

Thermal fluctuations in a hyperscaling-violation background

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Abstract In this paper, we study the effect of thermal fluctuations on the thermodynamics of a black geometry with hyperscaling violation. These thermal fluctuations in the thermodynamics of this system are produced from quantum corrections of geometry describing this system. We discuss the stability of this system using specific heat and the entire Hessian matrix of the free energy. We will analyze the effects of thermal fluctuations on the stability of this system. We also analyze the effects of thermal fluctuations on the criticality of the hyperscaling-violation background.

1 Introduction

It is important to associate an entropy with black holes to prevent the violation of the second law of thermodynamics. This is because if black holes were not maximum entropy objects, then the entropy of the universe would spontaneously reduce, whenever an object with a finite entropy crossed the horizon. So, black holes are maximum entropy objects, and they have more entropy than any other object with the same volume [1–5]. The scaling of this maximum entropy with the area of the horizon has led to the development of the holographic principle [6, 7]. The holographic principle equates the degrees of freedom in any region of space with the degrees of freedom on the boundary of that region.

The holographic principle is expected to be corrected near Planck scale, as quantum gravity corrections modify the manifold structure of space-time at Planck scale [8, 9]. As the

holographic principle was motivated from the entropy–area relation, it can be argued that the quantum gravity corrections would also modify this entropy–area relation. Now for a black hole with area A and entropy S_0 , original entropy–area relation in natural units is given by $S_0 = A/4$. However, the corrected entropy–area relation can be written as $S = S_0 + \alpha \log A + \gamma_1 A^{-1} + \gamma_2 A^{-2} \dots$, where $\alpha, \gamma_1, \gamma_2 \dots$, are coefficients which depend on the details of the model. The general structure of the corrections and their dependence on the area is a universal feature, and it occurs in almost all approaches to quantum gravity. The corrections to the thermodynamics of black holes have been studied using non-perturbative quantum general relativity [10]. In this formalism, the conformal blocks of a well defined conformal field theory were used to study the behavior of the density of states of a black hole. The quantum corrections to the thermodynamics of a black hole has also been studied using the Cardy formula [11]. The corrected thermodynamics of a black hole has also been studied by analyzing the effect of matter fields surrounding a black hole [12–14]. Such corrections have the general feature that they are represented by a logarithmic function of area.

As string theory is one of the most important approaches to quantum gravity, it is very important to understand the effects of quantum corrections produced by string theoretical effects. In fact, the corrections produced by string theoretical effects on the thermodynamics of a black hole have been studied, and it has been observed that they produce the same general form of the corrections as are produced by other approaches to quantum gravity [15–18]. The corrections to the thermodynamics of a dilatonic black hole have also been studied, and they have again been observed to have the same universal form [19]. The partition function of a black hole has also been used to analyze the corrections to the thermodynamics of a black hole [20]. Another universal feature of almost all theories of quantum gravity is the existence of a minimum

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measurable length scale, and this has motivated the generalized uncertainty principle, which in turn has been used to study corrections to the thermodynamics of the black holes [21,22]. It has been demonstrated that the black hole thermodynamics corrected by the generalized uncertainty principle again has the same universal form as produced by other approaches to quantum gravity.

In fact, the universality of this correction can be argued from purely thermodynamics arguments. This is because in the Jacobson formalism, the Einstein equations are thermodynamics identities [23,24]. So, space-time geometry is an emergent property from the thermodynamics. Thus, a thermal fluctuation in thermodynamics would produce a quantum fluctuation in the geometry of space-time. In fact, it has been demonstrated that the thermal fluctuations correct the thermodynamics of black holes, and this corrected thermodynamics has the same universal form as expected from the quantum gravitational effects [25–27]. As the coefficients of quantum gravity corrections depend on the details of the model, we will keep such a coefficient as a constant. Thus, we will analyze the corrections produced by thermal fluctuations on the near horizon geometry of a hyperscaling-violating background, with variable coefficients.

It may be noted that such thermal corrections have been studied for various different black geometries. Such corrections have been studied for Gödel black holes [28]. Such corrections for an AdS charged black hole have been studied, and it has been observed that the thermodynamics of this AdS black hole is modified by thermal fluctuations [29]. The effect of thermal fluctuations on the thermodynamics for a black Saturn have also been studied [30]. It has been demonstrated that the thermal fluctuations do not have any major effect on the stability of the black Saturn. The thermal fluctuations for a modified Hayward black hole have been studied, and it has been demonstrated that such thermal fluctuations reduce the pressure and internal energy of such a black hole [31]. The effect of thermal fluctuations on the thermodynamics of a charged dilatonic black Saturn has also been studied [32]. It was observed that the thermal fluctuations can be studied either using a conformal field theory or by analyzing the fluctuations in the energy of this system. However, it has been demonstrated that the fluctuations in the energy and the conformal field theory produce the same results for a charged dilatonic black Saturn. The effects of thermal fluctuations on the thermodynamics of a small singly spinning Kerr–AdS black hole have also been studied [33]. As dumb holes are analogs of black hole like solutions, it is possible to study such effects for dumb holes. In fact, the effects of thermal fluctuations on the thermodynamics of dumb holes has been studied [34]. The thermal fluctuations can affect the critical behaviors of black holes. Such corrections to a dyonic charged anti-de Sitter black hole have been studied, and it has been observed that

