Eur. Phys. J. C (2017) 77:463
DOI 10.1140/epjc/s10052-017-5028-x

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Warm inflation with an oscillatory inflaton in the non-minimal kinetic coupling model

Parviz Goodarzi[1,a], **H. Mohseni Sadjadi**[2,b]

[1] Department of Science, University of Ayatollah Ozma Borujerdi, Boroujerd, Iran
[2] Department of Physics, University of Tehran, Tehran, Iran

Received: 12 February 2017 / Accepted: 26 June 2017 / Published online: 12 July 2017

Abstract In the cold inflation scenario, the slow roll inflation and reheating via coherent rapid oscillation, are usually considered as two distinct eras. When the slow roll ends, a rapid oscillation phase begins and the inflaton decays to relativistic particles reheating the Universe. In another model dubbed warm inflation, the rapid oscillation phase is suppressed, and we are left with only a slow roll period during which the reheating occurs. Instead, in this paper, we propose a new picture for inflation in which the slow roll era is suppressed and only the rapid oscillation phase exists. Radiation generation during this era is taken into account, so we have warm inflation with an oscillatory inflaton. To provide enough e-folds, we employ the non-minimal derivative coupling model. We study the cosmological perturbations and compute the temperature at the end of warm oscillatory inflation.

1 Introduction

In the standard inflation model, the accelerated expansion and the reheating epochs are two distinct eras [1–3]. But in the warm inflation, relativistic particles are produced during the slow roll. Therefore, the warm inflation explains the slow roll and onset of the radiation dominated era in a unique framework [4–7]. Warm inflation is a good model for large scale structure formation, in which the density fluctuations arise from thermal fluctuation [8,9]. Various models have been proposed for warm inflation, e.g. tachyon warm inflation, warm inflation in loop quantum cosmology, etc. [10–12].

Oscillating inflation was first introduced in [13], where it was proposed that the inflation may continue, after the slow roll, during rapid coherent oscillation in the reheating era. An expression for the corresponding number of e-folds was obtained in [3].

Scalar field oscillation in inflationary model was also pointed out briefly in [14], where the decay of scalar fields during their oscillations to inflaton particles was proposed.

A brief investigation of the adiabatic perturbation in the oscillatory inflation can be found in [15]. The formalism used in [13] was extended in [16], by considering a coupling between inflaton and the Ricci scalar curvature. The shape of the potential, required to end the oscillatory inflation, was investigated in [17]. The rapid oscillatory phase provides a few e-folds so we cannot ignore the slow roll era in this formalism. Due to small few number of e-folds, a detailed study of the evolution of quantum fluctuations has not been performed. To cure this problem, one can consider a non-minimal derivative coupling model. The cosmological aspects of this model have been widely studied in the literature [18–39].

The oscillatory inflation in the presence of a non-minimal kinetic coupling was studied in [40] and it was shown that in the high-friction regime, the non-minimal coupling increases the number of e-folds and so can remedy the problem of the smallness of the number of e-folds arising in [13]. Scalar and tensor perturbations and power spectrum and spectral index for scalar and tensor modes in oscillatory inflation were derived in [40], in agreement with Planck 2013 data. However, it is not clear from this scenario how reheating occurs or the Universe becomes radiation dominated after the end of inflation. For a non-minimal derivative coupling model, the reheating process after the slow roll and warm slow roll inflation are studied in [41–48], respectively.

In the present work, inspired by the models mentioned above, we will consider oscillatory inflation in non-minimal derivative coupling model.

We will assume that the inflaton decays to the radiation during the oscillation, providing a new scenario: warm oscillatory inflation. Equivalently, this can be viewed as an oscillatory reheating phase which is not preceded by the slow roll.

[a] e-mail: goodarzi@ut.ac.ir
[b] e-mail: mohsenisad@ut.ac.ir

In Sect. 2, we examine conditions for warm oscillatory inflation and study the evolution of energy density of the scalar field and radiation. In Sect. 3, the thermal fluctuation is considered and spectral index and power spectrum are computed. We will consider observational constraints on oscillatory warm inflation parameters by using Planck 2015 data [49–53]. In Sect. 4, the temperature at the end of warm inflation is calculated. We will compute tensor perturbation in Sect. 5 and in Sect. 6, we conclude our results.

We use units $\hbar = c = 1$ throughout this paper.

2 Oscillatory warm inflation

In this section, based on our previous work [40–42], we will introduce the rapid oscillatory inflaton decaying to radiation in a non-minimal kinetic coupling model. We start with the action [54]

$$S = \int \left(\frac{M_{\rm P}^2}{2} R - \frac{1}{2} \Delta^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi) \right) \sqrt{-g} \mathrm{d}^4 x + S_{\rm int} + S_{\rm r}, \quad (1)$$

where $\Delta^{\mu\nu} = g^{\mu\nu} + \frac{1}{M^2} G^{\mu\nu}$, $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} R g^{\mu\nu}$ is the Einstein tensor, M is a coupling constant with mass dimension, $M_{\rm P} = 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $S_{\rm r}$ is the radiation action and $S_{\rm int}$ describes the interaction of the scalar field with radiation. There are no terms containing more than two times the derivative, so we have no additional degrees of freedom in this theory. We can calculate the energy-momentum tensor by variation of action with respect to the metric,

$$T_{\mu\nu} = T_{\mu\nu}^{(\varphi)} + \frac{1}{M^2} \Theta_{\mu\nu} + T_{\mu\nu}^{(r)}. \quad (2)$$

The energy-momentum tensor for radiation is

$$T_{\mu\nu}^{(r)} = (\rho_{\rm r} + P_{\rm r}) u_\mu u_\nu + P_{\rm r} g_{\mu\nu}, \quad (3)$$

where u^μ is the four-velocity of the radiation and $T_{\mu\nu}^{(\varphi)}$ is the minimal coupling counterpart of the energy-momentum tensor,

$$T_{\mu\nu}^{(\varphi)} = \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{2} g_{\mu\nu} (\nabla\varphi)^2 - g_{\mu\nu} V(\varphi). \quad (4)$$

The energy-momentum tensor corresponding to the non-minimal coupling term is

$$\begin{aligned}\Theta_{\mu\nu} = & -\frac{1}{2} G_{\mu\nu} (\nabla\varphi)^2 - \frac{1}{2} R \nabla_\mu \varphi \nabla_\nu \varphi + R_\mu^\alpha \nabla_\alpha \varphi \nabla_\nu \varphi \\ & + R_\nu^\alpha \nabla_\alpha \varphi \nabla_\mu \varphi + R_{\mu\alpha\nu\beta} \nabla^\alpha \varphi \nabla^\beta \varphi + \nabla_\mu \nabla^\alpha \varphi \nabla_\nu \nabla_\alpha \varphi \\ & - \nabla_\mu \nabla^\nu \varphi \Box \varphi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \nabla^\beta \varphi \nabla_\alpha \nabla_\beta \varphi + \frac{1}{2} g_{\mu\nu} (\Box\varphi)^2 \\ & - g_{\mu\nu} \nabla_\alpha \varphi \nabla_\beta \varphi R^{\alpha\beta}. \quad (5)\end{aligned}$$

Energy transfer between the scalar field and radiation is assumed to be

$$Q_\mu = -\Gamma u^\nu \partial_\mu \varphi \partial_\nu \varphi, \quad (6)$$

where

$$\nabla^\mu T_{\mu\nu}^{(r)} = Q^\nu \quad \text{and} \quad \nabla^\mu \left(T_{\mu\nu}^{(\varphi)} + \frac{1}{M^2} \Theta_{\mu\nu} \right) = -Q_\nu. \quad (7)$$

The scalar field equation of motion, in the Friedmann–Lemaître–Robertson–Walker (FLRW) metric, is

$$\left(1 + \frac{3H^2}{M^2}\right) \ddot{\varphi} + 3H \left(1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}\right) \dot{\varphi} + V'(\varphi) + \Gamma \dot{\varphi} = 0, \quad (8)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, a dot is differentiation with respect to cosmic time t, and a prime denotes differentiation with respect to the scalar field φ.

