 B_2^4 B_4^5 B_5^2 B_6^5 B_3^7 + 599001480 B_1^2 B_2^5 B_4^4 B_5^2 B_6^5 B_3^7$$
$$+ 114345000 B_1 B_2^4 B_4^6 B_5^3 B_6^4 B_3^7 + 224116200 B_1 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 19609868880 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^7 + 9158882040 B_1^2 B_2^5 B_4^4 B_5^3 B_6^4 B_3^7$$
$$+ 2858625000 B_1^3 B_2^4 B_4^4 B_5^3 B_6^4 B_3^7 + 1400726250 B_1^2 B_2^4 B_4^6 B_5^2 B_6^4 B_3^7$$
$$+ 59798117736 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7 + 9158882040 B_1^3 B_2^4 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 1400726250 B_1^2 B_2^6 B_4^4 B_5^2 B_6^4 B_3^7 + 19609868880 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7$$
$$+ 224116200 B_1^3 B_2^5 B_4^5 B_5 B_6^4 B_3^7 + 114345000 B_1^3 B_2^6 B_4^4 B_5 B_6^4 B_3^7$$
$$+ 30187080 B_1^2 B_2^4 B_4^5 B_5^4 B_6^3 B_3^7 + 966735000 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+ 17946116496 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7 + 3421632060 B_1^3 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 3421632060 B_1^3 B_2^5 B_4^4 B_5^3 B_6^3 B_3^7 + 2096325000 B_1^2 B_2^5 B_4^6 B_5^2 B_6^3 B_3^7$$
$$+ 2096325000 B_1^2 B_2^6 B_4^5 B_5^2 B_6^3 B_3^7 + 17946116496 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+ 966735000 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^7 + 30187080 B_1^4 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 439267752 B_1^3 B_2^5 B_4^5 B_5^3 B_6^2 B_3^7 + 16734009600 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^6$$
$$+ 5020202880 B_1^3 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6 + 5020202880 B_1^3 B_2^5 B_4^4 B_5^3 B_6^4 B_3^6$$
$$+ 16734009600 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^6$$
$$+ 6754454784 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6)\mathbf{Z}^{25}$$
$$+ (557800320 B_1 B_2^4 B_4^5 B_5^2 B_6^4 B_3^8 + 1859334400 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^8$$
$$+ 557800320 B_1^2 B_2^5 B_4^4 B_5 B_6^4 B_3^8 + 3067901760 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^8$$
$$+ 3067901760 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^8 + 221852400 B_1^2 B_2^4 B_4^4 B_5^2 B_6^5 B_3^7$$
$$+ 2212838320 B_1 B_2^4 B_4^5 B_5^3 B_6^4 B_3^7 + 1985156250 B_1^2 B_2^3 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 6097637700 B_1^2 B_2^4 B_4^4 B_5^3 B_6^4 B_3^7 + 520396800 B_1 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 40830215370 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^7 + 40830215370 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7$$
$$+ 6097637700 B_1^3 B_2^4 B_4^4 B_5^2 B_6^4 B_3^7 + 1985156250 B_1^3 B_2^5 B_4^3 B_5^2 B_6^4 B_3^7$$
$$+ 520396800 B_1^2 B_2^5 B_4^5 B_5 B_6^4 B_3^7 + 2212838320 B_1^3 B_2^5 B_4^4 B_5 B_6^4 B_3^7$$
$$+ 69877500 B_1 B_2^4 B_4^6 B_5^3 B_6^3 B_3^7 + 136959900 B_1 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 16980581310 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7 + 5281747240 B_1^2 B_2^5 B_4^4 B_5^3 B_6^3 B_3^7$$
$$+ 2862182400 B_1^3 B_2^4 B_4^4 B_5^3 B_6^3 B_3^7 + 855999375 B_1^2 B_2^4 B_4^6 B_5^2 B_6^3 B_3^7$$
$$+ 41763159900 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7 + 5281747240 B_1^3 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7$$

$$+ 855999375 B_1^2 B_2^6 B_4^4 B_5^2 B_6^3 B_3^7 + 16980581310 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 136959900 B_1^3 B_2^5 B_4^5 B_5 B_6^3 B_3^7 + 69877500 B_1^3 B_2^6 B_4^4 B_5 B_6^3 B_3^7$$
$$+ 1220188200 B_1^2 B_2^5 B_4^5 B_5^3 B_6^2 B_3^7 + 1220188200 B_1^3 B_2^5 B_4^5 B_5^2 B_6^2 B_3^7$$
$$+ 18993314340 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6 + 10676646750 B_1^2 B_2^5 B_4^4 B_5^3 B_6^4 B_3^6$$
$$+ 3890016900 B_1^3 B_2^4 B_4^4 B_5^3 B_6^4 B_3^6 + 38856294400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^6$$
$$+ 10676646750 B_1^3 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6 + 18993314340 B_1^3 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+ 3913140 B_1^2 B_2^4 B_4^5 B_5^4 B_6^3 B_3^6 + 81523750 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^6$$
$$+ 16798204500 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6 + 5039461350 B_1^3 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+ 5039461350 B_1^3 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6 + 16798204500 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+ 81523750 B_1^3 B_2^6 B_4^4 B_5^2 B_6^3 B_3^6 + 3913140 B_1^4 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+ 1423552900 B_1^3 B_2^5 B_4^5 B_5^3 B_6^2 B_3^6)\mathbf{Z}^{24} + (457380000 B_1 B_2^3 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+ 896464800 B_1 B_2^4 B_4^4 B_5^3 B_6^4 B_3^7 + 4201797600 B_1 B_2^4 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 5602905000 B_1^2 B_2^3 B_4^5 B_5^2 B_6^4 B_3^7 + 268939440 B_1 B_2^5 B_4^4 B_5^2 B_6^4 B_3^7$$
$$+ 32647658400 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^7 + 5602905000 B_1^2 B_2^5 B_4^3 B_5^2 B_6^4 B_3^7$$
$$+ 268939440 B_1^2 B_2^4 B_4^5 B_5 B_6^4 B_3^7 + 4201797600 B_1^2 B_2^5 B_4^4 B_5 B_6^4 B_3^7$$
$$+ 896464800 B_1^3 B_2^4 B_4^4 B_5 B_6^4 B_3^7 + 457380000 B_1^3 B_2^5 B_4^3 B_5 B_6^4 B_3^7$$
$$+ 1537683840 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7 + 2515590000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 6940533600 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^7 + 402494400 B_1 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 38328942960 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7 + 38328942960 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 6940533600 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 2515590000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^7$$
$$+ 402494400 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^7 + 1537683840 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^7$$
$$+ 1075757760 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^7 + 3585859200 B_1^2 B_2^5 B_4^5 B_5^2 B_6^2 B_3^7$$
