

Unimodular $f(G)$ gravity

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Received: 28 July 2017 / Accepted: 27 August 2017 / Published online: 14 September 2017

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Abstract In this paper we study a modified version of unimodular general relativity in the context of $f(G)$, G denoting the Gauss–Bonnet invariant. We focus on Bianchi-type I and Friedmann–Robertson–Walker universes and search for unimodular $f(G)$ models according to the de Sitter and power-law solutions. Assuming unimodular $f(G)$ gravity as a perfect fluid and making use of the slow-roll parameters, the inflationary model has been reconstructed in concordance with the Planck observational data. Moreover, we investigate the realization of the bounce and loop quantum cosmological ekpyrotic paradigms. Assuming suitable and appropriate scale factors, unimodular $f(G)$ models able to reproduce superbounce and ekpyrotic scenarios have been reconstructed.

1 Introduction

It is well known nowadays that interest in gravity theories derived from Lagrangians has extended beyond the general relativity (GR). The first step is introducing the cosmological constant, originating from the vacuum expectation value of the quantum field [1, 2]. However, there is no intrinsic mechanism in the theory that can dynamically induce the cosmological constant [3–5]. On the other hand, unimodular gravity [6–30] is an interesting gravitational theory that can be considered as a specific case of the GR, and in which the cosmological constant appears as the trace-free part of the gravitational field equations, fixing the determinant of the metric tensor as a number or a function [2, 5]. In other words, it would be more appropriate to state that unimodular gravity, in a sense, can offer a proposal to solve the cosmological problem. It is important to mention that in the context of unimodular gravity the problem of late-time acceleration of the universe can be developed, where the metric can be decomposed in the unimodular metric part and a scalar field

part [23–25]. Instead of considering GR, different kinds of modified gravity based on the curvature scalar have been performed in the recent years, as $f(R)$ [31–39] where R is the curvature scalar, $f(R, T)$, T being the trace of the energy-momentum tensor [40–49], and $f(G)$, G denoting the Gauss–Bonnet (GB) invariant [50–84].

In this paper we focus our attention to the $f(G)$ and the purpose here is to realize some cosmological evolutions with specific and realistic Hubble rate, and to investigate which unimodular $f(G)$ model can yield such cosmological evolutions of the universe. We fundamentally adopt two kinds of metric in this paper; the Bianchi type-I (BI) metric and the Friedman–Robertson–Walker (FRW) metric. It is obvious that for both metrics there is no compatibility with the unimodular constraint, and so we fix suitably the metrics in order to satisfy this constraint. With the BI metric we search for $f(G)$ models able to reproduce inflation through the de Sitter and power-law form of the scale factor. Through the use of the FRW metric, we still search for $f(G)$ models that agree with inflation. As we are addressing inflationary models we find it important to formulate the related observables, like the spectral index, the tensor-to-tensor ratio and the running of the spectral index. By considering the $f(G)$ model coming from power-law solutions of the scale factor, with non-specified integration constants in the perfect fluid description, we determine the slow-roll indices and confront them with the observational recent Planck data [85, 86] in order to correctly calculate the constants, and then obtaining the effective unimodular $f(G)$ model able to realize the inflationary epoch of our universe.

An alternative to the standard description of the acceleration of inflation is provided by bouncing cosmologies [87, 118], in which the appearance of the initial singularity is prevented. This feature can be represented by the use of scalar fields [92–118], by the models of modified gravity [93–105], and also by the use of the Loop Quantum Cosmology (LQC) [106–114]. In the general case, the realization of the bounce is related to the coupling of matter fields with an

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equation of state in order to violate the null energy condition [87–89, 118]. Bouncing cosmologies have been developed in several works [115–117, 119], but essentially it has been shown that bouncing cosmologies may present primordial instabilities [90, 91], which can be solved by the ekpyrotic scenario. In this paper we also pay attention to the superbounce and the LQC ekpyrotic scenarios in the context of unimodular $f(G)$ gravity by reconstructing a suitable model characteristic of each scenario. This kind of work has been performed in [119], but not in the unimodular context. For other reviews as regards bouncing cosmologies see [120, 121] for $f(R)$ theory, [122–125] for $f(G)$ theory and [126–151] for $f(T)$ theory, where T denotes the torsion scalar.

The manuscript is organized as follows: in Sect. 2 we present the general description of unimodular gravity. Section 3 is devoted to the formalism of $f(G)$ and unimodular equations of motion. The reconstruction of $f(G)$ gravity in an inhomogeneous universe is performed in Sect. 4, for both de Sitter solutions in Sect. 4.1 and power-law solutions in Sect. 4.2. In the FRW metric context, the reconstructions of unimodular $f(G)$ models have been developed within the de Sitter solutions in Sect. 5.1 and power-law solutions in Sect. 5.2, and the latter has been achieved according to Planck results in Sect. 6, finding the integration constants accordingly. The ekpyrotic scenario reconstruction and superbounce reconstruction from unimodular $f(G)$ gravity is performed in Sect. 7. We present our conclusion in Sect. 8.

2 General description of unimodular gravity

In this section we point out the generalization of the GR gravity formalism in order to provide the unimodular $f(G)$ gravity formalism. The unimodular gravity approach is based on the assumption that the determinant of the metric tensor is fixed, expressed by the relation $g_{\mu\nu}\delta g^{\mu\nu} = 0$. Throughout this paper the components of the metric are chosen in such a way that [2]

$$\sqrt{-g} = 1. \tag{1}$$

Let us start our study by using the Bianchi type-I metric, from which the usual and practical FRW metric can be recovered,

$$ds^2 = dt^2 - A(t)^2 dx^2 - B(t)^2 dy^2 - C(t)^2 dz^2. \tag{2}$$

It is easy to check that the unimodular constraint, expressed by Eq. (1) is not satisfied by the Bianchi-type I metric (2). In order to meet this condition we need to redefine the cosmic time coordinate as follows:

$$d\tau = A(t)B(t)C(t)dt, \tag{3}$$

in such a way that the metric (2) becomes

$$ds^2 = [A(t(\tau))B(t(\tau))C(t(\tau))]^{-2} d\tau^2 - A(t(\tau))^2 dx^2 - B(t(\tau))^2 dy^2 - C(t(\tau))^2 dz^2. \tag{4}$$

This metric obviously satisfies the constraint (1) and we shall refer to the latter as the unimodular Bianchi-type I metric from which the FRW metric can be recovered.