Γ is a positive constant, first introduced in [55] as a phenomenological term which describes the decay of φ to the radiation during reheating era. This term was vastly used in the subsequent literature studying the inflaton decay in the reheating era (see [56,57] and the references therein), where like our model the inflaton experiences a rapid oscillation phase. In [58,59], it is shown that the production of particles during high-frequency regime in reheating era can be expressed by adding a polarization term to the inflaton mass. To do so, a Lagrangian comprising the inflaton field and its interactions with bosonic and fermionic fields was employed. It was shown that the phenomenological term proposed in [56] can be derived in this context. The precise form of the dissipative term depends on the coupling between the inflaton and the relativistic particles it decays to, and also on the interactions of relativistic particles. As the nature of the inflaton and these relativistic particles are not yet completely known, the precise form of Γ is not clear.

However, one may employ a phenomenological effective field theory, or also thermal field theory [60], to study the effective dependency of Γ on temperature and dynamical fields. Thermal effect can also be inserted by including the thermal correction in the equations of motion [60]. In the period where the inflaton is dominant over relativistic thermal particles, it is safe to approximately take Γ as $\Gamma = \Gamma\Big|_{T=0}$ (like [55]), as explained in [61].

Similarly, in the framework of slow roll warm inflation, the possibility that Γ is a function of φ and temperature was discussed in the literature [7,62].

The Friedmann equations are given by

$$H^2 = \frac{1}{3M_{\rm P}^2} \left(\rho_\varphi + \rho_{\rm r}\right)$$

$$\dot{H} = -\frac{1}{2M_{\rm P}^2}\left(\rho_\varphi + \rho_{\rm r} + P_\varphi + P_{\rm r}\right). \qquad (9)$$

The energy density and the pressure of the inflaton can be expressed as

$$\rho_\varphi = \left(1 + \frac{9H^2}{M^2}\right)\frac{\dot{\varphi}^2}{2} + V(\varphi), \qquad (10)$$

and

$$P_\varphi = \left(1 - \frac{3H^2}{M^2} - \frac{2\dot{H}}{M^2}\right)\frac{\dot{\varphi}^2}{2} - V(\varphi) - \frac{2H\dot{\varphi}\ddot{\varphi}}{M^2}, \qquad (11)$$

respectively. Energy density of radiation is $\rho_{\rm r} = \frac{3}{4}TS$ [7]. S is the entropy density and T is the temperature. The equation of state parameter for radiation is $\frac{1}{3}$, hence the rate of radiation production is given by

$$\dot{\rho}_{\rm r} + 4H\rho_{\rm r} = \Gamma\dot{\varphi}^2. \qquad (12)$$

We assume that the potential is even, $V(-\varphi) = V(\varphi)$, and consider a rapid oscillating solution (around $\varphi = 0$) to (8), which in the high-friction regime, $H^2 \gg M^2$, reduces to

$$\ddot{\varphi} + 3H\left(1 + \frac{2}{3}\frac{\dot{H}}{H^2}\right)\dot{\varphi} + \frac{M^2 V_{,\varphi}}{3H^2} + \frac{M^2\Gamma}{3H^2} = 0. \qquad (13)$$

In our formalism the inflation has a quasi-periodic evolution,

$$\varphi(t) = \phi(t)\cos\left(\int A(t)\mathrm{d}t\right), \qquad (14)$$

with time dependent amplitude $\phi(t)$. The rapid oscillation (or high-frequency oscillation) is characterized by

$$\left|\frac{\dot{H}}{H}\right| \ll A, \quad \left|\frac{\dot{\phi}}{\phi}\right| \ll A, \quad \left|\frac{\dot{\rho}_\varphi}{\rho_\varphi}\right| \ll A. \qquad (15)$$

The existence of such a solution is verified in [41]. It is worth to note that for a power-law potential $V(\varphi) = \lambda\varphi^q$, (15) holds provided that

$$\Phi \ll \left(\frac{q^2 M_{\rm P}^4 M^2}{\lambda}\right)^{\frac{1}{q+2}}, \qquad (16)$$

which is opposite to the slow roll condition $\varphi^{q+2} \gg \left(\frac{M_{\rm P}^4 M^2}{\lambda}\right)$ [41].

The period of oscillation is

$$\tau(t) = 2\int_{-\phi}^{\phi}\frac{\mathrm{d}\varphi(t)}{\dot{\varphi}(t)}, \qquad (17)$$

and the rapid oscillation occurs for $H \ll \frac{1}{\tau}$ and $\frac{\dot{H}}{H} \ll \frac{1}{\tau}$. The inflaton energy density may estimated as $\rho_\varphi = V(\phi(t))$. In this epoch ρ_φ and H change insignificantly during a period of oscillation in the sense indicated in (15).

In the rapid oscillatory phase, the time average of the adiabatic index, defined by $\gamma = \frac{\rho_\varphi + P_\varphi}{\rho_\varphi}$, is given by $\gamma = \left\langle\frac{\rho_\varphi + P_\varphi}{\rho_\varphi}\right\rangle$, where the bracket denotes time averaging over one oscillation,

$$\langle O(t)\rangle = \frac{\int_t^{t+\tau} O(t')\mathrm{d}t'}{\tau}. \qquad (18)$$

For a power-law potential

$$V(\varphi) = \lambda\varphi^q, \qquad (19)$$

and in the high-friction limit ($\frac{H^2}{M^2} \gg 1$), the adiabatic index becomes [40]

$$\gamma \approx \frac{2q}{3q+6}. \qquad (20)$$

By averaging the continuity equation, we obtain [41]

$$<\dot{\rho_\varphi>} + 3H\gamma < \rho_\varphi > + \frac{\gamma\Gamma M^2}{3H^2} < \rho_\varphi >= 0. \qquad (21)$$

When the Universe is dominated by φ-particles, we take

$$\Gamma \ll \frac{9H^3}{M^2}. \qquad (22)$$

By this assumption the radiation may be still in equilibrium, and besides we can neglect the third term in (21). But as H decreases, and the radiation production term becomes more relevant, this approximation fails and the third terms in (21) get the same order of magnitude as the second term at a time $t_{\rm rh}$. Note that at $t_{\rm rh}$ the radiation and inflaton densities have the same order of magnitude, $\rho_\varphi(t_{\rm rh}) \sim \rho_{\rm r}(t_{\rm rh})$ [41,57]. When $t < t_{\rm rh}$, the average of the energy density of the scalar field can be approximated as

$$\langle\rho_\varphi\rangle \propto a(t)^{-3\gamma}. \qquad (23)$$

By using Eq. (23) and the Friedmann equation ($H^2 \approx \frac{1}{3M_{\rm P}^2}\rho_\varphi$), in the φ dominated era, we can easily obtain

$$a(t) \propto t^{\frac{q+2}{q}} \propto t^{\frac{2}{3\gamma}}. \qquad (24)$$

Therefore the Hubble parameter in the inflaton dominated era can be estimated as $H \approx \frac{2}{3\gamma t}$. In the rapid oscillation phase and with the power-law potential (19) we can write the amplitude of the oscillation as

$$\phi(t) \propto a(t)^{-\frac{2}{q+2}} \propto t^{-\frac{2}{q}}. \tag{25}$$

Our formalism is similar to methods used in the papers studying the reheating era after inflation in the minimal case [57]. But in the minimal case, for $\Gamma << 3H$ until $\Gamma \sim H$, where the Universe is dominated by the oscillating inflaton, instead of (24), we have $a(t) \propto t^{\frac{2}{3}}$.

In the high-friction limit, time averaging over one oscillation gives

$$\langle \dot{\varphi}^2(t) \rangle = \frac{2M^2}{9H^2(t)} \langle \rho_\varphi(t) - V(\varphi(t)) \rangle, \tag{26}$$

where we have used the fact that the Hubble parameter changes insignificantly during one period of oscillation. But

$$\begin{aligned} \langle \rho_\varphi(t) - V(\varphi(t)) \rangle &= \frac{\int_{-\phi(t)}^{\phi(t)} \sqrt{\rho_\varphi - V(\varphi)} \mathrm{d}\varphi}{\int_{-\phi(t)}^{\phi(t)} \frac{\mathrm{d}\varphi}{\sqrt{\rho_\varphi - V(\varphi)}}} \\ &= \lambda \phi^q(t) \frac{\int_0^1 \sqrt{1-x^q} \mathrm{d}x}{\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^q}}} \\ &= \lambda \phi^q(t) \frac{q}{q+2}, \end{aligned} \tag{27}$$

therefore

$$\langle \dot{\varphi}^2 \rangle \approx \gamma M_{\mathrm{P}}^2 M^2. \tag{28}$$