$$+ 1075757760 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^7 + 2561328000 B_1 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6$$
$$+ 4802490000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^4 B_3^6 + 14386125600 B_1^2 B_2^4 B_4^4 B_5^3 B_6^4 B_3^6$$
$$+ 48870138240 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6 + 48870138240 B_1^2 B_2^5 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+ 14386125600 B_1^3 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 4802490000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^4 B_3^6$$
$$+ 2561328000 B_1^3 B_2^5 B_4^4 B_5 B_6^4 B_3^6 + 28131531120 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+ 12664602720 B_1^2 B_2^5 B_4^4 B_5^3 B_6^3 B_3^6 + 6411081600 B_1^3 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6$$
$$+ 45964195200 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6 + 12664602720 B_1^3 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+ 28131531120 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 + 4183502400 B_1^2 B_2^5 B_4^5 B_5^3 B_6^2 B_3^6$$
$$+ 1255050720 B_1^3 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6 + 1255050720 B_1^3 B_2^5 B_4^4 B_5^3 B_6^2 B_3^6$$
$$+ 4183502400 B_1^3 B_2^5 B_4^5 B_5^2 B_6^2 B_3^6)\mathbf{Z}^{23}$$
$$+ (1400726250 B_1 B_2^3 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+ 3186932364 B_1 B_2^4 B_4^4 B_5^2 B_6^4 B_3^7 + 4669087500 B_1^2 B_2^3 B_4^4 B_5^2 B_6^4 B_3^7$$
$$+ 4669087500 B_1^2 B_2^4 B_4^3 B_5^2 B_6^4 B_3^7 + 3186932364 B_1^2 B_2^4 B_4^4 B_5 B_6^4 B_3^7$$
$$+ 1400726250 B_1^2 B_2^5 B_4^3 B_5 B_6^4 B_3^7 + 628897500 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^7$$
$$+ 1232639100 B_1 B_2^4 B_4^4 B_5^3 B_6^3 B_3^7 + 3421632060 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+ 7703994375 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^7 + 369791730 B_1 B_2^5 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 38630489436 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 7703994375 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^7$$
$$+ 369791730 B_1^2 B_2^4 B_4^5 B_5 B_6^3 B_3^7 + 3421632060 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^7$$
$$+ 1232639100 B_1^3 B_2^4 B_4^4 B_5 B_6^3 B_3^7 + 628897500 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^7$$
$$+ 3294508140 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^7 + 3294508140 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^7$$
$$+ 1200622500 B_1 B_2^3 B_4^5 B_5^3 B_6^4 B_3^6 + 2353220100 B_1 B_2^4 B_4^4 B_5^3 B_6^4 B_3^6$$
$$+ 4002075000 B_1^2 B_2^3 B_4^4 B_5^3 B_6^4 B_3^6 + 6556999680 B_1 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6$$
$$+ 14707625625 B_1^2 B_2^3 B_4^5 B_5^2 B_6^4 B_3^6 + 76881768516 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+ 14707625625 B_1^2 B_2^5 B_4^3 B_5^2 B_6^4 B_3^6 + 4002075000 B_1^3 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6$$
$$+ 6556999680 B_1^2 B_2^5 B_4^4 B_5 B_6^4 B_3^6 + 2353220100 B_1^3 B_2^4 B_4^4 B_5 B_6^4 B_3^6$$
$$+ 1200622500 B_1^3 B_2^5 B_4^3 B_5 B_6^4 B_3^6 + 3447314640 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+ 10017315000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 + 27874845306 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6$$
$$+ 80030488230 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 80030488230 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+ 27874845306 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6 + 10017315000 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6$$
$$+ 3447314640 B_1^3 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 8519617260 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6$$
$$+ 3843592830 B_1^2 B_2^5 B_4^4 B_5^3 B_6^2 B_3^6 + 1967099904 B_1^3 B_2^4 B_4^4 B_5^3 B_6^2 B_3^6$$
$$+ 13340250000 B_1^2 B_2^5 B_4^5 B_5^2 B_6^2 B_3^6 + 3843592830 B_1^3 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6$$
$$+ 8519617260 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6 + 2629630080 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^5$$
$$+ 788889024 B_1^3 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5 + 2629630080 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5)\mathbf{Z}^{22}$$
$$+ (1328096000 B_1 B_2^3 B_4^4 B_5^2 B_6^4 B_3^7 + 398428800 B_1 B_2^4 B_4^4 B_5 B_6^4 B_3^7$$
$$+ 1328096000 B_1^2 B_2^4 B_4^3 B_5 B_6^4 B_3^7 + 2191358400 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+ 4881115008 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^7 + 7304528000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^7$$
$$+ 7304528000 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^7 + 4881115008 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^7$$
$$+ 2191358400 B_1^2 B_2^5 B_4^3 B_5 B_6^3 B_3^7 + 3123681792 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^7$$
$$+ 1138368000 B_1 B_2^3 B_4^4 B_5^3 B_6^4 B_3^6 + 3890016900 B_1 B_2^3 B_4^5 B_5^2 B_6^4 B_3^6$$
$$+ 10875161528 B_1 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6 + 28921662000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+ 28921662000 B_1^2 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6 + 10875161528 B_1^2 B_2^4 B_4^4 B_5 B_6^4 B_3^6$$
$$+ 3890016900 B_1^2 B_2^5 B_4^3 B_5 B_6^4 B_3^6 + 1138368000 B_1^3 B_2^4 B_4^3 B_5 B_6^4 B_3^6$$
$$+ 2752521200 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6 + 5394941552 B_1 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6$$
$$+ 10284615000 B_1^2 B_2^3 B_4^4 B_5^3 B_6^3 B_3^6 + 97828500 B_1^2 B_2^4 B_4^3 B_5^3 B_6^3 B_3^6$$
$$+ 10284615000 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6 + 33718384700 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+ 97828500 B_1 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6 + 163830961008 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+ 97828500 B_1^3 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + 33718384700 B_1^2 B_2^5 B_4^3 B_5^2 B_6^3 B_3^6$$
$$+ 10284615000 B_1^3 B_2^4 B_4^3 B_5^2 B_6^3 B_3^6 + 97828500 B_1^2 B_2^4 B_4^5 B_5 B_6^3 B_3^6$$
