Eur. Phys. J. C (2017) 77:321
DOI 10.1140/epjc/s10052-017-4892-8

THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

Open charm-bottom axial-vector tetraquarks and their properties

S. S. Agaev[1], **K. Azizi**[2,a], **H. Sundu**[3]

[1] Institute for Physical Problems, Baku State University, Az 1148 Baku, Azerbaijan
[2] Department of Physics, Doğuş University, Acibadem-Kadiköy, 34722 Istanbul, Turkey
[3] Department of Physics, Kocaeli University, 41380 Izmit, Turkey

Received: 12 March 2017 / Accepted: 5 May 2017 / Published online: 18 May 2017
© The Author(s) 2017. This article is an open access publication

Abstract The charged axial-vector $J^P = 1^+$ tetraquarks $Z_q = [cq][\bar{b}\bar{q}]$ and $Z_s = [cs][\bar{b}\bar{s}]$ with the open charm-bottom contents are studied in the diquark–antidiquark model. The masses and meson–current couplings of these states are calculated by employing a QCD two-point sum rule approach, where the quark, gluon and mixed condensates up to eight dimensions are taken into account. These parameters of the tetraquark states Z_q and Z_s are used to analyze the vertices $Z_q B_c \rho$ and $Z_s B_c \phi$ to determine the strong $g_{Z_q B_c \rho}$ and $g_{Z_s B_c \phi}$ couplings. For these purposes, the QCD light-cone sum rule method and its soft-meson approximation are utilized. The couplings $g_{Z_q B_c \rho}$ and $g_{Z_s B_c \phi}$, extracted from this analysis, are applied for evaluating the strong $Z_q \to B_c \rho$ and $Z_s \to B_c \phi$ decays' widths, which are essential results of the present investigation. Our predictions for the masses of the Z_q and Z_s states are confronted with similar results available in the literature.

1 Introduction

Charmonium-like states discovered during the last years mainly in the exclusive B-meson decays as resonances in the relevant mass distributions became interesting objects for both experimental and theoretical studies in high energy physics. Conventional hadrons, composed of two and three quarks, and investigated in a rather detailed form, constitute the main part of the known particles. At the same time, the theory of the strong interactions—Quantum Chromodynamics—does not contain principles excluding the existence of the multi-quark states. The tetraquark and pentaquark states composed of four and five valence quarks, respectively, and hybrids built by quarks and gluons are among the most promising candidates to occupy the vacant shelves in the multi-quark spectroscopy. Due to joint efforts of experimentalists and theorists considerable progress in understanding of the quark–gluon structure of the multi-quark—exotic states and explaining their properties—was achieved, but remaining questions are more numerous than answered ones (for the latest reviews, see Refs. [1–4]).

The main source of problems, which complicates the study of the charmonium-like tetraquarks, is the existence of conventional charmonium states in the energy ranges of the decay processes to be explored. Charmonia generates difficulties in the interpretation of experimental results, because the pure $c\bar{c}$ states may emerge as the resonances in the mass distributions of the processes, or generate background effects due to states dynamically connected with $c\bar{c}$ levels. Only after eliminating effects of the charmonium states in forming of the experimental data, the observed resonances can be considered as real exotic particles. The well-known $X(3872)$ state is the best sample to illustrate existing problems. It was discovered as a very narrow resonance in B-meson decay $B \to KX \to KJ/\psi\rho \to KJ/\psi\pi^+\pi^-$ by the Belle Collaboration [5], and it was later confirmed in CDF, D0 and BaBar experiments (see Refs. [6–8]). Its other production mechanisms running through decay chains $B \to KX \to KJ/\psi\omega \to KJ/\psi\pi^+\pi^-\pi^0$, $B \to KX \to KJ/\psi\gamma$ and $B \to KX \to K\psi(2S)\gamma$ were also experimentally measured and comprehensively studied [9,10]. The gathered information poses severe restrictions on theoretical models claiming to describe a behavior of the $X(3872)$ state. Attempts were made to explain the collected data by treating $X(3872)$ as the excited conventional charmonium $\chi_{c1}(2^3P_1)$ [11], or as the state formed due to dynamical coupled-channel effects [12]. It was considered in the context of four-quark compounds, both as the $D\bar{D}^\star$ molecule or its admixtures with the charmonium states [13–16], and as the diquark–antidiquarks states [17–21].