This relation shows that, for non-minimal derivative coupling model and in the rapid oscillation phase, when the Universe is φ dominated, $\langle \dot{\varphi}^2 \rangle$ is approximately a constant. By inserting (28) into Eq. (12) we obtain

$$\rho_{\mathrm{r}} = \frac{3\Gamma \gamma^2 M^2 M_{\mathrm{P}}^2}{(8+3\gamma)} t \left[1 - \left(\frac{t_0}{t} \right)^{\left(1+\frac{8}{3\gamma}\right)} \right], \tag{29}$$

where t_0 is the time at which $\rho_{\mathrm{r}} = 0$. The number of e-folds from a specific time $t_* \in (t_0, t_{\mathrm{RD}})$ in inflation until radiation dominated epoch is given by

$$\mathcal{N}_{\mathrm{I}} = \int_{t_*}^{t_{\mathrm{RD}}} H \mathrm{d}t \approx \int_{t_*}^{t_{\mathrm{RD}}} \frac{2}{3\gamma t} \mathrm{d}t \approx \frac{2}{3\gamma} \ln \left(\frac{t_{\mathrm{RD}}}{t_*} \right), \tag{30}$$

where t_{RD} is the time at which the Universe becomes radiation dominated and inflation ceases. At this time

$$\rho_{\mathrm{r}}(T_{\mathrm{RD}}) \approx \rho_\varphi(t_{\mathrm{RD}}). \tag{31}$$

We can calculate the temperature at the end of warm inflation by [4–6]

$$\rho_{\mathrm{r}}(t_{\mathrm{RD}}) = g_{\mathrm{RD}} \frac{\pi^2}{30} T_{\mathrm{RD}}^4, \tag{32}$$

where g_{RD} is number of degree of freedom of relativistic particles and T_{RD} is the temperature of radiation at the beginning of radiation dominated era.

3 Cosmological perturbations

In this section, we study the evolution of thermal fluctuation during oscillatory warm inflation. We use the framework used in [8] and ignore the possible viscosity terms and shear viscous stress [65]. To investigate cosmological perturbations, we split the metric into two components: the background and the perturbations. The background is described by homogeneous and isotropic FLRW metric with oscillatory scalar field and the perturbed sector of the metric determines anisotropy. We assume that the radiation is in thermal equilibrium during warm inflation. The thermal fluctuations arising in warm inflation evolve gradually via cosmological perturbations equations. Until the freeze out time, the thermal noise has not a significant effect on perturbations development [8]. We consider the evolution equation of the first order cosmological perturbations for a system containing inflaton and radiation. In the longitudinal gauge the metric can be written as [63].

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1-2\Psi)\delta_{ij}\mathrm{d}x^i \mathrm{d}x^j. \tag{33}$$

As mentioned before, the energy-momentum tensor splits into radiation part $T_{\mathrm{r}}^{\mu\nu}$ and inflaton part $T_\varphi^{\mu\nu}$ as

$$T^{\mu\nu} = T_{\mathrm{r}}^{\mu\nu} + T_\varphi^{\mu\nu}. \tag{34}$$

The unperturbed parts of four-velocity components of the radiation fluid satisfy $\overline{u}_{ri} = 0$ and $\overline{u}_{r0} = -1$. By using normalization condition $g^{\mu\nu}u_\mu u_\nu = -1$, the perturbed part of the time component of the four-velocity becomes

$$\delta u^0 = \delta u_0 = \frac{h_{00}}{2}. \tag{35}$$

The space components, δu^i, are independent dynamical variables and $\delta u_i = \partial_i \delta u$ [63]. Energy transfer is described by [64]

$$Q_\mu = -\Gamma u^\nu \partial_\mu \varphi \partial_\nu \varphi. \tag{36}$$

We have also

$$\nabla_\mu T_{\rm r}^{\mu\nu} = Q^\nu, \quad (37)$$

and

$$\nabla_\mu T_{\varphi}^{\mu\nu} = -Q^\nu. \quad (38)$$

Equation (36) gives $Q_0 = \Gamma\dot{\varphi}^2$ and the unperturbed Eq. (37) becomes $Q_0 = \dot{\rho}_{\rm r} + 3H(\rho_{\rm r} + P_{\rm r})$, which is the continuity equation for the radiation field. In the same way Eq. (38) becomes $-Q_0 = \dot{\rho}_\varphi + 3H(\rho_\varphi + P_\varphi)$. Perturbations to the energy momentum transfer are described by (there is no perturbation for the dissipation factor Γ, which we have assumed to be a constant)

$$\delta Q_0 = -\delta\Gamma\dot{\varphi}^2 + \Phi\Gamma\dot{\varphi}^2 - 2\Gamma\dot{\varphi}\dot{\delta\varphi} \quad (39)$$

and

$$\delta Q_i = -\Gamma\dot{\varphi}\partial_i\delta\varphi. \quad (40)$$

The variation of Eq. (37) is $\delta(\nabla_\mu T_{\rm r}^{\mu\nu}) = \delta Q^\nu$, so its (0-0) component is

$$\dot{\delta\rho_{\rm r}} + 4H\delta\rho_{\rm r} + \frac{4}{3}\rho_{\rm r}\nabla^2\delta u - 4\dot{\Psi}\rho_{\rm r} = -\Phi\Gamma\dot{\varphi}^2 + \delta\Gamma\dot{\varphi}^2 + 2\Gamma\dot{\delta\varphi}\dot{\varphi}. \quad (41)$$

Similarly, for the ith component we derive

$$4\rho_{\rm r}\dot{\delta u^i} + 4\dot{\rho}_{\rm r}\delta u^i + 20H\rho_{\rm r}\delta u^i = -[3\Gamma\dot{\varphi}\partial_i\delta\varphi + \partial_i\delta\rho_{\rm r} + 4\rho_{\rm r}\partial_i\Phi]. \quad (42)$$

The equation of motion for $\delta\varphi$, computed by variation of (38), is $\delta(\nabla_\mu T_\varphi^{\mu\nu}) = -\delta Q^\nu$. The zero component of this equation is

$$\begin{aligned}
&\left(1 + \frac{3H^2}{M^2}\right)\ddot{\delta\varphi} + \left[(1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2})3H + \Gamma\right]\dot{\delta\varphi} + \delta V'(\varphi) \\
&\quad + \dot{\varphi}\delta\Gamma - \left(1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}\right)\frac{\nabla^2\delta\varphi}{a^2} \\
&= -\left[2V'(\varphi) + 3\Gamma\dot{\varphi} - \frac{6H\dot{\varphi}}{M^2}(3H^2 + 2\dot{H})\right. \\
&\quad \left. - \frac{6H^2\ddot{\varphi}}{M^2}\right] + \left(1 + \frac{9H^2}{M^2}\right)\dot{\varphi}\dot{\Phi}\Phi \\
&\quad + \frac{2H\dot{\varphi}}{M^2}\frac{\nabla^2\Phi}{a^2} + 3\left(1 + \frac{9H^2}{M^2} + \frac{2\dot{H}}{M^2}\right. \\
&\quad \left. + \frac{2H\ddot{\varphi}}{M^2}\right)\dot{\Psi} + \frac{6H\dot{\varphi}}{M^2}\ddot{\Psi} - \frac{2(\ddot{\varphi} + H\dot{\varphi})}{M^2}\frac{\nabla^2\Psi}{a^2}.
\end{aligned} \quad (43)$$