$$+ 10284615000 B_1^2 B_2^5 B_4^4 B_5 B_6^3 B_3^6 + 5394941552 B_1^3 B_2^4 B_4^4 B_5 B_6^3 B_3^6$$
$$+ 2752521200 B_1^3 B_2^5 B_4^3 B_5 B_6^3 B_3^6 + 1138368000 B_1 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6$$
$$+ 3890016900 B_1^2 B_2^3 B_4^5 B_5^3 B_6^2 B_3^6 + 10875161528 B_1^2 B_2^4 B_4^4 B_5^3 B_6^2 B_3^6$$
$$+ 28921662000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6 + 28921662000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^2 B_3^6$$
$$+ 10875161528 B_1^3 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6 + 3890016900 B_1^3 B_2^5 B_4^3 B_5^2 B_6^2 B_3^6$$
$$+ 1138368000 B_1^3 B_2^5 B_4^4 B_5 B_6^2 B_3^6 + 3123681792 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^5$$
$$+ 2191358400 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5 + 4881115008 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5$$
$$+ 7304528000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^5 + 7304528000 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^5$$
$$+ 4881115008 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5 + 2191358400 B_1^3 B_2^5 B_4^3 B_5^2 B_6^3 B_3^5$$
$$+ 1328096000 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^5 + 398428800 B_1^3 B_2^4 B_4^4 B_5^3 B_6^2 B_3^5$$
$$+ 1328096000 B_1^3 B_2^5 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{21}$$
$$+ (2629630080 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^7 + 788889024 B_1 B_2^4 B_4^4 B_5 B_6^3 B_3^7$$
$$+ 2629630080 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^7 + 8519617260 B_1 B_2^3 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+ 3843592830 B_1 B_2^4 B_4^3 B_5^2 B_6^4 B_3^6 + 13340250000 B_1^2 B_2^3 B_4^3 B_5^2 B_6^4 B_3^6$$
$$+ 1967099904 B_1 B_2^4 B_4^4 B_5 B_6^4 B_3^6 + 3843592830 B_1^2 B_2^3 B_4^4 B_5 B_6^4 B_3^6$$
$$+ 8519617260 B_1^2 B_2^4 B_4^3 B_5 B_6^4 B_3^6 + 3447314640 B_1 B_2^3 B_4^4 B_5^3 B_6^3 B_3^6$$
$$+ 10017315000 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6 + 27874845306 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+ 80030488230 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6 + 80030488230 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^6$$
$$+ 27874845306 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^6 + 10017315000 B_1^2 B_2^5 B_4^3 B_5 B_6^3 B_3^6$$
$$+ 3447314640 B_1^3 B_2^4 B_4^3 B_5 B_6^3 B_3^6 + 1200622500 B_1 B_2^3 B_4^5 B_5^3 B_6^2 B_3^6$$
$$+ 2353220100 B_1 B_2^4 B_4^4 B_5^3 B_6^2 B_3^6 + 6556999680 B_1^2 B_2^3 B_4^4 B_5^3 B_6^2 B_3^6$$
$$+ 4002075000 B_1 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6 + 14707625625 B_1^2 B_2^3 B_4^5 B_5^2 B_6^2 B_3^6$$
$$+ 76881768516 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6 + 14707625625 B_1^2 B_2^5 B_4^3 B_5^2 B_6^2 B_3^6$$

$$+6556999680B_1^3B_2^4B_4^3B_5^2B_6^2B_3^6+4002075000B_1^2B_2^5B_4^4B_5B_6^2B_3^6$$
$$+2353220100B_1^3B_2^4B_4^4B_5B_6^2B_3^6+1200622500B_1^3B_2^5B_4^3B_5B_6^2B_3^6$$
$$+3294508140B_1^2B_2^3B_4^4B_5^2B_6^4B_3^5+3294508140B_1^2B_2^4B_4^3B_5^2B_6^4B_3^5$$
$$+628897500B_1B_2^3B_4^5B_5^3B_6^3B_3^5+1232639100B_1B_2^4B_4^4B_5^3B_6^3B_3^5$$
$$+3421632060B_1^2B_2^3B_4^4B_5^3B_6^3B_3^5+369791730B_1^2B_2^4B_4^3B_5^3B_6^3B_3^5$$
$$+7703994375B_1^2B_2^3B_4^5B_5^2B_6^3B_3^5+38630489436B_1^2B_2^4B_4^4B_5^2B_6^3B_3^5$$
$$+369791730B_1^3B_2^3B_4^4B_5^2B_6^3B_3^5+7703994375B_1^2B_2^5B_4^3B_5^2B_6^3B_3^5$$
$$+3421632060B_1^3B_2^4B_4^3B_5^2B_6^3B_3^5+1232639100B_1^3B_2^4B_4^4B_5B_6^3B_3^5$$
$$+628897500B_1^3B_2^5B_4^3B_5B_6^3B_3^5+1400726250B_1^2B_2^3B_4^5B_5^3B_6^2B_3^5$$
$$+3186932364B_1^2B_2^4B_4^4B_5^3B_6^2B_3^5+4669087500B_1^2B_2^4B_4^5B_5^2B_6^2B_3^5$$
$$+4669087500B_1^2B_2^5B_4^4B_5^2B_6^2B_3^5+3186932364B_1^3B_2^4B_4^4B_5^2B_6^2B_3^5$$
$$+1400726250B_1^3B_2^5B_4^3B_5^2B_6^2B_3^5)\mathbf{Z}^{20}+(4183502400B_1B_2^3B_4^3B_5^2B_6^4B_3^6$$
$$+1255050720B_1B_2^3B_4^4B_5B_6^4B_3^6+1255050720B_1B_2^4B_4^3B_5B_6^4B_3^6$$
$$+4183502400B_1^2B_2^3B_4^3B_5B_6^4B_3^6+28131531120B_1B_2^3B_4^4B_5^2B_6^3B_3^6$$
$$+12664602720B_1B_2^4B_4^3B_5^2B_6^3B_3^6+45964195200B_1^2B_2^3B_4^3B_5^2B_6^3B_3^6$$
$$+6411081600B_1B_2^4B_4^4B_5B_6^3B_3^6+12664602720B_1^2B_2^3B_4^4B_5B_6^3B_3^6$$
$$+28131531120B_1^2B_2^4B_4^3B_5B_6^3B_3^6+2561328000B_1B_2^3B_4^4B_5^3B_6^2B_3^6$$
$$+4802490000B_1B_2^3B_4^5B_5^2B_6^2B_3^6+14386125600B_1B_2^4B_4^4B_5^2B_6^2B_3^6$$
$$+48870138240B_1^2B_2^3B_4^4B_5^2B_6^2B_3^6+48870138240B_1^2B_2^4B_4^3B_5^2B_6^2B_3^6$$
$$+14386125600B_1^2B_2^4B_4^4B_5B_6^2B_3^6+4802490000B_1^2B_2^5B_4^3B_5B_6^2B_3^6$$
$$+2561328000B_1^3B_2^4B_4^3B_5B_6^2B_3^6+1075757760B_1B_2^3B_4^4B_5^2B_6^4B_3^5$$
$$+3585859200B_1^2B_2^3B_4^3B_5^2B_6^4B_3^5+1075757760B_1^2B_2^4B_4^3B_5B_6^4B_3^5$$
$$+1537683840B_1B_2^3B_4^4B_5^3B_6^3B_3^5+402494400B_1^2B_2^3B_4^3B_5^3B_6^3B_3^5$$
$$+2515590000B_1B_2^3B_4^5B_5^2B_6^3B_3^5+6940533600B_1B_2^4B_4^4B_5^2B_6^3B_3^5$$
$$+38328942960B_1^2B_2^3B_4^4B_5^2B_6^3B_3^5+38328942960B_1^2B_2^4B_4^3B_5^2B_6^3B_3^5$$
$$+402494400B_1^3B_2^3B_4^3B_5^2B_6^3B_3^5+6940533600B_1^2B_2^4B_4^4B_5B_6^3B_3^5$$
$$+2515590000B_1^2B_2^5B_4^3B_5B_6^3B_3^5+1537683840B_1^3B_2^4B_4^3B_5B_6^3B_3^5$$
$$+457380000B_1B_2^3B_4^5B_5^3B_6^2B_3^5+896464800B_1B_2^4B_4^4B_5^3B_6^2B_3^5$$
$$+4201797600B_1^2B_2^3B_4^4B_5^3B_6^2B_3^5+268939440B_1^2B_2^4B_4^3B_5^3B_6^2B_3^5$$
$$+5602905000B_1^2B_2^3B_4^5B_5^2B_6^2B_3^5+32647658400B_1^2B_2^4B_4^4B_5^2B_6^2B_3^5$$
$$+268939440B_1^3B_2^3B_4^4B_5^2B_6^2B_3^5+5602905000B_1^2B_2^5B_4^3B_5^2B_6^2B_3^5$$
$$+4201797600B_1^3B_2^4B_4^3B_5^2B_6^2B_3^5+896464800B_1^3B_2^4B_4^4B_5B_6^2B_3^5$$
$$+457380000B_1^3B_2^5B_4^3B_5B_6^2B_3^5)\mathbf{Z}^{19}$$
$$+(1423552900B_1B_2^3B_4^3B_5B_6^4B_3^6+3913140B_1^2B_2^4B_4^3B_6^3B_3^6$$
$$+3913140B_2^3B_4^4B_5^2B_6^3B_3^6+81523750B_1B_2^2B_4^4B_5^2B_6^3B_3^6$$
$$+16798204500B_1B_2^3B_4^3B_5^2B_6^3B_3^6+5039461350B_1B_2^3B_4^4B_5B_6^3B_3^6$$
$$+5039461350B_1B_2^4B_4^3B_5B_6^3B_3^6+16798204500B_1^2B_2^3B_4^3B_5B_6^3B_3^6$$
$$+81523750B_1^2B_2^2B_4^4B_5B_6^3B_3^6+18993314340B_1B_2^3B_4^4B_5^2B_6^2B_3^6$$