3 Formalism of $f(G)$ gravity and unimodular equations of motion

Let us introduce the general $R + f(G)$ action as follows:

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [R + f(G)] + S_m, \tag{5}$$

where R represents the Ricci scalar, and the modification function $f(G)$ corresponds to a generic globally differentiable Gauss–Bonnet topological invariant G function. The matter action S_m is the one which induces the energy-momentum tensor $T_{\mu\nu}$. We focus our attention on the metric formalism where the variation of the action (5) with respect to the metric tensor yields within the minimum principle the following general equation of motion [50–54]:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + 8 \left[R_{\mu\rho\nu\sigma} + R_{\rho\nu\sigma\mu} - R_{\rho\sigma}g_{\nu\mu} - R_{\mu\nu}g_{\sigma\rho} + R_{\mu\sigma}g_{\nu\rho} + \frac{1}{2}R(g_{\mu\nu}g_{\sigma\rho} - g_{\mu\sigma}g_{\nu\rho}) \right] \times \nabla^\rho \nabla^\sigma f_G + (Gf_G - f)g_{\mu\nu} = \kappa T_{\mu\nu}. \tag{6}$$

Here $f_G = \frac{df(G)}{dG}$, and the Gauss–Bonnet term is defined by $G = R^2 - R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma}$, $R_{\mu\nu}$ and $R_{\mu\nu\lambda\sigma}$ being the Ricci tensor and Riemann tensor, respectively. We also point out some adopted definitions as follows: the signature of the Riemannian metric is $(+---)$, $\nabla_\mu V_\nu = \partial_\mu V_\nu - \Gamma_{\mu\nu}^\lambda V_\lambda$ and $R_{\mu\nu\rho}^\sigma = \partial_\nu \Gamma_{\mu\rho}^\sigma - \partial_\rho \Gamma_{\mu\nu}^\sigma + \Gamma_{\mu\rho}^\omega \Gamma_{\omega\nu}^\sigma$ for the covariant derivative of a covariant vector and the Riemann tensor, respectively. Making use of the metric (4) the field equations read

$$-24A^3A'B^3B'C^3C'f'_G + \left(8A^3A'B^3B'C^3C'' + 24A^3A'B^3B'C^2C'^2 + 8A^3A'B^3B''C^3C' + 24A^3A'B^2B'^2C^3C' + 8A^3A''B^3B'C^3C' + 24A^2A'^2B^3B'C^3C' \right) f_G + A^2BB'CC' + AA'B^2CC' + AA'BB'C^2 - f = 8\pi\rho, \tag{7}$$

$$\begin{aligned}
 &8A^4B^3B'C^3C'f_G'' + \left(8A^4B^3B'C^3C'' + 24A^4B^3B'C^2C'^2 \right. \\
 &+ 8A^4B^3B''C^3C' + 24A^4B^2B'^2C^3C' \\
 &+ 24A^3A'B^3B'C^3C'\left. \right) f_G' + \left(-8A^3A'B^3B'C^3C'' \right. \\
 &- 24A^3A'B^3B'C^2C'^2 - 8A^3A'B^3B''C^3C' \\
 &- 24A^3A'B^2B'^2C^3C' - 8A^3A''B^3B'C^3C' \\
 &- 24A^2A'^2B^3B'C^3C'\left. \right) f_G \\
 &- A^2B^2CC'' - A^2B^2C'^2 \\
 &- 3A^2BB'CC' - AA'B^2CC' - A^2BB''C^2 \\
 &- A^2B'^2C^2 - AA'BB'C^2 + f = 8\pi p_r, \tag{8}
 \end{aligned}$$

$$\begin{aligned}
 &8A^3A'B^4C^3C'f_G'' + \left(8A^3A'B^4C^3C'' \right. \\
 &+ 24A^3A'B^4C^2C'^2 + 24A^3A'B^3B'C^3C' \\
 &+ 8A^3A''B^4C^3C'24A^2A'^2B^4C^3C'\left. \right) f_G' \\
 &+ \left(-A^3A'B^3B'C^3C'' - 24A^3A'B^3B'C^2C'^2 \right. \\
 &- 8A^3A'B^3B''C^3C' - 24A^3A'B^2B'^2C^3C' \\
 &- 8A^3A''B^3B'C^3C' - 24A^2A'^2B^3B'C^3C'\left. \right) f_G \\
 &- A^2B^2CC'' - A^2B^2C'^2 - A^2BB'CC' \\
 &- 3AA'B^2CC' - AA'BB'C^2 - AA''B^2C^2 \\
 &- A'^2B^2C^2 + f = 8\pi p_t, \tag{9}
 \end{aligned}$$

$$\begin{aligned}
 &8A^3A'B^3B'C^4f_G'' + \left(24A^3A'B^3B'C^3C' \right. \\
 &+ 8A^3A'B^3B''C^4 + 24A^3A'B^2B'^2C^4 + 8A^3A''B^3B'C^4 \\
 &+ 24A^2A'^2B^3B'C^4\left. \right) f_G' + \left(-8A^3A'B^3B'C^3C'' \right. \\
 &- 24A^3A'B^3B'C^2C'^2 - 8A^3A'B^3B''C^3C' \\
 &- 24A^3A'B^2B'^2C^3C' \\
 &- 8A^3A''B^3B'C^3C' - 24A^2A'^2B^3B'C^3C'\left. \right) f_G \\
 &- A^2BB'CC' - AA'B^2CC' - A^2BB''C^2 \\
 &- A^2B'^2C^2 - 3AA'BB'C^2 - AA''B^2C^2 \\
 &- A'^2B^2C^2 + f = 8\pi p_t, \tag{10}
 \end{aligned}$$

