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On new bulk singularity structures, RR couplings in the asymmetric picture and their all order α' corrections

Ehsan Hatefi[1,2,a]

[1] Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, UK
[2] Institute for Theoretical Physics, TU Wien, Wiedner Hauptstrasse 8-10/136, 1040 Vienna, Austria

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Abstract We have analyzed in detail four and five point functions of the string theory amplitudes, including a closed string Ramond–Ramond (RR) in an asymmetric picture and either two or three transverse scalar fields in both IIA and IIB. The complete forms of these S-matrices are derived and these asymmetric S-matrices are also compared with their own symmetric results. This leads us to explore two different kinds of bulk singularity structures as well as various new couplings in the asymmetric picture of the amplitude in type II string theory. All order α' higher derivative corrections to these new couplings have been discovered as well. Several remarks for these two new bulk singularity structures and for contact interactions of the S-matrix have also been made.

1 Introduction

By now it is widely known that D-branes in superstring theory play the most important role in this area of research. Indeed there is no doubt they continue to have more contributions even to other topics of high energy physics as well [1–4].[1]

Although we have referred in [7] to various fascinating papers about the subject of string theory's effective actions, for the entire self-completeness, we point out some of the remarks that are of high importance to the author as follows.

To the best of our knowledge Myers [8] discovered more or less the complete form of a single bosonic action that can be generalized for diverse D_p-brane systems. Not really consequently but after a while we started finding out the generalization of that action with emphasis on exploring all order α' corrections to D-brane effective actions. This involves dealing with both Chern–Simons, DBI effective actions and mixed open-RR S-matrices. Besides those things, some of the new couplings and/or Myers terms have been derived in [9]. It is also worth highlighting the fact that applications to some of the new Myers terms have already been released in the literature.

For instance we have shown in [10] that the so-called N^3 entropy behavior of near extremal M5 branes is reconstructed from super-Yang–Mills part with the particular role of Myers terms. In [10] it was seen that the presence of closed string interaction terms (Myers terms) is necessary to reproduce that N^3 entropy growth, basically we had regenerated the leading N^3 entropy behavior from D0 quantum mechanics with Myers terms. The leading growth just came from the classical contribution. In this paper we also revealed that Myers terms come from just the closed string coupling to some lower dimensional branes which lie inside of the branes that one started with. These lower dimensional branes could be thought of just soliton solutions of the branes that one dealt with. Hence these Myers terms that can be found by world-sheet computations are important to actually explore N^3 entropy growth production and potentially might play role in M-theory [11,12] as well.

We have also introduced a new sort of Kaluza–Klein reduction method based on ADM decomposition. In particular in [13] we discussed how the world volume theory appears from the supergravity side, where we applied the scheme to IIB supergravity that is reduced on a 5D hyperboloidal space, and clarified how one can gain either AdS or dS brane world solutions by making further reduction to 4D.

Various remarks for D-brane anti D-brane system [14,15] involving their corrections [16] have been given. In addition to the efforts in [17], the entire form of the supersymmetric Myers action has not been concluded yet. On the other hand,

[1] Both supersymmetric and non-supersymmetric branes must be regarded as $p+1$ world volume dimensions in a flat empty space background, for which two different kinds of boundary conditions should have been taken into account [5,6].

[a] e-mails: e.hatefi@qmul.ac.uk; ehsan.hatefi@tuwien.ac.at; ehsan.hatefi@cern.ch; ehsanhatefi@gmail.com

the action for a single brane was understood in [18] where its generalization could be found in [19–22].

We invite the reader for the complete review of Chern–Simons effective actions and their remarks to have a look at [23–25]. One can find a very brief review of all DBI and new Wess–Zumino terms of BPS branes in [26].

To observe all the three ways of exploring the couplings in Effective Field Theory (EFT) (which are either Taylor or pull-back, Myers terms) for both supersymmetric and non-supersymmetric cases we suggest [27].

Behind AdS/CFT there is a close relation between an open and a closed string so one might be interested in gaining the mixed open–closed string amplitudes, which of particular interest to them in string theory is indeed the mixed RR potential (C-field)-open strings. In fact one may hope to address various issues (involving the AdS/CFT) by dealing with these mixed higher point functions of RR-scalar fields of type II. Let us just consider some of the work that is in correspondence either with the S-matrix formalism in the presence of D_p-branes or is related to D-brane physics applications [28–40].

The paper is organized as follows. In Sect. 2, we talk about the conventions and then try to provide the complete calculations for the four and five point functions of type IIA,IIB of string theory including an RR in the asymmetric picture and either two or three real transverse scalar fields. Indeed for $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0}\rangle$ S-matrix, we modify the so-called Wick-like rule to have the gauge invariance, particularly we show that in the asymmetric picture of RR one finds a new term in the amplitude and hence new couplings in an EFT can be constructed. In order to be able to produce all string contact interactions in an EFT, one needs to employ mixing couplings where the first scalar comes from Taylor expansion and the second one comes from the pull-back method. We then find the all order α' higher derivative correction to those couplings as well.

In the next sections we perform the entire analysis of a five point function of a C-field with three transverse scalar fields in zero picture, that is, we deal with $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0} V_{\phi^0}\rangle$. We also compare the exact form of the S-matrix in asymmetric picture with its own result in the symmetric picture $\langle V_{C^{-1}} V_{\phi^{-1}} V_{\phi^0} V_{\phi^0}\rangle$ at both level of contact interaction and singularity structures. We obtain various new contact interactions as well as two kinds of new bulk singularity structures with various new couplings in $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0} V_{\phi^0}\rangle$, which not appear in $\langle V_{C^{-1}} V_{\phi^{-1}} V_{\phi^0} V_{\phi^0}\rangle$.

These two different kinds of bulk singularity structures of the string amplitude are t, s, u as well as $(t + s + u)$-channels bulk singularity structures that can just be explored in an asymmetric picture of the amplitude. These bulk singularity structures carry momentum of RR in transverse directions (that is, p^i, p^j, p^k terms). Note that these terms could have been derived if winding modes (w^i, w^j, w^k terms) were allowed in the vertex operator of RR. However, in the vertex of RR in ten dimensions of space-time there are no winding modes in both symmetric and asymmetric picture of RR. Indeed these terms of the amplitude whose momenta of RR are carried in transverse direction cannot be obtained even by a T-duality transformation in flat ten dimensions of space-time. Hence the presence of RR makes computations complicated as was explained in [41,42]. Hence we just explore these new bulk singularities as well as new couplings in the asymmetric picture of the amplitude. Indeed we are also able to produce these two different bulk singularity structures of the string amplitude in field theory by taking into account various new couplings on the effective field theory side as well.

We also generalize all order α' higher derivative corrections to those new couplings that appeared in an asymmetric picture of the amplitude.

The important point must be noted. These new couplings are discovered by just scattering amplitude formalism in an antisymmetric picture and not any other tools such as T-duality can be employed to get these couplings. Because these new couplings carry momentum of RR in transverse (or bulk) directions, while winding modes are not embedded in ten dimension of RR vertex operator. We then construct all order bulk singularity structures of t, s, u, $(t + s + u)$ channels in an EFT where the universal conjecture of corrections [43] plays the fundamental ingredient in producing all the infinite singularity structures.

These new bulk singularity structures of the string theory amplitude that are just shown up in $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0} V_{\phi^0}\rangle$ can be generated by taking into account various new EFT couplings. All those new terms do carry the scalar products of momentum of RR in the bulk with the polarization of scalar fields accordingly. We think that the importance of these results will be provided in future research topics, such as all order Myers effect and various other subjects in type II string theory [44]. We have also observed that at the level of EFT the supergravity background fields in DBI action must be some functions of super-Yang–Mills. Some particular Taylor expansion for the background fields should also be taken into account as was notified in Dielectric effect [8].

2 The $\phi^0 - \phi^0 - C^{-2}$ with all order α' corrections

In this section we take into account some Conformal Field Theory (CFT) tools to get the entire S-matrix $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0}\rangle$. Having expanded the elements of scattering amplitude and taken some patters for string corrections, we would start generating α' corrections. Our S-matrix computations are valid at world-sheet level of four and five point functions at the disk level which covers both transverse and world volume directions. Although it is impossible to address all the attempts that have been carried out in this area, we can highlight several efforts that are worth considering for supersymmetric and non-supersymmetric cases [45–57].

One first needs to apply the general structure of vertices where in this paper we just insist on employing RR potential (which is a C_{p+1}-form field in the asymmetric picture), therefore all the other two or three transverse scalars have to be considered at zero picture. The vertex of RR in the asymmetric picture was first proposed by [58]. A new paper about picture changing operators has been recently released [59], however, to the best of our knowledge it is not understood how to deal with all RR closed–open string amplitudes. That is why we try to come up with direct calculations, although the computations in an asymmetric picture are very long and tedious. It would be very nice to work out more to actually understand whether or not the proposal in [59] can be applied to higher point functions of string theory including RR (in an asymmetric picture) and scalar fields. The three point function of a closed string RR and a transverse scalar field (describing the oscillation of the brane) in both the symmetric and asymmetric picture of RR has been accordingly computed in detail [60]. Let us as a warm-up just mention the results:

$$\mathcal{A}^{C^{-1}\phi^{-1}} = 2^{-1/2}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\xi_{1i}. \tag{1}$$

Also

$$\mathcal{A}^{\phi^0,C^{-2}} = [-ip^i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p) + ik_{1a}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ia})]\xi_{1i} \tag{2}$$

where the definitions could be read off from [60]. We have chosen the following notations for the entire ten dimensional space-time, world volume and transverse directions, respectively,

$$\mu,\nu = 0,1,\ldots,9$$
$$a,b,c = 0,1,\ldots,p$$
$$i,j = p+1,\ldots,9.$$

To be able to produce $\mathcal{A}^{\phi^0,C^{-2}}$ in an EFT part, we first apply to the second term of (2) momentum conservation on world volume ($k_1^a + p^a = 0$) and in particular consider the following Bianchi identity:

$$p^a\epsilon^{a_0\cdots a_{p-1}a} = 0, \tag{3}$$

so that the second term in (2) has a zero contribution to the S-matrix. The first term in (2) can be produced in an EFT by considering the Taylor expansion of a real scalar field through RR coupling as follows:

$$S_1 = \frac{(2\pi\alpha')\mu_p}{(p+1)!}\int \mathrm{d}^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}\mathrm{Tr}\,(\phi^i)\partial_i C_{a_0\cdots a_p}.$$

3 $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}\rangle$ amplitude

The four point function in an asymmetric picture of a RR and two real scalar fields in zero picture can be done by

$$\mathcal{A}^{\phi^0\phi^0C^{-2}} \sim \int \mathrm{d}x_1\mathrm{d}x_2\mathrm{d}^2z\langle V_\phi^{(0)}(x_1)V_\phi^{(0)}(x_2)V_{RR}^{(-2)}(z,\bar{z})\rangle. \tag{4}$$