The 00 component of the perturbation of the Einstein equation $G_{\mu\nu} = -8\pi G T_{\mu\nu}$ is

$$\begin{aligned}
&-3H\dot{\Psi} - 3H^2\Phi + \frac{\nabla^2\Psi}{a^2} = 4\pi G\left[-\left(1 + \frac{18H^2}{M^2}\right)\dot{\varphi}^2\Phi\right. \\
&\quad - \frac{9H\dot{\varphi}^2}{M^2}\dot{\Psi} + \frac{\dot{\varphi}^2}{M^2}\frac{\nabla^2\Psi}{a^2} + V\acute{(}\varphi)\delta\varphi \\
&\quad \left. + \left(1 + \frac{9H^2}{M^2}\right)\dot{\varphi}\dot{\delta\varphi} - \frac{2H\dot{\varphi}}{M^2}\frac{\nabla^2(\delta\varphi)}{a^2} + \delta\rho_{\rm r}\right],
\end{aligned} \quad (44)$$

and its ii component is

$$\begin{aligned}
&(3H^2 + 2\dot{H})\Phi + H(3\dot{\Psi} + \dot{\Phi}) + \frac{\nabla^2(\Phi - \Psi)}{3a^2} + \ddot{\Psi} \\
&= 4\pi G\left[\left((3H^2 + 2\dot{H})\frac{2\dot{\varphi}^2}{M^2} - \dot{\varphi}^2 + \frac{8H\dot{\varphi}\ddot{\varphi}}{M^2}\right)\Phi + \frac{3H\dot{\varphi}^2}{M^2}\dot{\Phi}\right. \\
&\quad + \frac{\dot{\varphi}^2}{M^2}\frac{\nabla^2\Phi}{3a^2} + \left(\frac{3H\dot{\varphi}^2}{M^2} + \frac{2\dot{\varphi}\ddot{\varphi}}{M^2}\right)\dot{\Psi} + \frac{\dot{\varphi}^2}{M^2}\ddot{\Psi} + \frac{\dot{\varphi}^2}{M^2}\frac{\nabla^2\Psi}{3a^2} \\
&\quad - V\acute{(}\varphi)\delta\varphi - \left[\left(-1 + \frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}\right)\dot{\varphi} + \frac{2H\ddot{\varphi}}{M^2}\right]\dot{\delta\varphi} \\
&\quad \left. - \frac{2H\dot{\varphi}}{M^2}\ddot{\delta\varphi} + \frac{2(\ddot{\varphi} + H\dot{\varphi})}{M^2}\frac{\nabla^2(\delta\varphi)}{3a^2} + \delta P_{\rm r}\right].
\end{aligned} \quad (45)$$

By using $-H\partial_i\Phi - \partial_i\dot{\Psi} = 4\pi G(\rho + P)\partial_i\delta u$, we can obtain (from the $0i$ component of the field equation)

$$\begin{aligned}
H\Phi + \dot{\Psi} &= 4\pi G\left[\frac{3H\dot{\varphi}^2}{M^2}\Phi + \frac{\dot{\varphi}^2}{M^2}\dot{\Psi} + \left(1 + \frac{3H^2}{M^2}\right)\dot{\varphi}\delta\varphi\right. \\
&\quad \left. - \frac{2H\dot{\varphi}}{M^2}\dot{\delta\varphi} + (\rho_{\rm r} + P_{\rm r})\delta u\right].
\end{aligned} \quad (46)$$

Using (41–46) we can calculate the perturbation parameters.

Depending on the physical process, e.g. thermal noise, expansion, curvature fluctuations, three separate regimes for the evolution of the scalar field fluctuations may be considered [8]. But one can generalize this approach, by adding stochastic noise source and viscous terms to cosmological perturbations equations [65].

During inflation the background has two components, oscillatory scalar field and radiation. The energy density of the scalar field decreases due to expansion and radiation generation. Quantities related to the scalar field in the background have oscillatory behaviors. So we replace the background quantities with their average values over oscillation. Also, we consider non-minimal derivative coupling at the high-friction limit.

By going to the Fourier space, the spatial parts of perturbational quantities get e^{ikx} where k is the wave number. So $\partial_j \to ik_j$ and $\nabla^2 \to -k^2$. Also we define

$$\delta u = -\frac{a}{k}ve^{ikx}. \quad (47)$$

So (41) becomes

$$\delta\dot{\rho_r} + 4H\delta\rho_r + \frac{4}{3}ka\rho_r v - 4\rho_r\dot{\Phi} = -\Gamma M^2 M_P^2\Phi, \tag{48}$$

and (42) becomes

$$4\frac{a}{k}((\dot{\rho_r v}) + 4H(\rho_r v)) = -\delta\rho_r - 4\rho_r\Phi. \tag{49}$$

(43) reduces to

$$\left(\frac{3H^2}{M^2}\right)\delta\ddot{\varphi} + \left[\left(\frac{3H^2}{M^2} + \frac{2\dot{H}}{M^2}\right)3H + \Gamma\right]\delta\dot{\varphi} + \delta V'(\varphi) = -2V'(\varphi)\Phi + 3\left(\frac{9H^2}{M^2} + \frac{2\dot{H}}{M^2}\right)\dot{\Phi}. \tag{50}$$

From (44) we have

$$-3H\dot{\Phi}\left(1 - \frac{3\gamma}{2}\right) - 3H^2\Phi(1-3\gamma) = \frac{1}{2M_P^2}(V'(\varphi)\delta\varphi + \delta\rho_r), \tag{51}$$

and we rewrite (45) as

$$(3H^2 + 2\dot{H})\Phi(1-\gamma) + H\dot{\Phi}(4-3\gamma) + \ddot{\Phi}\left(1 - \frac{1}{2}\gamma\right) = \frac{1}{2M_P^2}(-V'(\varphi)\delta\varphi + \delta P_r). \tag{52}$$

Note that we have replaced $\dot{\phi}^2$ and $\dot{\phi}$ by their average values i.e. $<\dot{\phi}^2>=\gamma M^2 M_P^2$ and $<\dot{\phi}>=0$. We restrict ourselves to the high-friction regime $\frac{H^2}{M^2} \gg 1$ and the modes satisfying $\frac{k}{a} \ll H$ and the zero-shear gauge $\Phi = \Psi$ [8] are considered.

Equation (46) may be written as

$$H\Phi\left(1 - \frac{3}{2}\gamma\right) + \dot{\Phi}\left(1 - \frac{1}{2}\gamma\right) = -\frac{2}{3M_P^2}\frac{a}{k}(v\rho_r), \tag{53}$$

and the time derivative of (46) gives

$$(H\dot{\Phi} + \dot{H}\Phi)\left(1 - \frac{3}{2}\gamma\right) + \ddot{\Phi}\left(1 - \frac{1}{2}\gamma\right) = -\frac{2}{3M_P^2}\frac{a}{k}\left(H(v\rho_r) + (\dot{v\rho_r})\right). \tag{54}$$

By analyzing the above equations we find

$$\left[3H^2\left(1 - \frac{3}{2}\gamma - \frac{1}{3}\gamma\right) + \dot{H}\left(\frac{2}{3} - \frac{7}{6}\gamma\right)\right]\Phi + \frac{5}{6}(4-3\gamma)H\dot{\Phi} + \frac{5}{6}\left(1 - \frac{1}{2}\gamma\right)\ddot{\Phi} = 0. \tag{55}$$

During the rapid oscillation, the Hubble parameter is $H = \frac{2}{3\gamma t}$, therefore (55) becomes

$$\left(\frac{2}{3\gamma}\right)\left[\frac{2}{\gamma} - \frac{13}{3} + \frac{7}{6}\gamma\right]\frac{\Phi}{t^2} + \frac{5}{9\gamma}(4-3\gamma)\frac{\dot{\Phi}}{t} + \frac{5}{6}\left(1 - \frac{1}{2}\gamma\right)\ddot{\Phi} = 0. \tag{56}$$

This equation has the solution $\Phi \propto t^{\alpha_\pm}$, therefore

$$\left(\frac{2}{3\gamma}\right)\left[\frac{2}{\gamma} - \frac{13}{3} + \frac{7}{6}\gamma\right] + \frac{5}{9\gamma}(4-3\gamma)\alpha + \frac{5}{6}\left(1 - \frac{1}{2}\gamma\right)\alpha(\alpha-1) = 0. \tag{57}$$

$\alpha's$ are the roots of this quadratic equation. We denote the positive root by α_+. From Eqs. (51) and (52), we deduce

$$-\frac{1}{M_P^2}V'(\varphi)\delta\varphi = 2\left(3H^2(1-2\gamma) + \dot{H}(1-\gamma)\right)\Phi + \left(7 - \frac{3}{2}\gamma\right)H\dot{\Phi} + \left(1 - \frac{1}{2}\gamma\right)\ddot{\Phi}. \tag{58}$$

It is now possible to use the relation $\Phi \propto t^{\alpha_+}$ to obtain $\delta\varphi$,

$$-\frac{1}{M_P^2}V'(\varphi)\delta\varphi = \left[\frac{4}{3\gamma}\left(\frac{2}{\gamma} - 5 + \gamma\right) + \frac{2}{3\gamma}\left(7 - \frac{15\gamma}{2}\right)\times\alpha_+\left(1 - \frac{1}{2}\gamma\right)\alpha_+(\alpha_+ - 1)\right]\frac{\Phi}{t^2}. \tag{59}$$