such a corrected solution also describes a van der Waals fluid [35]. It is also expected that such corrections to the solutions in AdS can be used to study the modifications to the quark–gluon plasma, and this can be done using the AdS/CFT correspondence [36–39]. In fact, it has been demonstrated that such corrections can produce interesting modifications to the ratio between the viscosity to entropy of such a system [40].

Now, as this form of the corrections to the thermodynamics is universal, we will analyze the effects of such corrections on the thermodynamics of a near horizon geometry with hyperscaling violation. Hyperscaling-violating backgrounds are interesting geometries, and have been used to study interesting physical systems [41–43]. The possible boundary conditions of scalar fields in a hyperscaling-violation geometry have been studied [44]. Such backgrounds are also interesting for holographic models [45], such as holographic superconductor [46]. Singularities in hyperscaling-violating spacetimes have also been investigated [47]. The analytic solution of a Vaidya-charged black hole with a hyperscaling-violating factor (in an Einstein–Maxwell-dilaton model [48]) has been obtained [49]. The hyperscaling-violating solutions in Einstein–Maxwell-scalar theory have also been studied [50–52]. Thermalization of mutual information in hyperscaling-violating backgrounds has been analyzed, and it has been observed that the dynamical exponent is important for understanding the mutual information in such a system [53]. Entanglement temperature [54] and entanglement entropy [55] with hyperscaling-violating backgrounds have also been investigated.

The black brane geometry with hyperscaling-violating backgrounds can also be constructed, and it is possible to study the thermodynamics of such a system. In fact, the thermodynamics of nonlinear charged Lifshitz black branes with hyperscaling violation has been discussed [56]. In this system, the effect of a nonlinear electromagnetic field on the hyperscaling-violating Lifshitz black branes has been studied. It may be noted that as the hyperscaling-violating geometries have been used for analyzing various different physical systems [42,57–59], it would be interesting to analyze the effects of quantum corrections on such a hyperscaling-violating geometry. In this paper, we will analyze the corrections produced by the thermal fluctuations on the near horizon geometry of a black brane [60]. The Null–Melvin–Twist and KK reduction has been used to analyze a hyperscaling-violating geometry constructed from a black brane [61,62]. This is an interesting geometry, and it is both important and interesting to analyze the effects of thermal fluctuations on such a geometry. Thus, we will analyze the effects of thermal fluctuations on the thermodynamics of this geometry. It will be observed that such correction terms can have very interesting effects on this system.

2 Hyperscaling-violating background

In this paper, we will analyze the corrections to the thermodynamics of a near horizon geometry of a black brane with hyperscaling violation. Such a geometry is described by the following metric [62]:

$$ds_{d+2}^2 = \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{2\theta/d} \times \left(-\left(\frac{R^2}{r}\right)^{-2(z-1)} f dt^2 + dy^2 + dx_i^2\right) + \left(\frac{R}{r}\right)^2 \left(\frac{r_F}{r}\right)^{2\theta/d} \left(\frac{dr^2}{f} + R^2 d\Omega_{d+2}^2\right), \tag{1}$$

where R is the AdS scale, $x_i = (x_1, x_2)$, r_F is a scale describing this system [62], and

$$f = 1 - \left(\frac{r_h}{r}\right)^{d+z-\theta}, \tag{2}$$

with $r = r_h$ as the radius of the horizon. Now we can study the finite temperature effects of hyperscaling violation using the condition $r_F < r_h$. It may be noted that this analysis can be simplified by setting $z = 1$, and this corresponds to an asymptotically AdS space-time [61]. The Null–Melvin–Twist and KK reduction can now be used to obtain the following metric:

$$ds_{d+2}^2 = K^{-2/3} \left(\frac{r}{R}\right)^2 M \left[-(1 + b^2 r^2 M^2) f dt^2 - 2b^2 r^2 f M^2 dt dy + (1 - b^2 r^2 f M^2) dy^2 + K dx_i^2\right] + K^{1/3} M \left(\frac{R}{r}\right)^2 f^{-1} dr^2, \tag{3}$$

$$\phi = -\frac{1}{2} \ln K,$$

$$A = \frac{M^2}{K} \left(\frac{r}{R}\right)^2 b(f dt + dy),$$

where A is a one-form field, ϕ is the dilaton, $K = 1 - (f - 1)br^2 M^2$, b is a free parameter (inverse of length), and