where we undertook the matter content as an anisotropic fluid described by the following energy-momentum tensor:

$$T_{\mu\nu} = (\rho + p_t) u_\mu u_\nu - p_t g_{\mu\nu} + (p_r - p_t) v_\mu v_\nu, \tag{11}$$

with ρ being the energy density, u_μ the 4-velocity and v_μ the space-like vector in the radial direction; p_r (the radial pressure) is the pressure in the direction of v_μ and p_t (the

tangential pressure) the pressure in the direction orthogonal to v_μ . Due to the fact that one considers an anisotropic spherically symmetric matter one has $p_r \neq p_t$ (their equality corresponds to isotropic spherically symmetric matter).

4 Reconstructing $f(G)$ gravity in inhomogeneous universe

In this section we search for the $f(G)$ action through the reconstruction scheme for some particular solution of the class of the metric explored in the previous section. We will just consider the de Sitter and power-law solutions. In the general case, using (4) and (11) the conservation equation for the energy-momentum tensor can easily be obtained by

$$\dot{\rho} + (H_x + H_y + H_z) \rho + H_x p_r + (H_y + H_z) p_t = 0 \tag{12}$$

where the following definitions have been made: $H_x = \frac{\dot{A}}{A}$, $H_y = \frac{\dot{B}}{B}$ and $H_z = \frac{\dot{C}}{C}$

4.1 Searching for de Sitter solutions

The de Sitter solutions are well known in the context of cosmology due to the fact that they may approximately describe the early and current epochs of the universe where its expansion is accelerated. Here, for the three directions the scale factors present an exponential expansion yielding constant Hubble parameters in each direction. Then we assume the scale factors as follows:

$$A = A_0 e^{H_{x0}t}, \quad B = B_0 e^{H_{y0}t}, \quad quad C = C_0 e^{H_{z0}t}, \tag{13}$$

where H_{x0} , H_{y0} and H_{z0} are positive constants expressing the instantaneous rates of the expansion at $t = 0$. By simple derivations one can see that the rates of the expansion for each directions read

$$H_x = \frac{\dot{A}}{A} = H_{x0}, \quad H_y = \frac{\dot{B}}{B} = H_{y0}, \quad H_z = \frac{\dot{C}}{C} = H_{z0}. \tag{14}$$

Then, in the context of the cosmic time t the Gauss–Bonnet invariant reads

$$G = \frac{8}{ABC} (\dot{A}\dot{B}\dot{C} + \dot{A}\ddot{B}\dot{C} + \ddot{A}\dot{B}\dot{C}), \tag{15}$$

$$= 8H_{x0}H_{y0}H_{z0} (H_{x0} + H_{y0} + H_{z0}) \tag{16}$$

and the field equations become

$$8H_{x0}H_{y0}H_{z0} (H_{x0} + H_{y0} + H_{z0}) f_G + H_{x0}H_{y0} + H_{x0}H_{z0} + H_{y0}H_{z0} - f = 8\pi\rho, \tag{17}$$

$$-8H_{x0}H_{y0}H_{z0} (H_{x0} + H_{y0} + H_{z0}) f_G - H_{z0}^2 - H_{y0}H_{z0} - H_{y0}^2 + f = 8\pi p_r, \tag{18}$$

$$-8H_{x0}H_{y0}H_{z0} (H_{x0} + H_{y0} + H_{z0}) f_G - H_{z0}^2 - H_{x0}H_{z0} - H_{x0}^2 + f = 8\pi p_t, \tag{19}$$

$$-8H_{x0}H_{y0}H_{z0} (H_{x0} + H_{y0} + H_{z0}) f_G - H_{y0}^2 - H_{x0}H_{y0} - H_{x0}^2 + f = 8\pi p_t. \tag{20}$$

Since we address de Sitter solutions, by assuming $p_r = p_t = p$, one has $p = -\rho$, and the combination of any two of the field equations yields

$$G \frac{df(G)}{dG} - f(G) + K = 0, \tag{21}$$

where K is a constant depending on H_{x0} , H_{y0} and H_{z0} . The general solution of Eq. (21) reads

$$f(G) = \alpha G + K, \tag{22}$$

where K is an integration constant. Making use of (3) and (13) the expressions of the scale factors can be written as

$$A[t(\tau)] = \left(\frac{H_{x0} + H_{y0} + H_{z0}}{B_0 C_0} \tau \right)^{\frac{H_{x0}}{H_{x0} + H_{y0} + H_{z0}}}, \tag{23}$$

$$B[t(\tau)] = \left(\frac{H_{x0} + H_{y0} + H_{z0}}{A_0 C_0} \tau \right)^{\frac{H_{y0}}{H_{x0} + H_{y0} + H_{z0}}}, \tag{24}$$

$$A[t(\tau)] = \left(\frac{H_{x0} + H_{y0} + H_{z0}}{A_0 B_0} \tau \right)^{\frac{H_{z0}}{H_{x0} + H_{y0} + H_{z0}}}, \tag{25}$$

such that the metric (4) in the unimodular context becomes

$$ds^2 = Q\tau^{-2}d\tau^2 - Q_1\tau^{q_1}dx^2 - Q_2\tau^{q_2}dy^2 - Q_3\tau^{q_3}dz^2 \tag{26}$$

with

$$Q = (H_{x0} + H_{y0} + H_{z0})^{-2} (B_0 C_0)^{q_1} (A_0 C_0)^{q_2} (A_0 B_0)^{q_3}, \tag{27}$$

$$Q_1 = A_0^2 \left(\frac{H_{x0} + H_{y0} + H_{z0}}{A_0 B_0 C_0} \right)^{q_1}, \tag{28}$$