The scalar field and RR vertex operators in the symmetric and asymmetric pictures are given by

$$\begin{aligned}
V_\phi^{(0)}(x) &= \xi_i(\partial X^i(x) + \alpha' ik{\cdot}\psi\psi^i(x))\mathrm{e}^{\alpha' ik\cdot X(x)},\\
V_\phi^{(-2)}(x_1) &= \mathrm{e}^{-2\phi(x_1)}V_\phi^{(0)}(x_1)\\
V_\phi^{(-1)}(x) &= \mathrm{e}^{-\phi(x)}\xi_i\psi^i(x)e^{2iq\cdot X(x)}\\
V_{RR}^{(-1)}(z,\bar{z}) &= (P_-\not{H}_{(n)}M_p)^{\alpha\beta}\mathrm{e}^{-\phi(z)/2}\\
&\quad\times S_\alpha(z)\mathrm{e}^{i\frac{\alpha'}{2}p\cdot X(z)}\mathrm{e}^{-\phi(\bar{z})/2}S_\beta(\bar{z})\mathrm{e}^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}\\
V_{RR}^{(-2)}(z,\bar{z}) &= (P_-\not{C}_{(n-1)}M_p)^{\alpha\beta}\mathrm{e}^{-3\phi(z)/2}\\
&\quad\times S_\alpha(z)\mathrm{e}^{i\frac{\alpha'}{2}p\cdot X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})\mathrm{e}^{i\frac{\alpha'}{2}p\cdot D\cdot X(\bar{z})}
\end{aligned}$$

where $k^2 = p^2 = 0$, $k.\xi_1 = 0$. We also consider $x_4 \equiv z = x + iy$, $x_5 \equiv \bar{z} = x - iy$ and the definitions of the RR's field strength and projection operator are

$$P_- = \frac{1}{2}(1-\gamma^{11}),\quad \not{H}_{(n)} = \frac{a_n}{n!}H_{\mu_1\ldots\mu_n}\gamma^{\mu_1}\ldots\gamma^{\mu_n}.$$

For type IIA (IIB) $n = 2,4$, $a_n = i$ ($n = 1,3,5$, $a_n = 1$). Note that the spinor notation reads

$$(P_-\not{H}_{(n)})^{\alpha\beta} = C^{\alpha\delta}(P_-\not{H}_{(n)})_\delta{}^\beta.$$

To make use of the holomorphic propagators we apply the following doubling trick. Hence one uses the following change of variables:

$$\tilde{X}^\mu(\bar{z}) \to D^\mu_\nu X^\nu(\bar{z}),\quad \tilde{\psi}^\mu(\bar{z}) \to D^\mu_\nu\psi^\nu(\bar{z}),$$
$$\tilde{\phi}(\bar{z}) \to \phi(\bar{z}),\quad \text{and}\quad \tilde{S}_\alpha(\bar{z}) \to M_\alpha{}^\beta S_\beta(\bar{z}),$$

with

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1}\end{pmatrix},\quad \text{and}$$

$$M_p = \begin{cases} \frac{\pm i}{(p+1)!}\gamma^{i_1}\gamma^{i_2}\ldots\gamma^{i_{p+1}}\epsilon_{i_1\ldots i_{p+1}} & \text{for } p \text{ even},\\ \frac{\pm 1}{(p+1)!}\gamma^{i_1}\gamma^{i_2}\ldots\gamma^{i_{p+1}}\gamma_{11}\epsilon_{i_1\ldots i_{p+1}} & \text{for } p \text{ odd}.\end{cases}$$

Finally we can use the following correlation functions for the X^μ, ψ^μ, ϕ fields:

$$\begin{aligned}
\langle X^\mu(z)X^\nu(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}\log(z-w),\\
\langle\psi^\mu(z)\psi^\nu(w)\rangle &= -\frac{\alpha'}{2}\eta^{\mu\nu}(z-w)^{-1},\\
\langle\phi(z)\phi(w)\rangle &= -\log(z-w).
\end{aligned}\tag{5}$$

Using the Wick theorem, the amplitude would be written down as

$$\int dx_1 dx_2 dx_4 dx_5 (P_- \not{C}_{(n-1)} M_p)^{\alpha\beta} (x_{45})^{-3/4} \xi_{1i}\xi_{2j}$$
$$(K_1 + K_2 + K_3 + K_4) K_5.$$

The exponential factors are

$$K_5 = |x_{12}|^{\alpha'^2 k_1.k_2} |x_{14}x_{15}|^{\frac{\alpha'^2}{2} k_1.p} |x_{24}x_{25}|^{\frac{\alpha'^2}{2} k_2.p} |x_{45}|^{\frac{\alpha'^2}{4} p.D.p}.$$

In order to be able to explore all the above the fermionic correlators including the correlation functions of various currents with spin operators, one has to consider the Wick-like rule [52,53] and modify it as follows:

$$\begin{aligned}
&\langle \psi^{\mu_1}(x_1)...\psi^{\mu_n}(x_n) S_\alpha(x_4) S_\beta(x_5)\rangle \\
&= \frac{1}{2^{n/2}} \frac{(x_4 - x_5)^{n/2-5/4}}{|x_1 - x_4|...|x_n - x_4|} \Big[(\Gamma^{\mu_n...\mu_1} C^{-1})_{\alpha\beta} \\
&\quad + \langle \psi^{\mu_1}(x_1)\psi^{\mu_2}(x_2)\rangle (\Gamma^{\mu_n...\mu_3} C^{-1})_{\alpha\beta} \pm \text{perms} \\
&\quad + \langle \psi^{\mu_1}(x_1)\psi^{\mu_2}(x_2)\rangle \langle \psi^{\mu_3}(x_3)\psi^{\mu_4}(x_4)\rangle (\Gamma^{\mu_n...\mu_5} C^{-1})_{\alpha\beta} \\
&\quad \pm \text{perms} + \cdots \Big]
\end{aligned} \tag{6}$$

where one has to consider all various contractions to make sense of the corrected S-matrix elements. The important point one has to hold, is that in all the above equations, $\Gamma^{\mu_n \cdots \mu_1}$ must be antisymmetric with respect to all the gamma matrices. The $x_i's$ are real and other important point is as follows. Apart from considering co-cycles in the fermionic two point functions, the Wick-like formula must be modified with a minus sign, in order to respect the gauge invariance of the higher point function of BPS or non-BPS S-matrices. Hence, the corrected Wick-like formula for the definition of the two point function is

$$\begin{aligned}
\langle \psi^\mu(x_1)\psi^\nu(x_2)\rangle &= -\alpha' \eta^{\mu\nu} \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} \\
&= -\alpha' \eta^{\mu\nu} \frac{Re[(x_1 - x_4)(x_2 - x_5)]}{(x_1 - x_2)(x_4 - x_5)}.
\end{aligned}$$

To elaborate and explain how exactly these signs guarantee gauge invariance, let us not consider the minus sign in a Wick-like formula (and dropped out co-cycles in the fermionic functions) and focus on the amplitude of an RR, a gauge field and two real tachyons. Having said that, the correlation function $\langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : \psi^b(x_2) : \psi^c(x_3) :\rangle$ would be found:

$$\begin{aligned}
&2^{-3/2}(x_{14}x_{15}x_{24}x_{25}x_{34}x_{35})^{-1/2}(x_{45})^{1/4}\Big[(\Gamma^{cba} C^{-1})_{\alpha\beta} \\
&\quad + 2\eta^{ab}(\gamma^c C^{-1})_{\alpha\beta} \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} \\
&\quad + 2\eta^{ac}(\gamma^b C^{-1})_{\alpha\beta} \frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} \\
&\quad + 2\eta^{bc}(\gamma^a C^{-1})_{\alpha\beta} \frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} \Big]
\end{aligned} \tag{7}$$

where the Mandelstam variables are $s = -(k_1 + k_3)^2, t = -(k_1 + k_2)^2, u = -(k_2 + k_3)^2$. Given (7), the final form of the S-matrix is

$$\begin{aligned}
&\mathcal{A}^{C^{-1}A^{-1}T^0T^0} \\
&= \frac{i\mu_p}{2\sqrt{2\pi}} [\text{Tr}\,((P_- \not{H}_{(n)} M_p)(k_3.\gamma)(k_2.\gamma)(\xi.\gamma)) I \delta_{p,n+2} \\
&\quad + \text{Tr}\,((P_- \not{H}_{(n)} M_p)\gamma^a) J \delta_{p,n} \\
&\quad \times \{-k_{2a}(t + 1/4)(2\xi.k_3) \\
&\quad + k_{3a}(s + 1/4)(2\xi.k_2) - \xi_a(s + 1/4)(t + 1/4)\}].
\end{aligned} \tag{8}$$

Now clearly if we replace $\xi_{1a} \to k_{1a}$ in (7) then one gets to know that the amplitude does not vanish any more. Because after replacing $\xi_{1a} \to k_{1a}$ the amplitude is proportional to $(s + 1/4)(t + 1/4)(-k_{1a} - k_{3a} + k_{2a})$, which is not zero and hence it is not gauge invariant any more. So one needs to consider the minus sign in co-cycles as well as the minus sign in the Wick-like formula.[2]

Now we apply the generalisation of the Wick-like rule to the amplitude so that one is able to derive all the correlators as

$$\begin{aligned}
K_1 &= \left(-\eta^{ij} x_{12}^{-2} + \left(ip^i \frac{x_{54}}{x_{14}x_{15}}\right)\left(ip^j \frac{x_{54}}{x_{24}x_{25}}\right)\right)(x_{45})^{-5/4}(C^{-1})_{\alpha\beta}, \\
K_2 &= (ip^i \frac{x_{54}}{x_{14}x_{15}}) ik_{2b}(x_{24}x_{25})^{-1}(x_{45})^{-1/4}(\Gamma^{jb}C^{-1})_{\alpha\beta}, \\
K_3 &= (ip^j \frac{x_{54}}{x_{24}x_{25}}) ik_{1a}(x_{14}x_{15})^{-1}(x_{45})^{-1/4}(\Gamma^{ia}C^{-1})_{\alpha\beta}, \\
K_4 &= -k_{1a}k_{2b}(x_{14}x_{15}x_{24}x_{25})^{-1}(x_{45})^{3/4} \\
&\times\Big[(\Gamma^{jbia}C^{-1})_{\alpha\beta} + \frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} \\
&\times (2\eta^{ab}(\Gamma^{ji}C^{-1})_{\alpha\beta} + 2\eta^{ij}(\Gamma^{ba}C^{-1})_{\alpha\beta}) \\
&-4\eta^{ab}\eta^{ij}(C^{-1})_{\alpha\beta}(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}})^2\Big].
\end{aligned} \tag{9}$$