But tetraquarks, which do not contain $\bar{c}c$ or $\bar{b}b$ pairs might also exist, because the fundamental laws of QCD do not forbid production of such resonances in hadronic processes. These particles may appear in the exclusive reactions

[a] e-mail: azizi.hep.ph@gmail.com

as the open charm (i.e., as states containing c or $\bar{c}$ quarks) and open bottom resonances. The $D^{\star}_{s0}(2317)$ and $D_{s1}(2460)$ mesons, discovered by the BaBar and CLEO Collaborations [22,23], are now being considered as candidates for open charm tetraquark states. The $X(5568)$ resonance remains a unique candidate for the open bottom tetraquark, which is also a particle containing four different quarks. Unfortunately, the experimental situation formed around $X(5568)$ remains unclear. Indeed, the evidence for $X(5568)$ was first announced by the D0 Collaboration in Ref. [24]. Later it was seen again by D0 in the B_s^0 meson's semileptonic decays [25]. Nevertheless, the LHCb and CMS Collaborations could not see the same resonance by an analysis of their experimental data [26,27]. Theoretical investigations aiming to explain the nature of $X(5568)$ and calculate its parameters lead also to contradictory conclusions. Predictions obtained in some of this work are in a nice agreement with results of the D0 Collaboration, while in others even the existence of the $X(5568)$ state is an object of doubt. The detailed discussions of these and related questions of the physics of the $X(5568)$ state can be found in the original work (see Ref. [2] and the references therein).

The open charm-bottom tetraquarks belong to another type of exotic states. They already attracted the interest of physicists even though still they have not been observed experimentally. The original investigations of these particles started more than two decades ago, and, therefore, a considerable theoretical information on their expecting properties is available in the literature. For example, the open charm-bottom type tetraquarks with the contents $\{Qq\}\{Q'q\}$, $\{Qs\}\{Q's\}$ and molecule structures were considered in Refs. [28,29], respectively. In these papers the masses of these hypothetical states were calculated in the context of the QCD two-point sum rule approach using in the operator product expansion (OPE) operators up to dimension six. In the framework of the diquark–antidiquark model the open charm-bottom states were analyzed in Ref. [30]. In order to extract masses of these states, the authors again utilized the QCD sum rule method, interpolating currents of different color structure. Other aspects of these tetraquark systems can be found in Refs. [31–35].

In a previous article [36] we explored the charged scalar $J^P = 0^+$ tetraquark states $Z_q = [cq][\bar{b}\bar{q}]$ and $Z_s = [cs][\bar{b}\bar{s}]$ in the context of the diquark–antidiquark model, and we calculated their masses and the widths of some of their decay channels. In the present work we extend our investigations by including into analysis the axial-vector $J^P = 1^+$ $Z_q = [cq][\bar{b}\bar{q}]$ and $Z_s = [cs][\bar{b}\bar{s}]$ open charm-bottom tetraquarks, and their kinematically allowed decay modes.

We start from calculation of their masses and meson–current couplings. For these purposes, we employ the QCD two-point sum rule method, which was invented to calculate the parameters of the conventional hadrons [37], but it soon was applied to an analysis of the exotic states as well (see Refs. [38–43]). The parameters of the open charm-bottom tetraquarks obtained within this method are used to explore the strong vertices $Z_q B_c \rho$ and $Z_s B_c \phi$ and calculate the corresponding couplings $g_{Z_q B_c \rho}$ and $g_{Z_s B_c \phi}$. These couplings are required to evaluate the widths of the $Z_q \to B_c \rho$ and $Z_s \to B_c \phi$ decays. To this end, we apply the QCD light-cone sum method and the soft-meson approximation proposed in Refs. [44–46]. For analysis of the strong vertices of tetraquarks the method was, for the first time, examined in Ref. [47], and afterwards successfully used to investigate decay channels of some tetraquarks states (see Refs. [48–50]).

The present work is organized in the following manner. In Sect. 2 we calculate the masses and meson–current couplings of the axial-vector tetraquarks with open charm-bottom contents. Section 3 is devoted to the computation of the strong couplings $g_{Z_q B_c \rho}$ and $g_{Z_s B_c \phi}$. In this section we calculate the widths of the decays $Z_q \to B_c \rho$ and $Z_s \to B_c \phi$. In Sect. 4 we examine our results as part of the general tetraquark's physics and compare them with predictions of Ref. [30], where the masses of the axial-vector open charm-bottom tetraquarks were found. It contains also our concluding remarks.

2 Masses and meson–current couplings

In order to find the masses and meson–current couplings of the diquark–antidiquark type axial-vector states Z_q and Z_s, we use the two-point QCD sum rules. Below the explicit expressions for the Z_q state are written down. Their generalization to the Z_s tetraquark is straightforward.