$$q_1 = \frac{2H_{x0}}{H_{x0} + H_{y0} + H_{z0}},$$

$$Q_2 = B_0^2 \left(\frac{H_{x0} + H_{y0} + H_{z0}}{A_0 B_0 C_0} \right)^{q_2}, \tag{29}$$

$$q_2 = \frac{2H_{y0}}{H_{x0} + H_{y0} + H_{z0}},$$

$$Q_3 = C_0^2 \left(\frac{H_{x0} + H_{y0} + H_{z0}}{A_0 B_0 C_0} \right)^{q_3}, \tag{30}$$

$$q_3 = \frac{2H_{z0}}{H_{x0} + H_{y0} + H_{z0}}.$$

4.2 Power-law solutions

In this subsection we devote our attention to the cosmological evolution described by a power-law functions of cosmic time in each direction of the space. We assume the following expressions for the scale factors in terms of cosmic time:

$$A(t) = A_0 t^a, \quad B(t) = B_0 t^b, \quad C(t) = C_0 t^c, \tag{31}$$

where a, b, c and A_0, B_0, C_0 are constants that should be determined from the initial conditions. In this case the rate of the expansion for each direction is given by

$$H_x = \frac{a}{t}, \quad H_y = \frac{b}{t}, \quad H_z = \frac{c}{t}, \tag{32}$$

and the expression of Gauss–Bonnet invariant in the context of cosmic time reads

$$G = \frac{8abc^2}{t^4} + \frac{8ab^2c}{t^4} + \frac{8a^2bc}{t^4} - \frac{24abc}{t^4}. \tag{33}$$

Then the field equations take the following forms:

$$Gf_G + \frac{bc}{t^2} + \frac{ac}{t^2} + \frac{ab}{t^2} - f = 8\pi\rho, \tag{34}$$

$$-Gf_G - \frac{(c-1)c}{t^2} - \frac{bc}{t^2} - \frac{(b-1)b}{t^2} + f = 8\pi p_r, \tag{35}$$

$$-Gf_G - \frac{(c-1)c}{t^2} - \frac{ac}{t^2} - \frac{(a-1)a}{t^2} + f = 8\pi p_t, \tag{36}$$

$$-Gf_G - \frac{(b-1)b}{t^2} - \frac{ab}{t^2} - \frac{(a-1)a}{t^2} + f = 8\pi p_t. \tag{37}$$

Here we also assume for simplicity that $p_r = p_t = p = \omega\rho$, ω being the parameter of the equation of state. Considering the first two field equations one gets

$$G(1 + 8\pi\omega) f_G - (1 + 8\pi\omega) f + (K_2 + 8\pi\omega K_1) \frac{1}{t^2} = 0 \tag{38}$$

with $K_1 = bc + ac + ab$ and $K_2 = -(c-1)c - bc - (b-1)b$. By making use of (33) the previous equations take the following form:

$$Gf_G - f + \mathcal{K}G^{1/2} = 0, \tag{39}$$

where

$$\mathcal{K} = \frac{K_2 + 8\pi\omega K_1}{(1 + 8\pi\omega) (8abc^2 + 8ab^2c + 8a^2bc - 24abc)^{1/2}}. \tag{40}$$

The general solution of the previous equation reads

$$f(G) = 2\mathcal{K}\sqrt{G} + \beta G \tag{41}$$

with β a positive constant to be determined using the cosmological data. In this case the metric (4) in the unimodular context becomes

$$ds^2 = Q\tau^r d\tau^2 - Q_1\tau^{r_1} dx^2 - Q_2\tau^{r_2} dy^2 - Q_3\tau^{r_3} dz^2 \tag{42}$$

where

$$Q = \frac{1}{(A_0B_0C_0)^2} \left(\frac{a+B+C+1}{A_0B_0C_0} \right)^r, \tag{43}$$

$$r = -2 \frac{a+b+c}{a+b+c+1},$$

$$Q_1 = A_0^2 \left(\frac{a+B+C+1}{A_0B_0C_0} \right)^{r_1},$$

$$r_1 = \frac{2a}{a+b+c+1}, \quad Q_2 = B_0^2 \left(\frac{a+B+C+1}{A_0B_0C_0} \right)^{r_2},$$

$$r_2 = \frac{2b}{a+b+c+1}, \tag{44}$$

$$Q_3 = C_0^2 \left(\frac{a+B+C+1}{A_0B_0C_0} \right)^{r_3}, \quad r_3 = \frac{2c}{a+b+c+1}. \tag{45}$$

5 Lagrange multiplier formulation and reconstruction of unimodular $f(G)$ gravity

In this section we will present the Lagrange multiplier formulation of $f(G)$ gravity, according to the method performed in context of $f(R)$ in [152–154]. This procedure is just used to ensure that the unimodular condition is satisfied. To do so, we introduce the Lagrangian multiplier λ and the unimodular $f(G)$ gravity with matter can be expressed as

$$S_\lambda = \int d^4x \left\{ \sqrt{-g} \left[\frac{R + f(G)}{2\kappa^2} - \lambda \right] + \lambda \right\}. \tag{46}$$

From now we will assume the FRW metric and consider the matter energy-momentum tensor as corresponding to a perfect fluid with the energy density and pressure ρ and p , respectively. Thereby the GB invariant expressions in terms of t and τ read

$$G = 24 \frac{\dot{A}^2 \ddot{A}}{A^3}, \tag{47}$$

$$= 24A^9 A'^2 A'' + 72A^8 A'^4 = 24A^{12} (\mathcal{H}^2 \mathcal{H}' + 4\mathcal{H}^4) \tag{48}$$

where the parameter $\mathcal{H} \equiv \frac{1}{A(\tau)} \frac{dA(\tau)}{d\tau}$ is the relative Hubble parameter in this context. Hence the field equations become

$$-24A^{12} \mathcal{H}^3 f'_G + 24A^{12} (\mathcal{H}^2 \mathcal{H}' + 4\mathcal{H}^4) f_G + 3A^6 \mathcal{H}^2 - (f - \lambda) - \rho = 0, \tag{49}$$

$$8A^{12} \mathcal{H}^2 f''_G + (16A^{12} \mathcal{H} \mathcal{H}' + 88A^{12} \mathcal{H}^3) f'_G - 24A^{12} (\mathcal{H}^2 \mathcal{H}' + 4\mathcal{H}^4) f_G - A^6 (2\mathcal{H}' + 9\mathcal{H}^2) + (f - \lambda) - p = 0. \tag{50}$$

