

Quantum fluctuations from thermal fluctuations in Jacobson formalism

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Abstract In the Jacobson formalism general relativity is obtained from thermodynamics. This is done by using the Bekenstein–Hawking entropy–area relation. However, as a black hole gets smaller, its temperature will increase. This will cause the thermal fluctuations to also increase, and these will in turn correct the Bekenstein–Hawking entropy–area relation. Furthermore, with the reduction in the size of the black hole, quantum effects will also start to dominate. Just as the general relativity can be obtained from thermodynamics in the Jacobson formalism, we propose that the quantum fluctuations to the geometry can be obtained from thermal fluctuations.

The entropy of a black hole is equal to the quarter of the area of its horizon in natural units [1,2]. This observation establishes a connection between the thermodynamics and the geometry of spacetime. This entropy associated with a black hole is also the maximum entropy that can be associated with any object of the same volume [3,4]. It is interesting to observe that this maximum entropy of a region of space scales with its area and not with its volume [5]. In fact, it is this observation that has motivated the holographic principle [6,7]. Even though the holographic principle is a very important principle in physics, it is expected that this holographic principle will get modified near the Planck scale due to quantum fluctuations [8,9]. This can also be observed from the fact that the relation between the entropy and area of a black hole is expected to get modified due to quantum fluctuations. The leading order correction to the relation between the area and entropy of a black hole is a logarithmic correction in almost all approaches to quantum gravity. In

particular, such logarithmic corrections have been obtained using non-perturbative quantum gravity [11], the Cardy formula [12], matter fields surrounding a black hole [13–15], string theory [16–19], dilatonic black holes [20] the partition function of a black hole [21], and the generalized uncertainty principle [10,22]. Even though the form of the corrections from various approaches to quantum gravity are logarithmic corrections, the coefficient of such a logarithmic correction is different for all these approaches to quantum gravity.

It may be noted that such logarithmic corrections can also be obtained by considering the effects of thermal fluctuations on the entropy of a black hole [29–31]. Now it is well known that in the Jacobson formalism, spacetime emerges from thermodynamics [23], in that general relativity can be deduced from the Bekenstein–Hawking entropy–area relation combined with the first law of thermodynamics. Thus, the correction to the Bekenstein–Hawking entropy–area relation would generate corrections to the structure of spacetime. Furthermore, as the black hole becomes smaller due to Hawking radiation, its temperature would increase, and this in turn would increase the contribution coming from the thermal corrections. However, as the black hole becomes smaller, the effect of quantum fluctuations would also increase. Thus, in this paper, we propose that both these effects are related, due to the relation between the geometry and the black hole thermodynamics. Furthermore, as the Jacobson formalism explicitly relies on this connection, we will use the Jacobson formalism to obtain quantum corrections to the metric by analyzing the correction produced by thermal fluctuations to the thermodynamics of a black hole.

Thus, we start from the Jacobson formalism, and in this formalism the thermodynamics relation $\delta Q = TdS$ is used to obtain the geometry of spacetime. This is because it is

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possible to express Q in terms of the energy-momentum tensor T_{ab} , and one may use the Hawking–Bekenstein relation to relate S to the event horizon area A . Thus, we obtain a geometrical quantity which can be expressed in terms of the Riemann tensor R_{ab} , and so a relation between T_{ab} and R_{ab} is obtained, and this can be demonstrated to be the Einstein field equations.

More precisely, and following the conventions of [23], for any point p , one can choose a two-surface element \mathcal{P} to which orthogonal boosts are generated by a Killing field χ^a such that the temperature T is taken as the Unruh temperature [24] defined by $T = \hbar\kappa/2\pi$ where κ represents the acceleration of the Killing orbit, and the heat flow is then defined by the boost-energy current $T_{ab}\chi^a$. As for the area, we consider a local Rindler horizon through p generated by χ^a whose future points to the energy carried by matter. The past-pointing heat flux through \mathcal{P} , beyond which lies the horizon denoted by \mathcal{H} , is

$$\delta Q = \int_{\mathcal{H}} T_{ab}\chi^a d\Sigma^b, \tag{1}$$

where $d\Sigma^a = k^a d\lambda dA$ with k^a a tangent vector to the horizon, λ an affine parameter vanishing at \mathcal{P} with negative values to the past of \mathcal{P} , and dA is the area element. Thus, it is possible to write

$$\delta Q = -\kappa \int_{\mathcal{H}} \lambda T_{ab}k^a k^b d\lambda dA. \tag{2}$$