$$M = \left(\frac{r_F}{r}\right)^{(2\theta/d)}. \tag{4}$$

This can be simplified using the following light-cone coordinates:

$$x^+ = bR(t + y),$$

$$x^- = \frac{1}{2bR}(t - y). \tag{5}$$

Thus, we can obtain the following scaled extremal metric:

$$ds_{d+2}^2 = \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} \left[\left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{(4\theta/d)} H_B^2 dx^{+2} - 2iH_B dx^+ dx^- + G_B^2 dx_i^2\right] + \left(\frac{R}{r}\right)^2 \left(\frac{r_F}{r}\right)^{(2\theta/d)} dr^2, \tag{6}$$

where $G_B = K(r_B)^{1/6}$, and

$$H_B = \left[K(r_B)^{-2/3} \left(\frac{f(r_B) - 1}{(2bR)^2} + \left(\frac{r_B}{R}\right)^2 \left(\frac{r_F}{r_B}\right)^{(4\theta/d)} f(r_B)\right) \right]^{1/2} \times \left(\frac{r_B}{R}\right)^{-1} \left(\frac{r_F}{r_B}\right)^{(-2\theta/d)}. \tag{7}$$

In order to write the metric (6), we analytically continue x^+ to ix^+ , and we assume that the system is inside a box by using the cut-off $r = r_B$. Here, the finite cut-off r_B is larger than the scale R . The metric (6) can be obtained from the equations of motion that follow from the action [63],

$$S = \frac{1}{16\pi G_{d+2}} \int dx^{d+2} \sqrt{-g} \times \left[\mathcal{R} - \frac{4}{3} \partial_\mu \phi \partial^\mu \phi - \frac{1}{4} R^2 e^{-8\pi/3} F_{\mu\nu} F^{\mu\nu} - 4A_\mu A^\mu - 4e^{2\pi/3} \frac{(e^{2\pi} - 4)}{R^2} \right], \tag{8}$$

where G_{d+2} , $cal R$ and $F_{\mu\nu}$ are the $(d+2)$ dimensional Newton constant, the scalar curvature and field-strength tensor, respectively.

3 Corrected thermodynamics

In this section, we will analyze the effects of thermal fluctuations on the thermodynamics of such a system. So, first we will obtain the thermodynamic quantities like the entropy, the free energy and the temperature for such a geometry [61]. The Hawking temperature of this geometry can be expressed as

$$T_H = \frac{d + 1 - \theta}{4} \frac{r_h}{\pi b R^3}, \tag{9}$$

and the local temperature for this geometry is given by

$$T = \frac{T_H}{\sqrt{f}}, \tag{10}$$

where f given by Eq. (2). The entropy of this geometry is given by

$$S_0 = \frac{4\pi b(d - \theta)(4d - 3\theta)}{d^2 R^2 (d + 1 - \theta)} r_h^{\frac{3(d-\theta)}{d}}. \tag{11}$$

Here the original entropy is denoted by S_0 , and the entropy corrected by thermal fluctuations will be denoted by S . Since the above metric is stationary but not static and describes the black brane rotating in the compactified x^- direction, it is natural to interpret the angular momentum and velocity as the charge and the conjugate chemical potential of the

system, respectively. So, the constant chemical potential for this system can be written as [61]

$$\mu = \frac{1}{2b^2R^2}. \tag{12}$$

The Helmholtz free energy of this system can be written as

$$F = E - TS, \tag{13}$$

where E is the internal energy of this system.

These thermodynamic quantities are calculated by neglecting the effects of thermal fluctuations on the thermodynamics of this system. This is valid as long as the temperature is sufficiently small. However, as the temperature of the black geometry increases, due to the decrease in the radius of the black geometry, we have to also consider the effects of thermal fluctuations. If the thermal fluctuations are still small enough, then they can be analyzed as a perturbation around the equilibrium state. These thermal fluctuations for a black geometry can be expressed as [25–27]

$$S = S_0 - \alpha \ln(S_0 T^2) + \frac{\gamma_1}{S_0} + \frac{\gamma_2}{S_0^2} + \dots, \tag{14}$$

where $\alpha, \gamma_1, \gamma_2, \dots$, are correction parameters which depend on the details of the model being studied. Using (11) and (10), the explicit form of S can be written as

$$\begin{aligned} S = & \frac{4\pi b(d-\theta)(4d-3\theta)}{d^2R^2(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} \\ & - \alpha \log \left[\frac{4\pi b(d-\theta)(4d-3\theta)}{d^2R^2(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} \right] \\ & - 2\alpha \log \left[\frac{(d+1-\theta)r_h}{4\pi bR^3} \frac{1}{\sqrt{1-\left(\frac{r_h}{r}\right)^{d+z-\theta}}} \right] \\ & + \gamma_1 \frac{d^2R^2(d+1-\theta)}{4\pi b(d-\theta)(4d-3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\ & + \gamma_2 \frac{d^4R^4(d+1-\theta)^2}{16\pi^2b^2(d-\theta)^2(4d-3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}}. \end{aligned} \tag{15}$$