By applying (9) into this four point amplitude we can easily determine that the S-matrix is $SL(2, R)$ invariant. We do the proper gauge fixing as $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$, taking $t = -\frac{\alpha'}{2}(k_1 + k_2)^2$ to get the S-matrix as

$$\begin{aligned}
\mathcal{A}^{\phi^0\phi^0 C^{-2}} &= -(2i)^{-2t} \xi_{1i}\xi_{2j}(P_- \not{C}_{(n-1)} M_p)^{\alpha\beta} \\
&\times \int_{-\infty}^{\infty} dx (1 + x^2)^{2t-1}(2x)^{-2t} \\
&\times \Big\{ (2i)^{-1}(C^{-1})_{\alpha\beta}(-\eta^{ij}(1 + x^2)^2(2x)^{-2} + 4p^i p^j) \\
&+ 2i(p^i k_{2b}(\Gamma^{jb}C^{-1})_{\alpha\beta} + p^j k_{1a}(\Gamma^{ia}C^{-1})_{\alpha\beta}) \\
&- k_{1a}k_{2b}(2i)\Big[(\Gamma^{jbia}C^{-1})_{\alpha\beta} \\
&+ (2\eta^{ab}(\Gamma^{ji}C^{-1})_{\alpha\beta} + 2\eta^{ij}(\Gamma^{ba}C^{-1})_{\alpha\beta}) \frac{1 - x^2}{4ix} \\
&- 4\eta^{ab}\eta^{ij}(C^{-1})_{\alpha\beta}\left(\frac{1 - x^2}{4ix}\right)^2\Big]\Big\}
\end{aligned} \tag{10}$$

[2] I, J are also given in [15].

where the sixth and seventh terms do not have any contribution to the S-matrix because the integration is taken on the whole space meanwhile the integrand is odd so the result vanishes. More crucially, after using kinematical relations we come to know that the sum of the first term and the last term of (10) is zero.

One now explores the final answer for the third and fourth term of (10) as follows:

$$\mathcal{A}_1^{\phi^0\phi^0C^{-2}} = (2i)^{-2t+1}\xi_{1i}\xi_{2j}\sqrt{(\pi)}\frac{\Gamma(-t+\frac{1}{2})}{\Gamma(-t+1)} \times[p^i k_{2b}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jb}) + p^j k_{1a}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ia})]. \quad (11)$$

Eventually one can find the result for the second term of asymmetric S-matrix as follows:

$$\mathcal{A}_2^{\phi^0\phi^0C^{-2}} = -(2i)^{-2t}\xi_{1i}\xi_{2j}\sqrt{(\pi)}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p) \times\left\{-2ip^ip^j\frac{\Gamma(-t+\frac{1}{2})}{\Gamma(-t+1)}\right\}. \quad (12)$$

The trace is non-zero for $p+1=n-1$ case and (12) does not include any poles, because the expansion is a low energy expansion ($t\to 0$). It has been emphasized in [27] that for a particular string S-matrix involving scalar fields one needs to take into account three different ways to actually regenerate all the string couplings in an EFT side. The first way was imposed by Myers in [8], while the second and the third ways of EFT have been completely mentioned in [27] to be either pull-back method or Taylor expansion of the scalar fields.

This S-matrix in the symmetric picture can be readily computed as

$$\mathcal{A}^{C^{-1}\phi^{-1}\phi^0} = i(2i)^{-2t+1}\int_{-\infty}^{\infty}\mathrm{d}x\,(x^2+1)^{2t-1}(2x)^{-2t}(\xi_{1i}\xi_{2j}2^{-1/2}) \times[p^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i) - k_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jai})]. \quad (13)$$

Carrying out the integrals explicitly and using momentum conservation, we obtain

$$\mathcal{A}^{C^{-1}\phi^{-1}\phi^0} = (2\xi_{1i}\xi_{2j}2^{-1/2}\pi^{1/2})\frac{\Gamma(-t+\frac{1}{2})}{\Gamma(-t+1)} \times[p^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i) - k_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jai})]. \quad (14)$$

The closed form of the expansion is

$$\sqrt{\pi}\frac{\Gamma(-t+\frac{1}{2})}{\Gamma(-t+1)} = \pi\sum_{n=-1}^{\infty}c_n(t)^{n+1} \quad (15)$$

where all c_n are related to higher derivative corrections of the scalar fields through either Taylor expansions or Pull-back of branes.

$$c_{-1}=1,\quad c_0=2ln(2),\quad c_1=\frac{\pi^2}{6}+2ln(2)^2. \quad (16)$$

Now we work out the related super-Yang–Mills vertices to reconstruct all string couplings to all orders in α'. In order to produce all infinite contact interactions for first term of (14) and also to produce $\mathcal{A}_2^{\phi^0\phi^0C^{-2}}$ part, one needs to deal with the Taylor expansion of the two real scalar fields through an RR $(p+1)$-form field as

$$S_2 = i\frac{(2\pi\alpha')^2\mu_p}{2(p+1)!}\int\mathrm{d}^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}\mathrm{Tr}\,(\Phi^i\Phi^j)\partial_i\partial_jC^{(p+1)}_{a_0\cdots a_p},$$

and subsequently all order α' corrections can be derived as

$$S_3 = \frac{(2\pi\alpha')^2\mu_p}{2(p+1)!}\int\mathrm{d}^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}p^jH^{(p+2)}_{ia_0\cdots a_p} \times\sum_{n=-1}^{\infty}c_n(\alpha')^{n+1}\mathrm{Tr}\,(\partial_{a_1}\ldots\partial_{a_{n+1}}\Phi^i\partial^{a_1}\ldots\partial^{a_{n+1}}\Phi^j). \quad (17)$$

In the meantime to be able to produce at the leading order the second term of (14) as well as the fifth term of $\mathcal{A}^{\phi^0\phi^0C^{-2}}$, one must employ the so-called pull-back formalism as follows:

$$S_4 = i\frac{(2\pi\alpha')^2\mu_p}{2(p-1)!}\int\mathrm{d}^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}\mathrm{Tr}\,(D_{a_0}\Phi^iD_{a_1}\Phi^j)C^{(p+1)}_{ija_2\cdots a_p}. \quad (18)$$

Essentially one can insist on producing all infinite corrections by imposing the higher derivative corrections to the pull-back and fix all their coefficients without any ambiguities in string theory as below

$$S_5 = i\frac{(2\pi\alpha')^2\mu_p}{2(p)!}\int\mathrm{d}^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}\sum_{n=-1}^{\infty}c_n(\alpha')^{n+1} \times\mathrm{Tr}\,(\partial_{a_1}\ldots\partial_{a_{n+1}}D_{a_0}\Phi^i\partial^{a_1}\ldots\partial^{a_{n+1}}\Phi^j)H^{(p+2)}_{ija_1\cdots a_p}.$$

There is no external gauge field in our S-matrix so one could propose the covariant derivatives of scalar fields to above higher derivative couplings to be able to keep track of the gauge invariance of the S-matrix as well.

3.1 The other RR couplings of type II string theory

Now in order to be able to produce the term that has been explicitly appeared by asymmetric amplitude in (11), one needs to write down sort of new couplings, in the sense that in this turn, neither both scalar fields come from the pull-back nor Taylor expansion.

Indeed for the first time, we just confirm the presence of sort of mixed couplings in string theory so that the first scalar field comes from the pull-back and the second scalar comes through Taylor expansion. Thus the presence of mixed coupling in EFT is now being discovered. Note that this fact has become apparent by just dealing with $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}\rangle$ S-matrix in an asymmetric picture of RR.

Let us write down the effective coupling at leading order which is α'^2 and then generalize it to all orders in α'. To be able to produce (11), at leading order one has to write down the following new coupling:

$$S_6 = \frac{i(2\pi\alpha')^2\mu_p}{2p!}\int d^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p} \times(\mathrm{Tr}\,(\Phi^i D_{a_0}\Phi^j)\partial_i C^{(p+1)}_{ja_1\cdots a_p} + \mathrm{Tr}\,(\Phi^j D_{a_0}\Phi^i)\partial_j C^{(p+1)}_{ia_1\cdots a_p}). \quad (19)$$

As is clear from (15), we have many contact interactions, which are related to higher derivative corrections of the scalar fields. These corrections without any ambiguities can be fixed by just the S-matrix method. One can now apply the higher derivative corrections to the scalar fields to actually get the all order α' corrections of the above couplings as follows:

$$S_7 = i\frac{(2\pi\alpha')^2\mu_p}{2p!}\int d^{p+1}\sigma(\varepsilon^v)^{a_0\cdots a_p}\sum_{n=-1}^{\infty}c_n(\alpha')^{n+1} \times(\mathrm{Tr}\,(\partial_{a_1}\ldots\partial_{a_{n+1}}\Phi^i D_{a_0}\partial^{a_1}\ldots\partial^{a_{n+1}}\Phi^j)\partial_i C^{(p+1)}_{ja_1\cdots a_p} + \mathrm{Tr}\,(\partial_{a_1}\ldots\partial_{a_{n+1}}\Phi^j D_{a_0}\partial^{a_1}\ldots\partial^{a_{n+1}}\Phi^i)\partial_j C^{(p+1)}_{ia_1\cdots a_p}). \quad (20)$$

Once more one could restore the gauge invariance by replacing the covariant derivatives in (20). Now let us deal with the five point function of an RR and three scalar fields in an asymmetric picture.

4 $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$ amplitude

The S-matrix elements of three transverse scalar fields and one RR in an asymmetric picture (C-field) $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$ at disk level can be presented by the following correlation functions:

$$\mathcal{A}^{C^{-2}\phi^0\phi^0\phi^0} \sim \int dx_1dx_2dx_3dzd\bar{z}\,\langle V^{(0)}_\phi(x_1)V^{(0)}_\phi(x_2) \times V^{(0)}_\phi(x_3)V^{(-\frac{3}{2},-\frac{1}{2})}_{RR}(z,\bar{z})\rangle. \quad (21)$$

One can extract the whole S-matrix and divide it out to various correlation functions.