As the entropy S is assumed to be proportional to the horizon area, so $dS = \eta\delta A$. Denoting the expansion of the horizon generated by θ , we obtain

$$\delta A = -\kappa \int_{\mathcal{H}} \theta d\lambda dA. \tag{3}$$

In order now to obtain the Einstein equations, it suffices to neglect, near \mathcal{P} , the shear σ^2 and the expansion θ terms, which vanish at \mathcal{P} by a suitable choice of the local Rindler horizon, in the Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b. \tag{4}$$

Thus, by integrating this equation, we find $\theta = -\lambda R_{ab}k^a k^b$, and Eq. (3) can be expressed as

$$\delta A = - \int_{\mathcal{H}} \lambda R_{ab}k^a k^b d\lambda dA. \tag{5}$$

Comparing Eqs. (2) and (5), we observe that $\delta Q = TdS = (\hbar\kappa/2\pi)\eta\delta A$ holds provided $T_{ab}k^a k^b = (\hbar\eta/2\pi)R_{ab}k^a k^b$ for all null k^a , which leads to $(2\pi/\hbar\eta)T_{ab} = R_{ab} + fg_{ab}$ for some function f . Energy and momentum conservation, combined with the contracted Bianchi identities, leads to $f = -R/2 + \Lambda$ for some constant Λ , and thus we get the Einstein equations

$$R_{ab} - \frac{1}{2} + \Lambda g_{ab} = \frac{2\pi}{\hbar\eta} T_{ab}. \tag{6}$$

We would like to point out that the proportionality constant η between the entropy and the area is related to Newton’s constant as $G = (4\hbar\eta)^{-1}$, and hence to the Planck length, but the cosmological constant Λ cannot be related to other constants, and thus there remains a free parameter even in the Jacobson formalism.

We would like to apply the former approach to determine the quantum corrections on the black hole geometry due to thermal fluctuations. We will consider a BTZ black hole as an example, but the formalism developed here can be applied to any black hole geometry. We first observe that in the Jacobson formalism, the field equations near the horizon of a BTZ black hole can be expressed as a thermodynamical identity, $dE = TdS + P_r dA$, where $E = M$ is the mass of BTZ black hole, dA is the change in the area of the black hole horizon when the horizon is displaced by an infinitesimally small amount, and P_r is the radial pressure provided by the source of Einstein equations. It may be noted that since we have a $2 + 1$ dimensional black hole, its volume is in fact the area it encloses. So, the term $P_r dA$ actually corresponds to PdV , from the general first law. Furthermore, the pressure P_r is well defined for BTZ black holes, since they are embedded in AdS spacetime. This term has occurred in earlier work on such black holes [25–28].

Now we will analyze the corrections to the entropy of a BTZ black hole due to thermal fluctuations [30,31]. Such thermal fluctuations have been analyzed as perturbations around the equilibrium, and this has been done by considering the system to be very close to equilibrium. So, the approximation used in this approach is valid as long as the correction due to the thermal fluctuations are small compared to the original quantity, i.e., as long as $\Delta S_0/S_0 = (S - S_0)/S_0 \ll 1$, where S is the corrected entropy and S_0 is the original entropy of the system. Thus, the ratio of the corrections to the original quantity should be small, and so the temperature should not be large enough to produce very large thermal fluctuations [29].

