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Bulk viscosity, interaction and the viability of phantom solutions

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Abstract We study the dynamics of a bulk viscosity model in the Eckart approach for a spatially flat Friedmann–Robertson–Walker (FRW) Universe. We have included radiation and dark energy, assumed as perfect fluids, and dark matter treated as an imperfect fluid having bulk viscosity. We also introduce an interaction term between the dark matter and dark energy components. Considering that the bulk viscosity is proportional to the dark matter energy density and imposing a complete cosmological dynamics, we find bounds on the bulk viscosity in order to reproduce a matter-dominated era (MDE). This constraint is independent of the interaction term. Some late time phantom solutions are mathematically possible. However, the constraint imposed by a MDE restricts the interaction parameter, in the phantom solutions, to a region consistent with a null value, eliminating the possibility of late time stable solutions with $w < -1$. From the different cases that we study, the only possible scenario, with bulk viscosity and interaction term, belongs to the quintessence region. In the latter case, we find bounds on the interaction parameter compatible with latest observational data.

1 Introduction

Since the discovery of the present stage of acceleration of the Universe [1,2] many candidates have been proposed to explain such an observational result [3–8]. Among them, the cosmological constant, $w_\Lambda = -1$, remains not only as the simplest alternative but also as consistent with the latest observational data [9]. Despite this, the ΛCDM model is not able to explain the results that still point to a phantom Universe [9], $w < -1$.

An interesting way to recover accelerated solutions is by introducing dissipative processes in ordinary fluids. This approach has been explored in the literature through the modeling of bulk viscosity in ordinary matter fluids [10–25] in the context of Eckart [26] or linear [27] and non-linear [28,29] Israel–Stewart theories.

Following the dissipative approach, in [20] it was shown that phantom solutions can be obtained by accepting the existence of bulk viscosity within the Eckart theory in the ΛCDM model.[1] This result was obtained by using multiple observational tests and considering that the bulk viscosity of some fluid depends on its own energy density, namely $\zeta_j = \zeta_j(\rho_j)$. This ansatz avoids the degeneracy problem associated with the case when the bulk viscosity is taken as $\zeta_j = \zeta_j(H)$ [20]. However, in [22], the same scenario was studied, from the dynamical system point of view, finding that viscous phantom solutions with stable behavior are not allowed in the framework of complete cosmological dynamics [21,30]. In the present paper we work along these lines by including an interaction between the dark matter and the dark energy. This kind of interaction mechanism has shown to be compatible with the current data [31]. In the context of viscous fluids, the interaction between dark matter and dark energy was studied in [21]. It has been shown that, under the ansatz $\zeta_j = \zeta_j(H)$, low-redshift data favors a positive definite value of the bulk viscosity, whereas high-redshift data prefers a negative value of the bulk viscosity. This latter result is in tension with the local second law of thermodynamics (LSLT) [32,33], which states that for an expanding Universe $\zeta \geq 0$ [34].

In the present work we are interested in extending the results obtained in [20,22] by taking into account an interaction term between dark energy and dark matter and, at the same time, extending the results in [21] by exploring a different functional form for the bulk viscosity.[2]

The paper is organized as follows: in Sect. 2 we present the field equation of the model. We take into account the contri-

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[1] Either the bulk viscosity was acting on the radiation, or on the pressureless matter, and crossing of the phantom divide is possible.

[2] Recall that in [21] the ansatz $\zeta_j = \zeta_j(H)$ was used, whereas in [20,22] the bulk viscosity was taken as $\zeta_j = \zeta_j(\rho_j)$ in order to avoid the model degeneration.

bution of pressureless matter, radiation and dark energy. The first matter fluid is considered as an imperfect fluid, having bulk viscosity in the framework of the Eckart theory [26], the remaining fluid obeying the barotropic equation of state (EOS). The bulk viscosity coefficient is taken to be proportional to the dark matter energy density. In Sect. 3, we study the evolution of the field equations from the perspective of the equivalent autonomous system. We focus our attention on a particular form for the interaction term between the dark matter and dark energy components. A detailed discussion as regards the viability of a complete cosmological dynamics [21,30] is provided. Important constraints on the bulk viscosity and interaction parameter are obtained. Finally, Sect. 4 is devoted to our conclusions.

2 The model

We study a cosmological model in a spatially flat FRW background metric, in which the matter components are radiation, dark matter and dark energy. We assume that the dark matter fluid presents a bulk viscosity in the framework of the Eckart theory, whereas the radiation and dark energy are assumed to be perfect fluids. Following this set up, the Friedmann constraint, the conservation equations for the matter fluids and the Raychaudhuri equation can be written

$$3H^2 = (\rho_{\rm r} + \rho_{\rm dm} + \rho_{\rm de}), \tag{1}$$

$$\dot\rho_{\rm r} = -4H\rho_{\rm r}, \tag{2}$$

$$\dot\rho_{\rm dm} = -3H\rho_{\rm dm} + 9H^2\zeta + Q, \tag{3}$$

$$\dot\rho_{\rm de} = -3H\gamma_{\rm de}\rho_{\rm de} - Q, \tag{4}$$

$$\dot H = -\frac{1}{2}\left(\rho_{\rm dm} + \frac{4}{3}\rho_{\rm r} + \gamma_{\rm de}\rho_{\rm de} - 3H\zeta\right), \tag{5}$$

where G is the Newton gravitational constant, H the Hubble parameter, $(\rho_{\rm dm}, \rho_{\rm r}, \rho_{\rm de})$ are the energy densities of dark matter, radiation and dark energy (DE) fluid components, respectively. $\gamma_{\rm de}$ is the barotropic index of the EOS of DE, which is defined from the relationship $p_{\rm de} = (\gamma_{\rm de} - 1)\rho_{\rm de}$, where $p_{\rm de}$ is the pressure of DE. The term Q in (3) and (4) is the interaction term between the dark matter and the dark energy components, while $9H^2\zeta$ in Eq. (3) corresponds to the bulk viscous pressure of the dark matter fluid, with ζ the bulk viscous coefficient.