As the values of these coefficients depend on the specifics of the model, we will keep these values as a variable in this paper. We will now neglect higher order corrections ($\gamma_3 = 0$) to this system. So, the corrected internal energy, which can be calculated from the definition $E = \int T dS$, is given by

$$\begin{aligned} E = & \frac{(d+1-\theta)}{4\pi b} \frac{r_h}{\sqrt{1-\left(\frac{r_h}{r}\right)^{d+z-\theta}}} \left[\frac{8\pi b(d-\theta)(4d-3\theta)}{5d^3R^5(d+1-\theta)} \right. \\ & r_h^{\frac{3(d-\theta)}{d}} - \frac{8\alpha}{3R^3} - \gamma_1 \frac{d^2(d+1-\theta)}{2\pi bR(d-\theta)(4d-3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\ & \left. + \gamma_2 \frac{d^4R(d+1-\theta)^2}{4\pi^2b^2(d-\theta)^2(4d-3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}} \right]. \end{aligned} \tag{16}$$

The corrected Helmholtz free energy is calculated by

$$\begin{aligned} F = & \frac{(d+1-\theta)}{4\pi b} \frac{r_h}{\sqrt{1-\left(\frac{r_h}{r}\right)^{d+z-\theta}}} \\ & \times \left[\frac{8\pi b(d-\theta)(4d-3\theta)}{5d^3R^5(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} - \frac{8\alpha}{3R^3} \right. \\ & - \gamma_1 \frac{d^2(d+1-\theta)}{2\pi bR(d-\theta)(4d-3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\ & + \gamma_2 \frac{d^4R(d+1-\theta)^2}{4\pi^2b^2(d-\theta)^2(4d-3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}} \\ & \left. - \frac{4\pi b(d-\theta)(4d-3\theta)}{d^2R^2(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} \right. \\ & + \alpha \log \left[\frac{4\pi b(d-\theta)(4d-3\theta)}{d^2R^2(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} \right] \\ & + 2\alpha \log \left[\frac{(d+1-\theta)r_h}{4\pi bR^3} \frac{1}{\sqrt{1-\left(\frac{r_h}{r}\right)^{d+z-\theta}}} \right] \\ & - \gamma_1 \frac{d^2R^2(d+1-\theta)}{4\pi b(d-\theta)(4d-3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\ & \left. - \gamma_2 \frac{d^4R^4(d+1-\theta)^2}{16\pi^2b^2(d-\theta)^2(4d-3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}} \right]. \end{aligned} \tag{17}$$

In the plots given in Fig. 1, we plot the internal energy in terms of horizon radius for two cases, i.e., $\theta < d$ and $\theta > d$. We analyze such a geometry with $d = 2, 3, 8, 9$. Now, for $\theta < d$, the behavior of the system does not depend on the dimension, and the system in all these different dimensions has the same behavior. However, for $\theta > d$, there are two phases of the system, i.e., $d = 2, 3$ (middle plot of Fig. 1), and $d = 8, 9$ (right plot of Fig. 1). The Helmholtz free energy of this system also has a similar behavior.

4 Stability

In this section, we will analyze the stability of this system using both specific heat and the entire Hessian matrix of the free energy. Now, the corrected specific heat can be calculated using the corrected entropy of this system,

$$C = T \left(\frac{dS}{dT} \right). \tag{18}$$

This is given by

$$\begin{aligned} C = & \frac{1-\left(\frac{r_h}{r}\right)^{d+z-\theta}}{1+\frac{(d+z-\theta-2)}{2}\left(\frac{r_h}{r}\right)^{d+z-\theta}} \\ & \times \left[\frac{12\pi b(d-\theta)^2(4d-3\theta)}{d^3R^2(d+1-\theta)} r_h^{\frac{3(d-\theta)}{d}} - \alpha \left(\frac{5d-3\theta}{d} \right) \right] \end{aligned}$$