By directly summing Eqs. (49) and (85), the term $(f - \lambda)$ is eliminated, giving rise to the following equations:

$$8A^{12} \mathcal{H}^2 f''_G + 16A^{12} (\mathcal{H} \mathcal{H}' + 4\mathcal{H}^3) f'_G - 2A^6 (\mathcal{H}' + 3\mathcal{H}^2) - \rho - p = 0. \tag{51}$$

In order to perform a consistent analysis, it is important to make use of the continuity equation for the matter, namely

$$\rho' + 3\mathcal{H}(\rho + p) = 0. \tag{52}$$

Let us adopt the equation of state $p = w\rho$. Thus solving the previous equation, one gets

$$\rho(\tau) = \rho_0 \exp \left\{ -3(1+w) \int_0^\tau \mathcal{H}(\zeta) d\zeta \right\} = \rho_0 A(\tau)^{-3(1+w)}. \tag{53}$$

Now, in order to obtain cosmological $f(G)$ models in the unimodular context, one just needs a suitable expression of the scale depending on τ , solving Eq. (51).

5.1 Reconstruction of unimodular $f(G)$ model describing de Sitter universe

In this subsection we consider the FRW metric that describes a de Sitter expanding universe with the scale factor reading

$$A(t) = e^{H_0 t}, \tag{54}$$

where H_0 is an arbitrary positive constant. By using Eq. (3), setting $B = C = A$, one can extract the t in terms of τ as

$$t = \frac{1}{3H_0} \ln(3H_0\tau), \tag{55}$$

such that the scale factor in terms of the τ reads

$$A(t(\tau)) = (3H_0\tau)^{1/3}. \tag{56}$$

In this case, addressing the de Sitter universe, $\omega = -1$ and the differential equation (51) becomes

$$3\tau \frac{d^2 f_G(\tau)}{d\tau^2} + 2 \frac{df_G(\tau)}{d\tau} = 0, \tag{57}$$

whose general solutions are

$$f_G(\tau) = 3C_1\tau^{1/3} + C_2 \tag{58}$$

where \mathcal{C}_1 and \mathcal{C}_2 are integration constants. On the other hand, one can express the GB invariant (48) in terms of τ ,

$$G(\tau) = 24H_0, \tag{59}$$

which obviously is a constant. The task to be done now is to invert the function $G(\tau)$ in order to obtain $\tau = \tau(G)$, but since $G(\tau) = \text{Const}$ it is common to fix $\mathcal{C}_1 = 0$, such that $f_G = df(G)/dG = \mathcal{C}_2$, whose general solution takes the following form:

$$f(G) = \mathcal{C}_2 G + \mathcal{C}_3. \tag{60}$$

This result is consistent with the one obtained in (22), confusing (α, K) with $(\mathcal{C}_1, \mathcal{C}_3)$, characteristic of the de Sitter universe.

5.2 Reconstruction of unimodular $f(G)$ model according to power-law solution

In this case we assume the scale factor in terms of the cosmic time to be

$$A(t) = A_0 \left(\frac{t}{t_0}\right)^q \tag{61}$$

where t_0 and A_0 are the initial time and the initial value of the scale factor, respectively. According to the relation $d\tau = A(t)^3 dt$ one gets

$$A(t(\tau)) = \left(\frac{\tau}{\tau_0}\right)^\sigma, \quad \sigma = \frac{q}{1+3q}, \quad \tau_0 = \frac{t_0}{A_0^{1/q}(1+3q)}. \tag{62}$$

It is easy to see that, for $\sigma = 1/3$, the de Sitter universe is recovered, while the power-law inflation is obtained for $q > 1$ ($1/4 < \sigma < 1/3$). The accelerated expansion is realized for $0 < q \leq 1$ ($0 < \sigma \leq 1/4$), where inflation is possible.

The GB invariant in this case is

$$G = 24\sigma^3 \left(\frac{4\sigma - 1}{\tau^4}\right) \left(\frac{\tau}{\tau_0}\right)^{12\sigma}. \tag{63}$$

Thus the differential equation (51) becomes

$$J_{01} f_G'' + J_{02} f_G' + (J_{03} + J_{04}) = 0, \tag{64}$$

with

$$\begin{aligned} J_{01}(\tau) &= \frac{8\sigma^2}{\tau^2} \left(\frac{\tau}{\tau_0}\right)^{12\sigma}, \\ J_{02}(\tau) &= \frac{16\sigma^2(4\sigma - 1)}{\tau^3} \left(\frac{\tau}{\tau_0}\right)^{12\sigma}, \\ J_{03}(\tau) &= -\frac{2\sigma(3\sigma - 1)}{\tau^2} \left(\frac{\tau}{\tau_0}\right)^{6\sigma}, \\ J_{04}(\tau) &= -(1 + w)\rho_0 \left(\frac{\tau}{\tau_0}\right)^{-3\sigma(1+w)}. \end{aligned} \tag{65}$$