$\delta\varphi$ simplifies to

$$\delta\varphi = -C\frac{M_P^{2+\alpha_+}}{V'(\varphi)}t^{\alpha_+-2}\times\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma} - 5 + \gamma\right) + \frac{2}{3\gamma}\left(7 - \frac{15\gamma}{2}\right)\alpha_+ + \left(1 - \frac{1}{2}\gamma\right)\alpha_+(\alpha_+ - 1)\right] \tag{60}$$

where C is a numerical constant. Thus the density perturbation, from Eq. (60), becomes [66,67]

$$\delta_H \approx \frac{16\pi}{5M_P^{2+\alpha_+}} \times \frac{V'\delta\varphi}{\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma} - 5+\gamma\right) + \frac{2}{3\gamma}\left(7 - \frac{15\gamma}{2}\right)\alpha_+ + \left(1 - \frac{1}{2}\gamma\right)\alpha_+(\alpha_+ - 1)\right]t^{\alpha_+-2}}. \tag{61}$$

In this relation $\delta\varphi$ is the scalar field fluctuation during the warm inflation, which instead of quantum fluctuation, are generated by thermal fluctuation [4–6,60]. Due to the thermal fluctuations, φ satisfies the Langevin equation with a stochastic noise source, using which one finds [43–45]

$$\delta\varphi^2 = \frac{k_F T}{2\pi^2}, \tag{62}$$

where k_F is the freeze-out scale, containing also terms corresponding to the non-minimal coupling. To compute k_F, we must determine when the damping rate of Eq. (52) becomes less than the expansion rate H. At t_F (freeze-out time [8]), the freeze-out wave number $k_F = \frac{k}{a(t_F)}$ is given by [43–45]

$$k_F = \sqrt{\Gamma H + 3H^2\left(1+\frac{3H^2}{M^2}\right)}. \tag{63}$$

In the minimal case $\frac{H^2}{M^2} = 0$, and (63) gives the well known result [60].

$$\delta\varphi^2 = \frac{\sqrt{\Gamma H + 3H^2}T}{2\pi^2}, \tag{64}$$

which reduces to $\delta\varphi^2 = \frac{\sqrt{3}HT}{2\pi^2}$ [4–6] in weak dissipative regime $\Gamma \ll H$, and to $\delta\varphi^2 = \frac{\sqrt{\Gamma H}T}{2\pi^2}$, in the strong dissipative regime $\Gamma \gg H$. For a more detailed discussion as regards the scalar field fluctuations (64), based on quantum field theory first principles; see [14]. In our case, as we are restricted to the high-friction regime $\frac{H^2}{M^2} \gg 1$ and also use the approximation (22) before the radiation dominated era, we have

$$\delta\varphi^2 = \frac{3H^2T}{2M\pi^2}. \tag{65}$$

Note that our study is restricted to the region $H < \Gamma \lesssim \left(\frac{H^2}{9M^2}\right)H$. By using (65), the density perturbation

$$\delta_H^2 \approx \left(\frac{16\pi}{5M_P^{2+\alpha_+}}\right)^2 t^{4-2\alpha_+} \times \frac{V'^2\delta\varphi^2}{\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma}-5+\gamma\right)+\frac{2}{3\gamma}\left(7-\frac{15\gamma}{2}\right)\alpha_+ + \left(1-\frac{1}{2}\gamma\right)\alpha_+(\alpha_+-1)\right]^2} \tag{66}$$

can be rewritten as

$$\delta_H^2 \approx \left(\frac{128}{25M_P^{4+2\alpha_+}}\right)t^{4-2\alpha_+}\left(\frac{3H^2}{M}\right)T \times \frac{V'^2}{\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma}-5+\gamma\right)+\frac{2}{3\gamma}\left(7-\frac{15\gamma}{2}\right)\alpha_+ + \left(1-\frac{1}{2}\gamma\right)\alpha_+(\alpha_+-1)\right]^2}. \tag{67}$$

We can now calculate power spectrum from relation $P_s(k_0) = \frac{25}{4}\delta_H^2(k_0)$ [66,67]. k_0 is a pivot scale. The spectral index for scalar perturbation is

$$n_s - 1 = \frac{d\ln\delta_H^2}{d\ln k}. \tag{68}$$

The derivative is taken at horizon crossing $k \approx aH$. The spectral index may be written as

$$n_s - 1 = \frac{d\ln\delta_H^2}{d\ln(aH)} = \left(\frac{1}{H+\frac{\dot H}{H}}\right)\frac{d\ln\delta_H^2}{dt}. \tag{69}$$

From $H = \frac{2}{3\gamma t}$ we have

$$n_s - 1 \approx \left(\frac{t}{\frac{2}{3\gamma}-1}\right)\frac{d\ln\delta_H^2}{dt}, \tag{70}$$

therefore

$$n_s - 1 \approx \left(\frac{4}{3\gamma}-\frac{5}{2}-2\alpha_+\right)\left(\frac{1}{\frac{2}{3\gamma}-1}\right). \tag{71}$$

This relation gives the spectral index as a function of γ. From Planck 2015 data $n_s = 0.9645 \pm 0.0049$ (68% CL, Planck TT, TE, EE + low P) γ is determined as $\gamma = 0.55902 \pm 0.00016$.

4 Evolution of the Universe and temperature of the warm inflation

In this section, by using our previous results, we intend to calculate the temperature of warm inflation as a function of observational parameters for the power-law potential (19) and a constant dissipation coefficient Γ, in the high-friction limit. For this purpose we follow the steps introduced in [72], and we divide the evolution of the Universe from t_* (a time at which a pivot scale exited the Hubble radius) in inflation era until now into three parts

- I from $t_\star$ until the end of oscillatory warm inflation, denoted by t_{RD}. in this period energy density of the oscillatory scalar field is dominated.
- II from t_{RD} until recombination era, denoted by t_{rec}.
- III from t_{rec} until the present time t_0.

Therefore the number of e-folds from horizon crossing until now becomes

$$\mathcal{N} = \ln\left(\frac{a_0}{a_\star}\right) = \ln\left(\frac{a_0}{a_{rec}}\right) + \ln\left(\frac{a_{rec}}{a_{RD}}\right) + \ln\left(\frac{a_{RD}}{a_\star}\right) = \mathcal{N}_I + \mathcal{N}_{II} + \mathcal{N}_{III} \tag{72}$$

4.1 Oscillatory warm inflation

During the warm oscillatory inflation, the scalar field oscillates and decays into the ultra-relativistic particles. In this period the energy density of oscillatory scalar field is dominated and the Universe expansion is accelerated. The beginning time of radiation dominated era is determined by the condition $\rho_r(t_{RD}) \simeq \rho_\varphi(t_{RD})$, which gives [41,42]

$$t_{RD}{}^3 = \frac{4(8+3\gamma)}{9\Gamma\gamma^4 M^2}. \tag{73}$$

From equations (73) and (29) we can calculate energy density of radiation at t_{RD}

$$\rho_r(t = t_{RD}) \approx M_P^2 \left[\frac{12\Gamma^2\gamma^2 M^4}{(8+3\gamma)^2}\right]^{\frac{1}{3}}. \tag{74}$$

Note that $t_{RD} \sim t_{rh}$, where t_{rh} is defined after (22). The temperature of the Universe at the end of oscillatory warm inflation becomes

$$T_{RD}{}^4 \approx \frac{30M_P^2}{\pi^2 g_{RD}}\left[\frac{12\Gamma^2\gamma^2 M^4}{(8+3\gamma)^2}\right]^{\frac{1}{3}}. \tag{75}$$

4.2 Radiation dominated and recombination eras

At the end of the warm inflation the magnitude of radiation energy density equals the energy density of the scalar field. Thereafter the Universe enters a radiation dominated era. During this period, the Universe is filled with ultra-relativistic particles which are in thermal equilibrium. In this epoch the Universe undergoes an adiabatic expansion where the entropy per comoving volume is conserved: $dS = 0$ [57]. In this era the entropy density, $s = Sa^{-3}$, is [57]

$$s = \frac{2\pi^2}{45} g T^3. \tag{76}$$

So we have

$$\frac{a_{rec}}{a_{RD}} = \frac{T_{RD}}{T_{rec}}\left(\frac{g_{RD}}{g_{rec}}\right)^{\frac{1}{3}}. \tag{77}$$

In the recombination era, g_{rec} corresponds to degrees of freedom of photons, hence $g_{rec} = 2$. Thus

$$\mathcal{N}_{II} = \ln\left(\frac{T_{RD}}{T_{rec}}\left(\frac{g_{RD}}{2}\right)^{\frac{1}{3}}\right). \tag{78}$$