In the literature [10,17,18,25,28,35–38], the usual ansatz[3] for the bulk viscous coefficient ζ is

$$\zeta = \xi\left(\frac{\rho_v}{\rho_{v0}}\right)^s, \tag{6}$$

[3] In the cases of radiative fluid and Maxwell–Boltzmann gas, the bulk viscosity coefficient can be obtained accurately due to the dissipation coefficients and second-order coefficients are known [13,32,39–42]. In these cases, the bulk viscosity coefficient depends on the temperature, $\zeta = \zeta(T)$.

where s and ξ are arbitrary constants. ρ_v corresponds to the energy density of the bulk viscosity fluid and its present day value is denoted by the subscript 0. In general, this ansatz leads to a large amplification of the ISW signal [15,17,43]. However, this problem is less severe if $s = 0$ (ζ = const) and $s = -1/2$ [17]. From the dynamical systems point of view, the choice of s in (6) leads to the following scenarios: (a) a two-dimensional phase space (see next section) for $s = 1/2$, (b) a more complex three-dimensional phase space for $s \neq 1/2$. In order to extend the results obtained in [20,22], henceforth we will focus our attention in the first case, thus we assume the bulk viscous coefficient ζ to be proportional to the energy density of the dark matter component in the form

$$\zeta = \xi\left(\frac{\rho_{\rm dm}}{\rho_{\rm dm0}}\right)^{\frac{1}{2}}, \tag{7}$$

where $\rho_{\rm dm0}$ is the present day value of the dark matter energy density.

3 The autonomous system

In order to study the dynamical properties of the system (2)–(4) and (5), we introduce the following dimensionless phase space variables to build an autonomous dynamical system:

$$x = \Omega_{\rm de} \equiv \frac{\rho_{\rm de}}{3H^2}, \quad y = \Omega_{\rm dm} \equiv \frac{\rho_{\rm dm}}{3H^2}, \quad \Omega_{\rm r} \equiv \frac{\rho_{\rm r}}{3H^2}; \tag{8}$$

using the Friedmann constraint (1) it is possible to reduce one degree of freedom, namely $\Omega_{\rm r} = 1 - x - y$. Then the equation of motion can be written as

$$\frac{{\rm d}x}{{\rm d}N} = 3x^2\gamma_{\rm de} - 3x\gamma_{\rm de} - 4x^2 - 3\xi_0 x\sqrt{y} - xy + 4x - \frac{Q}{3H(t)^3}, \tag{9}$$

$$\frac{{\rm d}x}{{\rm d}N} = 3xy\gamma_{\rm de} - 4xy - 3\xi_0 y^{3/2} - y^2 + 3\xi_0\sqrt{y} + y + \frac{Q}{3H(t)^3}, \tag{10}$$

where the derivatives are with respect to the e-folding number $N \equiv \ln a$ and we have introduced the dimensionless parameter

$$\xi_0 = \frac{\xi}{H_0\sqrt{\Omega_{\rm dm0}}}, \tag{11}$$

where H_0 and $\Omega_{\rm dm0}$ are the present day values of the Hubble parameter and the dark matter dimensionless density parameter. In order to guarantee nonviolation of the LSLT [32–34], $\xi_0 \geq 0$.

In addition, in order to achieve an autonomous system from (9) and (10) we must define the interaction function

Table 1 Location, existence conditions according to the phase space (15), and stability of the critical points of the autonomous system (13) and (14) for $\gamma_{\rm de}=0$ and $z=3\beta x$. The eigenvalues of the linear perturbation matrix associated to each of the following critical points are displayed in Table 2. We have introduced the definitions $A=\sqrt{4\beta+\xi_0^2}$, $B=\sqrt{-A\xi_0+2\beta+\xi_0^2}$, $C=\sqrt{A\xi_0+2\beta+\xi_0^2}$ and $D=8-8\beta-\zeta_0^2$

P_i	x	y	Existence	Stability
P_1	0	0	Always	Unstable if $\beta<\frac{4}{3}$ Saddle if $\beta>\frac{4}{3}$
P_2	0	1	Always	Saddle if $\beta<1\wedge 0<\xi_0<1-\beta$ Stable if $(\beta\le 1\wedge\xi_0>1-\beta)$ $\vee(\beta>1\wedge\xi_0>0)$
P_3	$\frac{1}{8}(\xi_0(A-3\sqrt{2}B)+D)$	$\frac{1}{2}(-A\xi_0+2\beta+\xi_0^2)$	$\beta\le 0\wedge(4\beta+\xi_0^2\ge 0\wedge 0<\xi_0\le 2)\vee(\xi_0>2\wedge\beta+\xi_0\ge 1)$	See discussion in Sect. 3.1.1
P_4	$\frac{1}{8}(-\xi_0(A+3\sqrt{2}C)+D)$	$\frac{1}{2}(A\xi_0+2\beta+\xi_0^2)$	$(0<\xi_0<2\wedge-\frac{\xi_0^2}{4}\le\beta\le 1-\xi_0)$ $\vee(\xi_0=2\wedge\beta=-1)$	see discussion in Sect. 3.1.1

Q. If the interaction term is taken as $Q=3Hf(\rho_{\rm m},\rho_{\rm de})$ [21,44–49], then we can introduce a new function

$$z\equiv\frac{Q}{3H^3}=z(x,y), \tag{12}$$

hence, the system (9) and (10) can be written as a two-dimensional autonomous system,

$$\frac{{\rm d}x}{{\rm d}N}=3(x-1)x\gamma_{\rm de}-3\xi_0x\sqrt{y}-x(4x+y-4)-z, \tag{13}$$

$$\frac{{\rm d}y}{{\rm d}N}=3xy\gamma_{\rm de}-y(4x+y-1)-3\xi_0(y-1)\sqrt{y}+z. \tag{14}$$