In order to be able to explore all fermionic correlations including the correlations of various currents with spin operators, one has to reconsider the modified Wick-like rule as mentioned in the last section. Let us simplify the S-matrix further:[3]

$$\mathcal{A}^{C^{-2}\phi^0\phi^0\phi^0} \sim \int dx_1dx_2dx_3dx_4dx_5(P_-\mathcal{C}\!\!\!/_{(n-1)}M_p)^{\alpha\beta} \times I\xi_{1i}\xi_{2j}\xi_{3k}x_{45}^{-3/4} \times((x_{45}^{-5/4}C^{-1}_{\alpha\beta})[-\eta^{ij}x_{12}^{-2}a_3^k - \eta^{ik}x_{13}^{-2}a_2^j - \eta^{jk}x_{23}^{-2}a_1^i + a_1^ia_2^ja_3^k] + a_{13}^{ik}a_2^{ja} + a_{23}^{jk}a_3^{ic} + a_3^ka_4^{jaic} + a_{12}^{ij}a_5^{kb} + a_1^ia_6^{kbja} + a_2^ja_7^{kbic} - i\alpha'^3k_{1c}k_{2a}k_{3b}I_8^{kbjaic})\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3), \quad (22)$$

where $x_{ij} = x_i - x_j$, and also

$$I = |x_{12}|^{\alpha'^2k_1.k_2}|x_{13}|^{\alpha'^2k_1.k_3}|x_{14}x_{15}|^{\frac{\alpha'^2}{2}k_1.p}|x_{23}|^{\alpha'^2k_2.k_3} \times|x_{24}x_{25}|^{\frac{\alpha'^2}{2}k_2.p}|x_{34}x_{35}|^{\frac{\alpha'^2}{2}k_3.p}|x_{45}|^{\frac{\alpha'^2}{4}p.D.p},$$

$$a_1^i = ip^i\left(\frac{x_{54}}{x_{14}x_{15}}\right),$$

$$a_{12}^{ij} = (-\eta^{ij}x_{12}^{-2} + a_1^ia_2^j),$$

$$a_{13}^{ik} = (-\eta^{ik}x_{13}^{-2} + a_1^ia_3^k),$$

$$a_{23}^{jk} = (-\eta^{jk}x_{23}^{-2} + a_2^ja_3^k),$$

$$a_2^j = ip^j\left(\frac{x_{54}}{x_{24}x_{25}}\right),$$

$$a_3^k = ip^k\left(\frac{x_{54}}{x_{34}x_{35}}\right),$$

$$a_2^{ja} = \alpha' ik_{2a}2^{-1}x_{45}^{-1/4}(x_{24}x_{25})^{-1}\{(\Gamma^{ja}C^{-1})_{\alpha\beta}\},$$

$$a_3^{ic} = \alpha' ik_{1c}2^{-1}x_{45}^{-1/4}(x_{14}x_{15})^{-1}\{(\Gamma^{ic}C^{-1})_{\alpha\beta}\},$$

$$a_4^{jaic} = -\alpha'^2k_{1c}k_{2a}2^{-2}x_{45}^{3/4}(x_{14}x_{15}x_{24}x_{25})^{-1} \times\left\{(\Gamma^{jaic}C^{-1})_{\alpha\beta} + \alpha'n_1\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}} + \alpha'^2n_2\left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)^2\right\},$$

$$a_5^{kb} = \alpha' ik_{3b}2^{-1}x_{45}^{-1/4}(x_{34}x_{35})^{-1}\{(\Gamma^{kb}C^{-1})_{\alpha\beta}\},$$

$$a_6^{kbja} = -\alpha'^2k_{2a}k_{3b}2^{-2}x_{45}^{3/4}(x_{34}x_{35}x_{24}x_{25})^{-1} \times\left\{(\Gamma^{kbja}C^{-1})_{\alpha\beta} + \alpha'n_3\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} + \alpha'^2n_4\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)^2\right\},$$

$$a_7^{kbic} = -\alpha'^2k_{1c}k_{3b}2^{-2}x_{45}^{3/4}(x_{34}x_{35}x_{14}x_{15})^{-1} \times\left\{(\Gamma^{kbic}C^{-1})_{\alpha\beta} + \alpha'n_5\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} + \alpha'^2n_6\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)^2\right\},$$

where all $n_i's$ as well as I_8^{kbjaic} are given in Appendix 1. One finds that the amplitude is SL(2, R) invariant so to remove the volume of the conformal Killing group as well as for practical reasons we fix the position of open strings at zero, one and infinity, that is, $x_1 = 0$, $x_2 = 1$, $x_3 \to \infty$. Now if we come over the evaluation of all the integrals on the location of closed string RR on the upper half plane as explained in Appendix 1, then one finally reads off all the elements of the string amplitude in an asymmetric picture of RR as follows:

$$\mathcal{A}^{C^{-2}\phi^0\phi^0\phi^0} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_{41} + \mathcal{A}_{42} + \mathcal{A}_{43} + \mathcal{A}_5 \; \mathcal{A}_{61} + \mathcal{A}_{62} + \mathcal{A}_{63} + \mathcal{A}_{71} + \mathcal{A}_{72} + \mathcal{A}_{73} + \mathcal{A}_{81} + \mathcal{A}_{82} \; \mathcal{A}_{83} + \mathcal{A}_{84} + \mathcal{A}_{85} + \mathcal{A}_{86} + \mathcal{A}_{87} + \mathcal{A}_{88} + \mathcal{A}_{89} + \mathcal{A}_{810} \quad (23)$$

where

$$\mathcal{A}_1 \sim i\mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)}M_p)[p.\xi_1p.\xi_2p.\xi_3Q_1 + \xi_1.\xi_2p.\xi_3Q_2 + \xi_1.\xi_3p.\xi_2Q_3 + \xi_3.\xi_2p.\xi_1Q_4],$$

[3] Here in type II we set $\alpha' = 2$.

$$
\begin{aligned}
\mathcal{A}_2 &\sim ik_{2a}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ja})\\
&\quad\times\{-\xi_1.\xi_3Q_3-p.\xi_1p.\xi_3Q_1\},\\
\mathcal{A}_3 &\sim ik_{1c}\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ic})\\
&\quad\times\{-\xi_2.\xi_3Q_4-p.\xi_2p.\xi_3Q_1\},\\
\mathcal{A}_{41} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jaic})\xi_{2j}\xi_{1i}p.\xi_3k_{1c}k_{2a}Q_1,\\
\mathcal{A}_{42} &\sim ip.\xi_3L_2\{t\xi_{1i}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ji})\\
&\quad-2\xi_1.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ac})k_{1c}k_{2a}\},\\
\mathcal{A}_{43} &\sim 2itp.\xi_3\xi_1.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)Q_5\left\{us-\frac{u+s+t}{2}\right\},
\end{aligned}\tag{24}
$$

$$
\begin{aligned}
\mathcal{A}_5 &\sim ik_{3b}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kb})\{-\xi_2.\xi_1Q_2-p.\xi_2p.\xi_1Q_1\},\\
\mathcal{A}_{61} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbja})\xi_{2j}\xi_{3k}p.\xi_1k_{3b}k_{2a}Q_1,\\
\mathcal{A}_{62} &\sim ip.\xi_1L_5\{u\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kj})\\
&\quad-2\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ba})k_{2a}k_{3b}\},\\
\mathcal{A}_{63} &\sim 2iup.\xi_1\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)Q_6\left\{st-\frac{u+s+t}{2}\right\},\\
\mathcal{A}_{71} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbic})\xi_{1i}\xi_{3k}p.\xi_2k_{3b}k_{1c}Q_1,\\
\mathcal{A}_{72} &\sim ip.\xi_2L_3\{-s\xi_{1i}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ki})\\
&\quad+2\xi_1.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bc})k_{1c}k_{3b}\},\\
\mathcal{A}_{73} &\sim 2isp.\xi_2\xi_1.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)Q_7\left\{ut-\frac{u+s+t}{2}\right\},\\
\mathcal{A}_{81} &\sim -i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbjaic})\xi_{1i}\xi_{2j}\xi_{3k}k_{1c}k_{2a}k_{3b}Q_1,\\
\mathcal{A}_{82} &\sim -iL_2\{t\xi_{1i}\xi_{2j}\xi_{3k}k_{3b}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbji})\\
&\quad-2\xi_1.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbac})\xi_{3k}k_{1c}k_{2a}k_{3b}\},\\
\mathcal{A}_{83} &\sim -iL_3\{-s\xi_{1i}\xi_{2j}\xi_{3k}k_{2a}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kjai})\\
&\quad+2\xi_1.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bjac})\xi_{2j}k_{1c}k_{2a}k_{3b}\},\\
\mathcal{A}_{84} &\sim iL_5\{-u\xi_{1i}\xi_{2j}\xi_{3k}k_{1c}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kjic})\\
&\quad+2\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{baic})\xi_{1i}k_{1c}k_{2a}k_{3b}\},\\
\mathcal{A}_{85} &\sim -2it\xi_{3k}k_{3b}\xi_1.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kb})\\
&\quad\times Q_5\left\{us-\frac{u+s+t}{2}\right\},\\
\mathcal{A}_{86} &\sim -iuL_6\{2t\xi_1.\xi_3k_{3b}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bj})\\
&\quad-2s\xi_1.\xi_2k_{2a}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ka})\},\\
\mathcal{A}_{87} &\sim isL_6\{-2t\xi_2.\xi_3k_{3b}\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bi})\\
&\quad+2u\xi_1.\xi_2k_{1c}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kc})\},\\
\mathcal{A}_{88} &\sim -2is\xi_{2j}k_{2a}\xi_1.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ja})\\
&\quad Q_7\{ut-\frac{u+s+t}{2}\},\\
\mathcal{A}_{89} &\sim -2iu\xi_{1i}k_{1c}\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ic})\\
&\quad\times Q_6\left\{st-\frac{u+s+t}{2}\right\},\\
\mathcal{A}_{810} &\sim -itL_6\{2s\xi_2.\xi_3k_{2a}\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ai})\\
&\quad-2u\xi_1.\xi_3k_{1c}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jc})\},
\end{aligned}\tag{25}
$$

where the functions $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, L_2, L_3, L_5, L_6$ are given in Appendix 2. Let us write down the same amplitude in its symmetric picture, that is, $\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle$, which means that we consider the amplitude in terms of the field strength of RR and start comparing these two results. Working out details and making use of the integrals presented in [27,61], one constructs the S-matrix in the symmetric picture of RR as follows:

$$
\begin{aligned}
\mathcal{A}^{C^{-1}\phi^{-1}\phi^0\phi^0} &= \mathcal{A}'_1+\mathcal{A}'_2+\mathcal{A}'_3+\mathcal{A}'_4\\
&\quad+\mathcal{A}'_5+\mathcal{A}'_6+\mathcal{A}'_7+\mathcal{A}'_8+\mathcal{A}'_9+\mathcal{A}'_{10}
\end{aligned}\tag{26}
$$

with

$$
\begin{aligned}
\mathcal{A}'_1 &\sim -2^{-1/2}\xi_{1i}\xi_{2j}\xi_{3k}[k_{3b}k_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kbjai})\\
&\quad-k_{3b}p^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kbi})\\
&\quad-k_{2a}p^k\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jai})\\
&\quad+p^jp^k\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)]Q_1,\\
\mathcal{A}'_2 &\sim 2^{-1/2}\{2\xi_1.\xi_2k_{2a}k_{3b}\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kba})\}L_2,\\
\mathcal{A}'_3 &\sim 2^{-1/2}\{\xi_{1i}\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kji})\}(-uL_5),\\
\mathcal{A}'_4 &\sim -2^{-1/2}\{2\xi_3.\xi_1k_{2a}k_{3b}\xi_{2j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bja})\}L_3,\\
\mathcal{A}'_5 &\sim 2^{-1/2}\{2\xi_3.\xi_2k_{2a}k_{3b}\xi_{1i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bai})\}L_5,\\
\mathcal{A}'_6 &\sim 2^{1/2}L_2\{-k_{2a}p^k\xi_1.\xi_2\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a)\},\\
\mathcal{A}'_7 &\sim 2^{1/2}L_3\{k_{3b}p^j\xi_1.\xi_3\xi_{2j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)\},\\
\mathcal{A}'_8 &\sim 2^{1/2}L_6\{\xi_{2j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^j)(ut\xi_1.\xi_3)\},\\
\mathcal{A}'_9 &\sim 2^{1/2}L_6\{\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^k)(us\xi_1.\xi_2)\},\\
\mathcal{A}'_{10} &\sim 2^{1/2}L_6\{\xi_{1i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)(ts\xi_3.\xi_2)\}
\end{aligned}\tag{27}
$$

where L_2, L_3, L_5, L_6 are introduced in (A.3). Let us compare the amplitudes in the asymmetric picture with its symmetric result and explore all new bulk singularity structures as well as new couplings (with their all order α' corrections) that just appeared in the asymmetric picture of the amplitude.