By the expansion of the Universe, the temperature decreases via $T(z) = T(z=0)(1+z)$, where z is the redshift parameter. Hence T_{rec} in terms of T_{CMB} is

$$T_{rec} = (1 + z_{rec})T_{CMB}. \tag{79}$$

We have also

$$\frac{a_0}{a_{rec}} = (1 + z_{rec}). \tag{80}$$

Therefore

$$\mathcal{N}_{II} + \mathcal{N}_{III} = \ln\left(\frac{T_{RD}}{T_{CMB}}\left(\frac{g_{RD}}{2}\right)^{\frac{1}{3}}\right). \tag{81}$$

4.3 Temperature of the warm oscillatory inflation

To obtain temperature of the warm inflation we must determine $\mathcal{N}$ in (72). We take $a_0 = 1$, so the number of e-folds from the horizon crossing until the present time is $\Delta = \exp(\mathcal{N})$, where

$$\Delta = \frac{1}{a_*} = \frac{H_*}{k_0} \approx \frac{2}{3\gamma t_* k_0}. \tag{82}$$

By Eqs. (72, 81, 82) we can derive T_{RD},

$$T_{RD} = T_{CMB}\left(\frac{2}{g_{RD}}\right)^{\frac{1}{3}} \frac{2}{3\gamma k_0}\left[\frac{4(8+3\gamma)}{9\Gamma\gamma^4 M^2}\right]^{-\frac{2}{9\gamma}} \times t_*^{\left(\frac{2}{3\gamma}-1\right)}. \tag{83}$$

We can remove ΓM^2 in this relation by (75),

$$T_{RD}^{\left(1-\frac{4}{3\gamma}\right)} \approx \frac{2T_{CMB}}{3\gamma k_0}\left(\frac{2}{g_{RD}}\right)^{\frac{\gamma-1}{3\gamma}}\left[\frac{2\sqrt{5}M_P}{\pi\gamma}\right]^{-\frac{2}{3\gamma}} \times t_*^{\left(\frac{2}{3\gamma}-1\right)}. \tag{84}$$

By using relation $\mathcal{P}_s(k_0) = \frac{25}{4}\delta_H^2(k_0)$ and Eq. (67), power spectrum becomes

$$\mathcal{P}_s(k_0) \approx \left(\frac{32}{M_P^{4+2\alpha_+}}\right) \times \frac{\langle V'(\varphi_*)\rangle^2}{\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma}-5+\gamma\right)+\frac{2}{3\gamma}\left(7-\frac{15\gamma}{2}\right)\alpha_+ + \left(1-\frac{1}{2}\gamma\right)\alpha_+(\alpha_+-1)\right]^2} \times t_*^{4-2\alpha_+}\sqrt{\Gamma H_* + 3H_*^2\left(1+\frac{3H_*^2}{M^2}\right)}T_*. \tag{85}$$

In this relation $T_\star$ is the temperature of the Universe at the horizon crossing. By Eq. (29) we can calculate temperature at horizon crossing as a function of t_*

$$T_* = \left[\frac{90\Gamma\gamma^2 M^2 M_P^2}{(8+3\gamma)\pi^2 g_*}\right]^{\frac{1}{4}} t_*^{\frac{1}{4}}. \tag{86}$$

We can remove ΓM^2 in Eq. (86) by (75)

$$T_* = \left[\frac{\pi\gamma g_{RD}^{\frac{1}{2}}}{2\sqrt{10}M_P}\right]^{\frac{1}{4}} T_{RD}^{\frac{3}{2}} t_*^{\frac{1}{4}}. \tag{87}$$

We have taken $g_{\rm RD} \sim g_*$. The time average of the potential derivative may computed as follows:

$$\langle V' \rangle = q\lambda \frac{\int_{-\phi}^{\phi} \varphi^{q-1} \frac{\mathrm{d}\varphi}{\dot\varphi}}{\int_{-\phi}^{\phi} \frac{\mathrm{d}\varphi}{\dot\varphi}} = q\lambda\phi^{n-1} \frac{\int_0^1 \frac{x^{q-1}\mathrm{d}x}{\sqrt{1-x^q}}}{\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^q}}} = 2q\lambda \frac{\Gamma\left(\frac{2+q}{2q}\right)}{\Gamma\left(\frac{1}{q}\right)} \phi^{q-1}. \tag{88}$$

In the inflationary regime we have $H^2 \approx \frac{1}{3M_{\rm P}^2}\rho_\varphi \approx \frac{1}{3M_{\rm P}^2}\lambda\phi^q$ and $H^2 \approx \frac{4}{9\gamma^2 t^2}$, therefore

$$\langle V'(\varphi_*) \rangle = 12\lambda \frac{\gamma}{2-3\gamma} \frac{\Gamma\left(\frac{1}{3\gamma}\right)}{\Gamma\left(\frac{1}{3\gamma}-\frac{1}{2}\right)} \left(\frac{4M_{\rm P}^2}{3\lambda\gamma^2}\right)^{\frac{9\gamma-2}{6\gamma}} t_*^{\frac{2-9\gamma}{3\gamma}}. \tag{89}$$

Thus we can write (85) as

$$\mathcal{P}_{\rm s}(k_0) \approx M_{\rm P}^{\left(\frac{5}{2}-\frac{4}{3\gamma}-2\alpha_+\right)} \lambda^{\left(\frac{2}{3\gamma}-1\right)} \Gamma^{\frac{1}{4}} M^{\frac{1}{2}} g_{\rm RD}^{-\frac{1}{4}} \beta t_*^{\left(-\frac{11}{4}+\frac{4}{3\gamma}-2\alpha_+\right)}. \tag{90}$$

β is given by

$$\beta = \frac{\frac{2048}{\sqrt{\pi}} \left(\frac{90\gamma^2}{8+3\gamma}\right)^{\frac{1}{4}} \left(\frac{4}{3\gamma^2}\right)^{\frac{9\gamma-2}{3\gamma}} \left(\frac{\Gamma\left(\frac{1}{3\gamma}\right)}{\Gamma\left(\frac{1}{3\gamma}-\frac{1}{2}\right)}\right)^2}{(2-3\gamma)^2\left[\frac{4}{3\gamma}\left(\frac{2}{\gamma}-5+\gamma\right)+\frac{2}{3\gamma}\left(7-\frac{15\gamma}{2}\right)\alpha_+ + \left(1-\frac{1}{2}\gamma\right)\alpha_+(\alpha_+-1)\right]^2}. \tag{91}$$

From Eq. (90), we derive t_* as

$$t_* = \left[\frac{\mathcal{P}_{\rm s}(k_0) g_{\rm RD}^{\frac{1}{4}}}{M_{\rm P}^{\left(\frac{5}{2}-\frac{4}{3\gamma}-2\alpha_+\right)} \lambda^{\left(\frac{2}{3\gamma}-1\right)} M^{\frac{1}{2}} \Gamma^{\frac{1}{4}} \beta}\right]^{\frac{12\gamma}{16-33\gamma-24\gamma\alpha_+}}. \tag{92}$$

By substituting t_* from Eq. (92) into Eq. (83), the temperature at the end of warm oscillatory inflation or beginning of the radiation domination is obtained:

$$T_{\rm RD} = T_{\rm CMB} \left(\frac{2}{g_{\rm RD}}\right)^{\frac{1}{3}} \frac{2}{3\gamma k_0} \left[\frac{4(8+3\gamma)}{9\Gamma\gamma^4 M^2}\right]^{-\frac{2}{9\gamma}} \times \left[\frac{\mathcal{P}_{\rm s}(k_0) g_{\rm RD}^{\frac{1}{4}}}{M_{\rm P}^{\left(\frac{5}{2}-\frac{4}{3\gamma}-2\alpha_+\right)} \lambda^{\left(\frac{2}{3\gamma}-1\right)} M^{\frac{1}{2}} \Gamma^{\frac{1}{4}} \beta}\right]^{\frac{4(2-3\gamma)}{16-33\gamma-24\gamma\alpha_+}}. \tag{93}$$