Imposing the conditions that radiation, dark matter and DE components be positive, definite, and bounded at all times, we can define the phase space of Eqs. (13) and (14) as

$$\Psi=\{(x,y):0\le 1-x-y\le 1,0\le x\le 1,0\le y\le 1\}. \tag{15}$$

Moreover, we can introduce another cosmological parameter of interest, like the deceleration parameter ($q=-(1+\dot{H}/H^2)$) and the total effective EOS ($w_{\rm eff}$) in terms of the dimensionless phase space variables (8):

$$q=\frac{1}{2}\left(3x\gamma_{\rm de}-4x-3\xi_0\sqrt{y}-y+2\right), \tag{16}$$

$$w_{\rm eff}=\frac{1}{3}\left(3x\gamma_{\rm de}-4x-3\xi_0\sqrt{y}-y+1\right). \tag{17}$$

3.1 Dynamics of the autonomous system

The autonomous system (13) and (14) allows one to study the dynamics of (2)–(4) and (5) for general interaction functions of the form $Q=3Hf(\rho_{\rm m},\rho_{\rm de})$. However, we will focus our attention on those interaction functions that lead to recovery of a critical point associated with a MDE, in order to explain the structure formation. The latter requirement implies the existence of a critical point of the form $(x,y)=(0,1)$. Thus, simple inspection of (13) and (14) shows that $(x,y)=(0,1)$ leads to

$$z(x=0,y=1)=0. \tag{18}$$

Some proposed forms of the interaction functions compatible with (18) are:

(a) $Q=3H\lambda\frac{\rho_{\rm m}\rho_{\rm de}}{\rho_{\rm m}+\rho_{\rm de}}\to z(x,y)=3\lambda\frac{xy}{x+y}$ [47]
(b) $Q=3H(\alpha\rho_{\rm m}+\beta\rho_{\rm de})\to z(x,y)=3(\beta x+\alpha y)$ [44,45]

where, in case (b), $\alpha=0$ in order to fulfill condition (18) and recover a MDE.[4] For mathematical simplicity, henceforth we will only study the second case that leads to [44,45,49,50]

$$z=3\beta x. \tag{19}$$

We also will restrict our analysis to the case $\gamma_{\rm de}=0$. The full set of critical points of (13) and (14) are summarized in Table 1, whereas the corresponding eigenvalues of the linear perturbation matrix are given in Table 2.

3.1.1 Critical points and stability

P_1 represents a decelerating solution ($q=1$, $w_{\rm eff}=1/3$) dominated by the radiation component, $\Omega_{\rm r}=1$, and it exists, unrestricted by the sign/value of the interaction and bulk viscosity parameters. However, its stability behavior depends on the value of the interaction parameter β, it namely being (i) unstable if $\beta<4/3$ or (ii) saddle if $\beta>4/3$.

Critical point P_2 corresponds to a pure dark matter-domination period ($\Omega_{\rm m}=1$) and always exists.[5] If the condition $\xi_0^2\ll 0$ is satisfied, then this point corresponds to the

[4] See a similar analysis in the case of the ansatz $\zeta_i=\zeta_i(H)$ in [21].

[5] Recall that P_3, like P_1, exists independently of the value/sign of the bulk viscosity and the existence of interaction between the dark components. This point has a similar behavior to points $2a$ in [21] and P_2 in [22].

Table 2 Eigenvalues and some basic physical parameters for the critical points listed in Table 1; see also Eqs. (16) and (17)

P_i	λ_1	λ_2	Ω_r	$w_{\rm eff}$	q
P_1	$4-3\beta$	$\mathrm{sgn}(\xi)\infty$	1	$\frac{1}{3}$	1
P_2	$-3\xi_0-1$	$-3(\beta+\xi_0-1)$	0	$-\xi_0$	$\frac{1}{2}(1-3\xi_0)$
P_3	See Appendix A	See Appendix A	0	$-1+\beta$	$\frac{3\beta}{2}-1$
P_4	See Appendix B	See Appendix B	0	$-1+\beta$	$\frac{3\beta}{2}-1$

standard matter-domination period, namely $w_{\rm eff}\approx 0$ and $q\approx 1/2$. Otherwise $w_{\rm eff}$ is negative and can behave as an accelerated solution if $\xi_0>\frac{1}{3}$ or even as a phantom solution if $\xi_0>1$. As Tables 1 and 2 show, these accelerated solutions are possible in the absence of dark energy ($x=\Omega_{\rm de}=0$). From the stability point of view, P_3 displays two different behaviors, that is: (i) saddle if $\beta<1\wedge 0<\xi_0<1-\beta$ or (ii) stable if $(\beta\le 1\wedge \xi_0>1-\beta)\vee(\beta>1\wedge\xi_0>0)$.

P_3 represents a scaling solution between dark matter and dark energy components and exists when

$$(\beta\le 0\wedge 4\beta+\xi_0^2\ge 0\wedge 0<\xi_0\le 2)\\ \vee(\beta\le 0\wedge\xi_0>2\wedge\beta+\xi_0\ge 1).$$

A background level, P_3 is able to mimic accelerated solutions[6] in the phantom and de Sitter regions, namely:

(i) Phantom region ($w_{\rm eff}<-1$)

1. Saddle if $0<\xi_0\le 2\wedge -\frac{\xi_0^2}{4}\le\beta<0$; see Fig. 1 for more details.
2. Saddle if $\xi_0>2\wedge 1-\xi_0\le\beta<0$; see Fig. 2 for more details.