Now for a canonical ensemble with partition function [30, 31],

$$Z = \int_0^\infty dE \rho(E) \exp(-\beta E), \tag{7}$$

the density of states for a system can be written as

$$\rho(E) = \frac{1}{2\pi i} \int_{\beta_0 - i\infty}^{\beta_0 + i\infty} d\beta \exp[S(\beta)], \tag{8}$$

where $S = \beta E + \ln Z$. It may be noted that usually the entropy is measured around the equilibrium temperature β_0 , and all thermal fluctuations are neglected. This is done by making the identification $T = \beta^{-1}$. However, it is possible to consider the thermal fluctuations, and expand the entropy $S(\beta)$ around the equilibrium temperature β_0 [30,31],

$$S = S_0 + \frac{1}{2}(\beta - \beta_0)^2 \left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0}, \tag{9}$$

where β is a temperature close to the equilibrium temperature β_0 , and

$$S_0 = [S(\beta)]_{\beta=\beta_0}, \quad S_0'' = \left[\frac{\partial^2 S(\beta)}{\partial \beta^2} \right]_{\beta=\beta_0}. \tag{10}$$

Now the density of states can be expressed as

$$\rho(E) = \frac{\exp(S_0)}{2\pi i} \int_{\beta_0-i\infty}^{\beta_0+i\infty} d\beta \times \exp \left(\frac{1}{2}(\beta - \beta_0)^2 \left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0} \right). \tag{11}$$

Furthermore, by a change of variables, we obtain

$$\rho(E) = \frac{\exp(S_0)}{\sqrt{2\pi}} \left[\left(\frac{\partial^2 S(\beta)}{\partial \beta^2} \right)_{\beta=\beta_0} \right]^{-1/2}. \tag{12}$$

Thus, it is possible to express S as

$$S = S_0 - \frac{\ln S_0''}{2}, \tag{13}$$

It is also possible to express the second derivative of the entropy in terms of fluctuations of the energy, and so the corrected entropy can be written as [31]

$$S = S_0 - \frac{1}{2} \ln(S_0 T^2). \tag{14}$$

So, the thermal fluctuations decrease entropy of the BTZ black hole. It may be noted that black holes have a negative heat capacity; therefore, as the temperature of a black hole increases, its entropy decreases. This unusual behavior of black holes even occurs when thermal fluctuations are neglected. Furthermore, the behavior of the original [32], and the correct entropy [31], is well known, and this is not the aim of this paper. The main aim of this paper is to use this corrected entropy to obtain quantum corrections using the Jacobson formalism.

Let us consider a non-rotating BTZ black hole in three-dimensions with metric [43]

$$ds^2 = - \left(\frac{r^2}{l^2} - 8G_3 M \right) dt^2 + \left(\frac{r^2}{l^2} - 8G_3 M \right)^{-1} dr^2 + r^2 d\theta^2. \tag{15}$$

Its Bekenstein–Hawking entropy and Hawking temperature are given by

$$S_0 = \frac{2\pi r}{4G_3} \tag{16}$$

and

$$T = \frac{r}{2\pi l^2} = \left[\frac{G_3}{\pi^2 l^2} \right] S_0 \tag{17}$$

where $r = \sqrt{8G_3 M} l$ is the radius of the horizon ($G_3 = 3$ -dimensional Newton constant), M being the mass of the black hole and l is related to the cosmological constant by $\Lambda = -1/l^2$. Now, let us suppose that the corrected metric has the form

$$ds'^2 = -F(r)dt^2 + (F(r))^{-1}dr^2 + r^2d\theta^2, \tag{18}$$

where $F(r)$ is a function of (r) . Here we have assumed that the fluctuations are only r dependent, and they do not have any angular dependence. The justification for this is that any fluctuation that causes a shear to the black hole surface is highly unstable [50,51]. Since we have started with a spherically symmetric BTZ black holes, it is justified to assume that the relevant fluctuations are only r dependent. In fact, as we consider a black hole near equilibrium, and one that is non-rotating, it is necessary for its stability to only consider spherically symmetric fluctuations [52].