$$+10676646750B_1B_2^4B_4^3B_5^2B_6^2B_3^6+38856294400B_1^2B_2^3B_4^3B_5^2B_6^2B_3^6$$
$$+3890016900B_1B_2^4B_4^4B_5B_6^2B_3^6+10676646750B_1^2B_2^3B_4^4B_5B_6^2B_3^6$$
$$+18993314340B_1^2B_2^4B_4^3B_5B_6^2B_3^6+1220188200B_1B_2^3B_4^3B_5^2B_6^4B_3^5$$
$$+1220188200B_1^2B_2^3B_4^3B_5B_6^4B_3^5+69877500B_1B_2^2B_4^4B_5^3B_6^3B_3^5$$
$$+136959900B_1B_2^3B_4^3B_5^3B_6^3B_3^5+16980581310B_1B_2^3B_4^4B_5^2B_6^3B_3^5$$
$$+855999375B_1^2B_2^2B_4^4B_5^2B_6^3B_3^5+5281747240B_1B_2^4B_4^3B_5^2B_6^3B_3^5$$
$$+41763159900B_1^2B_2^3B_4^3B_5^2B_6^3B_3^5+855999375B_1^2B_2^4B_4^2B_5^2B_6^3B_3^5$$
$$+2862182400B_1B_2^4B_4^4B_5B_6^3B_3^5+5281747240B_1^2B_2^3B_4^4B_5B_6^3B_3^5$$
$$+16980581310B_1^2B_2^4B_4^3B_5B_6^3B_3^5+136959900B_1^3B_2^3B_4^3B_5B_6^3B_3^5$$
$$+69877500B_1^3B_2^4B_4^2B_5B_6^3B_3^5+2212838320B_1B_2^3B_4^4B_5^3B_6^2B_3^5$$
$$+520396800B_1^2B_2^3B_4^3B_5^3B_6^2B_3^5+1985156250B_1B_2^3B_4^5B_5^2B_6^2B_3^5$$
$$+6097637700B_1B_2^4B_4^4B_5^2B_6^2B_3^5+40830215370B_1^2B_2^3B_4^4B_5^2B_6^2B_3^5$$
$$+40830215370B_1^2B_2^4B_4^3B_5^2B_6^2B_3^5+520396800B_1^3B_2^3B_4^3B_5^2B_6^2B_3^5$$
$$+6097637700B_1^2B_2^4B_4^4B_5B_6^2B_3^5+1985156250B_1^2B_2^5B_4^3B_5B_6^2B_3^5$$
$$+2212838320B_1^3B_2^4B_4^3B_5B_6^2B_3^5+221852400B_1^2B_2^4B_4^4B_5^2B_6B_3^5$$
$$+3067901760B_1^2B_2^3B_4^4B_5^2B_6^3B_3^4+3067901760B_1^2B_2^4B_4^3B_5^2B_6^3B_3^4$$
$$+557800320B_1^2B_2^3B_4^4B_5^3B_6^2B_3^4+1859334400B_1^2B_2^4B_4^4B_5^2B_6^2B_3^4$$
$$+557800320B_1^3B_2^4B_4^3B_5^2B_6^2B_3^4)\mathbf{Z}^{18}$$
$$+(6754454784B_1B_2^3B_4^3B_5B_6^3B_3^6+16734009600B_1B_2^3B_4^3B_5^2B_6^2B_3^6$$
$$+5020202880B_1B_2^3B_4^4B_5B_6^2B_3^6+5020202880B_1B_2^4B_4^3B_5B_6^2B_3^6$$
$$+16734009600B_1^2B_2^3B_4^3B_5B_6^2B_3^6+439267752B_1B_2^3B_4^3B_5B_6^4B_3^5$$
$$+30187080B_1^2B_2^4B_4^3B_6^3B_3^5+30187080B_2^3B_4^4B_5^2B_6^3B_3^5$$
$$+966735000B_1B_2^2B_4^4B_5^2B_6^3B_3^5+17946116496B_1B_2^3B_4^3B_5^2B_6^3B_3^5$$
$$+2096325000B_1^2B_2^2B_4^3B_5^2B_6^3B_3^5+2096325000B_1^2B_2^3B_4^2B_5^2B_6^3B_3^5$$
$$+3421632060B_1B_2^3B_4^4B_5B_6^3B_3^5+3421632060B_1B_2^4B_4^3B_5B_6^3B_3^5$$
$$+17946116496B_1^2B_2^3B_4^3B_5B_6^3B_3^5+966735000B_1^2B_2^4B_4^2B_5B_6^3B_3^5$$
$$+114345000B_1B_2^2B_4^4B_5^3B_6^2B_3^5+224116200B_1B_2^3B_4^3B_5^3B_6^2B_3^5$$
$$+19609868880B_1B_2^3B_4^4B_5^2B_6^2B_3^5+1400726250B_1^2B_2^2B_4^4B_5^2B_6^2B_3^5$$
$$+9158882040B_1B_2^4B_4^3B_5^2B_6^2B_3^5+59798117736B_1^2B_2^3B_4^3B_5^2B_6^2B_3^5$$
$$+1400726250B_1^2B_2^4B_4^2B_5^2B_6^2B_3^5+2858625000B_1B_2^4B_4^4B_5B_6^2B_3^5$$
$$+9158882040B_1^2B_2^3B_4^4B_5B_6^2B_3^5+19609868880B_1^2B_2^4B_4^3B_5B_6^2B_3^5$$
$$+224116200B_1^3B_2^3B_4^3B_5B_6^2B_3^5+114345000B_1^3B_2^4B_4^2B_5B_6^2B_3^5$$
$$+599001480B_1^2B_2^3B_4^4B_5^2B_6B_3^5+599001480B_1^2B_2^4B_4^3B_5^2B_6B_3^5$$
$$+2253968640B_1B_2^3B_4^4B_5^2B_6^3B_3^4+8611029504B_1^2B_2^3B_4^3B_5^2B_6^3B_3^4$$
$$+2253968640B_1^2B_2^4B_4^3B_5B_6^3B_3^4+409812480B_1B_2^3B_4^4B_5^3B_6^2B_3^4$$
$$+122943744B_1^2B_2^3B_4^3B_5^3B_6^2B_3^4+7991343360B_1^2B_2^3B_4^4B_5^2B_6^2B_3^4$$
$$+7991343360B_1^2B_2^4B_4^3B_5^2B_6^2B_3^4+122943744B_1^3B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+409812480B_1^3B_2^4B_4^3B_5B_6^2B_3^4)\mathbf{Z}^{17}$$
$$+(8199664704B_1B_2^3B_4^3B_5B_6^2B_3^6+49305564B_1^2B_2^3B_4^3B_6^3B_3^5$$
$$+25155900B_1^2B_2^4B_4^2B_6^3B_3^5+25155900B_2^2B_4^4B_5^2B_6^3B_3^5$$
$$+49305564B_2^3B_4^3B_5^2B_6^3B_3^5+1445944500B_1B_2^2B_4^3B_5^2B_6^3B_3^5$$
$$+1027199250B_1B_2^3B_4^2B_5^2B_6^3B_3^5+308159775B_1B_2^2B_4^4B_5B_6^3B_3^5$$
$$+8951787306B_1B_2^3B_4^3B_5B_6^3B_3^5+1027199250B_1^2B_2^2B_4^3B_5B_6^3B_3^5$$
$$+308159775B_1B_2^4B_4^2B_5B_6^3B_3^5+1445944500B_1^2B_2^3B_4^2B_5B_6^3B_3^5$$
$$+34303500B_1^2B_2^4B_4^3B_6^2B_3^5+34303500B_2^3B_4^4B_5^2B_6^2B_3^5$$
$$+1413304200B_1B_2^2B_4^4B_5^2B_6^2B_3^5+30824064054B_1B_2^3B_4^3B_5^2B_6^2B_3^5$$
$$+3902976000B_1^2B_2^2B_4^3B_5^2B_6^2B_3^5+3902976000B_1^2B_2^3B_4^2B_5^2B_6^2B_3^5$$
$$+6565308750B_1B_2^3B_4^4B_5B_6^2B_3^5+6565308750B_1B_2^4B_4^3B_5B_6^2B_3^5$$
$$+30824064054B_1^2B_2^3B_4^3B_5B_6^2B_3^5+1413304200B_1^2B_2^4B_4^2B_5B_6^2B_3^5$$
$$+305613000B_1B_2^3B_4^4B_5^2B_6B_3^5+1669054464B_1^2B_2^3B_4^3B_5^2B_6B_3^5$$
$$+305613000B_1^2B_2^4B_4^3B_5B_6B_3^5+264136950B_1B_2^2B_4^4B_5^2B_6^3B_3^4$$
$$+4790284884B_1B_2^3B_4^3B_5^2B_6^3B_3^4+880456500B_1^2B_2^2B_4^3B_5^2B_6^3B_3^4$$
$$+880456500B_1^2B_2^3B_4^2B_5^2B_6^3B_3^4+4790284884B_1^2B_2^3B_4^3B_5B_6^3B_3^4$$
$$+264136950B_1^2B_2^4B_4^2B_5B_6^3B_3^4+48024900B_1B_2^2B_4^4B_5^3B_6^2B_3^4$$
$$+94128804B_1B_2^3B_4^3B_5^3B_6^2B_3^4+5042614500B_1B_2^3B_4^4B_5^2B_6^2B_3^4$$
$$+588305025B_1^2B_2^2B_4^4B_5^2B_6^2B_3^4+1961016750B_1B_2^4B_4^3B_5^2B_6^2B_3^4$$

$$+27835512516B_1^2B_2^3B_4^3B_5^2B_6^2B_3^4+588305025B_1^2B_2^4B_4^2B_5^2B_6^2B_3^4$$
$$+1961016750B_1^2B_2^3B_4^4B_5B_6^2B_3^4+5042614500B_1^2B_2^4B_4^3B_5B_6^2B_3^4$$
$$+94128804B_1^3B_2^3B_4^3B_5B_6^2B_3^4+48024900B_1^3B_2^4B_4^2B_5B_6^2B_3^4$$
$$+356548500B_1^2B_2^3B_4^4B_5^2B_6B_3^4+356548500B_1^2B_2^4B_4^3B_5^2B_6B_3^4)\mathbf{Z}^{16}$$
$$+(21913584B_1B_2^3B_4^3B_6^3B_3^5+73045280B_1^2B_2^3B_4^2B_6^3B_3^5$$
$$+73045280B_2^2B_4^3B_5^2B_6^3B_3^5+21913584B_2^3B_4^3B_5B_6^3B_3^5$$
$$+1016898960B_1B_2^2B_4^3B_5B_6^3B_3^5+1016898960B_1B_2^3B_4^2B_5B_6^3B_3^5$$
$$+99607200B_1^2B_2^3B_4^3B_6^2B_3^5+50820000B_1^2B_2^4B_4^2B_6^2B_3^5$$
$$+50820000B_2^2B_4^4B_5^2B_6^2B_3^5+99607200B_2^3B_4^3B_5^2B_6^2B_3^5$$
$$+3252073440B_1B_2^2B_4^3B_5^2B_6^2B_3^5+2075150000B_1B_2^3B_4^2B_5^2B_6^2B_3^5$$
$$+622545000B_1B_2^2B_4^4B_5B_6^2B_3^5+19480302576B_1B_2^3B_4^3B_5B_6^2B_3^5$$
$$+2075150000B_1^2B_2^2B_4^3B_5B_6^2B_3^5+622545000B_1B_2^4B_4^2B_5B_6^2B_3^5$$
$$+3252073440B_1^2B_2^3B_4^2B_5B_6^2B_3^5+887409600B_1B_2^3B_4^3B_5^2B_6B_3^5$$
$$+887409600B_1^2B_2^3B_4^3B_5B_6B_3^5+871627680B_1B_2^2B_4^3B_5^2B_6^3B_3^4$$
$$+766975440B_1B_2^3B_4^2B_5^2B_6^3B_3^4+2414513024B_1B_2^3B_4^3B_5B_6^3B_3^4$$
$$+766975440B_1^2B_2^2B_4^3B_5B_6^3B_3^4+871627680B_1^2B_2^3B_4^2B_5B_6^3B_3^4$$
$$+11383680B_1B_2^2B_4^3B_5^3B_6^2B_3^4+830060000B_1B_2^2B_4^4B_5^2B_6^2B_3^4$$
$$+18476731056B_1B_2^3B_4^3B_5^2B_6^2B_3^4+4063973760B_1^2B_2^2B_4^3B_5^2B_6^2B_3^4$$