(ii) de Sitter region ($w_{\rm eff}=-1$)

1. Only if $\beta=0$. We do not consider this case here because that means a null interaction between dark matter and dark energy.[7]

The critical point P_4 corresponds to a scaling solution between dark matter and dark energy. This point exists in the region

$$\left(0<\xi_0<2\wedge -\frac{\xi_0^2}{4}\le\beta\le 1-\xi_0\right)\vee(\xi_0=2\wedge\beta=-1)\,.$$

In the existence regions, P_4 is able to mimic only accelerated solutions, namely

(iii) Phantom region ($w_{\rm eff}<-1$)

1. Stable if $0<\xi_0\le 1\wedge -\frac{\xi_0^2}{4}\le\beta<0$.

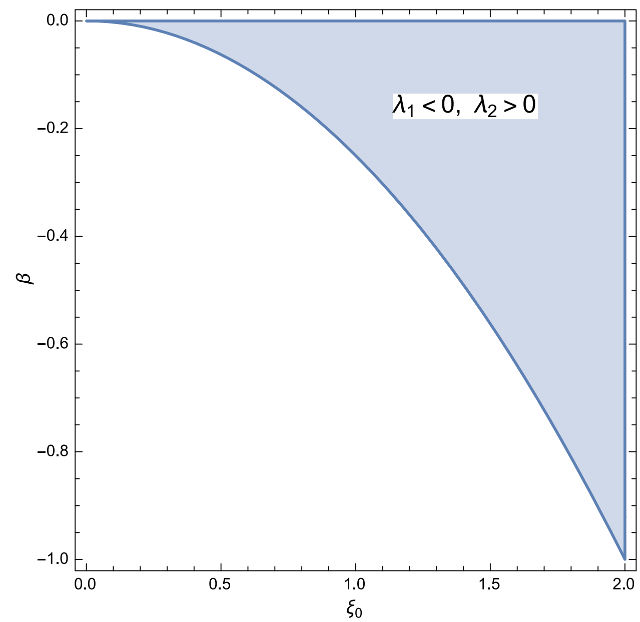

Fig. 1 Saddle ($\lambda_1<0$ and $\lambda_2>0$) region for P_3 in the phantom region case 1. See the corresponding eigenvalues (λ_1,λ_2) in Appendix A

2. Saddle if $1<\xi_0<2\wedge -\frac{\xi_0^2}{4}\le\beta\le 1-\xi_0$ in a narrow region in the parameter space (ξ_0,β), as Fig. 3 shows, otherwise it is stable.
3. If $\xi_0=2\wedge\beta=-1$ then $w_{\rm eff}=-2$, being an unrealistic value for the effective EOS parameter.

(iv) de Sitter region ($w_{\rm eff}=-1$)

1. As in P_3, $\beta=0$ leads to a de Sitter solution. As we mentioned before, this is discarded because it requires a null interaction between dark matter and dark energy.[8]

(v) Quintessence region ($-1<w_{\rm eff}<-1/3$)

1. Stable if $0<\xi_0\le\frac{1}{3}\wedge 0<\beta<\frac{2}{3}$.
2. Saddle if $\frac{1}{3}<\xi_0<1\wedge 0<\beta\le 1-\xi_0$ in a narrow region in the parameter space (ξ_0,β), as Fig. 4 shows, otherwise is stable.

[6] Unlike the previous critical points (P_1–P_2), it is not possible to reproduce, in the region of existence, decelerated solutions such as pressureless matter ($w_{\rm eff}=0$) or radiation ($w_{\rm eff}=1/3$).

[7] The case with $\beta=0$ was studied in [22].

[8] Recall that the null interaction case was developed in [22].

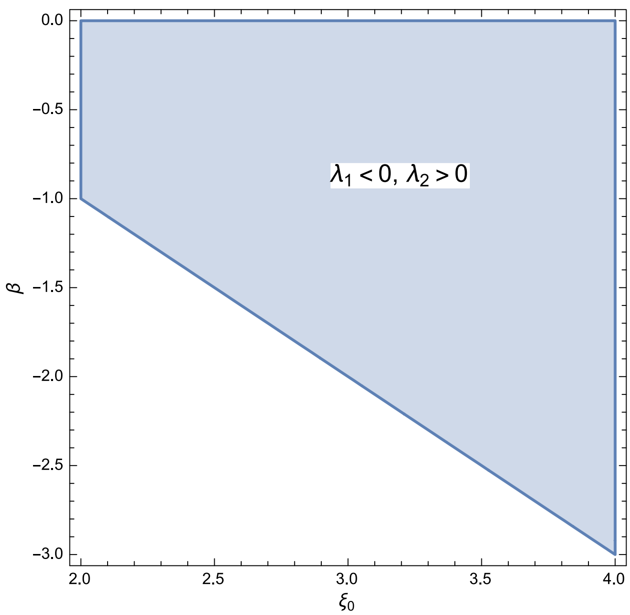

Fig. 2 Saddle ($\lambda_1 < 0$ and $\lambda_2 > 0$) region for P_3 in the phantom region case 2. See the corresponding eigenvalues (λ_1, λ_2) in Appendix A

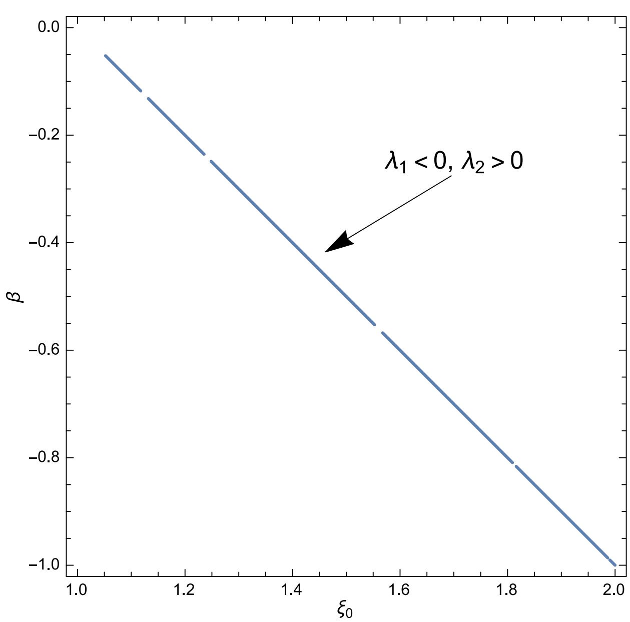

Fig. 3 Saddle ($\lambda_1 < 0$ and $\lambda_2 > 0$) region for P_4 in the phantom region case 2. See the corresponding eigenvalues (λ_1, λ_2) in Appendix B

