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Non-local deformation of a supersymmetric field theory

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Abstract In this paper, we will analyze a supersymmetric field theory deformed by generalized uncertainty principle and Lifshitz scaling. It will be observed that this deformed supersymmetric field theory contains non-local fractional derivative terms. In order to construct such a deformed $\mathcal{N} = 1$ supersymmetric theory, a harmonic extension of functions will be used. However, the supersymmetry will only be preserved for a free theory and will be broken by the inclusion of interaction terms.

1 Introduction

Three dimensional supersymmetry is important as it has been observed in the Kondo effect [1,2]. The original Kondo effect describes a defect interacting with a free Fermi liquid of itinerant electrons, and the supersymmetry is introduced if the ambient theory is an interacting CFT. In fact, this introduces qualitatively new features into the system. A meta-magnetic transition in models for heavy fermions has been analyzed using a doped Kondo lattice model in two dimensions [3]. It has been demonstrated that such a system exhibits field-driven quantum phase transitions due to a breakdown of the Kondo effect [4,5]. Such systems are analyzed using Lifshitz theories which are theories based on an anisotropic scaling between space and time. The second order quantum phase transition has also been analyzed using Lifshitz theories [6–9]. The location of a Fermi-surface-changing Lifshitz transition is determined by carrier doping in some heavy fermion compounds [10]. The chemical potential does not cause a heavy band to shift rigidly due to a strong correlation. This is determined by the interplay of heavy and additional light bands crossing the Fermi level.

Three dimensional supersymmetry have also been observed in graphene [11,12]. Furthermore, the van der Waals and Casimir interaction, between graphene and a material plate, between a single-wall carbon nanotube and a plate, and between graphene and an atom or a molecule, have been analyzed using Lifshitz scaling [13]. It may be noted that by generalizing the usual Lifshitz theory, it is possible to describe such materials which could not be described with the local dielectric response [14]. The Casimir–Lifshitz free energy, between two parallel plates made of dielectric material possessing a constant conductivity at low temperatures, has been studied; and the temperature correction for this system has also been analyzed [15]. Many properties of narrow heavy fermion bands can be described by a Zeeman-driven Lifshitz transition [16]. The fermionic theories with $z = 3$ have been analyzed [17,18]. In fact, the Nambu–Jona-Lasinio type four-fermion coupling at the $z = 3$ Lifshitz fixed point in four dimensions is asymptotically free and generates a mass scale dynamically [19]. Furthermore, fermionic theories with $z = 2$ have been constructed, and it has been demonstrated that the construction of such fermionic theories requires a non-local differential operator [20]. However, it is possible to analyze this non-local differential operator using the harmonic extension of functions [21–25].

The Lifshitz theories based on the generalized uncertainty principle have also been constructed [26]. The generalized uncertainty principle is motivated by the existence of a minimum length scale, which in turn is predicted in almost all approaches to quantum gravity. According to most quantum gravity theories, the classical picture of spacetime as a continuous differential manifold breaks down below the Planck

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length. This is because fluctuations in the geometry of order one at the Planck scale impose a minimum length scale below which space cannot be probed. Such a minimum measurable length scale occurs in string theory, as space cannot be probed below the string length scale in perturbative string theory [27–31]. In loop quantum gravity, the existence of a minimum length turns the big bang into a big bounce [32]. Even though the existence of a minimum measurable length scale in predicted in almost all theories of quantum gravity, it is not consistent with the usual Heisenberg uncertainty principle. This is because according to the usual Heisenberg uncertainty principle, the length can in principle be measured with arbitrary precision, if the momentum is not measured [33–45]. So, according to the usual Heisenberg uncertainty principle, a minimum measurable length scale does not exist. Therefore, it is necessary to modify the Heisenberg uncertainty principle to make it consistent with the existence of a minimum measurable length scale. This modified uncertainty principle is called the generalized uncertainty principle. The modification of the Heisenberg uncertainty principle leads to a deformation of the usual Heisenberg algebra.