$$+4063973760B_1^2B_2^3B_4^2B_5^2B_6^2B_3^4+1778700000B_1B_2^3B_4^4B_5B_6^2B_3^4$$
$$+1778700000B_1B_2^4B_4^3B_5B_6^2B_3^4+18476731056B_1^2B_2^3B_4^3B_5B_6^2B_3^4$$
$$+830060000B_1^2B_2^4B_4^2B_5B_6^2B_3^4+11383680B_1^3B_2^3B_4^2B_5B_6^2B_3^4$$
$$+323400000B_1B_2^3B_4^4B_5^2B_6B_3^4+2253071744B_1^2B_2^3B_4^3B_5^2B_6B_3^4$$
$$+323400000B_1^2B_2^4B_4^3B_5B_6B_3^4+4079910912B_1^2B_2^3B_4^3B_5^2B_6^2B_3^3)\mathbf{Z}^{15}$$
$$+(41087970B_1B_2^3B_4^2B_6^3B_3^5+41087970B_2^2B_4^3B_5B_6^3B_3^5$$
$$+136959900B_1B_2^2B_4^2B_5B_6^3B_3^5+56029050B_1B_2^3B_4^3B_6^2B_3^5$$
$$+186763500B_1^2B_2^3B_4^2B_6^2B_3^5+186763500B_2^2B_4^3B_5^2B_6^2B_3^5$$
$$+56029050B_2^3B_4^3B_5B_6^2B_3^5+2655776970B_1B_2^2B_4^3B_5B_6^2B_3^5$$
$$+2655776970B_1B_2^3B_4^2B_5B_6^2B_3^5+499167900B_1B_2^3B_4^3B_5B_6B_3^5$$
$$+35218260B_1^2B_2^3B_4^2B_6^3B_3^4+35218260B_2^2B_4^3B_5^2B_6^3B_3^4$$
$$+117394200B_1B_2^2B_4^2B_5^2B_6^3B_3^4+607296690B_1B_2^2B_4^3B_5B_6^3B_3^4$$
$$+607296690B_1B_2^3B_4^2B_5B_6^3B_3^4+117394200B_1^2B_2^2B_4^2B_5B_6^3B_3^4$$
$$+48024900B_1^2B_2^3B_4^3B_6^2B_3^4+24502500B_1^2B_2^4B_4^2B_6^2B_3^4$$
$$+24502500B_2^2B_4^4B_5^2B_6^2B_3^4+48024900B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+4609356210B_1B_2^2B_4^3B_5^2B_6^2B_3^4+3054383640B_1B_2^3B_4^2B_5^2B_6^2B_3^4$$
$$+446054400B_1^2B_2^2B_4^2B_5^2B_6^2B_3^4+300155625B_1B_2^2B_4^4B_5B_6^2B_3^4$$
$$+14283282150B_1B_2^3B_4^3B_5B_6^2B_3^4+3054383640B_1^2B_2^2B_4^3B_5B_6^2B_3^4$$
$$+300155625B_1B_2^4B_4^2B_5B_6^2B_3^4+4609356210B_1^2B_2^3B_4^2B_5B_6^2B_3^4$$
$$+54573750B_1B_2^2B_4^4B_5^2B_6B_3^4+1671169500B_1B_2^3B_4^3B_5^2B_6B_3^4$$
$$+298045440B_1^2B_2^2B_4^3B_5^2B_6B_3^4+298045440B_1^2B_2^3B_4^2B_5^2B_6B_3^4$$
$$+1671169500B_1^2B_2^3B_4^3B_5B_6B_3^4+54573750B_1^2B_2^4B_4^2B_5B_6B_3^4$$
$$+3585859200B_1B_2^3B_4^3B_5^2B_6^2B_3^3+1075757760B_1^2B_2^2B_4^3B_5^2B_6^2B_3^3$$
$$+1075757760B_1^2B_2^3B_4^2B_5^2B_6^2B_3^3+3585859200B_1^2B_2^3B_4^3B_5B_6^2B_3^3$$
$$+651974400B_1^2B_2^3B_4^3B_5^2B_6B_3^3)\mathbf{Z}^{14}+(119528640B_1B_2^3B_4^2B_6^2B_3^5$$
$$+119528640B_2^2B_4^3B_5B_6^2B_3^5+398428800B_1B_2^2B_4^2B_5B_6^2B_3^5$$
$$+35218260B_1B_2^3B_4^2B_6^3B_3^4+35218260B_2^2B_4^3B_5B_6^3B_3^4$$
$$+180457200B_1B_2^2B_4^2B_5B_6^3B_3^4+48024900B_1B_2^3B_4^3B_6^2B_3^4$$
$$+268939440B_1^2B_2^3B_4^2B_6^2B_3^4+268939440B_2^2B_4^3B_5^2B_6^2B_3^4$$
$$+133402500B_1B_2B_4^3B_5^2B_6^2B_3^4+672348600B_1B_2^2B_4^2B_5^2B_6^2B_3^4$$
$$+48024900B_2^3B_4^3B_5B_6^2B_3^4+4320547560B_1B_2^2B_4^3B_5B_6^2B_3^4$$
$$+4320547560B_1B_2^3B_4^2B_5B_6^2B_3^4+672348600B_1^2B_2^2B_4^2B_5B_6^2B_3^4$$
$$+133402500B_1^2B_2^3B_4B_5B_6^2B_3^4+413887320B_1B_2^2B_4^3B_5^2B_6B_3^4$$
$$+356548500B_1B_2^3B_4^2B_5^2B_6B_3^4+1189465200B_1B_2^3B_4^3B_5B_6B_3^4$$
$$+356548500B_1^2B_2^2B_4^3B_5B_6B_3^4+413887320B_1^2B_2^3B_4^2B_5B_6B_3^4$$
$$+1644128640B_1B_2^2B_4^3B_5^2B_6^2B_3^3+1075757760B_1B_2^3B_4^2B_5^2B_6^2B_3^3$$
$$+292723200B_1^2B_2^2B_4^2B_5^2B_6^2B_3^3+3585859200B_1B_2^3B_4^3B_5B_6^2B_3^3$$
$$+1075757760B_1^2B_2^2B_4^3B_5B_6^2B_3^3+1644128640B_1^2B_2^3B_4^2B_5B_6^2B_3^3$$
$$+651974400B_1B_2^3B_4^3B_5^2B_6B_3^3+195592320B_1^2B_2^2B_4^3B_5^2B_6B_3^3$$
$$+195592320B_1^2B_2^3B_4^2B_5^2B_6B_3^3+651974400B_1^2B_2^3B_4^3B_5B_6B_3^3)\mathbf{Z}^{13}$$
$$+(6391462B_1B_2^2B_4^2B_6^3B_3^4+6391462B_2^2B_4^2B_5B_6^3B_3^4$$
$$+8715630B_1B_2^2B_4^3B_6^2B_3^4+253998360B_1B_2^3B_4^2B_6^2B_3^4$$
$$+29052100B_1^2B_2^2B_4^2B_6^2B_3^4+14822500B_2B_4^3B_5^2B_6^2B_3^4$$
$$+29052100B_2^2B_4^2B_5^2B_6^2B_3^4+14822500B_1^2B_2^3B_4B_6^2B_3^4$$
$$+253998360B_2^2B_4^3B_5B_6^2B_3^4+181575625B_1B_2B_4^3B_5B_6^2B_3^4$$
$$+8715630B_2^3B_4^2B_5B_6^2B_3^4+1554121926B_1B_2^2B_4^2B_5B_6^2B_3^4$$
$$+181575625B_1B_2^3B_4B_5B_6^2B_3^4+20212500B_1^2B_2^3B_4^2B_6B_3^4$$
$$+20212500B_2^2B_4^3B_5^2B_6B_3^4+110387200B_1B_2^2B_4^2B_5^2B_6B_3^4$$
$$+388031490B_1B_2^2B_4^3B_5B_6B_3^4+388031490B_1B_2^3B_4^2B_5B_6B_3^4$$
$$+110387200B_1^2B_2^2B_4^2B_5B_6B_3^4+5478396B_1B_2^2B_4^2B_5B_6^3B_3^3$$
$$+111143340B_1^2B_2^3B_4^2B_6^2B_3^3+111143340B_2^2B_4^3B_5^2B_6^2B_3^3$$
$$+155636250B_1B_2B_4^3B_5^2B_6^2B_3^3+542666124B_1B_2^2B_4^2B_5^2B_6^2B_3^3$$
$$+1941993130B_1B_2^2B_4^3B_5B_6^2B_3^3+1941993130B_1B_2^3B_4^2B_5B_6^2B_3^3$$
$$+542666124B_1^2B_2^2B_4^2B_5B_6^2B_3^3+155636250B_1^2B_2^3B_4B_5B_6^2B_3^3$$
$$+353089660B_1B_2^2B_4^3B_5^2B_6B_3^3+247546530B_1B_2^3B_4^2B_5^2B_6B_3^3$$
$$+60555264B_1^2B_2^2B_4^2B_5^2B_6B_3^3+707437500B_1B_2^3B_4^3B_5B_6B_3^3$$
$$+247546530B_1^2B_2^2B_4^3B_5B_6B_3^3+353089660B_1^2B_2^3B_4^2B_5B_6B_3^3)\mathbf{Z}^{12}$$
$$+(66594528B_1B_2^2B_4^2B_6^2B_3^4+21344400B_1B_2^3B_4B_6^2B_3^4$$
$$+21344400B_2B_4^3B_5B_6^2B_3^4+66594528B_2^2B_4^2B_5B_6^2B_3^4$$
$$+71148000B_1B_2B_4^2B_5B_6^2B_3^4+71148000B_1B_2^2B_4B_5B_6^2B_3^4$$
$$+29106000B_1B_2^3B_4^2B_6B_3^4+29106000B_2^2B_4^3B_5B_6B_3^4$$
$$+191866752B_1B_2^2B_4^2B_5B_6B_3^4+137214000B_1B_2^3B_4^2B_6^2B_3^3$$
$$+35858592B_1^2B_2^2B_4^2B_6^2B_3^3+18295200B_2B_4^3B_5^2B_6^2B_3^3$$
$$+35858592B_2^2B_4^2B_5^2B_6^2B_3^3+60984000B_1B_2B_4^2B_5^2B_6^2B_3^3$$
$$+18295200B_1^2B_2^3B_4B_6^2B_3^3+137214000B_2^2B_4^3B_5B_6^2B_3^3$$
$$+224116200B_1B_2B_4^3B_5B_6^2B_3^3+1339753968B_1B_2^2B_4^2B_5B_6^2B_3^3$$
$$+224116200B_1B_2^3B_4B_5B_6^2B_3^3+60984000B_1^2B_2^2B_4B_5B_6^2B_3^3$$
$$+24948000B_1^2B_2^3B_4^2B_6B_3^3+24948000B_2^2B_4^3B_5^2B_6B_3^3$$
$$+40748400B_1B_2B_4^3B_5^2B_6B_3^3+177031008B_1B_2^2B_4^2B_5^2B_6B_3^3$$
$$+467082000B_1B_2^2B_4^3B_5B_6B_3^3+467082000B_1B_2^3B_4^2B_5B_6B_3^3$$
$$+177031008B_1^2B_2^2B_4^2B_5B_6B_3^3+40748400B_1^2B_2^3B_4B_5B_6B_3^3)\mathbf{Z}^{11}$$
$$+(2614689B_2^2B_4^2B_6^2B_3^4+8715630B_1B_2^2B_4B_6^2B_3^4$$
$$+8715630B_2B_4^2B_5B_6^2B_3^4+11884950B_1B_2^2B_4^2B_6B_3^4$$
$$+11884950B_2^2B_4^2B_5B_6B_3^4+87268104B_1B_2^2B_4^2B_6^2B_3^3$$
$$+7470540B_2B_4^2B_5^2B_6^2B_3^3+28586250B_1B_2^3B_4B_6^2B_3^3$$
$$+7470540B_1^2B_2^2B_4B_6^2B_3^3+28586250B_2B_4^3B_5B_6^2B_3^3$$

$$+87268104 B_2^2 B_4^2 B_5 B_6^2 B_3^3+202848030 B_1 B_2 B_4^2 B_5 B_6^2 B_3^3$$
$$+202848030 B_1 B_2^2 B_4 B_5 B_6^2 B_3^3+38981250 B_1 B_2^3 B_4^2 B_6 B_3^3$$