3.1.2 *Cosmological evolution*

According to current observational data, any model that aims to make a complete description of the evolution of the Universe must follow the complete cosmological paradigm [21, 22,30]. This paradigm imposes transitions between three different evolution eras from early times to late times, namely: (i) radiation-dominated era (RDE), (ii) matter-dominated era (MDE) at intermediate stage of evolution, and (iii) era of

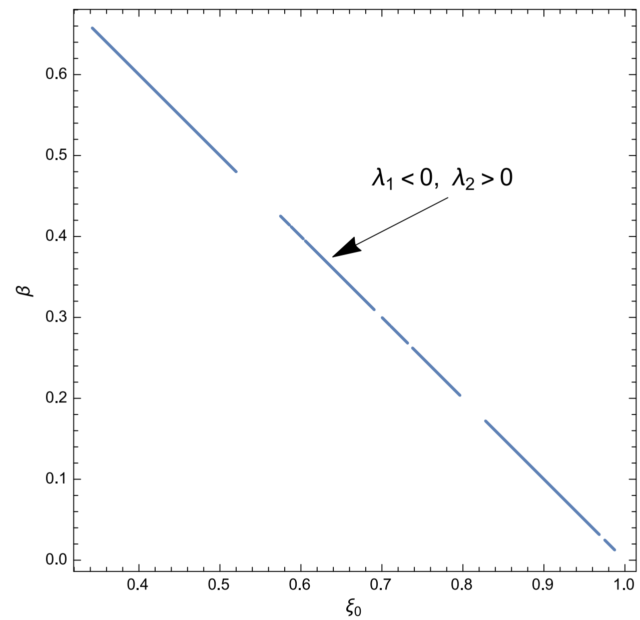

Fig. 4 Saddle ($\lambda_1 < 0$ and $\lambda_2 > 0$) region for P_4 in the quintessence region case 2. See the corresponding eigenvalues (λ_1, λ_2) in Appendix B

accelerated expansion. Every one of these statements can be translated into a critical point connected by heteroclinic orbits [51–54].

The condition for a purely RDE ($\Omega_r = 1$) is always satisfied by P_1, independently of the value of the bulk viscosity parameter ξ_0. Its unstable behavior, given that $\beta < 4/3$, guarantees that it can be the source of any solution in the phase space.

For intermediate stages of cosmic evolution, the presence of MDE is needed in order to describe the formation of structures. This matter-dominated period can be recovered by P_2. This critical point exists independently of the value of the bulk viscosity parameter but a background level, for a non-null value of ξ_0, it behaves as a decelerating solution[9] if $0 < \xi_0 < 1/3$. If the bulk viscosity takes a sufficiently small value, $\xi_0 \approx 0$, it is possible to recover $w_{\mathrm{eff}} \approx 0$ and $q \approx 1/2$. In order to bound the possible values for ξ_0 to fulfill the latter statement, we will use recent constraints on the dark matter EOS, which state that $-0.000896 < w_{\mathrm{dm}} < 0.00238$ at the 3σ level [55] using the latest Planck data release [9].[10] Thus, only a tiny contribution of bulk viscosity is allowed in order to recover a true MDE with P_2:

$$0 \leq \xi_0 < 0.000896, \tag{20}$$

[9] As the existence of this critical point is also independent of the interaction between dark matter and dark energy, these results recover the behavior of P_2 in [22].

[10] Stricter limits on w_{dm} were placed by [56] using large-scale cosmological observations. Among others, similar constraints on w_{dm} are found in [57,58].

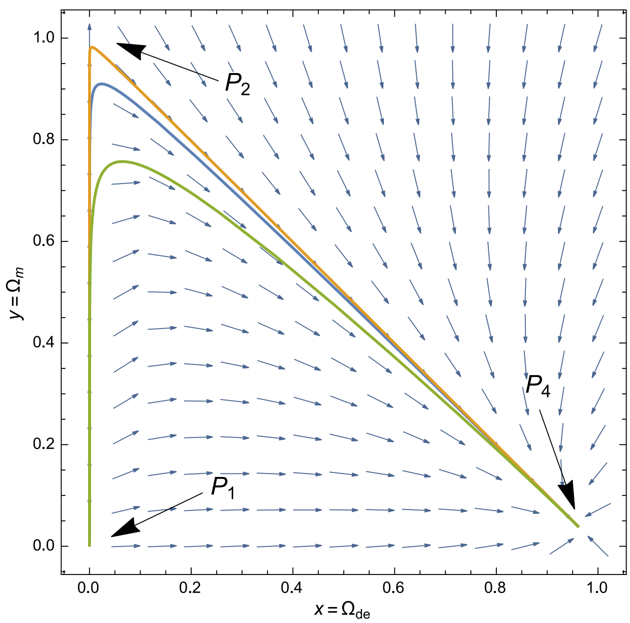

Fig. 5 Vector field in the plane (x, y) for the autonomous system (13) and (14) with $\gamma_{de} = 0$. The free parameters have been chosen as $(\xi_0, \beta) = (0.0008, 0.038)$. In this case, the quintessence solution, P_4, is the late time attractor of the system, representing an accelerated solution ($w_{eff} = -0.96$). The transition from the RDE (P_1) to P_4 allows for the selection of appropriate initial conditions to recover a true MDE (P_3) with $w_{eff} \simeq 0$ according to condition (20) [55]

these constraints on ξ_0 are also consistent with those obtained in [18,19] in the absence of interaction between dark matter and dark energy. As we mentioned in Sect. 3.1.1, P_2 is able to reproduce an accelerated solution given that $\xi_0 > 1/3$. However, as Tables 1 and 2 show, this possible behavior has to be ruled out because of the impossibility of finding another critical point.