Even though the generalized uncertainty principle is motivated from quantum gravity, a modification of this principle can have low energy effects which can be detected in the laboratory [46]. In fact, such effects are expected to be observed in Lamb shift, Landau levels, and the tunneling current in a scanning tunneling microscope [47]. Furthermore, as it has been recently studied in graphene, it is expected that such a low energy effect from the generalized uncertainty principle can be observed in graphene. Thus, it is both interesting and important to analyze supersymmetric theories in three dimensions, with Lifshitz scaling based on the generalized uncertainty principle. Such an analysis would be important in analyzing the low energy effect of the generalized uncertainty principle on the Kondo effect in heavy metals, and the van der Waals and Casimir interaction in graphene. It will be possible to construct a free supersymmetric theory based on generalized uncertainty principle and Lifshitz scaling. Even though the introduction of interactions will break the supersymmetry of such a theory, such a theory might be interesting as free field theories are also very important as effective field theories in describing materials like graphene. It may be noted that four dimensional supersymmetric theories with Lifshitz scaling have been studied [48,49], but so far three dimensional theories with Lifshitz scaling have not been studied. Furthermore, the generalized uncertainty principle has never been combined with supersymmetric field theories based on Lifshitz scaling. However, such a construction is important for the analysis of condensed matter systems. So, in this paper, we will analyze three dimensional supersymmetric field Lifshitz theories based on the generalized uncertainty principle.

2 Deformed superspace

In this paper, we shall analyze supersymmetric Lifshitz theories with the existence of a minimum measurable length scale.

Let us first introduce these two concepts. First, the existence of a minimum measurable length scale becomes manifest by deforming the usual uncertainty principle to a generalized uncertainty principle,

$$\Delta x \Delta p = \frac{1}{2}[1 + \beta(\Delta p)^2], \tag{1}$$

where $\beta = \beta_0 \ell_{Pl}^2$, β_0 is a constant normally assumed to be of order one, and $\ell_{Pl} \approx 10^{-35}$ m. This deformation of the uncertainty principle in turn deforms the usual Heisenberg algebra to

$$[x^i, p_j] = i[\delta^i_j + \beta p^2 \delta^i_j + 2\beta p^i p_j]. \tag{2}$$

Correspondingly, the coordinate representation of the momentum operator is modified to the first order in β as

$$p_i = -i\partial_i(1 - \beta\partial^i\partial_i). \tag{3}$$

Second, in theories with Lifshitz scaling, space and time scale differently. Thus, we can write the scaling of space and time as

$$\begin{aligned} x &\to bx, \\ t &\to b^z t, \end{aligned} \tag{4}$$

where z is called the degree of anisotropy and b is called the scaling factor. In this paper, we shall consider $z = 2$. It may be noted that this transformation reduces to the usual conformal transformation for $z = 1$.

Now we will incorporate the generalized uncertainty principle into a theory with Lifshitz scaling. Such a deformed three dimensional Lifshitz bosonic action is given by [26]

$$S_b = \frac{1}{2}\int d^3x \left(\phi\partial^0\partial_0\phi - \kappa^2\partial^i\phi\mathcal{T}_\partial^2\partial_i\phi\right), \tag{5}$$

where the non-local fractional derivative operator $\mathcal{T}_\partial$ is given by

$$\begin{aligned} \mathcal{T}_\partial &= T_\partial(1 - \beta\partial^j\partial_j) \\ &= \sqrt{-\partial^i\partial_i}(1 - \beta\partial^j\partial_j). \end{aligned} \tag{6}$$

Such an incorporation breaks the Lifshitz scaling, as β does not scale with the space and time. However, it is possible to preserve the Lifshitz scaling by promoting the parameter β to a background field which scales as [26]

$$\beta \to b^2\beta. \tag{7}$$

It may be noted that the non-local differential operator used in the construction of the Lifshitz bosonic action based on the generalized uncertainty principle can be analyzed using the harmonic extension of functions from R^2 to $R^2 \times (0, \infty)$ [20–25]. In fact, it can be effectively viewed as a local differential operator by using this harmonic extension of functions. The operator T_∂ can be defined by its action on the functions $f : R^2 \to R$. In this case, its harmonic extension $u : R^2 \times (0, \infty) \to R$ satisfies $T_\partial f(x) = -\partial_y u(x, y)|_{y=0}$. Now let $u : R^2 \times (0, \infty) \to R$ be the harmonic extension of $f : R^2 \to R$, such that its restriction to R^2 coincides with $f : R^2 \to R$. Now the solution of the Dirichlet problem defined by $u(x, 0) = f(x)$ and $\partial^2 u(x, y) = 0$ can be used to find u, where ∂^2 is the Laplacian on R^3. There exists a unique harmonic extension $u \in C^\infty(R^2 \times (0, \infty))$ for a smooth function $C_0^\infty(R^2)$. Now we can write $T_\partial^2 f(x) = \partial_y^2 u(x, y)|_{y=0} = -\partial^i \partial_i u(x, y)|_{y=0}$, because $T_\partial f(x)$ also has a harmonic extension to $R^2 \times (0, \infty)$. Furthermore, it is possible to write $T_\partial = \sqrt{-\partial^i \partial_i}$, as $T_\partial^2 f(x) = -\partial^i \partial_i f(x)$. Thus, we obtain $T_\partial \exp ikx = |k| \exp ikx$, as $T_\partial^2 \exp ikx = |k|^2 \exp ikx$.