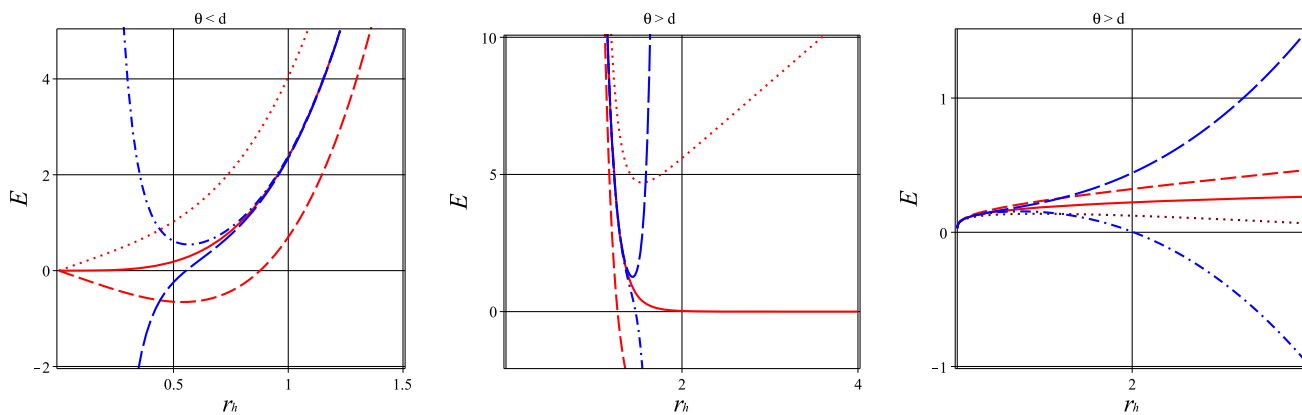


Fig. 1 Internal energy in terms of r_h . We set unit values for all parameters in this model. *Left* $\theta < d$ for $\theta = 1, d = 9$. *Middle* $\theta > d$ for $\theta = 10, d = 2$. *Right* $\theta > d$ for $\theta = 10, d = 8$. $\alpha = 0.5, \gamma_1 = 0$ (red dash), $\alpha = -0.5, \gamma_1 = 0$ (red dot), $\alpha = 0, \gamma_1 = 0$ (red solid), $\alpha = 0, \gamma_1 = 0.5$ (blue dash dot), $\alpha = 0, \gamma_1 = -0.5$ (blue long dash)

dash, $\alpha = -0.5, \gamma_1 = 0$ (red dot), $\alpha = 0, \gamma_1 = 0$ (red solid), $\alpha = 0, \gamma_1 = 0.5$ (blue dash dot), $\alpha = 0, \gamma_1 = -0.5$ (blue long dash)

$$\begin{aligned}
 & -\alpha \left[\frac{(d+z-\theta)}{1 - \left(\frac{r_h}{r}\right)^{d+z-\theta}} \right] \left(\frac{r_h}{r}\right)^{d+z-\theta} \\
 & -3\gamma_1 \frac{dR^2(d+1-\theta)}{4\pi b(4d-3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\
 & -3\gamma_2 \frac{d^4 R^4 (d+1-\theta)^2}{8\pi^2 b^2 \theta (d-\theta)(4d-3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}} \Big]. \tag{19}
 \end{aligned}$$

We can analyze the behavior of the specific heat for this system using numerical techniques. We observe that, for $\theta = d/2$, a thermodynamically stable phase exists for this system [61]. However, there are phases in this system where the specific heat is negative. For such phases, it is possible to use the corrected entropy to obtain a positive specific heat. The case of $\theta > d$ is very interesting. In this case, the specific heat can become negative when ($d = 8, \theta = 10$). In Fig. 2, we can see the behavior of the specific heat. A solid red line of Fig. 2 shows that the specific heat is completely negative for $d = 8$ and $\theta = 10$. It may be noted that $\alpha = 0.5$ produces a negative specific heat, while a negative value of α produces a stable region. Similar positive regions can be obtained by using a positive value of γ_1 (also for positive γ_2). We find that an infinitesimal positive value of α also produces a negative specific heat. Without logarithmic corrections, we can always obtain positive specific heat with infinitesimal γ_1 . However, we can still obtain instability for the small black geometries (small r_h). Hence, as we can see from the dash dotted line of Fig. 2, corresponding to $\gamma_1 > 0$ and $\alpha = 0$, there is an asymptotic behavior for the specific heat which shows an unstable/stable black hole phase transition. This is different for phase transitions in hyperscaling-violating geometries [58].

Due to the presence of the chemical potential, we can analyze this system using the matrix of second derivatives of free energy with respect to temperature T and chemical potential μ , which is given by