It also known that these thermal fluctuations become dominant at high temperatures. As the temperature of a black holes increases as its size decreases, these thermal fluctuations also increase as the black hole reduces its size. However, as the black hole becomes smaller, quantum effects also become dominant. As these thermal fluctuations scale with the quantum fluctuations, in the Jacobson formalism (where spacetime emerges from thermodynamics), we can argue that these thermal fluctuations actually produce quantum corrections to this emergent spacetime [23]. It may be noted that the correction to the entropy produced by these thermal fluctuations is a logarithmic correction, and it scales as $\ln(S_0 T^2)$. This also indicates that these thermal corrections in the thermodynamics are related to the quantum corrections to the geometry of spacetime. This is because in almost all approaches to quantum gravity, we obtain a logarithmic correction term as the leading order quantum correction to the black hole entropy [10–22]. Even though a logarithmic correction term is produced in almost all approaches to quantum gravity, this term is proportional to a constant, and that constant depends on the details of the various approaches to quantum gravity. In fact, this constant is usually proportional to some new constant in that theory. As this constant depends on the details of the model used, we will use an arbitrary constant α and define the corrected microcanonical entropy as [33–42]

$$S = S_0 + \alpha \ln(S_0 T^2). \tag{19}$$

It may be noted that the corrected metric should reduce to the original metric, as $\alpha \rightarrow 0$, and this occurs if thermal fluctuations are neglected. If we neglect the higher order terms, we write

$$dS = \left(\frac{2\pi}{4G_3} + \frac{\alpha}{r} \right) dr. \tag{20}$$

As $M = \frac{r^2}{8G_3l^2}$, we find that

$$dM = \frac{r}{4G_3l^2} dr. \tag{21}$$

By substituting Eqs. (17), (20), and (21) in the first law of thermodynamics,

$$dM = TdS + P_r dA, \tag{22}$$

we find

$$\frac{r}{4G_3l^2} = \frac{r}{2\pi l^2} \left(\frac{2\pi}{4G_3} + \frac{\alpha}{r} \right) + P_r (8\pi r) \tag{23}$$

where $A = 4\pi r^2$. By simplifying the last equation we find

$$-8G_3\pi P_r = -\frac{1}{4l^2} + \frac{1}{4l^2} + \alpha \frac{G_3}{\pi l^2 (2r)}. \tag{24}$$

Now, the Einstein equation is given by

$$G_{ab} + \Lambda g_{ab} = -8\pi G_3 T_{ab}. \tag{25}$$

The Einstein equation for the metric (18) when evaluated at the radius of the horizon reads [44]

$$-8G_3\pi T_0^0 = \frac{1}{2r} F'(r) - \frac{1}{l^2} \tag{26}$$

where the prime indicates the derivative with respect to r . If we suppose that the static BTZ black hole for the entropy (19) is consistent with Jacobson’s approach, then we can find the corrected metric by identifying between (24) and Eq. (26). Then we get the differential equation

$$\frac{1}{2r} F'(r) - \frac{1}{l^2} = \alpha \frac{G_3}{2\pi l^2 r}. \tag{27}$$

By solving it we find

$$F(r) = \frac{r^2}{l^2} + \alpha \frac{G_3 r}{2\pi l^2} + C \tag{28}$$

where C is the integral constant which is equal to $-8MG_3$ as $F(r)$ is equal to $f(r)$ for $\alpha \rightarrow 0$. Thus, the corrected metric of (15) is given by

$$ds^2 = - \left(\frac{r^2}{l^2} + \alpha \frac{G_3 r}{2\pi l^2} - 8MG_3 \right) dt^2 + \left(\frac{r^2}{l^2} + \alpha \frac{G_3 r}{2\pi l^2} - 8MG_3 \right)^{-1} dr^2 + r^2 d\theta^2. \tag{29}$$

Thus, the thermal fluctuations to the thermodynamics can give rise to quantum corrections to the metric in the Jacobson formalism. Furthermore, by considering the logarithmic corrections to the thermodynamics, we only analyze the first order corrections to the metric.

It would be interesting to study the corrected thermodynamics based on this quantum corrected metric. Even though α is a constant, its value depends on the details of

the approach. So, here we will analyze the corrected thermodynamics for different values of α . The corrected outer and inner horizons are given by

$$r'_{\pm} = \frac{\sqrt{G_3} \sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M} \pm \alpha G_3}{4\pi}. \tag{30}$$

Now using the formula for the temperature [46],

$$T = \frac{r_+^2 - r_-^2}{2\pi r_+}, \tag{31}$$

the corrected temperature can be written as

$$T' = \frac{\sqrt{G_3} (\alpha^2 G_3 + 64\pi^2 l^2 M)}{2\pi^2 (\sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M} - \alpha \sqrt{G_3})}. \tag{32}$$