5 Singularity comparisons between asymmetric and symmetric pictures

In this section we try to produce all the singularity structures of symmetric picture by dealing with the S-matrix element in asymmetric picture of RR. We add the first term of $\mathcal{A}_{810}$ of asymmetric amplitude with first term of $\mathcal{A}_{87}$ and apply momentum conservation along the world volume of the brane to actually obtain the following couplings:

$$
\begin{aligned}
&-2istL_6\xi_2.\xi_3\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bi})(k_{3b}+k_{2b})\\
&\quad=-2istL_6\xi_2.\xi_3\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bi})(-k_{1b}-p_b);
\end{aligned}
$$

now if we use $p\not{C}=\not{H}$ then we see that above coupling precisely generates $\mathcal{A}'_{10}$ of the symmetric picture $(\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle)$.[4] The first term of the above equation will be precisely canceled out by the sum of the first term of $\mathcal{A}_3$ of the asymmetric picture and the whole $\mathcal{A}_{89}$.

[4] Up to a normalisation constant $2^{1/2}i$.

Likewise what we did previously, here we try to add the second terms of $\mathcal{A}_{86}$ and $\mathcal{A}_{87}$ and in particular we take into account the momentum conservation to actually arrive at the following singularities:

$$2isuL_6\xi_2.\xi_1\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kc})(-k_{3c}-p_c). \tag{28}$$

Using $p\not{C} = \not{H}$ we are able to reconstruct $\mathcal{A}'_9$ of $\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle$. The first term of (28) will be removed by the sum of the first term of $\mathcal{A}_5$ and the whole $\mathcal{A}_{85}$ of the asymmetric S-matrix. The same holds as follows.

Adding the second term of $\mathcal{A}_{810}$ and the first term of $\mathcal{A}_{86}$ also paying particular attention to momentum conservation give rise to the following elements:

$$-2ituL_6\xi_3.\xi_1\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bj})(-k_{2b}-p_b), \tag{29}$$

which is precisely $\mathcal{A}'_8$. Notice to the point that the first term of (29) has been equivalently canceled out by the sum of first term of $\mathcal{A}_2$ and the entire $\mathcal{A}_{88}$ of $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$.

Having taken the second term of $\mathcal{A}_{72}$ and having applied the momentum conservation to it we seem to get

$$2ip.\xi_2\xi_3.\xi_1L_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bc})k_{3b}(-k_{3c}-p_c-k_{2c}); \tag{30}$$

evidently the first term above has no contribution, because there is an antisymmetric ϵ tensor while the whole element is symmetric with respect to interchanging k_3 so the answer turns out to be zero while the second term in (30) precisely generates $\mathcal{A}'_7$. The the last term in (30) remains to be explored later on.

By applying the same tricks to the second term of $\mathcal{A}_{42}$ we obtain

$$-2ip.\xi_3\xi_2.\xi_1L_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ac}) \times k_{2a}(-k_{2c}-p_c-k_{3c}), \tag{31}$$

obviously the first term above has a zero contribution to the S-matrix and the second term reconstructs $\mathcal{A}'_6$, meanwhile the last term will be considered in the next section. We also need to take into account the second term of $\mathcal{A}_{84}$ and apply the momentum conservation to it to get

$$2iL_5\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{baic}) \times \xi_{1i}k_{2a}k_{3b}(-k_{2c}-k_{3c}-p_c). \tag{32}$$

The first and second term in (32) have zero contribution and the third term produces $\mathcal{A}'_5$. Once more one needs to deal with the second term of $\mathcal{A}_{83}$ and draw attention to momentum conservation in such a way that the following singularities turn out to be produced:

$$-2iL_3\xi_1.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{bjac}) \times \xi_{2j}k_{2a}k_{3b}(-k_{2c}-k_{3c}-p_c), \tag{33}$$

clearly the first and second term of (33) have no contribution to the amplitude as there is an antisymmetric ϵ tensor while the whole singularity is symmetric with respect to interchanging k_2, k_3 so the answer turns out to be zero while the third term in the above singularity re-builds $\mathcal{A}'_4$.

We need to carry out the same tricks to the second term of $\mathcal{A}_{82}$ to be able to recreate exactly $\mathcal{A}'_2$ of symmetric amplitude.

Eventually we need to add the first terms of $\mathcal{A}_{82}$, $\mathcal{A}_{83}$ and $\mathcal{A}_{84}$ to actually derive the following singularities:

$$iL_{22}\xi_{1i}\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbji})(k_{1b}+k_{2b}+k_{3b}) = iL_{22}\xi_{1i}\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbji})(-p_b),$$

which is nothing but exactly the $\mathcal{A}'_3$ part of the symmetric S-matrix and $L_{22} = -uL_5$. From the above comparisons we come to the following points.

By doing a careful analysis of asymmetric elements we were able to re-generate all order α' singularity structures of symmetric amplitude ($\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle$). However, the important point that must be emphasized is as follows. Regarding our true comparisons and the remarks that have already been pointed out in this section, we have got some extra contact interactions as well as two extra kinds of bulk singularity structures in the asymmetric amplitude that cannot be shown up by symmetric analysis and all of them will be highlighted in the next section. We also try to introduce new couplings in an EFT to be able to produce all those new bulk singularities as well.

5.1 Bulk singularity structures in the asymmetric picture

As we have already argued, the S-matrix of an RR and three transverse scalars in its asymmetric picture (in addition to all the singularities of the symmetric picture) generates two different kinds of bulk singularity structures. For instance the second term of $\mathcal{A}_{62}$ of the asymmetric picture ($\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$) has got a new kind of infinite u-channel bulk singularities which cannot be obtained from the symmetric picture of $\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle$. Indeed we would have expected these bulk singularities in the asymmetric picture, because of the symmetries with respect to interchanging all three scalars as well as symmetries of the string amplitude.

Therefore let us point out the first kind of u-channel bulk singularity structure in asymmetric analysis (which is the second term of $\mathcal{A}_{62}$) of ($\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$) as follows:

$$-2ip.\xi_1L_5\xi_2.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ba})k_{3b}k_{2a} \tag{34}$$

as well as t, s-channel bulk singularities coming from the second terms of $\mathcal{A}_{42}$ or (31) and $\mathcal{A}_{72}$ or (30) appropriately as follows:

$$2ip.\xi_3 L_2 \xi_2.\xi_1 \mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p \Gamma^{ac}) k_{3c} k_{2a}$$
$$-2ip.\xi_2 L_3 \xi_1.\xi_3 \mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p \Gamma^{bc}) k_{3b} k_{2c} \quad (35)$$

where in the first and second equations one can use momentum conservation to actually write k_{3c} (k_{2c}) in terms of k_{1c}. Regarding the symmetries of S-matrix we just produce u-channel bulk singularities then by interchanging momenta and polarizations we can easily produce t, s-channel bulk singularities as well.

Let us first generate these new u-channel bulk singularities of the S-matrix. We need to consider the following rule:

$$\mathcal{A} = V_\alpha^a(C_{p-1}, \phi_1, A) G_{\alpha\beta}^{ab}(A) V_\beta^b(A, \phi_2, \phi_3). \quad (36)$$

The kinetic terms of scalar field and gauge field have been taken into account to obtain the following vertex and propagator:

$$V_\beta^b(A, \phi_2, \phi_3) = iT_p(2\pi\alpha')^2 \xi_2.\xi_3 (k_2 - k_3)^b \mathrm{Tr}\,(\lambda_2\lambda_3\lambda_\beta),$$
$$G_{\alpha\beta}^{ab}(A) = \frac{i\delta_{\alpha\beta}\delta^{ab}}{(2\pi\alpha')^2 T_p u}. \quad (37)$$

In order to find $V_\alpha^a(C_{p-1}, \phi_1, A)$, one has to employ a Taylor expansion as follows:

$$S_8 = i(2\pi\alpha')^2 \mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p-1)!} (\varepsilon^v)^{a_0\cdots a_p} \times [\partial_i C_{a_0\ldots a_{p-2}} F_{a_{p-1}a_p} \phi^i] \quad (38)$$

then take the integration by parts. The gauge field here is Abelian; taking (38) to momentum space and considering the following equation:

$$-p.\xi_1 C_{a_0\ldots a_{p-2}} (p + k_1)_{a_{p-1}} = p.\xi_1 C_{a_0\ldots a_{p-2}} (k_2 + k_3)_{a_{p-1}},$$

we can obtain

$$V_\alpha^a(C_{p-1}, \phi_1, A) = i(2\pi\alpha')^2 \mu_p (\varepsilon^v)^{a_0\cdots a_{p-1}a} \times p.\xi_1 C_{a_0\ldots a_{p-2}} (k_2 + k_3)_{a_{p-1}} \mathrm{Tr}\,(\lambda_1\lambda_\alpha). \quad (39)$$

Both the propagator and $V_\beta^b(A, \phi_2, \phi_3)$ are derived from kinetic terms; thus there is not any correction to these terms. Therefore in order to explore all infinite u-channel bulk poles one should impose the all infinite higher derivative correction to (38) as

$$S_9 = i(2\pi\alpha')^2 \mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p-1)!} (\varepsilon^v)^{a_0\cdots a_p} \sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} \times [\partial_i C_{a_0\ldots a_{p-2}} D_{a_0} \cdots D_{a_n} F_{a_{p-1}a_p} D^{a_0} \cdots D^{a_n} \phi^i] \quad (40)$$

to get the all order extension of the above vertex operator as

$$V_\alpha^a(C_{p-1}, \phi_1, A) = \sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} (k_1.k)^{n+1} i(2\pi\alpha')^2 \mu_p (\varepsilon^v)^{a_0\cdots a_{p-1}a} \times p.\xi_1 C_{a_0\ldots a_{p-2}} (k_2 + k_3)_{a_{p-1}} \mathrm{Tr}\,(\lambda_1\lambda_\alpha). \quad (41)$$

Now if we replace (41) and (37) inside (36) then we are able to precisely produce all infinite u-channel bulk singularities of this amplitude as follows:

$$-2ip.\xi_1 \frac{16\mu_p\pi^2}{u(p-1)!} \sum_{n=-1}^{\infty} b_n (t+s)^{n+1} \xi_2.\xi_3 k_{3b} k_{2a} \times C_{a_0\ldots a_{p-2}} (\varepsilon^v)^{a_0\cdots a_{p-2}ba}. \quad (42)$$

The second kind of new bulk singularity structure is as follows.