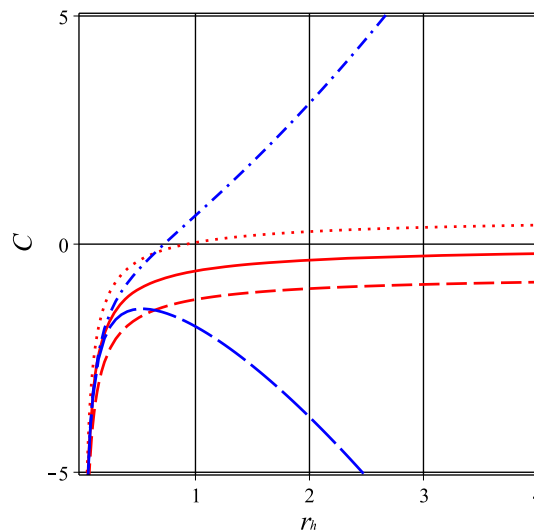


Fig. 2 Specific heat in terms of r_h . We set unit value for all model parameters except $\theta = 10, d = 8$. $\alpha = 0.5, \gamma_1 = 0$ (red dash), $\alpha = -0.5, \gamma_1 = 0$ (red dot), $\alpha = 0, \gamma_1 = 0$ (red solid), $\alpha = 0, \gamma_1 = 0.5$ (blue dash dot), $\alpha = 0, \gamma_1 = -0.5$ (blue long dash)

$$\begin{aligned}
 H_{11} &= \frac{\partial^2 F}{\partial T^2}, & H_{12} &= \frac{\partial^2 F}{\partial T \partial \mu}, \\
 H_{21} &= \frac{\partial^2 F}{\partial \mu \partial T}, & H_{22} &= \frac{\partial^2 F}{\partial \mu^2}. \tag{20}
 \end{aligned}$$

Now $H_{11}H_{22} - H_{12}H_{21} = 0$ implies that one of the eigenvalues is zero, and so we should have to use the other. This can be expressed as the trace of the matrix,

$$Tr(H) = H_{11} + H_{22}. \tag{21}$$

In Fig. 3, we plot the variation of the Hessian trace with respect to the horizon. Now we have a negative region for $\theta < d$ (specially $\theta = 1$, and $d = 8$). By using the higher order corrections, we can now obtain positive regions. Thus,

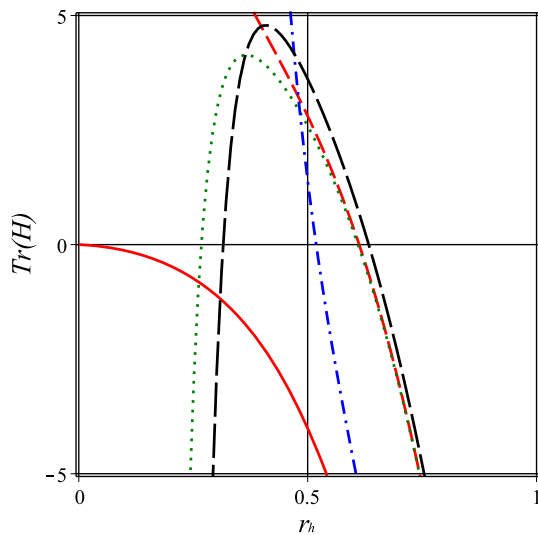


Fig. 3 Hessian trace in terms of r_h . We set unit values for all parameters in the model except $\theta = 1, d = 8. \alpha = 0.5, \gamma_1 = 0$ (red dash), $\alpha = 0.5, \gamma_1 = -0.02$ (green dot), $\alpha = 0, \gamma_1 = 0$ (red solid), $\alpha = 0, \gamma_1 = 0.5$ (blue dash dot), $\alpha = 0.5, \gamma_1 = -0.1, \gamma_2 = 0.5$ (black long dash)

we can consider the positive α , negative γ_1 , and positive γ_2 , in this system.

5 PV-criticality

It is possible to study PV-criticality for a system using the extended phase space [64]. In this formalism, it is possible to define a thermodynamic volume and pressure for a geometry [65,66]. Then this thermodynamic volume and pressure can be used to study the critical phenomena for such a geometry. Thus, using the extended phase space thermodynamic, the volume for a hyperscaling-violation background can be written as

$$V = A r_h = \frac{\pi b(4d - 3\theta)(d - \theta)r_h^{4 - \frac{3\theta}{d}}}{d^2 R^2(d - \theta + 1)}. \tag{22}$$

The pressure for such a background can be calculated using [67]

$$P = \frac{T}{v}, \tag{23}$$

where $v = 2\sqrt{V/\pi}$. So, we can write the pressure for a hyperscaling-violation background as

$$P_0 = \frac{(d - \theta + 1)}{8\pi b R^3} \sqrt{\frac{b(4d^2 - 7d\theta + 3\theta^2)r_h^{4 - \frac{3\theta}{d}}}{d^2 R^2(d - \theta + 1)}} \frac{r_h}{\sqrt{1 - \left(\frac{r_h}{r}\right)^{d+z-\theta}}}. \tag{24}$$