$$+10187100 B_1^2 B_2^2 B_4^2 B_6 B_3^3+5197500 B_2 B_4^3 B_5^2 B_6 B_3^3$$
$$+10187100 B_2^2 B_4^2 B_5^2 B_6 B_3^3+28385280 B_1 B_2 B_4^2 B_5^2 B_6 B_3^3$$
$$+5197500 B_1^2 B_2^3 B_4 B_6 B_3^3+38981250 B_2^2 B_4^3 B_5 B_6 B_3^3$$
$$+63669375 B_1 B_2 B_4^3 B_5 B_6 B_3^3+440527626 B_1 B_2^2 B_4^2 B_5 B_6 B_3^3$$
$$+63669375 B_1 B_2^3 B_4 B_5 B_6 B_3^3+28385280 B_1^2 B_2^2 B_4 B_5 B_6 B_3^3$$
$$+30735936 B_1 B_2^2 B_4^2 B_5 B_6^2 B_3^2+5588352 B_1 B_2^2 B_4^2 B_5^2 B_6 B_3^2$$
$$+5588352 B_1^2 B_2^2 B_4^2 B_5 B_6 B_3^2)\mathbf{Z}^{10}+(6225450 B_2^2 B_4^2 B_6^2 B_3^3$$
$$+13280960 B_1 B_2 B_4^2 B_6^2 B_3^3+27544440 B_1 B_2^2 B_4 B_6^2 B_3^3$$
$$+27544440 B_2 B_4^2 B_5 B_6^2 B_3^3+13280960 B_2^2 B_4 B_5 B_6^2 B_3^3$$
$$+41164200 B_1 B_2 B_4 B_5 B_6^2 B_3^3+205800 B_1 B_2^2 B_4^2 B_5 B_3^3$$
$$+37560600 B_1 B_2^2 B_4^2 B_6 B_3^3+3773000 B_2 B_4^2 B_5^2 B_6 B_3^3$$
$$+9240000 B_1 B_2^3 B_4 B_6 B_3^3+3773000 B_1^2 B_2^2 B_4 B_6 B_3^3$$
$$+9240000 B_2 B_4^3 B_5 B_6 B_3^3+37560600 B_2^2 B_4^2 B_5 B_6 B_3^3$$
$$+82222140 B_1 B_2 B_4^2 B_5 B_6 B_3^3+82222140 B_1 B_2^2 B_4 B_5 B_6 B_3^3$$
$$+11383680 B_1 B_2 B_4^2 B_5 B_6^2 B_3^2+11383680 B_1 B_2^2 B_4 B_5 B_6^2 B_3^2$$
$$+2069760 B_1 B_2 B_4^2 B_5^2 B_6 B_3^2+24147200 B_1 B_2^2 B_4^2 B_5 B_6 B_3^2$$
$$+2069760 B_1^2 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^9+(1867635 B_2 B_4^2 B_6^2 B_3^3$$
$$+1867635 B_2^2 B_4 B_6^2 B_3^3+6225450 B_1 B_2 B_4 B_6^2 B_3^3$$
$$+6225450 B_2 B_4 B_5 B_6^2 B_3^3+2546775 B_2^2 B_4^2 B_6 B_3^3$$
$$+8489250 B_1 B_2 B_4^2 B_6 B_3^3+13222440 B_1 B_2^2 B_4 B_6 B_3^3$$
$$+13222440 B_2 B_4^2 B_5 B_6 B_3^3+8489250 B_2^2 B_4 B_5 B_6 B_3^3$$
$$+23654400 B_1 B_2 B_4 B_5 B_6 B_3^3+1600830 B_1 B_2^2 B_4 B_6^2 B_3^2$$
$$+1600830 B_2 B_4^2 B_5 B_6^2 B_3^2+5336100 B_1 B_2 B_4 B_5 B_6^2 B_3^2$$
$$+396900 B_1 B_2^2 B_4^2 B_5 B_3^2+2182950 B_1 B_2^2 B_4^2 B_6 B_3^2$$
$$+291060 B_2 B_4^2 B_5^2 B_6 B_3^2+291060 B_1^2 B_2^2 B_4 B_6 B_3^2$$
$$+2182950 B_2^2 B_4^2 B_5 B_6 B_3^2+9168390 B_1 B_2 B_4^2 B_5 B_6 B_3^2$$
$$+9168390 B_1 B_2^2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^8+(996072 B_2 B_4 B_6^2 B_3^3$$
$$+1358280 B_2 B_4^2 B_6 B_3^3+1358280 B_2^2 B_4 B_6 B_3^3$$
$$+4527600 B_1 B_2 B_4 B_6 B_3^3+4527600 B_2 B_4 B_5 B_6 B_3^3$$
$$+853776 B_1 B_2 B_4 B_6^2 B_3^2+853776 B_2 B_4 B_5 B_6^2 B_3^2$$
$$+211680 B_1 B_2 B_4^2 B_5 B_3^2+211680 B_1 B_2^2 B_4 B_5 B_3^2$$
$$+1164240 B_1 B_2 B_4^2 B_6 B_3^2+1853280 B_1 B_2^2 B_4 B_6 B_3^2$$
$$+1853280 B_2 B_4^2 B_5 B_6 B_3^2+1164240 B_2^2 B_4 B_5 B_6 B_3^2$$
$$+6044544 B_1 B_2 B_4 B_5 B_6 B_3^2)\mathbf{Z}^7+(916839 B_2 B_4 B_6 B_3^3$$
$$+148225 B_2 B_4 B_6^2 B_3^2+36750 B_1 B_2^2 B_4 B_3^2$$
$$+36750 B_2 B_4^2 B_5 B_3^2+200704 B_1 B_2 B_4 B_5 B_3^2$$
$$+26950 B_1 B_2^2 B_6 B_3^2+202125 B_2 B_4^2 B_6 B_3^2$$
$$+202125 B_2^2 B_4 B_6 B_3^2+1539384 B_1 B_2 B_4 B_6 B_3^2$$
$$+26950 B_4^2 B_5 B_6 B_3^2+1539384 B_2 B_4 B_5 B_6 B_3^2$$
$$+369600 B_1 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^6+(44100 B_1 B_2 B_4 B_3^2$$
$$+44100 B_2 B_4 B_5 B_3^2+32340 B_1 B_2 B_6 B_3^2+407484 B_2 B_4 B_6 B_3^2$$
$$+32340 B_4 B_5 B_6 B_3^2+24192 B_1 B_2 B_4 B_5 B_3$$
$$+133056 B_1 B_2 B_4 B_6 B_3$$
$$+133056 B_2 B_4 B_5 B_6 B_3)\mathbf{Z}^5+(11025 B_2 B_4 B_3^2+8085 B_2 B_6 B_3^2$$
$$+8085 B_4 B_6 B_3^2+9450 B_1 B_2 B_4 B_3+9450 B_2 B_4 B_5 B_3$$
$$+6930 B_1 B_2 B_6 B_3$$
$$+51975 B_2 B_4 B_6 B_3+6930 B_4 B_5 B_6 B_3)\mathbf{Z}^4+(560 B_1 B_2 B_3$$
$$+4200 B_2 B_4 B_3+560 B_4 B_5 B_3+3080 B_2 B_6 B_3$$
$$+3080 B_4 B_6 B_3)\mathbf{Z}^3$$
$$+(315 B_2 B_3+315 B_4 B_3+231 B_6 B_3)\mathbf{Z}^2+42 B_3 \mathbf{Z}+1,$$

$$\boxed{H_4}= B_1^3 B_2^6 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{30}$$
$$+30 B_1^3 B_2^5 B_3^8 B_4^6 B_5^3 B_6^4 \mathbf{Z}^{29}+(120 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^8$$
$$+315 B_1^3 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7)\mathbf{Z}^{28}+(2240 B_1^2 B_2^5 B_4^6 B_5^3 B_6^4 B_3^7$$
$$+1050 B_1^3 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7+770 B_1^3 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{27}$$
$$+(4200 B_1^2 B_2^4 B_4^6 B_5^3 B_6^4 B_3^7+9450 B_1^2 B_2^5 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+1050 B_1^3 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7+6930 B_1^2 B_2^5 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+5775 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7)\mathbf{Z}^{26}+(31500 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^7$$
$$+10752 B_1^2 B_2^5 B_4^5 B_5^2 B_6^4 B_3^7+23100 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^7$$
$$+59136 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^7+8316 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+9702 B_1^3 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6)\mathbf{Z}^{25}+(45360 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^7$$
$$+249480 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^7+92400 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+36750 B_1^2 B_2^4 B_4^5 B_5^3 B_6^4 B_3^6+26950 B_1^2 B_2^4 B_4^6 B_5^3 B_6^3 B_3^6$$
$$+107800 B_1^2 B_2^5 B_4^5 B_5^3 B_6^3 B_3^6+8085 B_1^3 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+26950 B_1^3 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{24}+(443520 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^7$$
$$+94080 B_1^2 B_2^4 B_4^5 B_5^2 B_6^4 B_3^6+1132560 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+316800 B_1^2 B_2^5 B_4^5 B_5^2 B_6^3 B_3^6+32340 B_1^3 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+16500 B_1^3 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6)\mathbf{Z}^{23}+(44100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^4 B_3^6$$
$$+32340 B_1 B_2^4 B_4^5 B_5^3 B_6^3 B_3^6+495000 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6$$
$$+242550 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^6+3256110 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+202125 B_1^2 B_2^5 B_4^4 B_5^2 B_6^3 B_3^6+44550 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+177870 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^6+1358280 B_1^2 B_2^4 B_4^5 B_5^3 B_6^3 B_3^5)\mathbf{Z}^{22}$$