The number of e-folds during warm oscillatory inflation becomes

$$\mathcal{N}_{\rm I} \approx \frac{2}{3\gamma} \ln\left(\left(\frac{4(8+3\gamma)}{9\Gamma\gamma^4 M^2}\right)^{\frac{1}{3}} \times \left(\frac{\mathcal{P}_{\rm s}(k_0) g_{\rm RD}^{\frac{1}{4}}}{M_{\rm P}^{(\frac{5}{2}-\frac{4}{3\gamma})-2\alpha_+} \lambda^{(\frac{2}{3\gamma}-1)} M^{\frac{1}{2}} \Gamma^{\frac{1}{4}} \beta}\right)^{\frac{12\gamma}{-16+33\gamma+24\gamma\alpha_+}}\right). \tag{94}$$

We set $g_{\rm RD} = 106.75$, which is for the ultra-relativistic degrees of freedom at the electroweak energy scale. Also, from Planck 2015 data, at the pivot scale $k_0 = 0.002\,\mathrm{Mpc}^{-1}$ and in one sigma level, we have $\mathcal{P}_{\rm s}(k_0) = (2.014 \pm 0.046) \times 10^{-9}$ and $n_{\rm s} = 0.9645 \pm 0.0049$ (68% CL, Planck TT, TE, EE + low P) [49–53]. By using $\gamma = 0.55902$, $M = 10^{-16} M_{\rm P}$, $\lambda = (10^{-8} M_{\rm P})^{4-q}$, and $\Gamma = 10^{-4} M_{\rm P}$ in Eq. (93) the temperature of the Universe at the end of warm inflation and the number of e-folds become $T_{\rm end} \approx 3.83 \times 10^{12}$ GeV, and $\mathcal{N} = 61.42$ respectively.

5 Tensorial perturbation

In this section, we follow the method used in [68] to study tensorial perturbation. The power spectrum for tensorial perturbation is given by [68]

$$P_{\rm t}(k) = \frac{k^3}{2\pi^2} \left|\frac{v_k}{z}\right|^2 \coth\left(\frac{k}{2T}\right), \tag{95}$$

where v_k can be calculated from the Mukhanov equation [69–71]

$$\frac{\mathrm{d}^2 v_k}{\mathrm{d}\eta^2} + \left(c^2 k^2 - \frac{1}{z}\frac{\mathrm{d}^2 z}{\mathrm{d}\eta^2}\right) v_k = 0. \tag{96}$$

η is the conformal time, $c_{\rm t}$ is the sound speed for the tensor mode and k is the wave number for the mode function v_k [69–71] and z is given by

$$z = a(t) M_{\rm p} \frac{\sqrt{e_{ij}^{\lambda} e_{ij}^{\lambda}}}{2} \sqrt{1-\alpha}. \tag{97}$$

The polarization tensor is normalized as $e^{\lambda}_{ij}e^{\lambda'}_{ij}=2\delta_{\lambda\lambda'}$. For our model, with a quasi-periodic scalar inflaton background, we have $\alpha=\frac{\dot{\varphi}^2}{2M^2M_p^2}$ and c is given by relation

$$c^2=\frac{1+\alpha}{1-\alpha}. \qquad (98)$$

Therefore

$$\frac{1}{z}\frac{d^2z}{d\eta^2}=\left(\frac{q}{2}+1\right)\left(\frac{q}{2}+2\right)\eta^{-2}. \qquad (99)$$

By using this relation, the equation for the mode function becomes

$$\frac{d^2v_k}{d\eta^2}+\left(c^2k^2-\left(\frac{q}{2}+1\right)\left(\frac{q}{2}+2\right)\eta^{-2}\right)v_k=0. \qquad (100)$$

Solutions to this mode equation are the Hankel functions of the first and second kind,

$$v_k(\eta)=\eta^{\frac{1}{2}}\left[C^{(1)}(k)H^{(1)}_{\nu}(ck\eta)+C^{(2)}(k)H^{(2)}_{\nu}(ck\eta)\right]. \qquad (101)$$

Well within the horizon, the modes satisfy $k\gg aH$, and they can be approximated by flat waves. Therefore

$$v_k(\eta)\approx\frac{\sqrt{\pi}}{2}e^{i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}}(-\eta)^{\frac{1}{2}}H^{(1)}_{\nu}(-ck\eta). \qquad (102)$$

On the other hand, when we want to compute power spectrum, we need to have modes that are outside the horizon. So by taking the limit $\frac{k}{aH}\to 0$, we obtain the asymptotic form of mode function as

$$v_k(\eta)\to e^{i\left(\nu+\frac{1}{2}\right)\frac{\pi}{2}}2^{\left(\nu-\frac{3}{2}\right)}\frac{\Gamma(\nu)}{\Gamma(\frac{3}{2})}\frac{1}{\sqrt{2ck}}(-ck\eta)^{\left(-\nu+\frac{1}{2}\right)}. \qquad (103)$$

By using this relation we can write the power spectrum as

$$P_t(k)=\frac{k^3}{2\pi^2}\frac{2^{(2\nu-3)}}{\beta^2a^2}\left(\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right)^2\frac{1}{2ck}(-ck\eta)^{(-2\nu+1)}\coth\left(\frac{k}{2T}\right). \qquad (104)$$

In the rapid oscillation epoch $\epsilon=\frac{\dot{H}}{H^2}=\frac{3\gamma}{2}$ (see (24)), so we can write the conformal time as

$$\eta=-\frac{1}{aH}\frac{1}{1-\epsilon}. \qquad (105)$$

At the horizon crossing $c_sk=aH$, we can write (104) as

$$P_t=A_t^2(q)\left(\frac{H}{M_p}\right)^2\coth\left(\frac{k}{2T}\right)|_{ck=aH}, \qquad (106)$$

where

$$A_t(q)=\frac{3^{(\frac{1}{2})}2^{(q-\frac{1}{2})}\Gamma(\frac{3}{2}+\frac{q}{2})(q+2)^{(-\frac{q+1}{2})}}{\pi\Gamma(\frac{3}{2})(q+3)^{(\frac{1}{4})}(2q+3)^{(\frac{3}{4})}}. \qquad (107)$$

The ratio of tensor to scalar spectrum [from Eqs. (90) and (106)] becomes

$$r=\frac{P_t(k_0)}{\mathcal{P}_s(k_0)}\approx\left[\frac{4A_t^2(q)g_{RD}^{\frac{1}{4}}t_*^{\left(\frac{4}{3\gamma}-\frac{11}{4}-2\alpha_+\right)}}{9\gamma^2M_p^{\left(\frac{9}{2}-\frac{4}{3\gamma}-2\alpha_+\right)}\lambda^{\left(\frac{2}{3\gamma}-1\right)}M^{\frac{1}{2}}\Gamma^{\frac{1}{4}}\beta}\right]\times\coth\left(\frac{k}{2T}\right)\Bigg|_{ck=aH}. \qquad (108)$$

From Eq. (108) by using $\gamma=0.55902$, $g_{RD}=106.75$, $M=10^{-16}M_p$ and $\Gamma=10^{-4}M_p$, the ratio of tensor to scalar at the pivot scale $k_*=0.002\,\mathrm{Mpc}^{-1}$ becomes $r\approx 0.081$, which is consistent with Planck 2015 data, $r_{0.002}<0.10$ (95% CL, Planck TT, TE, EE + lowP).

6 Conclusion

In the standard model of inflation, the inflaton begins a coherent rapid oscillation after the slow roll. During this stage, the inflaton decays to radiation and reheats the Universe. In this paper, we considered a rapid oscillatory inflaton during the inflation era. This scenario does not work in minimal coupling model due to the fewness of e-folds during rapid oscillation. But the non-minimal derivative coupling can remedy this problem in the high-friction regime. Therefore we proposed a new model in which inflation and rapid oscillation are unified without considering the slow roll. The number of e-folds was calculated. We investigated cosmological perturbations and the temperature of the Universe was determined as a function of the spectral index.

We used a phenomenological approach to describing the interaction term between the inflaton and the radiation, but a precise study of thermal radiation productions must be performed based on quantum field theory principles. An attempt in this regard may be found in [14].

To complete our study one may consider quantum and thermal corrections to the parameters of the system such as the inflaton mass and its coupling to the radiation. In the slow roll model, the role of theses corrections on the observed spectrum is studied in the literature [73,74]. Recently, in [75], it was shown that in warm slow roll inflation, it is possible to sustain the flatness of the potential against the thermal and loop quantum corrections. In the non-minimal derivative coupling model, by power counting analysis, and unitarity constraint which implies $H \ll \Lambda$, where $\Lambda = (H^2 M_P)^{\frac{1}{3}}$ is the cutoff of the theory, it was shown that for power-law potentials quantum radiation corrections are subleading [54]. However, it may be interesting to study in detail the effect of loop quantum corrections and the corresponding renormalization on the behavior of our model and on its spectral index and power spectrum.