We may calculate the corrected entropy from the area law,

$$S' = 4\pi r'_+ = \sqrt{G_3} \sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M} - \alpha G_3. \tag{33}$$

We may also study the PV criticality from this corrected metric in the extended phase space [47]. So we can use the definition of the thermodynamics pressure,

$$P = \frac{T}{v}, \tag{34}$$

with $v = 2(V/\pi)^{1/2}$, where V is the BTZ black hole volume, given by

$$V = 16\pi r_+^2 = \frac{(\alpha G_3 - \sqrt{G_3} \sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M})^2}{\pi}. \tag{35}$$

Thus, the thermodynamics pressure is

$$P = \frac{\sqrt{G_3} (\alpha^2 G_3 + 64\pi^2 l^2 M)}{4\pi (\zeta - \alpha \sqrt{G_3}) \sqrt{(\alpha G_3 - \sqrt{G_3} \zeta)^2}} \tag{36}$$

with $\zeta = \sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M}$.

Now we are able to calculate the Gibbs free energy $G(T, l, \alpha)$, from the definition

$$G = M + PV - TS. \tag{37}$$

The Gibbs free energy determines the critical behavior of the BTZ black hole; if $G > 0$ we say the black hole is critical. However, if $G < 0$, the black hole is not critical, and the saddle points of the $G(M, \alpha)$ plot indicate the phase transition. The explicit formula of the Gibbs free energy is given by

$$G = -\frac{\alpha^2 G_3^2}{2\pi^2} - 32G_3 l^2 M + M + \frac{\sqrt{G_3} (\alpha^2 G_3 + 64\pi^2 l^2 M) \sqrt{(\alpha G_3 - \sqrt{G_3} \sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M})^2}}{4\pi^2 (\sqrt{\alpha^2 G_3 + 128\pi^2 l^2 M} - \alpha \sqrt{G_3})}. \tag{38}$$

As we can see from the plot of $G(M, \alpha)$, for this black hole no critical phenomena exist, and the black hole always remains uncritical (as G remains less than zero).

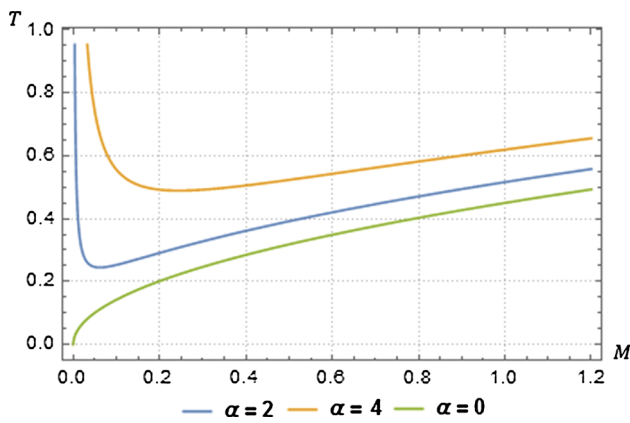


Fig. 1 A plot of the BTZ temperature and its mass for a fixed $l = 0.5$ in Planck units. Observe how quantum corrections lead to a significant increase of temperature at low masses (small scale). Here quantum fluctuations are expected to be most prominent

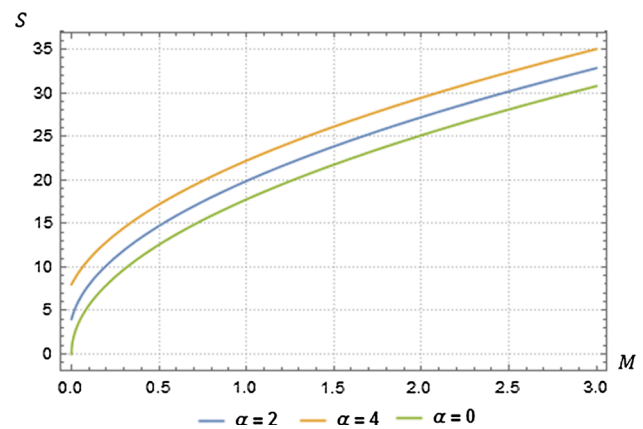


Fig. 2 A plot of the BTZ entropy and its mass for a fixed $l = 0.5$ in Planck units. Quantum corrections are mainly relevant for small mass