We consider the following bulk singularity structures of asymmetric picture as well, which can be eventually simplified, by various algebraic calculations, as follows.

Indeed if we add the second term of $\mathcal{A}_1$ and the entire $\mathcal{A}_{43}$ of $\langle V_{C^{-2}} V_{\phi^0} V_{\phi^0} V_{\phi^0}\rangle$ we derive

$$\mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p) ip.\xi_3 \xi_2.\xi_1 \left(Q_2 + 2tQ_5\left\{us - \frac{u+s+t}{2}\right\}\right)$$
$$= \mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p)(-2iusp.\xi_3\xi_2.\xi_1) L_6. \quad (43)$$

Likewise if we add up the third term of $\mathcal{A}_1$ and the entire $\mathcal{A}_{73}$ of the asymmetric picture we obtain

$$\mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p) ip.\xi_2 \xi_3.\xi_1 \left(Q_3 + 2sQ_7\left\{ut - \frac{u+s+t}{2}\right\}\right)$$
$$= \mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p)(-2iutp.\xi_2\xi_3.\xi_1) L_6. \quad (44)$$

Finally we must add up the fourth term of $\mathcal{A}_1$ and the whole $\mathcal{A}_{63}$ of asymmetric S-matrix to be able to gain

$$\mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p) ip.\xi_1 \xi_3.\xi_2 \left(Q_4 + 2uQ_6\left\{st - \frac{u+s+t}{2}\right\}\right)$$
$$= \mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p)(-2istp.\xi_1\xi_3.\xi_2) L_6. \quad (45)$$

All the above terms must be added up as follows:

$$\mathrm{Tr}\,(P_-\mathcal{C}\!\!\!/_{(n-1)} M_p) p^i (-2ius\xi_{3i}\xi_2.\xi_1 - 2iut\xi_{2i}\xi_3.\xi_1 - 2ist\xi_{1i}\xi_3.\xi_2) L_6. \quad (46)$$

Having extracted the trace and replacing L_6 expansion in string amplitude, one finds out the second kind of new bulk singularity structure of BPS branes as follows:

$$16\pi\mu_p \frac{1}{(p+1)!(s+t+u)} \mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3) \sum_{n,m=0}^{\infty} (a_{n,m} + b_{n,m})$$
$$(-2ius\xi_{3i}\xi_1.\xi_2[s^m u^n + s^n u^m]$$
$$-2iut\xi_{2i}\xi_1.\xi_3[t^m u^n + t^n u^m]$$
$$-2its\xi_{1i}\xi_3.\xi_2[s^m t^n + s^n t^m])\epsilon^{a_0\cdots a_p} p^i C_{a_0\cdots a_p}. \quad (47)$$

This second new bulk singularity structure is related to new structures of an infinite $(t+s+u)$ bulk singularities. Indeed L_6 does have infinite $(t+s+u)$ channel singularities and in order to produce them in an EFT one has to consider the following rule:

$$\mathcal{A} = V_\alpha^i(C_{p+1}, \phi) G_{\alpha\beta}^{ij}(\phi) V_\beta^j(\phi, \phi_1, \phi_2, \phi_3). \quad (48)$$

The following coupling in an EFT is needed:

$$S_{10} = i(2\pi\alpha')\mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} D_{a_0}\phi^i C^{(p+1)}_{ia_1\cdots a_p} \quad (49)$$

where in (49) the scalar field has been taken from the pull-back. The trace in (46) shows the RR potential has to be $p+1$ form field. (49) is a new coupling that plays the crucial rule for matching all the infinite new bulk singularity structures of the string amplitude with effective field theory. If we take into account (49) and $p_{a_0}C^i_{a_1\cdots a_p} = p^i C_{a_0\cdots a_p}$ as well as the kinetic term of the scalar field $((2\pi\alpha')^2/2)D_a\phi^i D^a\phi_i$ we obtain

$$V^i_\alpha(C_{p+1},\phi) = i(2\pi\alpha')\mu_p \frac{1}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} p^i C_{a_0\cdots a_p}\mathrm{Tr}\,(\lambda_\alpha),$$
$$G^{ij}_{\alpha\beta}(\phi) = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2(t+s+u)}. \quad (50)$$

One needs to also impose the infinite higher derivative corrections to four real scalar field couplings that are derived in [41] as

$$(2\pi\alpha')^4\frac{1}{4\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}^{nm}_{11}+\mathcal{L}^{nm}_{12}+\mathcal{L}^{nm}_{13}), \quad (51)$$

$$\begin{aligned}
\mathcal{L}^{nm}_{11} &= -\mathrm{Tr}\,(\,a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D_b\phi_i D^b\phi^j D^a\phi_j] \\
&\quad + b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i D^b\phi^j D_b\phi_i D^a\phi_j] + h.c.\,),\\
\mathcal{L}^{nm}_{12} &= -\mathrm{Tr}\,(\,a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D_b\phi_i D^a\phi^j D^b\phi_j] \\
&\quad + b_{n,m}\mathcal{D}'_{nm}[D_b\phi^i D^b\phi^j D_a\phi_i D^a\phi_j] + h.c.\,),\\
\mathcal{L}^{nm}_{13} &= \mathrm{Tr}\,(\,a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^a\phi_i D_b\phi^j D^b\phi_j] \\
&\quad + b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i D_b\phi^j D^a\phi_i D^b\phi_j] + h.c.\,),
\end{aligned}$$

to actually obtain the following vertex:

$$\begin{aligned}
V^j_\beta(\phi,\phi_1,\phi_2,\phi_3) &= \frac{1}{4\pi^2}(\alpha')^{n+m}(a_{n,m}+b_{n,m})(2\pi\alpha')^4 T_p \\
&\quad \times(2ts\xi^j_1\xi_2.\xi_3(s^m t^n + s^n t^m) \\
&\quad +2ut\xi^j_2\xi_1.\xi_3(t^m u^n + t^n u^m) \\
&\quad +2us\xi^j_3\xi_1.\xi_2(s^m u^n + s^n u^m)) \\
&\quad \times\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3\lambda_\beta). \quad (52)
\end{aligned}$$

Now if we replace (52), (50) inside (48) then we are exactly able to produce all order $(t+s+u)$ channel bulk singularity structures of an RR and three scalar fields of the string amplitude (47) on the effective field theory side as well. Note that the important point was to derive the new coupling of a scalar field and a $p+1$ potential RR field as explored in (49).

Note that there had been another kind of $(t+s+u)$ channel poles which have already been discovered in [43]. For completeness let us very briefly just produce all infinite u- and t-channel poles accordingly through the field strength of RR. Eventually one can exchange the momenta and polarisations to get all infinite s-channel poles as well. We have

$$\begin{aligned}
\mathcal{A}_u &= \frac{32}{p!}\pi^2\mu_p\sum_{n=-1}^{\infty}\frac{1}{u}b_n(t+s)^{n+1}\xi_2 \\
&\quad .\xi_3 k_{2a}k_{3b}\xi_{1k}\epsilon^{a_0\cdots a_{p-2}ba}H^k_{a_0\cdots a_{p-2}}.
\end{aligned}$$

Now if we impose the mixed couplings of RR's field strength, a gauge field and a scalar field through the pull-back as follows:

$$\begin{aligned}
S_{11} &= (2\pi\alpha')^2\mu_p\int d^{p+1}\sigma\frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p} \\
&\quad \times F_{a_0a_1}D_{a_2}\phi^k C^{(p-1)}_{ka_3\ldots a_p} \quad (53)
\end{aligned}$$

and if we take integration by parts on the scalar field we come to know that D_{a_2} can just act on the C-field; because of the antisymmetric property of (ε^v) it cannot act on F. Thus one can explore the following vertex operator:

$$\begin{aligned}
V^a_\alpha(C_{p-1},\phi_1,A) &= \frac{(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a} \\
&\quad \times(H^{(p)})^k{}_{a_0\cdots a_{p-2}}\xi_{1k}k_{a_{p-1}}\mathrm{Tr}\,(\lambda_1\lambda_\alpha).
\end{aligned}$$

k is the momentum of the off-shell gauge field $k = k_2 + k_3$ and one imposes higher derivative corrections [5] to be able to get the all order extension of the vertex as follows:

$$\begin{aligned}
V^a_\alpha(C_{p-1},\phi_1,A) &= \frac{(2\pi\alpha')^2\mu_p}{(p)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a} \\
&\quad \times H^k{}_{a_0\cdots a_{p-2}}\xi_{1k}k_{a_{p-1}}\mathrm{Tr}\,(\lambda_1\lambda_\alpha) \\
&\quad \times\sum_{n=-1}^{\infty}b_n(t+s)^{n+1}. \quad (54)
\end{aligned}$$

Implementing (54) and (37) inside (36) one can exactly generate these infinite u-channel poles. Eventually one reads off all infinite t-channel poles as follows:

[5] We have

$$S_{12} = i(2\pi\alpha')^2\mu_p\int \mathrm{d}^{p+1}\sigma\frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}$$
$$\times\sum_{n=-1}^{\infty}b_n(\alpha')^{n+1}[\mathrm{Tr}\,(\partial_{a_{m_0}}\cdots\partial_{a_{m_n}}F_{a_0a_1}\partial^{a_{m_0}}\cdots\partial^{a_{m_n}}D_{a_2}\phi^k)C^{(p-1)}_{ka_3\ldots a_p}].$$

$$\mathcal{A}_2 = \pm\frac{16}{p!}\pi^2\mu_p \sum_{n=-1}^{\infty}\frac{1}{t}b_n(u+s)^{n+1} \times(\epsilon^{a_0\cdots a_{p-2}ba}H^k_{a_0\cdots a_{p-2}}(2\xi_1.\xi_2k_{2a}k_{3b}\xi_{3k}) + p^k\epsilon^{a_0\cdots a_{p-1}a}H_{a_0\cdots a_{p-1}}(-2k_{2a}\xi_1.\xi_2\xi_{3k}))\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3). \quad (55)$$