Now using this scalar product, we can write the bosonic action as

$$S_b = \frac{1}{2} \int d^3x \; i\partial^\mu \phi \; G_{\mu\nu} \partial^\nu \phi, \tag{8}$$

where $G_{\mu\nu}$ is a matrix. It is also possible to define a set of local gamma matrices such that they satisfy

$$\{\Gamma_\mu, \Gamma_\nu\} = 2G_{\mu\nu}. \tag{9}$$

It is possible to write a Lifshitz fermionic operator based on the generalized uncertainty principle as

$$\Gamma^\mu \partial_\mu = \gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i. \tag{10}$$

This is because if $\{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}$, then it is possible to write $\Gamma_0 = \gamma_0$ and $\Gamma_i = \kappa \mathcal{T}_\partial \gamma_i$. Furthermore, we can also write

$$\Gamma^\mu \partial_\mu \Gamma^\nu \partial_\nu = \partial^0 \partial_0 - \kappa^2 (\partial^i \partial_i (1 - \beta \partial^k \partial_k))^2. \tag{11}$$

We can write a Lifshitz fermionic action based on generalized uncertainty principle using three dimensional spinor fields, $\psi_a = \psi^b C_{ba}$, and $\psi^a = C^{ab} \psi_b$. Here we have $C_{ab} C^{cd} = \delta_a^c \delta_b^d - \delta_b^c \delta_a^d$. The square of these spinor fields is given by $\psi^2 = \psi^a \psi_a / 2$. Now the Lifshitz fermionic action based on generalized uncertainty principle can be written as

$$\begin{aligned} S_f &= \frac{1}{2} \int d^3x \; \psi^a (\Gamma^\mu \partial_\mu)_a^b \psi_b \\ &= \frac{1}{2} \int d^3x \; \psi^a (\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)_a^b \psi_b. \end{aligned} \tag{12}$$

We have the Lifshitz bosonic and Lifshitz fermionic theories based on the generalized uncertainty principle, so we can construct a free supersymmetric theory with $\mathcal{N} = 1$ supersymmetry using these actions. Thus, motivated by the definition of the generator of ordinary $\mathcal{N} = 1$ supersymmetry, we can write the generator of $\mathcal{N} = 1$ supersymmetry for a Lifshitz theory based on the generalized uncertainty as

$$Q_a = \partial_a - (\gamma^0 \partial_0 \theta + \gamma^i \kappa \mathcal{T}_\partial \partial_i \theta)_a. \tag{13}$$

Now let $u(x, y)$ be the harmonic extension of $f(x)$, and so $\partial_i u(x, y)$ will be the harmonic extension of $\partial_i f(x)$,

$$\begin{aligned} T_\partial \partial_i f(x) &= -\partial_y \partial_i u(x, y)|_{y=0} \\ &= -\partial_i u_y(x, y)|_{y=0}. \end{aligned} \tag{14}$$

Furthermore, we have $-\partial_i u_y(x, y)|_{y=0} = \partial_i T_\partial f(x)$ as $T_\partial f(x) = -u_y(x, 0)$. So, the operator T_∂ commutes with an ordinary derivative ∂_i,

$$T_\partial \partial_i f(x) = \partial_i T_\partial f(x). \tag{15}$$

Thus, we can now construct a super-derivative D_a which will commute with the generator of $\mathcal{N} = 1$ supersymmetry,

$$D_a = \partial_a - (\gamma^0 \partial_0 \theta - \gamma^i \kappa \mathcal{T}_\partial \partial_i \theta)_a. \tag{16}$$

Furthermore, we have the following non-local supersymmetric algebra:

$$\begin{aligned} &\{Q_a, Q_b\} = 2(\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab}, \\ &\{D_a, D_b\} = -2(\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab}, \\ &\{Q_a, D_b\} = 0. \end{aligned} \tag{17}$$

The states in this theory that are invariant under a symmetry are annihilated by generators of that symmetry. So, by taking the trace of $\langle E | \{Q_a, Q_b\} | E \rangle$, it is possible to demonstrate that the energy of the ground state vanishes even for this deformed supersymmetric theory. Furthermore, as the Lifshitz momentum deformed by the generalized uncertainty principle again commutes with the generators of the supersymmetry, there occurs a degeneracy in the mass of two states that are related to each other by these generators of supersymmetry.