$$+(178200 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^6+168960 B_1 B_2^4 B_4^5 B_5^2 B_6^3 B_3^6$$
$$+2182950 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6+2674100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+711480 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^6+1478400 B_1^2 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5$$
$$+1131900 B_1^2 B_2^4 B_4^4 B_5^3 B_6^3 B_3^5+4928000 B_1^2 B_2^4 B_4^5 B_5^2 B_6^3 B_3^5$$
$$+23100 B_1^3 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5+830060 B_1^2 B_2^4 B_4^5 B_5^3 B_6^2 B_3^5)\mathbf{Z}^{21}$$
$$+(970200 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^6+349272 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+3234000 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6+155232 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^6$$
$$+853776 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^6+577500 B_1 B_2^3 B_4^5 B_5^3 B_6^3 B_3^5$$
$$+1559250 B_1^2 B_2^3 B_4^4 B_5^3 B_6^3 B_3^5+7074375 B_1^2 B_2^3 B_4^5 B_5^2 B_6^3 B_3^5$$
$$+9315306 B_1^2 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5+1143450 B_1^2 B_2^3 B_4^5 B_5^3 B_6^2 B_3^5$$
$$+996072 B_1^2 B_2^4 B_4^4 B_5^3 B_6^2 B_3^5+3811500 B_1^2 B_2^4 B_4^5 B_5^2 B_6^2 B_3^5$$
$$+5082 B_1^3 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{20}+(2069760 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^6$$
$$+693000 B_1 B_2^3 B_4^4 B_5^3 B_6^3 B_3^5+3326400 B_1 B_2^3 B_4^5 B_5^2 B_6^3 B_3^5$$
$$+369600 B_1 B_2^4 B_4^4 B_5^2 B_6^3 B_3^5+21801780 B_1^2 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5$$
$$+4331250 B_1^2 B_2^4 B_4^3 B_5^2 B_6^3 B_3^5+1478400 B_1^2 B_2^4 B_4^4 B_5 B_6^3 B_3^5$$
$$+508200 B_1 B_2^3 B_4^5 B_5^3 B_6^2 B_3^5+2439360 B_1^2 B_2^3 B_4^4 B_5^3 B_6^2 B_3^5$$
$$+6225450 B_1^2 B_2^3 B_4^5 B_5^2 B_6^2 B_3^5+11384100 B_1^2 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5)\mathbf{Z}^{19}$$
$$+(14314300 B_1 B_2^3 B_4^4 B_5^2 B_6^3 B_3^5+14437500 B_1^2 B_2^3 B_4^3 B_5^2 B_6^3 B_3^5$$
$$+3056130 B_1^2 B_2^3 B_4^4 B_5 B_6^3 B_3^5+1559250 B_1^2 B_2^4 B_4^3 B_5 B_6^3 B_3^5$$
$$+1372140 B_1 B_2^3 B_4^4 B_5^3 B_6^2 B_3^5+3176250 B_1 B_2^3 B_4^5 B_5^2 B_6^2 B_3^5$$
$$+127050 B_1 B_2^4 B_4^4 B_5^2 B_6^2 B_3^5+28420210 B_1^2 B_2^3 B_4^4 B_5^2 B_6^2 B_3^5$$

$$+8575875B_1^2B_2^4B_4^3B_5^2B_6^2B_3^5+2032800B_1^2B_2^4B_4^4B_5B_6^2B_3^5$$
$$+6338640B_1^2B_2^3B_4^4B_5^2B_6^3B_3^4+711480B_1^2B_2^3B_4^4B_5^3B_6^2B_3^4$$
$$+2371600B_1^2B_2^4B_4^4B_5^2B_6^2B_3^4)\mathbf{Z}^{18}+(577500B_1B_2^2B_4^4B_5^2B_6^3B_3^5$$
$$+10187100B_1B_2^3B_4^3B_5^2B_6^3B_3^5+1774080B_1B_2^3B_4^4B_5B_6^3B_3^5$$
$$+5913600B_1^2B_2^3B_4^3B_5B_6^3B_3^5+18705960B_1B_2^3B_4^4B_5^2B_6^2B_3^5$$
$$+32524800B_1^2B_2^3B_4^3B_5^2B_6^2B_3^5+7470540B_1^2B_2^3B_4^4B_5B_6^2B_3^5$$
$$+3811500B_1^2B_2^4B_4^3B_5B_6^2B_3^5+8279040B_1B_2^3B_4^4B_5^2B_6^3B_3^4$$
$$+8731800B_1^2B_2^3B_4^3B_5^2B_6^3B_3^4+711480B_1B_2^3B_4^4B_5^3B_6^2B_3^4$$
$$+16625700B_1^2B_2^3B_4^4B_5^2B_6^2B_3^4+4446750B_1^2B_2^4B_4^3B_5^2B_6^2B_3^4)\mathbf{Z}^{17}$$
$$+(4527600B_1B_2^3B_4^3B_5B_6^3B_3^5+508200B_1B_2^2B_4^4B_5^2B_6^2B_3^5$$
$$+24901800B_1B_2^3B_4^3B_5^2B_6^2B_3^5+5488560B_1B_2^3B_4^4B_5B_6^2B_3^5$$
$$+18295200B_1^2B_2^3B_4^3B_5B_6^2B_3^5+2182950B_1B_2^2B_4^4B_5^2B_6^3B_3^4$$
$$+11884950B_1B_2^3B_4^3B_5^2B_6^3B_3^4+3880800B_1^2B_2^3B_4^3B_5B_6^3B_3^4$$
$$+108900B_1B_2^2B_4^4B_5^3B_6^2B_3^4+18478980B_1B_2^3B_4^4B_5^2B_6^2B_3^4$$
$$+1334025B_1^2B_2^2B_4^4B_5^2B_6^2B_3^4+45530550B_1^2B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+4446750B_1^2B_2^3B_4^4B_5B_6^2B_3^4+2268750B_1^2B_2^4B_4^3B_5B_6^2B_3^4$$
$$+1584660B_1^2B_2^3B_4^4B_5^2B_6B_3^4)\mathbf{Z}^{16}+(15937152B_1B_2^3B_4^3B_5B_6^2B_3^5$$
$$+3234000B_1B_2^2B_4^3B_5^2B_6^3B_3^4+5588352B_1B_2^3B_4^3B_5B_6^3B_3^4$$
$$+6203600B_1B_2^2B_4^4B_5^2B_6^2B_3^4+53742416B_1B_2^3B_4^3B_5^2B_6^2B_3^4$$
$$+5808000B_1^2B_2^2B_4^3B_5^2B_6^2B_3^4+5808000B_1B_2^3B_4^4B_5B_6^2B_3^4$$
$$+34036496B_1^2B_2^3B_4^3B_5B_6^2B_3^4+3234000B_1B_2^3B_4^4B_5^2B_6B_3^4$$
$$+5588352B_1^2B_2^3B_4^3B_5^2B_6B_3^4+15937152B_1^2B_2^3B_4^3B_5^2B_6^2B_3^3)\mathbf{Z}^{15}$$
$$+(1584660B_1B_2^2B_4^3B_5B_6^3B_3^4+108900B_2^2B_4^4B_5^2B_6^2B_3^4$$
$$+18478980B_1B_2^2B_4^3B_5^2B_6^2B_3^4+1334025B_1B_2^2B_4^4B_5B_6^2B_3^4$$
$$+45530550B_1B_2^3B_4^3B_5B_6^2B_3^4+4446750B_1^2B_2^2B_4^3B_5B_6^2B_3^4$$
$$+2268750B_1^2B_2^3B_4^2B_5B_6^2B_3^4+2182950B_1B_2^2B_4^4B_5^2B_6B_3^4$$
$$+11884950B_1B_2^3B_4^3B_5^2B_6B_3^4+3880800B_1^2B_2^3B_4^3B_5B_6B_3^4$$
$$+508200B_1B_2^2B_4^4B_5^2B_6^2B_3^3+24901800B_1B_2^3B_4^3B_5^2B_6^2B_3^3$$
$$+5488560B_1^2B_2^2B_4^3B_5^2B_6^2B_3^3+18295200B_1^2B_2^3B_4^3B_5B_6^2B_3^3$$
$$+4527600B_1^2B_2^3B_4^3B_5^2B_6B_3^3)\mathbf{Z}^{14}+(711480B_2^2B_4^3B_5^2B_6^2B_3^4$$
$$+16625700B_1B_2^2B_4^3B_5B_6^2B_3^4+4446750B_1B_2^3B_4^2B_5B_6^2B_3^4$$
$$+8279040B_1B_2^2B_4^3B_5^2B_6B_3^4+8731800B_1B_2^3B_4^3B_5B_6B_3^4$$
$$+18705960B_1B_2^2B_4^3B_5^2B_6^2B_3^3+32524800B_1B_2^3B_4^3B_5B_6^2B_3^3$$
$$+7470540B_1^2B_2^2B_4^3B_5B_6^2B_3^3+3811500B_1^2B_2^3B_4^2B_5B_6^2B_3^3$$
$$+577500B_1B_2^2B_4^4B_5^2B_6B_3^3+10187100B_1B_2^3B_4^3B_5^2B_6B_3^3$$
$$+1774080B_1^2B_2^2B_4^3B_5^2B_6B_3^3+5913600B_1^2B_2^3B_4^3B_5B_6B_3^3)\mathbf{Z}^{13}$$
$$+(711480B_2^2B_4^3B_5B_6^2B_3^4+2371600B_1B_2^2B_4^2B_5B_6^2B_3^4$$
$$+6338640B_1B_2^2B_4^3B_5B_6B_3^4+1372140B_2^2B_4^3B_5^2B_6^2B_3^3$$
$$+3176250B_1B_2B_4^3B_5^2B_6^2B_3^3+127050B_1B_2^2B_4^2B_5^2B_6^2B_3^3$$
$$+28420210B_1B_2^2B_4^3B_5B_6^2B_3^3+8575875B_1B_2^3B_4^2B_5B_6^2B_3^3$$
$$+2032800B_1^2B_2^2B_4^2B_5B_6^2B_3^3+14314300B_1B_2^2B_4^3B_5^2B_6B_3^3$$
$$+14437500B_1B_2^3B_4^3B_5B_6B_3^3+3056130B_1^2B_2^2B_4^3B_5B_6B_3^3$$
$$+1559250B_1^2B_2^3B_4^2B_5B_6B_3^3)\mathbf{Z}^{12}+(508200B_2B_4^3B_5^2B_6^2B_3^3$$
$$+2439360B_2^2B_4^3B_5B_6^2B_3^3+6225450B_1B_2B_4^3B_5B_6^2B_3^3$$
$$+11384100B_1B_2^2B_4^2B_5B_6^2B_3^3+693000B_2^2B_4^3B_5^2B_6B_3^3$$