Note that our model is an initial study of warm oscillatory inflation. Further studies may be performed by considering thermal correction to the effective potential [76], and also by taking into account the temperature dependency of the dissipative factor and checking all consistency conditions. We leave these problems for future work.

Funded by SCOAP^3.

References

1. A.H. Guth, Phys. Rev. D **23**, 347 (1981)
2. A. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, 1990)
3. A. Liddle, A. Mazumder, Phys. Rev. D **58**, 083508 (1996)
4. A. Berera, Phys. Rev. Lett. **75**, 3218 (1995)
5. A. Berera, L.Z. Fang, Phys. Rev. Lett. **74**, 1912 (1995)
6. Y. Gim, W. Kim, arXiv:1608.07466
7. A. Berera, Nucl. Phys. B **585**, 666 (2000)
8. L.M.H. Hall, I.G. Moss, A. Berera, Phys. Rev. D **69**, 083525 (2004)
9. M. Bastero-Gil, A. Berera, R.O. Ramos, J.G. Rosa, Phys. Lett. B **712**, 425 (2012)
10. S. del Campo, R. Herrera, D. Pavon, Phys. Rev. D **75**, 083518 (2007)
11. R. Herrera, S. del Campo, C. Campuzano, JCAP **10**, 009 (2006)
12. X.M. Zhang, J.Y. Zhu, Phys. Rev. D **87**, 043522 (2013)
13. T. Damour, V.F. Mukhanov, Phys. Rev. Lett. **80**, 3440 (1998)
14. S. Bartruma, M. Gilb, A. Bereraa, R. Cerezob, R.O. Ramosc, J.G. Rosa, Phys. Lett. B **732**, 116 (2014)
15. A. Taruya, Phys. Rev. D **59**, 103505 (1999)
16. J. Lee, S. Koh, C. Park, S.J. Sin, C.H. Lee, Phys. Rev. D **61**, 027301 (1999)
17. M. Sami, Grav. Cosmol. **8**, 309 (2003)
18. S.V. Sushkov, Phys. Rev. D **80**, 103505 (2009)
19. E. Saridakis, S.V. Sushkov, Phys. Rev. D **81**, 083510 (2010)
20. H.M. Sadjadi, Phys. Rev. D **83**, 107301 (2011)
21. Y. Cai, Y.S. Piao, JHEP **03**, 134 (2016)
22. Y. Huang, Y. Gong, Sci. China Phys. Mech. Astron. **59**, 640402 (2016)
23. Z. Yi, Y. Gong, arXiv:1512.05555
24. N. Yang, Q. Gao, Y. Gong, Int. J. Mod. Phys. A **30**, 1545004 (2015)
25. Y.S. Myung, T. Moon, B.H. Lee, arXiv:1505.04027
26. A. Cisterna, M. Cruz, T. Delsate, J. Saavedra, Phys. Rev. D **92**, 104018 (2015)
27. M. Minamitsuji, Phys. Rev. D **89**, 064017 (2014)
28. A. Ghalee, Phys. Lett. B **724**, 198 (2013)
29. K. Nozari, N. Behrouz, Phys. Dark Universe **13**, 92 (2016)
30. B. Gumjudpai, P. Rangdee, Gen. Relativ. Gravit. **47**, 140 (2015)
31. H.M. Sadjadi, Gen. Relativ. Gravit. **46**, 1817 (2014)
32. B. Gumjudpai, N. Kaewkhao, arXiv:1608.04014 [gr-qc]
33. H. Sheikhahmadi, E.N. Saridakis, A. Aghamohammadi, K. Saaidi, arXiv:1603.03883
34. J.B. Dent, S. Dutta, E.N. Saridakis, J.Q. Xia, JCAP **1311**, 058 (2013)
35. T. Harko, F.S.N. Lobo, E.N. Saridakis, M. Tsoukalas, arXiv:1609.01503
36. A. Banijamali, B. Fazlpour, Phys. Lett. B **703**, 366 (2011)
37. K. Feng, T. Qiu, Phys. Rev. D **90**, 123508 (2014)
38. K. Feng, T. Qiu, Y.S. Piao, Phys. Lett. B **729**, 99 (2014)
39. Y.S. Myung, T. Moon, arXiv:1601.03148
40. H.M. Sadjadi, P. Goodarzi, Phys. Lett. B **732**, 278 (2014)
41. H.M. Sadjadi, P. Goodarzi, JCAP **02**, 038 (2013)
42. H.M. Sadjadi, P. Goodarzi, JCAP **07**, 039 (2013)
43. K. Nozari, M. Shoukrani, N. Rashidi, Adv. High Energy Phys. **2014**, 343819 (2014)
44. K. Nozari, M. Shoukrani, Astrophys. Space Sci. **361**, 289 (2016)
45. H.M. Sadjadi, P. Goodarzi, Eur. Phys. J. C **75**, 513 (2015)
46. T. Qiu, K. Feng, arXiv:1608.01752 [hep-ph]
47. I. Dalianis, G. Koutsoumbas, K. Ntrekis, E. Papantonopoulos, arXiv:1608.04543
48. R. Herrera, J. Saavedra, C. Campuzano, arXiv:1609.03957
49. Planck collaboration, P. Ade et al., Planck 2015 results. XX. Constraints on inflation. arXiv:1502.02114
50. Planck collaboration, P. Ade et al., Planck 2015 results. XIII. Cosmological parameters—Planck Collaboration (Ade, P.A.R. et al.). arXiv:1502.01589
51. Planck collaboration, P. Ade et al., Planck 2015 results. I. Overview of products and scientific results—Planck Collaboration (Adam, R. et al.). arXiv:1502.01582
52. WMAP collaboration, G. Hinshaw et al., Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results. arXiv:1212.5226
53. WMAP collaboration, C. Bennett et al., Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results. arXiv:1212.5225
54. C. Germani, A. Kehagias, Phys. Rev. Lett. **105**, 011302 (2011)
55. A. Albrecht, P.J. Steinhardt, M.S. Turner, F. Wilczek, Phys. Rev. Lett. **48**, 1437 (1982)
56. A. Linde, Contemp. Concepts Phys. **5**, 1 (2005)
57. E. Kolb, M. Turner, *The Early Universe* (Addison-Wesley Publishing Company, Redwood City, 1990)
58. L. Kofman, A. Linde, A.A. Starobinsky, Phys. Rev. D **56**, 3258 (1997)
59. L. Kofman, arXiv:astro-ph/9605155
60. A. Berera, I.G. Moss, R.O. Ramos, Rep. Prog. Phys. **72**, 026901 (2009)
61. M. Drewes, J.U. Kang, Nucl. Phys. B **875**, 315 (2013). arXiv:1305.0267
62. X.M. Zhang, J.Y. Zhu, Phys. Rev. D **87**, 043522 (2013)
63. S. Weinberg, Cosmology (Oxford University Press, Oxford, 2008)
64. I.G. Moss, C. Xiong, JCAP **0704**, 007 (2007). astro-ph/0701302
65. L. Visinelli, arXiv:1410.1187 [astro-ph.CO]
66. R. Herrera, S. del Campo, C. Campuzano, JCAP **10**, 009 (2006)
67. S. del Campo, R. Herrera, D. Pavon, Phys. Rev. D **75**, 083518 (2007)
68. K. Bhattacharya, S. Mohanty, A. Nautiyal, Phys. Rev. Lett. **97**, 251301 (2006)
69. C. Germani, Y. Watanabe, JCAP **07**, 031 (2011)
70. C. Germani, L. Martucci, P. Moyassari, Phys. Rev. D **85**, 103501 (2012)

71. C. Germani, arXiv:1112.1083v1 [astro-ph.CO]
72. J. Mielczarek, Phys. Rev. D **83**, 023502 (2011)
73. D. Boyanovsky, H.J. de Vega, N.G. Sanchez, Nucl. Phys. B **747**, 25 (2006). arXiv:astro-ph/0503669
74. A. Bilandzic, T. Prokopec, Phys. Rev. D **76**, 103507 (2007)
75. M. Bastero-Gil, A. Berera, R.O. Ramos, J.G. Rosa. Phys. Rev. Lett. **117**, 151301 (2016)
76. L.M.H. Hall, I.G. Moss, Phys. Rev. D **71**, 023514 (2005)