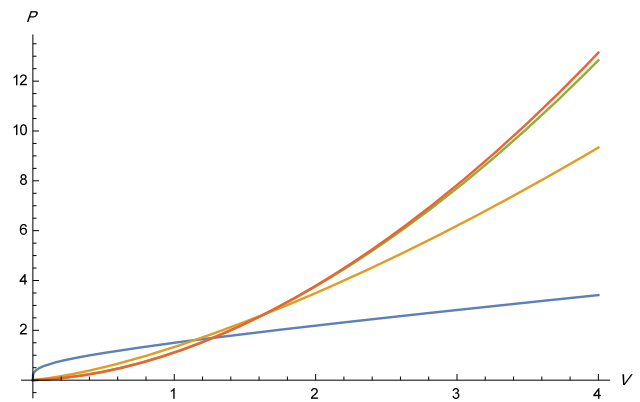


Fig. 4 P-V diagram for the hyperscaling-violating geometry for $d = 2, 3, 8$ and 9 , showing different behaviors for $d = 2$ from the other dimensions. We set $\alpha = 0.5$ and $\gamma_1 = \gamma_2 = 0$

Figure 4 shows a plot of the PV-diagram for the hyperscaling-violating black geometry (for fixed R, B , and for different dimensions). Now, we are able to calculate the enthalpy for the system as

$$\begin{aligned} \mathcal{H} = & \frac{(d + 1 - \theta)}{4\pi b} \frac{r_h}{\sqrt{1 - \left(\frac{r_h}{r}\right)^{d+z-\theta}}} \\ & \times \left[\frac{8\pi b(d - \theta)(4d - 3\theta)}{5d^3 R^5(d + 1 - \theta)} r_h^{\frac{3(d-\theta)}{d}} - \frac{8\alpha}{3R^3} \right. \\ & - \gamma_1 \frac{d^2(d + 1 - \theta)}{2\pi b R(d - \theta)(4d - 3\theta)} r_h^{-\frac{3(d-\theta)}{d}} \\ & \left. + \gamma_2 \frac{d^4 R(d + 1 - \theta)^2}{4\pi^2 b^2(d - \theta)^2(4d - 3\theta)^2} r_h^{-\frac{6(d-\theta)}{d}} \right] \\ & + \frac{1}{8R^4 d} \sqrt{\frac{(d + 1 - \theta)(4d - 3\theta)(d - \theta)}{1 - \left(\frac{r_h}{r}\right)^{d+z-\theta}}} r_h^{6 - \frac{3d}{\theta}}. \tag{25} \end{aligned}$$

We may use this PV relation to study the criticality. This can be done by using the Gibbs free energy $G(T) = E + PV - TS$. The Gibbs free energy for the hyperscaling-violating black geometry in the extended phase space can be written as

$$\begin{aligned} G_0 = & \frac{(d - \theta)(4d - 3\theta)r_h^{5 - \frac{3\theta}{d}}}{8d^2 R^5} \sqrt{\frac{b(4d^2 - 7d\theta + 3\theta^2)r_h^{4 - \frac{3\theta}{d}}}{d^2 R^2(d - \theta + 1)}} \\ & - \frac{(4d - 3\theta)(d - \theta)r_h^{\frac{3(d-\theta)}{d} + 1}}{d^2 R^5} + r_h. \tag{26} \end{aligned}$$

It is expected that the thermal fluctuations will correct this expression for the Gibbs free energy, and it is possible analyze the effects of thermal fluctuations on criticality using this corrected Gibbs free energy. In order to see the effects of the correction terms, we set $\alpha = 0.5$ and $\gamma_1 = \gamma_2 = 0$. The Gibbs free energy corrected by thermal fluctuations can be written as