The previous equation can be rewritten by dividing the whole equation by $J_{01} \neq 0$,

$$f_G'' + J_1 f_G' + J_2 = 0, \tag{66}$$

where

$$\begin{aligned} J_1(\tau) &= \frac{1 - 3\sigma}{\tau}, \\ J_2(\tau) &= \frac{1 - 3\sigma}{4\sigma} \left(\frac{\tau}{\tau_0}\right)^{-6\sigma} \\ &\quad + \frac{\rho_0(1 + w)\tau^2}{8\sigma^2} \left(\frac{\tau}{\tau_0}\right)^{-3\sigma(5+w)}, \end{aligned} \tag{67}$$

whose general solution reads

$$\begin{aligned} f_G(\tau) &= \frac{\tau^2 \left(\frac{\tau}{\tau_0}\right)^{-6\sigma}}{8\sigma(1 - 2\sigma)} \\ &\quad + \frac{\rho_0(1 + w)\tau^4 \left(\frac{\tau}{\tau_0}\right)^{-3\sigma(5+w)}}{8\sigma^2 [4 - 3(5 + w)\sigma] [-1 + (7 + 3w)\sigma]} \\ &\quad + \mathcal{K}_1 \frac{\tau^{3-8\sigma}}{3 - 8\sigma} + \mathcal{K}_2, \end{aligned} \tag{68}$$

\mathcal{K}_1 and \mathcal{K}_2 being integration constants. From Eq. (63), one can express τ and $\frac{\tau}{\tau_0}$ in terms of G ,

$$\begin{aligned} \tau &= \chi_1 G^{\frac{1}{4(3\sigma-1)}}, \quad \frac{\tau}{\tau_0} = \chi_2 G^{\frac{1}{4(3\sigma-1)}}, \\ \chi_1 &= \left[\frac{\tau_0^{12\sigma}}{24\sigma^3(4\sigma - 1)} \right]^{\frac{1}{4(3\sigma-1)}}, \\ \chi_2 &= \left[\frac{\tau_0^4}{24\sigma^3(4\sigma - 1)} \right]^{\frac{1}{4(3\sigma-1)}}. \end{aligned} \tag{69}$$

Hence, Eq. (68) can now be written in terms of G as

$$\begin{aligned} f_G(G) &= \frac{\chi_1^2 \chi_2^{-6\sigma}}{8\sigma(1 - 2\sigma)} G^{-\frac{1}{2}} \\ &\quad + \frac{\rho_0(1 + w)\chi_1^4 \chi_2^{-3\sigma(5+w)}}{8\sigma^2 [4 - 3(5 + w)\sigma] [-1 + (7 + 3w)\sigma]} G^{\frac{4-3\sigma(5+w)}{4(3\sigma-1)}} \\ &\quad + \frac{\mathcal{K}_1 \chi_1^{3-8\sigma}}{3 - 8\sigma} G^{\frac{3-8\sigma}{4(3\sigma-1)}} + \mathcal{K}_2. \end{aligned} \tag{70}$$

By integrating the expression in the right hand side of the (70) with respect to G , one gets

$$\begin{aligned} f(G) &= \frac{\chi_1^2 \chi_2^{-6\sigma} G^{\frac{1}{2}}}{4\sigma(1 - 2\sigma)} \\ &\quad + \frac{\rho_0(1 - 3\sigma)\chi_1^4 \chi_2^{-3\sigma(5+w)} G^{\frac{3\sigma(1+w)}{4(1-3\sigma)}}}{6\sigma^3 [4 - 3(5 + w)\sigma] [-1 + (7 + 3w)\sigma]} \\ &\quad + \frac{4\mathcal{K}_1(3\sigma - 1)\chi_1^{3-8\sigma} G^{\frac{4\sigma-1}{4(3\sigma-1)}}}{(4\sigma - 1)(3 - 8\sigma)} + \mathcal{K}_2 G + \mathcal{K}_3. \end{aligned} \tag{71}$$

6 Unimodular inflationary cosmological $f(G)$ model

In the previous section we performed the reconstruction of the specific $f(G)$ form that generates scale-factor evolution. The obtained model will be explored in the present section to investigate inflation realization in unimodular $f(G)$ gravity. Moreover, we will focus our attention on the investigation of inflationary observables such as scalar and tensor spectral indices, the tensor-to-tensor ratio and the running spectral index confronting them with the cosmological observational data.

6.1 Slow-roll parameters and inflationary observables

The exploration of any inflationary scenario is essentially based on the values of related observables such as the scalar spectral index of the curvature perturbations n_s , its running $\alpha_s \equiv dn_s/d \ln k$, k being the absolute value of the wave number \vec{k} , the tensor spectral index n_T and the tensor-to-tensor ratio r . The determination of the values of the observables would be by detailed perturbation analysis and we propose to escape from these complicated procedures by making use of the Einstein frame, where all the inflationary information is driven by the so-called effective scalar potential $V(\phi)$. To do so it is common to define the slow-roll parameters ϵ , η and ξ in terms of this potential and its derivatives [155–157, 161],

$$\begin{aligned} \epsilon &\equiv \frac{M_p^2}{2} \left(\frac{1}{V} \frac{dV}{d\phi} \right)^2, \quad \eta \equiv \frac{M_p^2}{V} \frac{d^2V}{d\phi^2}, \\ \xi^2 &\equiv \frac{M_p^4}{V^2} \frac{dV}{d\phi} \frac{d^3V}{d\phi^3}. \end{aligned} \tag{72}$$

We point out that the calculation of the spectral index of the primordial curvature perturbations and of the scalar-to-tensor ratio relies upon the technique of the perfect fluid approximation in modified gravity, and this was developed in [158, 159]. It is important to mention that a more direct approach is to calculate the spectral index directly via the direct calculation of the power spectrum, as was done in [160], in the context of standard $f(G)$ gravity; this is, however, very tedious. Also it has been shown in [5] that the inflation ends when $\epsilon = 1$. The approximated expressions for the observables are presented in [161]:

$$\begin{aligned} r &\approx 16\epsilon, \quad n_s \approx 1 - 6\epsilon + 2\eta, \\ \alpha_s &\approx 16\epsilon\eta - 24\epsilon^2 - 2\xi^2, \quad n_T \approx -2\epsilon. \end{aligned} \tag{73}$$