Concerning the late time evolution of the Universe, the model has two more critical point capable of providing accelerated solutions, namely P_3 and P_4. Both represent scaling solutions between dark matter and dark energy. As was discussed in Sect. 3.1.1, from the mathematical point of view, it is possible to obtain phantom, de Sitter and quintessence solutions with saddle or stable behaviors depending of the values of the free parameters (ξ_0, β). If the interaction parameter is negative ($\beta < 0$), meaning energy transfer from dark matter to dark energy, it is possible to obtain a late time transition between two phantom solutions: P_3 case (i)1 (saddle) $\rightarrow$ P_4 case (iii)1 (stable). This transition requires $-\xi_0^2/4 \leq \beta < 0$, but if we also demand previous stages of RDE and MDE we must impose condition (20), leading to an almost null value for the interaction parameter,

$$-2.00704 \times 10^{-7} < \beta < 0, \tag{21}$$

thus the phantom solutions P_3 and P_4 tend to de Sitter solutions $w_{eff} = -1$ ($\beta = 0$). The rest of the late time phantom solutions demand very large values of the bulk viscosity parameter, $\xi_0 > 1$, compared to those allowed by (20) in order to recover a true MDE; hence they are ruled out.

The only possible late time scenario with a non-null value of the interaction parameter corresponds to a stable quintessence solution (P_4). This solution requires

$$0 < \xi_0 \leq \frac{1}{3} \wedge 0 < \beta < \frac{2}{3}.$$

If we impose the condition (20) to ensure a true MDE and take into account the latest constraint on the value of the dark energy EOS [9], the following tiny region is obtained for the interaction parameter:

$$0 < \beta \leq 0.039. \tag{22}$$

Figure 5 shows some example orbits in the plane ($x = \Omega_{de}$, $y = \Omega_m$) to illustrate the above scenario.

4 Concluding remarks

In this work we studied the dynamics of model of the Universe filled with radiation, dark matter and dark energy. The dark matter component was treated as an imperfect fluid having bulk viscosity, whereas the remaining fluids were considered as perfect fluids. The bulk viscosity was taken as proportional to the dark matter density $\zeta \propto \rho_m^{\frac{1}{2}}$ [20] and we introduce an interaction term between the dark matter and the dark energy components with the objective of extending the previous results developed in [22]. This new term was taken as $Q = 3H\rho_{de}$ [44,45,49,50].

Recall that the ansatz on the bulk viscosity used in [21,50] ($\zeta \propto H$) is different from the one used in this work. Thus the results obtained now are new compared with those obtained in [21,50] and an extension to those obtained in [22] by the introduction of the interaction term.

We performed a dynamical system analysis of the model in order to investigate its asymptotic evolution and behavior. The imposition of a transition from a RDE to an accelerated dominated solution, passing through a true MDE reduces the possible values of the bulk viscosity parameter to a tiny region $0 \leq \xi_0 < 0.000896$. This finding extends those obtained in [18,19,22] with no interaction between dark matter and dark energy.

The presence of an interaction between dark matter and dark energy allows one, from the mathematical point of view, to obtain stable (saddle) late time accelerated solutions in the phantom, de Sitter and quintessence regions. However, the requirement of a true MDE imposes strong constraints on the interaction parameter β in the case of late time phantom solutions. In both cases, regardless of the direction of energy transfer between dark matter and dark energy, the interaction parameter is consistent with a null value; hence the de Sitter solution will be the late time attractor. Moreover, the impossibility of having late time accelerated solutions, caused solely by the viscous matter (P_2), found in [22] with $\beta = 0$, was extended to this new scenario with interaction between dark matter and dark energy.

The only favorable scenario with a non-null value of the interaction parameter, $0 < \beta \leq 0.039$, is described by the late time stable quintessence solution P_4. This solution is capable of fulfilling the complete cosmological paradigm, that is, a transition P_1(RDE) $\rightarrow$ P_2 (MDE) $\rightarrow$ P_4. Recall that this quintessence solution is compatible with the latest constraint on the values of the dark energy EOS [9].

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Appendix A: Eigenvalues of critical point P_3

The eigenvalues of P_3 are

$$\lambda_1 = -\frac{\sqrt{-18A\xi_0^5 + 3F_5\xi_0^4 - 6F_4\xi_0^3 + 6F_3\xi_0^2 + 24F_2\xi_0 + 8F_1 + 9\xi_0^6}}{4\sqrt{2}B} - \frac{3A\xi_0^2}{4\sqrt{2}B} + 3\beta + \frac{3\beta\xi_0}{2\sqrt{2}B} + \frac{3\xi_0^3}{4\sqrt{2}B} + \frac{3\xi_0}{2\sqrt{2}B} - \frac{7}{2} \tag{23}$$

$$\lambda_2 = \frac{\sqrt{-18A\xi_0^5 + 3F_5\xi_0^4 - 6F_4\xi_0^3 + 6F_3\xi_0^2 + 24F_2\xi_0 + 8F_1 + 9\xi_0^6}}{4\sqrt{2}B} - \frac{3A\xi_0^2}{4\sqrt{2}B} + 3\beta + \frac{3\beta\xi_0}{2\sqrt{2}B} + \frac{3\xi_0^3}{4\sqrt{2}B} + \frac{3\xi_0}{2\sqrt{2}B} - \frac{7}{2} \tag{24}$$

where

$$A = \sqrt{4\beta + \xi_0^2}, \tag{25}$$

$$B = \sqrt{-A\xi_0 + 2\beta + \xi_0^2}, \tag{26}$$

$$F_1 = (7 - 6\beta)^2 B^2 - 24\beta(3\beta^2 - 7\beta + 4), \tag{27}$$

$$F_2 = A(15\beta^2 - 32\beta + 16) + \sqrt{2}(3\beta^2 - 3\beta + 1)B, \tag{28}$$