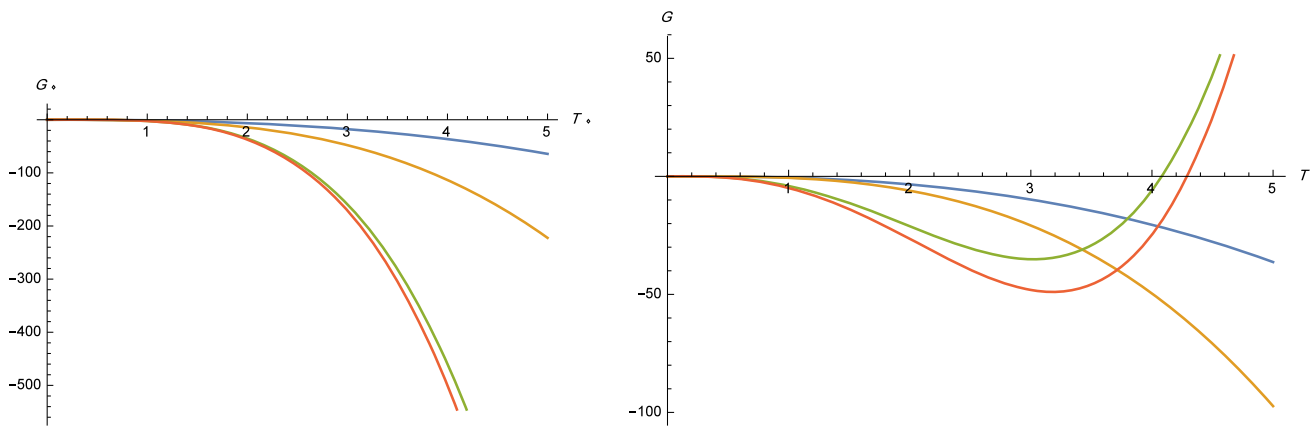


Fig. 5 Gibbs free energy as a function of temperature, for different space dimensions $d = 2$ blue, $d = 3$ yellow, $d = 8$ green and $d = 9$ red. On the left, we have the Gibbs free energy calculated without thermal fluctuations. On the right, thermal fluctuations are considered ($\alpha = 0.5, \gamma_1 = \gamma_2 = 0$)

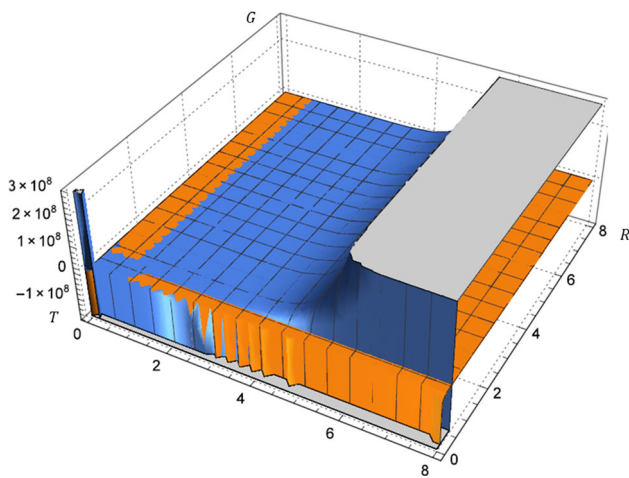


Fig. 6 Gibbs free energy $G(T, R)$, for $d = 3$ in orange and $d = 9$ in blue. Showing criticality for black hyperscaling-violating geometries for $d = 9$, while it there is no criticality for the $d = 3$ case ($\alpha = 0.5, \gamma_1 = \gamma_2 = 0$)

$$G = \frac{(d - \theta)(4d - 3\theta)r_h^{5 - \frac{3\theta}{d}}}{8d^2 R^5 \sqrt{\frac{b(4d^2 - 7d\theta + 3\theta^2)r_h^{4 - \frac{3\theta}{d}}}{d^2 R^2(d - \theta + 1)}}} - \frac{(4d - 3\theta)(d - \theta)r_h^{\frac{3(d - \theta)}{d} + 1}}{d^2 R^5} + dr_h^{\frac{3(d - \theta)}{d} + 1}. \tag{27}$$

It may be noted from Fig. 5 that there is no criticality in any dimension and at any temperature, if thermal fluctuations are neglected. In Fig. 6, we consider a logarithmic correction, and we neglect higher order corrections. We analyze the Gibbs free energy as a function of T and R , for $d = 3$ and $d = 9$. We can see criticality for black hyperscaling-violating geometries for ($d = 9$), while there is no criticality for the $d = 3$. So, the Gibbs free energy also indicates that the criticality of this system does not change in two and three

dimensions due to thermal fluctuations. However, for higher dimensions there is a phase transformation at high temperature. Thus, the thermal fluctuations can change the critical phenomena in such geometries.

6 Conclusion

The hyperscaling-violating backgrounds are interesting subjects of many recent studies [68–71]. In this paper, we have studied the thermodynamics of a black geometry with hyperscaling violation. As this geometry will be corrected due to quantum gravitational effects, we expected corrections to the thermodynamics of such black geometries. This is because quantum fluctuations in this geometry will produce thermal fluctuations in the thermodynamics of this system. As the form of these corrections is universal, we have used this universal form to analyze its effects on such a black geometry. We have discussed the stability of this system. This was done by using the specific heat of this system and the entire Hessian matrix of the free energy for this system. It was demonstrated that the thermal fluctuations of this system can modify the behavior of this system, and this can also affect the stability of this system. Finally, we used Gibbs free energy to analyze criticality for this system.

It may be noted that the thermal fluctuations to the thermodynamics of various geometries has been recently studied [29–32]. However, most of the work on the corrected thermodynamics has been done using the logarithmic correction term. It is well known that the thermodynamics of a black geometry will get corrected from higher order correction terms [27]. It would be interesting to investigate such correction terms for these geometries. It would be possible that such correction terms will modify the stability of such systems. The stability of such system can be studied by using

the specific heat. It is also possible to analyze the stability of such a system using the entire Hessian matrix of the free energy. It would thus be interesting to perform this study for these geometries with higher order thermal fluctuations.

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