Since we are dealing with modified gravity, its obvious that the conformal transformation to the Einstein frame is not possible and one cannot define a scalar potential nor the potential slow-roll parameters. To do so, one can introduce the Hubble slow-roll parameters ϵ_n by

$$\epsilon_{n+1} \equiv \frac{d \ln |\epsilon_n|}{dN}, \tag{74}$$

where the initial value of this parameter is $\epsilon_0 \equiv H_{\text{ini}}/H$ and the parameter N , known as the e-folding number, is defined as $N \equiv \ln(a/a_{\text{ini}})$, a_{ini} being the scale factor at the beginning of the inflation and H_{ini} the corresponding Hubble parameter. Hence one can express the first three ϵ as

$$\begin{aligned} \epsilon_1 &= -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{H}}{H\dot{H}} - \frac{2\dot{H}}{H^2}, \quad \epsilon_3 = \left(H\ddot{H} - 2\dot{H}^2 \right)^{-1} \\ &\times \left[\frac{H\dot{H}\ddot{H} - \ddot{H}(\dot{H}^2 + H\dot{H})}{H\dot{H}} - \frac{2\dot{H}}{H^2} (H\ddot{H} - 2\dot{H}^2) \right], \end{aligned} \tag{75}$$

in such a way that the inflationary-related observables can now be written as

$$\begin{aligned} r &\approx 16\epsilon_1, \quad n_s \approx 1 - 2\epsilon_1 - 2\epsilon_2, \\ \alpha_s &\approx -2\epsilon_1\epsilon_2 - \epsilon_3 - 2\epsilon_3, \quad n_T \approx -2\epsilon_1. \end{aligned} \tag{76}$$

According to (61) and (62), one gets

$$\epsilon_1 = \frac{1}{q} = \frac{1 - 3\sigma}{\sigma}, \quad \epsilon_2 = 0, \quad \epsilon_3 = \frac{1}{q} = \frac{1 - 3\sigma}{\sigma}, \tag{77}$$

yielding

$$\begin{aligned} r &\approx \frac{16(1 - 3\sigma)}{\sigma}, \quad n_s \approx \frac{7\sigma - 2}{\sigma}, \\ \alpha_s &\approx 0, \quad n_T \approx \frac{2(3\sigma - 1)}{\sigma}. \end{aligned} \tag{78}$$

6.2 Obtaining inflationary unimodular $f(G)$ model according to the Planck results

According to the Planck results [85, 162] one has $0.962 \leq n_s \leq 0.974$, leading to $52631 \leq q \leq 76.923$. Thus, fixing $q = 60$, that is, $\sigma = 0.331$, one gets $n_s \approx 0.966$, $r \approx 0.266$, $\alpha_s = 0$ and $n_T = 0.033$, consistent with the Planck results. Now, still in agreement with the Planck results we will determine the constants appearing in (71) in order to obtain the $f(G)$ model that describes the inflation. We also note that according to the Planck results [85, 162] one has $66.9 \text{ km/s/Mpc} \leq H_0 \leq 68.7 \text{ km/s/Mpc}$ and we fix it to $H_0 = 67.8 \text{ km/s/Mpc}$ and we have the well-known relation between the current values of the cosmic time and the Hubble parameter pointing to the age of the universe, i.e., $t_0 \approx 2/3H_0$, and one gets $t_0 = 9.832 \times 10^{-3} \text{ Mpc s/km}$. Using Eq. (62) and $A_0 = 1$, one gets $\tau_0 = 5.43 \times 10^{-5} \text{ Mpc s/km}$. Using $\rho_0 = 3H_0^2$ and assuming that at this stage of inflation only the radiation is the dominated constituent of the universe (rigorously just after the inflation) i.e., $w = 1/3$, one gets

$$\begin{aligned} f(G) &= 1.186\sqrt{G} - 1.074 \times 10^{-779} G^{47.285} \\ &\quad - 2.858 \times 10^{205} G^{-11.571} + G \end{aligned} \tag{79}$$

where we fix for simplicity $\mathcal{K}_1 = \mathcal{K}_2 = 1$ and $\mathcal{K}_3 = 0$.

7 Superbounce and loop quantum ekpyrotic cosmology from unimodular $f(G)$ gravity

In this section we propose to reconstruct the unimodular $f(G)$ model able to realize the superbounce and loop quantum ekpyrotic cosmology. The first attempts to realize these cosmologies in modified gravity were performed in [163].

7.1 Ekpyrotic scenario reconstruction from unimodular $f(G)$ gravity

The task here is to find the $f(G)$ model in the unimodular context which, in the large cosmic time limit, corresponds to the late-time era of the ekpyrotic scenario. We assume the well-known gravitational action as in [124, 125],

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} + f(G) \right]. \tag{80}$$

By making use of the auxiliary scalar field ϕ the previous action becomes

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{2\kappa^2} - V(\phi) - \xi(\phi)G \right]. \tag{81}$$

The variation of the action (80) with respect to ϕ yields

$$V'(\phi) + \xi'(\phi)G = 0, \tag{82}$$

whose solution, if it exists, will be a functional dependence of G , i.e. $\phi = \phi(G)$ and now substituting it into (80) leads to

$$f(G) = -V(\phi(G)) - \xi(\phi(G))G, \tag{83}$$

and it is obvious that if we find $\phi(G)$, the algebraic $f(G)$ will be obtained straightforwardly. We have

$$24A^{12}\mathcal{H}^3\xi' - 24A^{12}(\mathcal{H}^2\mathcal{H}' + 4\mathcal{H}^4)\xi + 3A^6\mathcal{H}^2 + V + \xi G = 0, \tag{84}$$

$$-8A^{12}\mathcal{H}^2\xi'' - (16A^{12}\mathcal{H}\mathcal{H}' + 88A^{12}\mathcal{H}^3)\xi' + 24A^{12}(\mathcal{H}^2\mathcal{H}' + 4\mathcal{H}^4)\xi - A^6(2\mathcal{H}' + 9\mathcal{H}^2) - V - \xi G = 0. \tag{85}$$