$$F_3 = 3\sqrt{2}A(\beta - 2)B - 42\beta^2 + 124\beta - 58, \tag{29}$$

$$F_4 = 2A(6\beta - 1) + 3\sqrt{2}(\beta - 2)B, \tag{30}$$

$$F_5 = 3A^2 + 24\beta - 4. \tag{31}$$

Appendix B: Eigenvalues of critical point P_4

The eigenvalues of P_4 are

$$\lambda_1 = -\frac{\sqrt{18A\xi_0^5 + 3F_{10}\xi_0^4 + 6F_9\xi_0^3 - 6F_8\xi_0^2 - 24F_7\xi_0 + 8F_6 + 9\xi_0^6}}{4\sqrt{2}C} + \frac{3A\xi_0^2}{4\sqrt{2}C} + 3\beta + \frac{3\beta\xi_0}{2\sqrt{2}C} + \frac{3\xi_0^3}{4\sqrt{2}C} + \frac{3\xi_0}{2\sqrt{2}C} - \frac{7}{2}, \tag{32}$$

$$\lambda_2 = \frac{\sqrt{18A\xi_0^5 + 3F_{10}\xi_0^4 + 6F_9\xi_0^3 - 6F_8\xi_0^2 - 24F_7\xi_0 + 8F_6 + 9\xi_0^6}}{4\sqrt{2}C} + \frac{3A\xi_0^2}{4\sqrt{2}C} + 3\beta + \frac{3\beta\xi_0}{2\sqrt{2}C} + \frac{3\xi_0^3}{4\sqrt{2}C} + \frac{3\xi_0}{2\sqrt{2}C} - \frac{7}{2}, \tag{33}$$

where

$$A = \sqrt{4\beta + \xi_0^2}, \tag{34}$$

$$C = \sqrt{A\xi_0 + 2\beta + \xi_0^2}, \tag{35}$$

$$F_6 = (7 - 6\beta)^2 C^2 - 24\beta(3\beta^2 - 7\beta + 4), \tag{36}$$

$$F_7 = A(15\beta^2 - 32\beta + 16) + \sqrt{2}(-3\beta^2 + 3\beta - 1)C, \tag{37}$$

$$F_8 = 3\sqrt{2}A(\beta - 2)C + 42\beta^2 - 124\beta + 58, \tag{38}$$

$$F_9 = 2A(6\beta - 1) - 3\sqrt{2}(\beta - 2)C, \tag{39}$$

$$F_{10} = 3A^2 + 24\beta - 4. \tag{40}$$

References

1. A.G. Riess et al., Observational evidence from supernovae for an accelerating universe and a cosmological constant. Astron. J. **116**, 1009–1038 (1998)
2. S. Perlmutter et al., Measurements of Omega and Lambda from 42 high redshift supernovae. Astrophys. J. **517**, 565–586 (1999)
3. V. Sahni, A.A. Starobinsky, The case for a positive cosmological Lambda term. Int. J. Mod. Phys. D **9**, 373–444 (2000)
4. E.J. Copeland, M. Sami, S. Tsujikawa, Dynamics of dark energy. Int. J. Mod. Phys. D **15**, 1753–1936 (2006)
5. T. Buchert, Dark energy from structure: a status report. Gen. Relativ. Gravit. **40**, 467–527 (2008)
6. T. Clifton, P.G. Ferreira, A. Padilla, C. Skordis, Modified gravity and cosmology. Phys. Rep. **513**, 1–189 (2012)
7. M.J. Mortonson, D.H. Weinberg, M. White, *Dark Energy: A Short Review* (2013). arXiv:1401.0046
8. K. Koyama, Cosmological tests of modified gravity. Rep. Prog. Phys. **79**(4), 046902 (2016)
9. P.A.R. Ade et al., Planck 2015 results. XIII. Cosmological parameters. Astron. Astrophys. **594**, A13 (2016)
10. J.D. Barrow, The deflationary universe: an instability of the de Sitter universe. Phys. Lett. B **180**, 335–339 (1986)
11. I. Waga, R.C. Falcao, R. Chanda, A bulk viscosity driven inflationary model. Phys. Rev. D **33**, 1839 (1986)
12. R. Maartens, Dissipative cosmology. Class. Quantum Gravity **12**, 1455–1465 (1995)
13. M. Bastero-Gil, A. Berera, R. Cerezo, R.O. Ramos, G.S. Vicente, Stability analysis for the background equations for inflation with dissipation and in a viscous radiation bath. JCAP **1211**, 042 (2012)
14. M.R. Setare, V. Kamali, Warm-intermediate inflationary universe model with viscous pressure in high dissipative regime. Gen. Relativ. Gravit. **46**, 1698 (2014)