$$+3326400B_1B_2B_4^3B_5^2B_6B_3^3+369600B_1B_2^2B_4^2B_5^2B_6B_3^3$$
$$+21801780B_1B_2^2B_4^3B_5B_6B_3^3+4331250B_1B_2^3B_4^2B_5B_6B_3^3$$
$$+1478400B_1^2B_2^2B_4^2B_5B_6B_3^3+2069760B_1B_2^2B_4^3B_5^2B_6B_3^2)\mathbf{Z}^{11}$$
$$+(5082B_1B_2^2B_4^2B_6^2B_3^3+1143450B_2B_4^3B_5B_6^2B_3^3$$
$$+996072B_2^2B_4^2B_5B_6^2B_3^3+3811500B_1B_2B_4^2B_5B_6^2B_3^3$$
$$+577500B_2B_4^3B_5^2B_6B_3^3+1559250B_2^2B_4^3B_5B_6B_3^3$$
$$+7074375B_1B_2B_4^3B_5B_6B_3^3+9315306B_1B_2^2B_4^2B_5B_6B_3^3$$
$$+853776B_1B_2^2B_4^2B_5B_6^2B_3^2+970200B_1B_2B_4^3B_5^2B_6B_3^2$$
$$+349272B_1B_2^2B_4^2B_5^2B_6B_3^2+3234000B_1B_2^2B_4^3B_5B_6B_3^2$$
$$+155232B_1^2B_2^2B_4^2B_5B_6B_3^2)\mathbf{Z}^{10}+(830060B_2B_4^2B_5B_6^2B_3^3$$
$$+23100B_1B_2^2B_4^2B_6B_3^3+1478400B_2B_4^3B_5B_6B_3^3$$
$$+1131900B_2^2B_4^2B_5B_6B_3^3+4928000B_1B_2B_4^2B_5B_6B_3^3$$
$$+711480B_1B_2B_4^2B_5B_6^2B_3^2+178200B_2B_4^3B_5^2B_6B_3^2$$
$$+168960B_1B_2B_4^2B_5^2B_6B_3^2+2182950B_1B_2B_4^3B_5B_6B_3^2$$
$$+2674100B_1B_2^2B_4^2B_5B_6B_3^2)\mathbf{Z}^9+(1358280B_2B_4^2B_5B_6B_3^3$$
$$+177870B_2B_4^2B_5B_6^2B_3^2+44100B_1B_2^2B_4^2B_5B_3^2$$
$$+44550B_1B_2^2B_4^2B_6B_3^2+32340B_2B_4^2B_5^2B_6B_3^2$$
$$+495000B_2B_4^3B_5B_6B_3^2+242550B_2^2B_4^2B_5B_6B_3^2$$
$$+3256110B_1B_2B_4^2B_5B_6B_3^2+202125B_1B_2^2B_4B_5B_6B_3^2)\mathbf{Z}^8$$
$$+(94080B_1B_2B_4^2B_5B_3^2+32340B_1B_2B_4^2B_6B_3^2$$
$$+16500B_1B_2^2B_4B_6B_3^2+1132560B_2B_4^2B_5B_6B_3^2$$
$$+316800B_1B_2B_4B_5B_6B_3^2+443520B_1B_2B_4^2B_5B_6B_3)\mathbf{Z}^7$$
$$+(36750B_2B_4^2B_5B_3^2+8085B_2B_4^2B_6B_3^2+26950B_1B_2B_4B_6B_3^2$$
$$+26950B_4^2B_5B_6B_3^2+107800B_2B_4B_5B_6B_3^2+45360B_1B_2B_4^2B_5B_3$$
$$+249480B_2B_4^2B_5B_6B_3+92400B_1B_2B_4B_5B_6B_3)\mathbf{Z}^6$$
$$+(9702B_2B_4B_6B_3^2+31500B_2B_4^2B_5B_3+10752B_1B_2B_4B_5B_3$$
$$+8316B_1B_2B_4B_6B_3+23100B_4^2B_5B_6B_3+59136B_2B_4B_5B_6B_3)\mathbf{Z}^5$$
$$+(4200B_3B_5B_4^2+1050B_1B_2B_3B_4+9450B_2B_3B_5B_4$$
$$+5775B_2B_3B_6B_4+6930B_3B_5B_6B_4)\mathbf{Z}^4+(1050B_2B_3B_4$$
$$+2240B_3B_5B_4+770B_3B_6B_4)\mathbf{Z}^3+(315B_3B_4$$
$$+120B_5B_4)\mathbf{Z}^2+30B_4\mathbf{Z}+1,$$

$$\boxed{H_5}=B_1^2B_2^3B_3^4B_4^3B_5^2B_6^2\mathbf{Z}^{16}$$
$$+16B_1B_2^3B_3^4B_4^3B_5^2B_6^2\mathbf{Z}^{15}+120B_1B_2^2B_3^4B_4^3B_5^2B_6^2\mathbf{Z}^{14}$$
$$+560B_1B_2^2B_3^3B_4^3B_5^2B_6^2\mathbf{Z}^{13}+(1050B_1B_2^2B_4^2B_5^2B_6^2B_3^3$$
$$+770B_1B_2^2B_4^3B_5^2B_6B_3^3)\mathbf{Z}^{12}+(672B_1B_2^2B_4^2B_5B_6^2B_3^3$$
$$+3696B_1B_2^2B_4^2B_5^2B_6B_3^3)\mathbf{Z}^{11}+(3696B_1B_2^2B_4^2B_5B_6B_3^3$$
$$+4312B_1B_2^2B_4^2B_5^2B_6B_3^2)\mathbf{Z}^{10}+(2640B_1B_2B_3^2B_5^2B_6B_4^2$$
$$+8800B_1B_2^2B_3^2B_5B_6B_4^2)\mathbf{Z}^9+(660B_2B_4^2B_5^2B_6B_3^2$$
$$+8085B_1B_2B_4^2B_5B_6B_3^2+4125B_1B_2^2B_4B_5B_6B_3^2)\mathbf{Z}^8$$
$$+(2640B_2B_4^2B_5B_6B_3^2+8800B_1B_2B_4B_5B_6B_3^2)\mathbf{Z}^7$$
$$+(4312B_2B_4B_5B_6B_3^2+3696B_1B_2B_4B_5B_6B_3)\mathbf{Z}^6$$
$$+(672B_1B_2B_3B_4B_5+3696B_2B_3B_4B_6B_5)\mathbf{Z}^5$$
$$+(1050B_2B_3B_4B_5+770B_3B_4B_6B_5)\mathbf{Z}^4$$
$$+560B_3B_4B_5\mathbf{Z}^3+120B_4B_5\mathbf{Z}^2+16B_5\mathbf{Z}+1,$$

$$\boxed{H_6}=B_1^2B_2^4B_3^6B_4^4B_5^2B_6^4\mathbf{Z}^{22}$$
$$+22B_1^2B_2^4B_3^6B_4^4B_5^2B_6^3\mathbf{Z}^{21}+231B_1^2B_2^4B_3^5B_4^4B_5^2B_6^3\mathbf{Z}^{20}$$
$$+(770B_1^2B_2^3B_4^4B_5^2B_6^3B_3^5+770B_1^2B_2^4B_4^3B_5^2B_6^3B_3^5)\mathbf{Z}^{19}$$
$$+(770B_1B_2^3B_4^4B_5^2B_6^3B_3^5+5775B_1^2B_2^3B_4^3B_5^2B_6^3B_3^5$$
$$+770B_1^2B_2^4B_4^3B_5B_6^3B_3^5)\mathbf{Z}^{18}+(8316B_1B_2^3B_4^3B_5^2B_6^3B_3^5$$

$$
\begin{aligned}
&+8316B_1^2B_2^3B_4^3B_5B_6^3B_3^5+9702B_1^2B_2^3B_4^3B_5^2B_6^3B_3^4)\mathbf{Z}^{17}\\
&+(14784B_1B_2^3B_4^3B_5B_6^3B_3^5+26950B_1B_2^3B_4^3B_5^2B_6^3B_3^4\\
&+26950B_1^2B_2^3B_4^3B_5B_6^3B_3^4+5929B_1^2B_2^3B_4^3B_5^2B_6^2B_3^4)\mathbf{Z}^{16}\\
&+(16500B_1B_2^2B_4^3B_5^2B_6^3B_3^4+90112B_1B_2^3B_4^3B_5B_6^3B_3^4\\
&+16500B_1^2B_2^3B_4^2B_5B_6^3B_3^4+23716B_1B_2^3B_4^3B_5^2B_6^2B_3^4\\
&+23716B_1^2B_2^3B_4^3B_5B_6^2B_3^4)\mathbf{Z}^{15}+(72765B_1B_2^2B_4^3B_5B_6^3B_3^4\\
&+72765B_1B_2^3B_4^2B_5B_6^3B_3^4+32670B_1B_2^2B_4^3B_5^2B_6^2B_3^4\\
&+108900B_1B_2^3B_4^3B_5B_6^2B_3^4+32670B_1^2B_2^3B_4^2B_5B_6^2B_3^4)\mathbf{Z}^{14}\\
&+(107800B_1B_2^2B_4^2B_5B_6^3B_3^4+177870B_1B_2^2B_4^3B_5B_6^2B_3^4\\
&+177870B_1B_2^3B_4^2B_5B_6^2B_3^4+16940B_1B_2^2B_4^3B_5^2B_6^2B_3^3\\
&+16940B_1^2B_2^3B_4^2B_5B_6^2B_3^3)\mathbf{Z}^{13}+(379456B_1B_2^2B_4^2B_5B_6^2B_3^4\\
&+45276B_1B_2^2B_4^2B_5B_6^3B_3^3+5082B_1B_2^2B_4^2B_5^2B_6^2B_3^3\\
&+105875B_1B_2^2B_4^3B_5B_6^2B_3^3+105875B_1B_2^3B_4^2B_5B_6^2B_3^3\\
&+5082B_1^2B_2^2B_4^2B_5B_6^2B_3^3)\mathbf{Z}^{12}+705432B_1B_2^2B_3^3B_4^2B_5B_6^2\mathbf{Z}^{11}\\
&+(5082B_1B_2^2B_4^2B_6^2B_3^3+5082B_2^2B_4^2B_5B_6^2B_3^3\\
&+105875B_1B_2B_4^2B_5B_6^2B_3^3+105875B_1B_2^2B_4B_5B_6^2B_3^3\\
&+45276B_1B_2^2B_4^2B_5B_6B_3^3+379456B_1B_2^2B_4^2B_5B_6^2B_3^2)\mathbf{Z}^{10}\\
&+(16940B_1B_2^2B_4B_6^2B_3^3+16940B_2B_4^2B_5B_6^2B_3^3\\
&+177870B_1B_2B_4^2B_5B_6^2B_3^2+177870B_1B_2^2B_4B_5B_6^2B_3^2\\
&+107800B_1B_2^2B_4^2B_5B_6B_3^2)\mathbf{Z}^9+(32670B_1B_2^2B_4B_6^2B_3^2\\
&+32670B_2B_4^2B_5B_6^2B_3^2+108900B_1B_2B_4B_5B_6^2B_3^2\\
&+72765B_1B_2B_4^2B_5B_6B_3^2+72765B_1B_2^2B_4B_5B_6B_3^2)\mathbf{Z}^8\\
&+(23716B_1B_2B_4B_6^2B_3^2+23716B_2B_4B_5B_6^2B_3^2\\
&+16500B_1B_2^2B_4B_6B_3^2+16500B_2B_4^2B_5B_6B_3^2\\
&+90112B_1B_2B_4B_5B_6B_3^2)\mathbf{Z}^7+(5929B_2B_4B_6^2B_3^2\\
&+26950B_1B_2B_4B_6B_3^2+26950B_2B_4B_5B_6B_3^2\\
&+14784B_1B_2B_4B_5B_6B_3)\mathbf{Z}^6+(9702B_2B_4B_6B_3^2\\
&+8316B_1B_2B_4B_6B_3+8316B_2B_4B_5B_6B_3)\mathbf{Z}^5\\
&+(770B_1B_2B_3B_6+5775B_2B_3B_4B_6+770B_3B_4B_5B_6)\mathbf{Z}^4\\
&+(770B_2B_3B_6+770B_3B_4B_6)\mathbf{Z}^3\\
&+231B_3B_6\mathbf{Z}^2+22B_6\mathbf{Z}+1.
\end{aligned}
$$

It should be noted that five polynomials: H_1, H_2, H_4, H_5, H_6 were found earlier (in a non-ordered form) in Ref. [48]. The biggest key polynomial H_3 was not presented in Ref. [48]. We note that the "length" of the polynomial H_3 is more than $5/8$ of the total "length" of all polynomials.

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