It should be noted that as the thermal fluctuations increase we will have to include higher order corrections to this perturbative expansion [45]. However, near the Planck scale the temperature is expected to become sufficiently large to break this perturbative expansion, and at this stage the system cannot be analyzed as a perturbation around equilibrium temperature. This is expected as general relativity emerges from thermodynamics in the Jacobson formalism [23], and so we expect that thermal fluctuations will occur because of quantum fluctuations. It is also known that such logarithmic correction to the entropy are leading order corrections generated from various different approaches to quantum gravity [10–22]. Furthermore, we also expect that as the black hole will become small and its temperature increases, we have to consider higher order thermal corrections as seen from Fig. 1 [45], which will correspond to higher order quantum corrections in the Jacobson formalism [23]. However, near the Planck scale it is expected that the manifold description of the spacetime will break down, so we cannot analyze the system using a quantum correction to classical geometry [48,49]. Similarly, it is expected that the equilibrium description of the thermodynamics will break down at the Planck scale, and we cannot analyze the system using thermal fluctuations to the equilibrium thermodynamics (Figs. 2 and 3).

It would be interesting to analyze this connection between the breaking of the manifold structure of spacetime and the breaking of the equilibrium description of the thermodynamics. It might also be interesting to note that by using non-equilibrium thermodynamics, we might be able to analyze some purely quantum gravitational states of spacetime near the Planck scale. The effects of large fluctuations on the behavior of black holes has been studied [53], and it would be interesting to analyze such effects using the Jacobson formalism. However, in this paper, we have only analyzed the first order corrections to the equilibrium entropy from ther-

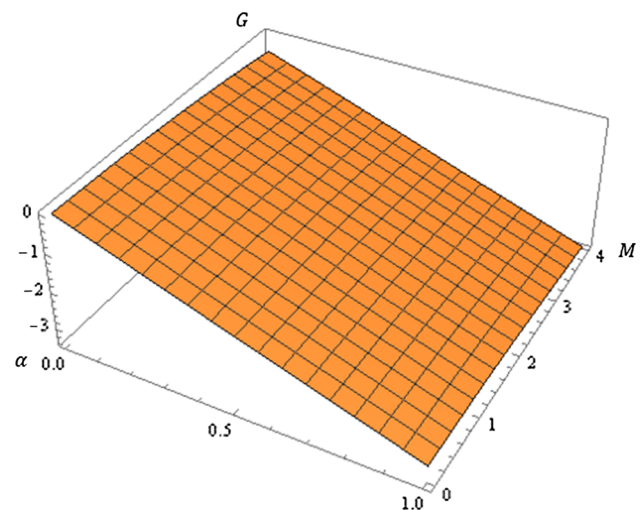


Fig. 3 A 3D plot of $G(M, \alpha)$ of a quantum corrected BTZ black hole for a fixed $l = 0.5$. We observe that, for different masses and perturbation parameters, the BTZ black hole does not show any critical behavior, $G < 0$

mal fluctuations around an equilibrium. The important thing to note here is that just as general relativity can emerge from thermodynamics in the Jacobson formalism, quantum gravity can emerge from thermal fluctuations to the thermodynamics. Furthermore, it is just possible to analyze small quantum fluctuations to the geometry by analyzing small thermal fluctuations to the metric. However, at the Planck scale, just as we expect a manifold description to spacetime to break down, we also expect the equilibrium description to the thermodynamics to break down. We would like to point out that such thermal fluctuations have been studied for various kinds of dynamical black objects [33–42]. It would be interesting to analyze the effects of such thermal fluctuations on the spacetime metric using the formalism developed in this paper. The black hole thermodynamics for time dependent Vaidya black holes has also been studied [54–57]. It would be interesting

to analyze the thermal fluctuations for such black holes, and then use the formalism of this paper to obtain the corrected quantum corrected metric for Vaidya black holes.

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