However, because of the non-local differential operator in the definition of Q_a, these variations do not obey the Leibniz rule and so the differentiation of a product of superfields is not the same as the differential of each of those superfields. This problem can be evaded for free theories. This is because for free theories we can always shift one differential operator at a time from one field to another in the Lagrangian. Thus, in the case of free theories, even theories with Lifshitz scaling deformed by generalized uncertainty principle, we can still construct a non-local supersymmetric field theory using the superspace formalism. But as soon as the interactions are introduced, they will tend to break this supersymmetry. Now we will analyze some properties of the superspace which is suitable to construct free non-local supersymmetric theories. First, we have

$$D_a D_b = -C_{ab} D^2 - (\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab}. \tag{18}$$

Furthermore, the complete anti-symmetrization of three two-dimensional indices vanishes,

$$2D_a D_b D_c = D_a\{D_b, D_c\} + D_b\{D_a, D_c\} + D_c\{D_a, D_b\}. \tag{19}$$

So we can write $D^a D_b D_a = 0$, and $D^2 D_a = -D_a D^2$, where $D^2 D_a = (\gamma^0 \partial_0 D + \gamma^i \kappa \mathcal{T}_\partial \partial_i D)_a$. These properties will be used to study various non-local Lifshitz supersymmetric field theories based on the generalized uncertainty principle.

3 Supersymmetric field theory

In this section, we will analyze Lifshitz supersymmetric field theories based on the generalized uncertainty principle. We will write an action for a generalized uncertainty principle deformed Lifshitz theory in $\mathcal{N} = 1$ superspace formalism, so that it has manifest $\mathcal{N} = 1$ supersymmetry. In order to do that, we first expand a superfield Φ as $\Phi = \phi + \psi^a \theta_a - \theta^2 F$. Now we can write $\phi = [\Phi]_|$, $\psi_a = [D_a \Phi]_|$, $F = [D^2 \Phi]_|$, where $'|'$ means that at the end of the calculations we set $\theta_a = 0$. The non-local supersymmetric transformations generated by $\epsilon^a Q_a$ can be written as

$$\begin{aligned} \epsilon^a Q_a \phi &= -\epsilon^a \psi_a, \\ \epsilon^a Q_a \psi_a &= -\epsilon^b [C_{ab} F + (\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab} \phi], \\ \epsilon^a Q_a F &= -\epsilon^a (\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)^b_a \psi_b. \end{aligned} \tag{20}$$

We can write a free action for the deformed supersymmetic theory in $\mathcal{N} = 1$ superspace as

$$\begin{aligned} S_{\text{free}}[\Phi] &= \frac{1}{2} \int d^3x D^2 [\Phi D^2 \Phi]_| \\ &= \frac{1}{2} \int d^3x [D^2 \Phi D^2 \Phi + D^a \Phi D_a D^2 \Phi + \Phi (D^2)^2 \Phi]_| \\ &= \frac{1}{2} \int d^3x \Big[F^2 + \phi(\partial^0 \partial_0 - \kappa^2 (\partial^i \partial_i (1 - \beta \partial^j \partial_j))^2 \phi \\ &\quad + \psi^a (\gamma^0 \partial_0 + \gamma^i \kappa \mathcal{T}_\partial \partial_i)^b_a \psi_b \Big] \\ &= S_a + S_b + S_f, \end{aligned} \tag{21}$$

where S_b is the deformed bosonic action, S_f is the deformed fermionic action, and S_a is the deformed action for the auxiliary field F.