By combining the above equations through a straightforward summation one gets the following equation:

$$-8A^{12}\mathcal{H}^2\xi'' - 16A^{12}(\mathcal{H}\mathcal{H}' + 4\mathcal{H}^3)\xi' - 2A^6(\mathcal{H}' + 3\mathcal{H}^2) = 0. \tag{86}$$

Since we are searching for an ekpyrotic model we have to take the large cosmic time limit of the scale factor assumed as $A(t) = A_0^{v/2}t^v$, corresponding, in the unimodular context, to

$$A(t(\tau)) = \bar{A}\tau^l, \quad l = \frac{v}{3v+1}, \quad \bar{A} = \left[(3v+1)\sqrt{A_0} \right]^l. \tag{87}$$

According to the previous expression of the scale factor in the unimodular context, Eq. (86) takes the following form:

$$\xi'' + \gamma_1(\tau)\xi' + \gamma_2(\tau) = 0, \tag{88}$$

with

$$\gamma_1(\tau) = 8\bar{A}l\tau^{l-1} + 2(l-1)\tau^{-1}, \quad \text{and} \\ \gamma_2(\tau) = \frac{3}{4}\bar{A}^{-6}\tau^{-6l} + \frac{1}{4l}\bar{A}^{-7}(l-1)\tau^{-7l}. \tag{89}$$

From this, according to the large cosmic time, we arrive at the following expression of $\xi(\tau)$ and $V(\tau)$:

$$\xi(\tau) = \Gamma_1\tau^{2(1-3l)}, \quad V(\tau) = \Gamma_2\tau^{-2(1-3l)}, \tag{90}$$

with

$$\Gamma_1 = \frac{(1-3l)^{6l-1}}{l(1-2l)\bar{A}^{3l}}, \quad \text{and} \quad \Gamma_2 = 24\frac{l^2(4l-1)(1-3l)^{2(3l-2)}}{1-2l}. \tag{91}$$

Making use of (48) and (87), one can express τ in terms of G as

$$\tau = \Gamma_3 G^{\frac{1}{4(3l-1)}}, \tag{92}$$

where

$$\Gamma_3 = \left[24\bar{A}l^3(4l-1) \right]^{\frac{1}{4(1-3l)}}. \tag{93}$$

Now injecting Eq. (92) of τ into (90), one obtains

$$\xi(G) = \Gamma_1\Gamma_3^{2(1-3l)}G^{-1/2}, \quad \text{and} \quad V(G) = \Gamma_2\Gamma_3^{2(3l-1)}G^{1/2}, \tag{94}$$

such that, from (83), the algebraic $f(G)$ ekpyrotic model reads

$$f(G) = -\Gamma_1\Gamma_3^{2(1-3l)}G^{-1/2} - \Gamma_2\Gamma_3^{2(3l-1)}G^{3/2}. \tag{95}$$

7.2 Superbounce reconstruction from unimodular $f(G)$ gravity

In this case the scale factor is given by [88]

$$A(t) \propto (t_* - t)^{2/c^2}, \tag{96}$$

where t_* denotes the big crunch time, and c is a parameter constrained to $c > \sqrt{6}$ [88]. In the context of unimodular gravity, one gets

$$A(t(\tau)) \propto (\tau_* - \tau)^{\frac{2}{6+c^2}}. \tag{97}$$

In this case, for simplicity and for large cosmic time t , i.e. near the bounce, the τ dependent functions $\xi(\tau)$ and $V(\tau)$

behave as

$$\xi(\tau) \propto (\tau_* - \tau)^{\frac{2c^2}{6+c^2}}, \quad V(\tau) = (\tau_* - \tau)^{\frac{-2}{6+c^2}}. \quad (98)$$

On the other hand, from (97) and (48), one expresses τ in terms of G as

$$\tau(G) = G^{-\frac{6+c^2}{4c^2}}, \quad (99)$$

such that the algebraic $f(G)$ model near the bounce reads

$$f(G) = \mathcal{Z}_1 G^{-1/2} + \mathcal{Z}_2 G^{3/2} \quad (100)$$

where \mathcal{Z}_1 and \mathcal{Z}_2 are integration constants depending on the parameter c .

8 Conclusion

In this paper we explored a type of modified GR theory, namely $f(G)$ theory of gravity in the unimodular context, where G denotes the GB invariant. In such a theory an important condition is applied to the metric tensor constraining its determinant to a number or a specific function. Here we work fixing it to 1 and consider both appropriate BI and FRW universes. In a first step the task is to reconstruct the unimodular $f(G)$ models considering de Sitter and power-law solutions for the scale factors. The resulting models are different from the standard $f(G)$ model (without considering unimodular formalism). In the second step we focused our attention to the model provided by the power-law solutions in the FRW universe and searched for the input constants that can constrain it to an inflationary model. To do so, we proceeded to the determination of the slow-roll parameters and the corresponding observational indices from the unimodular $f(G)$ field equations, which appear as functions of the input parameters. According to the recent Planck results we calculated accordingly the input parameters, obtaining an unimodular $f(G)$ inflationary model.

On the other hand we investigated, still in the unimodular $f(G)$ context, the superbounce and the loop quantum cosmology ekpyrotic paradigms. As is well known, bouncing solutions can be an alternative to inflation and thus it appears interesting to search for a related $f(G)$ model. To do so, we considered the well-known standard cosmic time depending scale factors able to lead to the bouncing cosmology in the one case and the ekpyrotic cosmology in the other. The corresponding scale factors in the unimodular context have been determined, namely in terms of the auxiliary time τ . Through the unimodular formalism and the related field equations, the bouncing and ekpyrotic $f(G)$ models have been reconstructed.

Acknowledgements The author thanks Profs. S. D. Odintsov, V. Oikonomou and A. Yu. Petrov for useful comments and suggestions.

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