Having considered the Taylor expansion for the scalar field

$$S_{13} = (2\pi\alpha')^2\mu_p\int \mathrm{d}^{p+1}\sigma\frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}F_{a_0a_1}\phi^k\partial_k C^{(p-1)}_{a_2\ldots a_p} \quad (56)$$

and by integration by parts on the location of the gauge field, we get the following action:

$$S_{13} = i(2\pi\alpha')^2\mu_p\int \mathrm{d}^{p+1}\sigma\frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p} \times(-A_{a_1}\partial_{a_0}\phi^k\partial_k C^{(p-1)}_{a_2\ldots a_p} - A_{a_1}\phi^k\partial_{a_0}\partial_k C^{(p-1)}_{a_2\ldots a_p}).$$

We now apply the higher derivative corrections to (56) so that the following vertex can be obtained:

$$V^a_\alpha(C_{p-1},\phi_3,A) = \frac{(2\pi\alpha')^2\mu_p}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_{p-1}a}\mathrm{Tr}\,(\lambda_3\lambda_\alpha) \times\sum_{n=-1}^{\infty}b_n(s+u)^{n+1} \times p^k\xi_{3k}(p+k_3)_{a_{p-1}}(C^{(p-1)})_{a_0\cdots a_{p-2}}. \quad (57)$$

Taking the rule $A = V^a_\alpha(C_{p-1},\phi_3,A)G^{ab}_{\alpha\beta}(A)V^b_\beta(A,\phi_1,\phi_2)$ and the vertices as

$$V^b_\beta(A,\phi_1,\phi_2) = iT_p(2\pi\alpha')^2\xi_1.\xi_2(k_1-k_2)^b\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_\beta),$$
$$G^{ab}_{\alpha\beta}(A) = \frac{i\delta_{\alpha\beta}\delta^{ab}}{(2\pi\alpha')^2T_pt},$$

and also considering momentum conservation we obtain

$$V^b_\beta(A,\phi_1,\phi_2) = iT_p(2\pi\alpha')^2\xi_1.\xi_2(-2k_2-k_3-p)^b\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_\beta). \quad (58)$$

Now replacing (58) and (57) inside the above rule and making use of the fact that $(k_3+p)^2 = (k_1+k_2)^2 = t$ we are exactly able to produce all infinite t-channel poles of (55). Exchanging the momenta and polarizations we are also able to construct all infinite s-channel poles as well. Let us discuss contact interactions.

5.2 All order α' contact interactions of asymmetric S-matrix

To begin with, we start generating the contact terms of symmetric picture by comparing them with contact terms of the asymmetric picture. We address other contact interactions that appeared just in asymmetric S-matrix and essentially we write down new EFT couplings and explore their all order α' higher derivative corrections accordingly. First let us apply the momentum conservation to $\mathcal{A}_{81}$. It is easy to show that the first term of $\mathcal{A}'_1$ of $\langle V_{C^{-1}}V_{\phi^{-1}}V_{\phi^0}V_{\phi^0}\rangle$ can be reconstructed by the prescribed $\mathcal{A}_{81}$. Once more we did apply momentum conservation to $\mathcal{A}_{71}$ of $\langle V_{C^{-2}}V_{\phi^0}V_{\phi^0}V_{\phi^0}\rangle$ to derive

$$ip.\xi_2\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbic})\xi_{1i}\xi_{3k}Q_1(-k_{3c}-k_{2c}-p_c)k_{3b} \quad (59)$$

where due to antisymmetric property of ϵ tensor the first term in (59) has no contribution to the contact interactions and the third term does generate the second contact term interaction of $\mathcal{A}'_1$, meanwhile the second term in (59) will be an extra contact interaction in the asymmetric picture for which reason we consider it in the next section.

By the same arguments as $\mathcal{A}_{41}$ we obtain

$$ip.\xi_3\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jaic})\xi_{1i}\xi_{2j}Q_1(-k_{2c}-k_{3c}-p_c)k_{2a}; \quad (60)$$

needless to say that the first term in (60) has no contribution to the amplitude and the third term above can produce precisely third contact term interaction of $\mathcal{A}'_1$ while the second term in the above equation should be regarded as an extra contact interaction in the asymmetric picture that will be taken into account in the next section.

Finally let us produce the last contact interaction of $\mathcal{A}'_1$ of the symmetric picture as follows. In order to do so, one has to apply momentum conservation to the second term of $\mathcal{A}_3$ of the asymmetric picture to obtain

$$-ip.\xi_2p.\xi_3Q_1\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ic})(-k_{2c}-k_{3c}-p_c) \quad (61)$$

where the last term in (61) produces the last contact interaction of $\mathcal{A}'_1$ of the symmetric picture (the fourth term of $\mathcal{A}'_1$), meanwhile the other terms in (61) remain to be extra contact interactions in the asymmetric amplitude.

Henceforth, up to now we have been able to precisely construct or generate the entire contact terms that have been appearing in symmetric picture. However, as we have revealed the other terms of asymmetric S-matrix elements lead us to conclude that those terms are extra contact interactions that can just be explored in asymmetric picture of the amplitude for which we are going to consider them in the next section.

5.3 All order contact terms of asymmetric S-matrix

In previous section we compared the contact terms of the S-matrix element of an RR and three transverse scalars in both symmetric and asymmetric pictures. This leads us to explore various new contact interactions in an asymmetric picture of the amplitude. Therefore without further explanations we first write down all the other contact interactions that just appeared in the asymmetric picture as follows:

$$\mathcal{A}^{C^{-2}\phi^0\phi^0\phi^0} = \mathcal{A}''_{11} + \mathcal{A}''_{22} + \mathcal{A}''_3 + \mathcal{A}''_{421} + \mathcal{A}''_{52} + \mathcal{A}''_5 \\ \mathcal{A}''_{61} + \mathcal{A}''_{621} + \mathcal{A}''_{71} + \mathcal{A}''_{721} \quad (62)$$

where

$$\begin{aligned}
\mathcal{A}''_{11} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p)[p.\xi_1 p.\xi_2 p.\xi_3 Q_1],\\
\mathcal{A}''_{22} &\sim ik_{2a}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ja})\{-p.\xi_1 p.\xi_3 Q_1\},\\
\mathcal{A}''_{3} &\sim i(-k_{2c}-k_{3c})\xi_{1i}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ic})\{-p.\xi_2 p.\xi_3 Q_1\},\\
\mathcal{A}''_{5} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{jaic})\xi_{2j}\xi_{1i}p.\xi_3(-k_{3c})k_{2a}Q_1,\\
\mathcal{A}''_{421} &\sim ip.\xi_3 L_2\{t\xi_{1i}\xi_{2j}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ji})\},\\
\mathcal{A}''_{52} &\sim ik_{3b}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kb})\{-p.\xi_2 p.\xi_1 Q_1\},\\
\mathcal{A}''_{61} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbja})\xi_{2j}\xi_{3k}p.\xi_1 k_{3b}k_{2a}Q_1,\\
\mathcal{A}''_{621} &\sim ip.\xi_1 L_5\{u\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kj})\},\\
\mathcal{A}''_{71} &\sim i\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{kbic})\xi_{1i}\xi_{3k}p.\xi_2 k_{3b}(-k_{2c})Q_1,\\
\mathcal{A}''_{721} &\sim ip.\xi_2 L_3\{-s\xi_{1i}\xi_{3k}\mathrm{Tr}\,(P_-\not{C}_{(n-1)}M_p\Gamma^{ki})\},
\end{aligned}$$

where these contact terms can be generated in a new standard form of effective field theory couplings that we address it now. Note that all the leading term of the above couplings appear to be at $(\alpha')^3$ order. One can obtain their all order α' contact interactions as well.

The expansion for Q_1 is given in Appendix 2. If we consider $\mathcal{A}''_{11}$, extract the trace and employ the Taylor expansion for all three scalar fields, one can show that the leading contact interaction can be precisely obtained by the following coupling:

$$S_{14} = i(2\pi\alpha')^3\mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times(\partial_i\partial_j\partial_k C^{(p+1)}_{a_0\ldots a_p}\phi^i\phi^j\phi^k). \quad (63)$$

Now all $\mathcal{A}''_{22}$, $\mathcal{A}''_{52}$ and $\mathcal{A}''_3$ have the same structure so we just produce $\mathcal{A}''_{22}$. Therefore to produce this term in an EFT, one needs to extract the trace and show that the second scalar comes from the pull-back, while the first and third scalar come from the Taylor expansion. Hence, the leading contact interaction can be precisely derived by the following coupling:

$$S_{15} = i(2\pi\alpha')^3\mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times(D_{a_0}\phi^j\partial_i\partial_k C^{(p+1)}_{ja_1\ldots a_p}\phi^i\phi^k). \quad (64)$$

All order α' corrections to (64) can be derived by considering all the infinite terms appearing in the expansion of Q_1 as shown in Appendix 2.

Note that all $\mathcal{A}''_5$, $\mathcal{A}''_{61}$ and $\mathcal{A}''_{71}$ have the same structure so we just produce $\mathcal{A}''_5$. Therefore for this term one needs to first extract the trace and show that the first and the second scalar come from pull-back, while this time the third scalar comes from the Taylor expansion. Thus the leading contact interaction can be explored by the following coupling:

$$S_{16} = i(2\pi\alpha')^3\mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times(D_{a_0}\phi^i D_{a_1}\phi^j\partial_k C^{(p+1)}_{ija_2\ldots a_p}\phi^k)$$

where its all order α' corrections have already been given in Appendix 2.

Finally all $\mathcal{A}''_{421}$, $\mathcal{A}''_{621}$ and $\mathcal{A}''_{721}$ have the same structure and also $tL_2 = sL_3 = uL_5 = L_{22}$, the expansion of L_{22} is also given is Appendix 2. We just produce $\mathcal{A}''_{421}$. For this term one needs to first extract the trace and show that the third scalar comes from Taylor expansion, while the first and second scalar neither come from Taylor nor pull-back. The leading contact interaction can be precisely derived by the following coupling:

$$S_{17} = i(2\pi\alpha')^3\mu_p \int \mathrm{d}^{p+1}\sigma \frac{1}{(p+1)!}(\varepsilon^v)^{a_0\cdots a_p} \\ \times(\phi^i\phi^j\partial_k C^{(p+3)}_{ija_0\ldots a_p}\phi^k) \quad (65)$$

where its all order α' corrections can also be derived as we did in (A.5).

Therefore in this section not only did we derive new couplings of RR in an EFT approach but also we have been able to find the all order α' higher derivative interactions on both world volume and transverse directions.

6 Conclusion

In this paper we have analyzed in detail the four and five point functions of the string theory, including an RR in an asymmetric picture and either two or three real transverse scalar fields. We also compared the exact form of the S-matrix in an asymmetric picture with its own result in the symmetric picture. We have obtained two different kinds of new bulk singularity structures as well as various new couplings in the asymmetric picture that are absent in its symmetric picture. We have also generalized their all order α' higher derivative interactions as well.