15. B. Li, J.D. Barrow, Does bulk viscosity create a viable unified dark matter model? Phys. Rev. D **79**, 103521 (2009)
16. A. Avelino, U. Nucamendi, Can a matter-dominated model with constant bulk viscosity drive the accelerated expansion of the universe? JCAP **0904**, 006 (2009)
17. H. Velten, D.J. Schwarz, Constraints on dissipative unified dark matter. JCAP **1109**, 016 (2011)
18. H. Velten, D. Schwarz, Dissipation of dark matter. Phys. Rev. D **86**, 083501 (2012)
19. H. Velten, D.J. Schwarz, J.C. Fabris, W. Zimdahl, Viscous dark matter growth in (neo-)Newtonian cosmology. Phys. Rev. D **88**(10), 103522 (2013)
20. H. Velten, J. Wang, X. Meng, Phantom dark energy as an effect of bulk viscosity. Phys. Rev. D **88**, 123504 (2013)
21. A. Avelino, Y. Leyva, L.A. Urena-Lopez, Interacting viscous dark fluids. Phys. Rev. D **88**, 123004 (2013)
22. N. Cruz, S. Lepe, Y. Leyva, F. Peña, J. Saavedra, No stable dissipative phantom scenario in the framework of a complete cosmological dynamics. Phys. Rev. D **90**(8), 083524 (2014)
23. G. Acquaviva, A. Beesham, Nonlinear bulk viscosity and the stability of accelerated expansion in FRW spacetime. Phys. Rev. D **90**(2), 023503 (2014)
24. G. Acquaviva, A. Beesham, Nonlinear bulk viscosity in FRW cosmology: a phase space analysis. Class. Quantum Gravity **32**(21), 215026 (2015)
25. G. Acquaviva, A. John, A. Pénin, Dark matter perturbations and viscosity: a causal approach. Phys. Rev. D **94**(4), 043517 (2016)
26. C. Eckart, The thermodynamics of irreversible processes. 3. Relativistic theory of the simple fluid. Phys. Rev. **58**, 919–924 (1940)
27. W. Israel, J.M. Stewart, Transient relativistic thermodynamics and kinetic theory. Ann. Phys. **118**, 341–372 (1979)
28. R. Maartens, V. Mendez, Nonlinear bulk viscosity and inflation. Phys. Rev. D **55**, 1937–1942 (1997)
29. L.P. Chimento, A.S. Jakubi, V. Mendez, R. Maartens, Cosmological solutions with nonlinear bulk viscosity. Class. Quantum Gravity **14**, 3363–3375 (1997)
30. G. Leon, Y. Leyva, J. Socorro, Quintom phase-space: beyond the exponential potential. Phys. Lett. B **732**, 285–297 (2014)
31. V. Salvatelli, N. Said, M. Bruni, A. Melchiorri, D. Wands, Indications of a late-time interaction in the dark sector. Phys. Rev. Lett. **113**(18), 181301 (2014)
32. R. Maartens, Causal thermodynamics in relativity (1996). arXiv:astro-ph/9609119
33. W. Zimdahl, D. Pavon, Expanding universe with positive bulk viscous pressures? Phys. Rev. D **61**, 108301 (2000)
34. C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973)
35. G.L. Murphy, Big-bang model without singularities. Phys. Rev. D **8**, 4231–4233 (1973)
36. S. Nojiri, S.D. Odintsov, S. Tsujikawa, Properties of singularities in (phantom) dark energy universe. Phys. Rev. D **71**, 063004 (2005)
37. H. Velten, T.R.P. Carams, J.C. Fabris, L. Casarini, R.C. Batista, Structure formation in a Λ viscous CDM universe. Phys. Rev. D **90**(12), 123526 (2014)
38. L. Chimento, A.S. Jakubi, Cosmological solutions of the Einstein equations with a causal viscous fluid. Class. Quantum Gravity **10**, 2047–2058 (1993)
39. W.A. Hiscock, J. Salmonson, Dissipative Boltzmann–Robertson–Walker cosmologies. Phys. Rev. D **43**, 3249–3258 (1991)
40. M. Zakari, D. Jou, Equations of state and transport equations in viscous cosmological models. Phys. Rev. D **48**(4), 1597 (1993)
41. S. Jeon, Hydrodynamic transport coefficients in relativistic scalar field theory. Phys. Rev. D **52**, 3591–3642 (1995)
42. S. Jeon, L.G. Yaffe, From quantum field theory to hydrodynamics: transport coefficients and effective kinetic theory. Phys. Rev. D **53**, 5799–5809 (1996)
43. O.F. Piattella, J.C. Fabris, W. Zimdahl, Bulk viscous cosmology with causal transport theory. JCAP **1105**, 029 (2011)
44. M. Quartin, M.O. Calvao, S.E. Joras, R.R.R. Reis, I. Waga, Dark interactions and cosmological fine-tuning. JCAP **0805**, 007 (2008)
45. G. Caldera-Cabral, R. Maartens, L.A. Urena-Lopez, Dynamics of interacting dark energy. Phys. Rev. D **79**, 063518 (2009)
46. W. Zimdahl, D. Pavon, Interacting quintessence. Phys. Lett. B **521**, 133–138 (2001)
47. W. Zimdahl, D. Pavn, Scaling cosmology. Gen. Relativ. Gravit. **35**, 413–422 (2003)
48. Z.-K. Guo, N. Ohta, S. Tsujikawa, Probing the coupling between dark components of the universe. Phys. Rev. D **76**, 023508 (2007)
49. B. Wang, E. Abdalla, F. Atrio-Barandela, D. Pavon, Dark matter and dark energy interactions: theoretical challenges, cosmological implications and observational signatures. Rep. Prog. Phys. **79**(9), 096901 (2016)
50. G.M. Kremer, O.A.S. Sobreiro, Bulk viscous cosmological model with interacting dark fluids. Braz. J. Phys. **42**, 77–83 (2012)
51. J. Wainwright, G.F.R. Ellis, *Dynamical Systems in Cosmology* (Cambridge University Press, Cambridge, 2005)
52. A.A. Coley, *Dynamical Systems and Cosmology* (Springer, Dordrecht, 2013)
53. J.M. Heinzle, C. Uggla, N. Rohr, The cosmological billiard attractor. Adv. Theor. Math. Phys. **13**(2), 293–407 (2009)
54. L.A. Urena-Lopez, Unified description of the dynamics of quintessential scalar fields. JCAP **1203**, 035 (2012)
55. D.B. Thomas, M. Kopp, C. Skordis, Constraining dark matter properties with cosmic microwave background observations. Astrophys. J. **830**(2), 155 (2016)
56. M. Kunz, S. Nesseris, I. Sawicki, Constraints on dark-matter properties from large-scale structure. Phys. Rev. D **94**(2), 023510 (2016)
57. C.M. Muller, Cosmological bounds on the equation of state of dark matter. Phys. Rev. D **71**, 047302 (2005)
58. J. Barranco, A. Bernal, D. Nunez, Dark matter equation of state from rotational curves of galaxies. Mon. Not. R. Astron. Soc. **449**(1), 403–413 (2015)