In this action, the supersymmetric variations of the temporal parts cancel out as in the ordinary supersymmetric field theories. Furthermore, the non-local supersymmetric variation of a part of the bosonic action generates $\epsilon^a \psi_a \kappa^2 (\partial^i \partial_i (1 - \beta \partial^j \partial_j))^2 \phi$, and this term exactly cancels with a term generated by the non-local supersymmetric variation of a part of fermionic action. The fermionic action contains a non-local part, $\epsilon^b (\gamma^j \kappa \mathcal{T}_\partial \partial_j)^a_b \phi . (\gamma^j \kappa \mathcal{T}_\partial \partial_j)^c_a \psi_c$. This does not directly cancel out with the non-local supersymmetric variation of the bosonic part. However, if we view the non-local operator in terms of harmonic extensions of functions, then this term can be written as $\epsilon^b \phi \kappa^2 (\partial^i \partial_i (1 - \beta \partial^j \partial_j))^2 \psi_b$. Here the derivatives only act on the fermionic part. Let $u_1(x, y)$ be the harmonic extension of $f_1(x)$ to $C = R^2 \times (0, \infty)$, and $u_2(x, y)$ be the harmonic extension of $f_2 : (x)$ to $C = R^2 \times (0, \infty)$. Now both of these harmonic extensions vanish for $|x| \to \infty$ and $|y| \to \infty$, and we can write [50]

$$\int_C u_1(x, y) \partial^2 u_2(x, y) \mathrm{d}x\mathrm{d}y - \int_C u_2(x, y) \partial^2 u_1(x, y) \mathrm{d}x\mathrm{d}y = 0. \tag{22}$$

Thus, we obtain

$$\int_{R^2} \big(u_1(x, y) \partial_y u_2(x, y) - u_2(x, y) \partial_y u_1(x, y) \big) \,|_{y=0} \,\mathrm{d}x = 0. \tag{23}$$

This can be expressed in terms of $f_1(x)$ and $f_2(x)$,

$$\int_{R^2} \big(f_1(x) \partial_y f_2(x) - f_2(x) \partial_y f_1(x) \big) \,\mathrm{d}x = 0. \tag{24}$$

Thus, $\mathcal{T}_\partial$ is moved from $f_2(x)$ to $f_1(x)$,

$$\int_{R^2} f_1(x) \mathcal{T}_\partial f_2(x) = \int_{R^2} f_2(x) \mathcal{T}_\partial f_1(x). \tag{25}$$

Now the non-local term generated by the non-local supersymmetric variation of the fermionic action can be expressed in terms of $\epsilon^a \phi \kappa^2 (\partial^i \partial_i (1 - \beta \partial^j \partial_j))^2 \psi_a$, and so it also cancels out with the non-local supersymmetric variation of the bososnic action. It may be noted that this can be done only formally by using the theory of harmonic extensions of functions from R^2 to $R^2 \times (0, \infty)$. Similarly, the remaining terms generated by a non-local supersymmetric variation of the fermionic part cancel with the terms generated by the non-local supersymmetric variation of the auxiliary field. This theory will have a generalized uncertainty principle associated with deformed Lifshitz scaling and $\mathcal{N} = 1$ supersymmetry, even after the mass term, $m D^2 [\Phi^2]_| /2 = m \psi^2 + m A F$, is added to its Lagrangian. It is possible to show that this mass term is also invariant under the non-local supersymmetric transformations. This is because the invariance of the temporal part is again similar to the usual non-local supersymmetric theories and the invariance of the remaining part can be demonstrated by using the theory of harmonic extensions of functions from R^2 to $R^2 \times (0, \infty)$, as in the previous case.

We can now use the standard method—the functional integral to quantize the supersymmetric Lifshitz free field theory deformed by the generalized uncertainty principle. If it was possible to have an extension to an interactive theory, we could also obtain the Feynman graphs using this method. However, it will be demonstrated that the interactions terms break the supersymmetry in those theories.

The generating functional integral for the free theory can be written as

$$Z_0[J] = \frac{D\Phi \exp i\, (S_{\text{free}}[\Phi] + J\Phi)}{D\Phi \exp i\, (S_{\text{free}}[\Phi])}, \tag{26}$$

where

$$J\Phi = \int d^3x D^2[J\Phi)]_|. \tag{27}$$

Thus, we obtain

$$Z[J] = \exp -i \int d^3x D^2[J(D^2+m)^{-1}J]_|. \tag{28}$$

Now the superfield propagator can be written as

$$\langle \Phi(p, \theta_1)\Phi(-p, \theta_2)\rangle = \frac{D^2 - m}{p^0 p_0 - \kappa^2 (p^i p_i (1 - \beta p^k p_k))^2 - m^2} \delta(\theta_1 - \theta_2). \tag{29}$$