These two different kinds of bulk singularity structures of the string amplitude are u, t, s (appeared in (34), (35)) as well as $(t+s+u)$-channel bulk singularity structures (47) that can just be explored in an asymmetric picture of the amplitude. These bulk singularity structures carry momentum of RR in transverse directions (that is, p^i, p^j, p^k terms). Note that these terms could have been derived if winding modes (w^i, w^j, w^k terms) were allowed in the vertex operator of RR. However, in the vertex of RR in ten dimensions of space-time there are no winding modes in both symmetric and asymmetric picture of RR.

Indeed these terms of the amplitude whose momenta of RR are carried in transverse direction cannot be obtained even by T-duality transformation in flat ten dimensions of space-time. Hence the presence of RR makes computations complicated as was explained in [41,42]. Hence we just explore these new bulk singularities as well as new couplings in the asymmetric picture of the amplitude. Indeed we are also able to produce these two different bulk singularity structures of the string amplitude in field theory by taking into account various new couplings in effective field theory side as well. We think that the importance of these results will be shown in future research topics, such as of the all order Myers effect and various other subjects in type II superstring theory [44].

We have also observed that at the level of EFT the supergravity background fields in DBI action must be some functions of super-Yang–Mills. We have also shown that some particular Taylor expansion for the background fields should be taken into account as was noticed in Dielectric effect [8]. These results might be important both in constructing higher point functions of string theory amplitudes as well as discovering symmetries or mathematical results behind scattering amplitudes. We hope to address these issues in near future.

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Appendix 1

All n_i's for the asymmetric amplitude are given by

$$
\begin{aligned}
n_1 &= (\eta^{ac}(\Gamma^{ji}C^{-1})_{\alpha\beta} + \eta^{ij}(\Gamma^{ac}C^{-1})_{\alpha\beta}),\\
n_2 &= (-\eta^{ac}\eta^{ij}(C^{-1})_{\alpha\beta}),\\
n_3 &= (\eta^{ab}(\Gamma^{kj}C^{-1})_{\alpha\beta} + \eta^{jk}(\Gamma^{ba}C^{-1})_{\alpha\beta}),\\
n_4 &= (-\eta^{ab}\eta^{jk}(C^{-1})_{\alpha\beta}),\\
n_5 &= (\eta^{bc}(\Gamma^{ki}C^{-1})_{\alpha\beta} + \eta^{ik}(\Gamma^{bc}C^{-1})_{\alpha\beta}),\\
n_6 &= (-\eta^{bc}\eta^{ik}(C^{-1})_{\alpha\beta}).
\end{aligned} \tag{A.1}
$$

Indeed the correlation function of three currents with two spin fields, does carry so many terms. One can further simplify it and write it down in a very compact formula as below

$$
I_8^{kbjaic} = \langle : S_\alpha(x_4) : S_\beta(x_5) :: \psi^c\psi^i(x_1) : \psi^a\psi^j(x_2) : \psi^b\psi^k(x_3)\rangle \tag{A.2}
$$

with the following ingredients:

$$
\begin{aligned}
I_8^{kbjaic} &= \Big\{(\Gamma^{kbjaic}C^{-1})_{\alpha\beta} + \alpha' n_7\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\\
&+\alpha' n_8\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}} + \alpha' n_9\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}} + \alpha'^2 n_{10}\\
&\times\left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)^2 + \alpha'^2 n_{11}\left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)\\
&+\alpha'^2 n_{12}\left(\frac{Re[x_{14}x_{25}]}{x_{12}x_{45}}\right)\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)\\
&+\alpha'^2 n_{13}\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)^2 + \alpha'^2 n_{14}\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)^2\\
&+\alpha'^2 n_{15}\left(\frac{Re[x_{14}x_{35}]}{x_{13}x_{45}}\right)\left(\frac{Re[x_{24}x_{35}]}{x_{23}x_{45}}\right)\Big\}\\
&\times 2^{-3}x_{45}^{7/4}(x_{14}x_{15}x_{24}x_{25}x_{34}x_{35})^{-1}
\end{aligned}
$$

where

$$
\begin{aligned}
n_7 &= (\eta^{ac}(\Gamma^{kbji}C^{-1})_{\alpha\beta} + \eta^{ij}(\Gamma^{kbac}C^{-1})_{\alpha\beta}),\\
n_8 &= (\eta^{bc}(\Gamma^{kjai}C^{-1})_{\alpha\beta} + \eta^{ik}(\Gamma^{bjac}C^{-1})_{\alpha\beta}),\\
n_9 &= (\eta^{ab}(\Gamma^{kjic}C^{-1})_{\alpha\beta} + \eta^{jk}(\Gamma^{baic}C^{-1})_{\alpha\beta}),\\
n_{10} &= (-\eta^{ac}\eta^{ij}(\Gamma^{kb}C^{-1})_{\alpha\beta}),\\
n_{11} &= (-\eta^{ac}\eta^{ik}(\Gamma^{bj}C^{-1})_{\alpha\beta} + \eta^{bc}\eta^{ij}(\Gamma^{ka}C^{-1})_{\alpha\beta}),\\
n_{12} &= (\eta^{ac}\eta^{jk}(\Gamma^{bi}C^{-1})_{\alpha\beta} - \eta^{ab}\eta^{ij}(\Gamma^{kc}C^{-1})_{\alpha\beta}),\\
n_{13} &= (-\eta^{bc}\eta^{ik}(\Gamma^{ja}C^{-1})_{\alpha\beta}),\\
n_{14} &= (-\eta^{ab}\eta^{jk}(\Gamma^{ic}C^{-1})_{\alpha\beta}),\\
n_{15} &= (-\eta^{bc}\eta^{jk}(\Gamma^{ai}C^{-1})_{\alpha\beta} + \eta^{ik}\eta^{ab}(\Gamma^{jc}C^{-1})_{\alpha\beta}).
\end{aligned}
$$

The integral could be carried over just in terms of Gamma functions and no longer any hypergeometric function appears, where one needs to employ the following sort of integrations on the upper half plane [61]:

$$
\int d^2z|1-z|^a|z|^b(z-\bar{z})^c(z+\bar{z})^d
$$

where a, b, c are arbitrary Mandelstam variables where for $d = 0, 1$ the result is given [61] and for $d = 2$ one needs to work things out [27] with the following Mandelstam definitions:

$$
\begin{aligned}
s &= \frac{-\alpha'}{2}(k_1+k_3)^2, \quad t = \frac{-\alpha'}{2}(k_1+k_2)^2,\\
u &= \frac{-\alpha'}{2}(k_2+k_3)^2.
\end{aligned}
$$

Appendix 2

The functions $Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_7, L_2, L_3, L_5, L_6$ are given by

$$\begin{aligned}
Q_1 &= (2)^{-2(t+s+u)+1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u+1)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
Q_2 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t-\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t)\Gamma(-t-s)\Gamma(-s-u+1)},\\
Q_3 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s-\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s)\Gamma(-s-u)},\\
Q_4 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u-\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t)\Gamma(-t-s+1)\Gamma(-s-u)},\\
L_2 &= (2)^{-2(t+s+u)}\pi \\
&\quad\times\frac{\Gamma(-u+1)\Gamma(-s+1)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
Q_5 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t-\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
Q_6 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u-\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
Q_7 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s-\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
L_3 &= (2)^{-2(t+s+u)}\pi \\
&\quad\times\frac{\Gamma(-u+1)\Gamma(-s)\Gamma(-t+1)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
L_5 &= (2)^{-2(t+s+u)}\pi \\
&\quad\times\frac{\Gamma(-u)\Gamma(-s+1)\Gamma(-t+1)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)},\\
L_6 &= (2)^{-2(t+s+u)-1}\pi \\
&\quad\times\frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}.
\end{aligned} \tag{A.3}$$

The expansion of Q_1 is

$$\begin{aligned}
Q_1 &= -\frac{2\pi^{5/2}}{3}\Bigg(3\sum_{n=0}^{\infty}c_n(s+t+u)^{n+1} \\
&\quad+\sum_{n,m=0}^{\infty}c_{n,m}[(s^n t^m+s^m t^n) \\
&\quad+(s^n u^m+s^m u^n)+(u^n t^m+u^m t^n)] \\
&\quad+\sum_{p,n,m=0}^{\infty}f_{p,n,m}(s+t+u)^{p+1}[(s+t)^n(st)^m \\
&\quad+(s+u)^n(su)^m+(u+t)^n(ut)^m]\Bigg).
\end{aligned} \tag{A.4}$$

One can obtain all order α' corrections to (63) by considering all the higher derivative terms appearing in the expansion of Q_1 as explained in the following:

$$\begin{aligned}
&(st)^m C\phi_1\phi_2\phi_3 = (\alpha')^{2m} C D_{a_1}\cdots D_{a_{2m}}\phi_1 D^{a_1}\cdots \\
&\quad D^{a_m}\phi_2 D^{a_{m+1}}\cdots D^{a_{2m}}\phi_3,\\
&(s+t)^n C\phi_1\phi_2\phi_3 = (\alpha')^n C D_{a_1}\cdots D_{a_n}\phi_1 D^{a_1}\cdots D^{a_n}(\phi_2\phi_3),\\
&(s)^m t^n C\phi_1\phi_2\phi_3 = (\alpha')^{n+m} C D_{a_1}\cdots D_{a_n} D_{a_1} \\
&\quad\cdots D_{a_m}\phi_1 D^{a_1}\cdots D^{a_n}\phi_2 D^{a_1}\cdots D^{a_m}\phi_3,\\
&(s)^n t^m C\phi_1\phi_2\phi_3 = (\alpha')^{n+m} C D_{a_1}\cdots D_{a_n} D_{a_1}\cdots D_{a_m}\phi_1 D^{a_1} \\
&\quad\cdots D^{a_m}\phi_2 D^{a_1}\cdots D^{a_n}\phi_3,\\
&(s+t+u)^{p+1}C,\phi_1\phi_2\phi_3 \\
&\quad=\left(\frac{\alpha'}{2}\right)^{p+1} C(D_a D^a)^{p+1}(\phi_1\phi_2\phi_3),
\end{aligned} \tag{A.5}$$

where the coefficients can be found in [9] and in the above equation all the commutator terms should not be considered. The expansion for L_{22} is

$$\begin{aligned}
L_{22} &= \frac{\pi^{3/2}}{3}\Bigg(\sum_{n=-1}^{\infty}b_n(s+u)^{n+1}+\sum_{p,n,m=0}^{\infty}e_{p,n,m}t^{p+1}(us)^n(u+s)^m \\
&\quad+\sum_{n=-1}^{\infty}b_n(s+t)^{n+1}+\sum_{p,n,m=0}^{\infty}e_{p,n,m}u^{p+1}(ts)^n(t+s)^m \\
&\quad+\sum_{n=-1}^{\infty}b_n(u+t)^{n+1}+\sum_{p,n,m=0}^{\infty}e_{p,n,m}s^{p+1}(tu)^n(t+u)^m\Bigg).
\end{aligned} \tag{A.6}$$

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