It may be noted that if we add any interaction term this will break the supersymmetry of this theory. This is because even though for a free field theory the non-local derivative can be shifted from one field to the another by using harmonic extensions of functions from R^2 to $R^2 \times (0, \infty)$, the Leibniz rule does not hold in general. Thus, when we have interacting theories, the non-local supersymmetric variation of a product of more than two fields is not equal to the individual non-local supersymmetric variation of those fields. In fact, if we take a simple interaction of the form

$$S[\Phi] = S_{\text{free}}[\Phi] + S_{\text{int}}[\Phi], \tag{30}$$

where

$$\begin{aligned} S_{\text{int}}[\Phi] &= \frac{\lambda}{6} \int d^3 D^2[\Phi^3]_| \\ &= \frac{\lambda}{2} \int d^3 (\phi \psi^a \psi_a + \phi^2 F), \end{aligned} \tag{31}$$

then it is not invariant under the non-local supersymmetric variation generated by $\epsilon^a Q_a$. This is because in ordinary supersymmetric field theories we need to show that $\epsilon^a \psi^b (\gamma^\mu \partial_\mu)_{ab} \phi^2 = 2\epsilon^a \psi^b \phi (\gamma^\mu \partial_\mu)_{ab} \phi$, however, for the non-local part of this deformed theory, we have $\epsilon^a \psi^b (\gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab} \phi^2 \neq 2\epsilon^a \psi^b \phi (\gamma^i \kappa \mathcal{T}_\partial \partial_i)_{ab} \phi$. Thus, the non-local supersymmetric variation of the interaction terms cannot cancel out.

4 Conclusion

In this paper, we analyzed a supersymmetric theory deformed by generalized uncertainty principle and Lifshitz scaling. The action of this deformed theory contains non-local fractional derivatives. Thus, even the generators of supersymmetry contain non-local fractional derivative terms. However, these fractional derivative terms can effectively be treated as a local operator by using harmonic extensions of functions from R^2 to $R^2 \times (0, \infty)$. Furthermore, this non-local operator commutes with the local derivatives, and so we could construct a super-derivative which commutes with the generator of the supersymmetry. This super-derivative was used in the construction of various non-local supersymmetric field theories. A free matter theory deformed by the generalized uncertainty principle and Lifshitz scaling was constructed such that it was invariant under non-local supersymmetric transformations. It was argued that any free non-local supersymmetric theory will be invariant under non-local supersymmetric transformations. However, it was demonstrated that even a simple interaction term will break the supersymmetry of this theory.

The effect of the generalized uncertainty principle on AdS/CFT has already been analyzed [51]. The AdS/CFT correspondence relates the supergravity solutions on AdS to a superconformal field theory on its boundary [52–56]. It would be interesting to analyze the AdS/CFT correspondence for Lifshitz theories based on the generalized uncertainty principle. The holographic dual to the Lifshitz field theory has also been analyzed [57–60]. In these Lifshitz theories, the dependence of physical quantities such as the energy density on the momentum scale is evaluated using the renormalization group flow at finite temperature [61]. In fact, gravity with anisotropic scaling is obtained from the holographic renormalization of asymptotically Lifshitz spacetimes [62]. The holographic counter-terms induced near anisotropic infinity take the form of the action for gravity at a Lifshitz point. It has been observed that the $z = 2$ anisotropic Weyl anomaly in dual field theories, in three dimensions, can be obtained from the holographic renormalization of Horava–Lifshitz gravity [63]. In fact, Lifshitz theories have also become important because of the development of Horava–Lifshitz gravity [64–68]. Even though the addition of higher order curvature terms to the gravitational action makes it renormalizable, it spoils the unitarity of this theory. However, it is possible to add higher order spatial derivatives without adding any higher order temporal derivatives. Even though we have this breaking of Lorentz symmetry in the Horava–Lifshitz theory of gravity, general relativity is recovered in the infrared limit.

It may be noted that a system at finite temperature and finite chemical potential with a Lifshitz black hole in place of a Lifshitz geometry has been used for analyzing the fermionic retarded Green's function with $z = 2$ [69]. In fact, the Hawking radiation for Lifshitz fermions has also been studied [70]. It would be interesting to analyze the effect that the generalized uncertainty principle can have on such systems.

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