The two-point sum rule can be extracted from an analysis of the correlation function

$$\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \left\langle 0|\mathcal{T}\{J_\mu(x) J_\nu^{\dagger}(0)\}|0\right\rangle, \qquad (1)$$

where J_μ is the interpolating current of the Z_q state.

The scalar and axial-vector open charm-bottom diquark–antidiquark states can be modeled using different type of interpolating currents [30]. Thus, the interpolating currents can be either symmetric or antisymmetric in the color indices. In our previous work we chose the symmetric interpolating current to find masses and decay widths of the scalar open charm-bottom tetraquarks [36]. In the present work to consider the axial-vector tetraquark states Z_q and Z_s we use again the interpolating currents, which are symmetric in the color indices. Such an axial-vector current has the following form:

$$J_\mu = q_a^T C\gamma_5 c_b \left(\overline{q}_a \gamma_\mu C \overline{b}_b^T + \overline{q}_b \gamma_\mu C \overline{b}_a^T \right), \qquad (2)$$

and it is symmetric under exchange of the color indices $a \leftrightarrow b$. Here by C we denote the charge conjugation matrix.

To derive QCD sum rules for the mass and meson–current coupling we follow standard prescriptions of the sum rule method and express the correlation function $\Pi_{\mu\nu}(p)$ in terms of the physical parameters of the Z_q state, which results in obtaining $\Pi_{\mu\nu}^{\rm Phys}(p)$. From another side the same function should be obtained in terms of the quark–gluon degrees of freedom $\Pi_{\mu\nu}^{\rm QCD}(p)$.

We start from the function $\Pi_{\mu\nu}^{\rm Phys}(p)$ and compute it by suggesting that the tetraquarks under consideration are the ground states in the relevant hadronic channels. After saturating the correlation function with a complete set of the Z_q states and performing in Eq. (1) integration over x, we get the required expression for $\Pi_{\mu\nu}^{\rm Phys}(p)$,

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{\langle 0|J_\mu|Z_q(p)\rangle\langle Z_q(p)|J_\nu^\dagger|0\rangle}{m_Z^2-p^2}+\cdots$$

where m_Z is the mass of the Z_q state, and dots indicate contributions coming from higher resonances and continuum states. We introduce the meson–current coupling f_Z by means of the equality

$$\langle 0|J_\mu|Z_q(p)\rangle=f_Z m_Z\varepsilon_\mu,$$

where ε_μ is polarization vector of the axial-vector tetraquark. In terms of m_Z and f_Z the correlation function takes the simple form

$$\Pi_{\mu\nu}^{\rm Phys}(p)=\frac{m_Z^2f_Z^2}{m_Z^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_Z^2}\right)+\cdots . \tag{3}$$

Having applied the Borel transformation to the function $\Pi_{\mu\nu}^{\rm Phys}(p)$ we get

$$\mathcal{B}_{p^2}\Pi_{\mu\nu}^{\rm Phys}(p^2)=m_Z^2f_Z^2e^{-m_Z^2/M^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{m_Z^2}\right)+\ldots \tag{4}$$

In order to obtain the function $\Pi_{\mu\nu}^{\rm QCD}(p)$ we substitute the interpolating current given by Eqs. (2) into (1), and we employ the light and heavy quark propagators in calculations. For $\Pi_{\mu\nu}^{\rm QCD}(p)$, as a result, we get

$$\begin{aligned}\Pi_{\mu\nu}^{\rm QCD}(p)=i\int d^4xe^{ipx}&\Big\{{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{b'b}(-x)\gamma_\nu S_q^{a'a}(-x)\right]\\ &\times{\rm Tr}\left[\gamma_5\widetilde{S}_q^{aa'}(x)\gamma_5S_c^{bb'}(x)\right]+{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{a'b}(-x)\right.\\ &\times\left.\gamma_\nu S_q^{b'a}(-x)\right]{\rm Tr}\left[\gamma_5\widetilde{S}_q^{aa'}(x)\gamma_5S_c^{bb'}(x)\right]\\ &+{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{b'a}(-x)\gamma_\nu S_q^{a'b}(-x)\right]{\rm Tr}\left[\gamma_5\widetilde{S}_q^{aa'}(x)\gamma_5S_c^{bb'}(x)\right]\\ &+{\rm Tr}\left[\gamma_\mu\widetilde{S}_b^{a'a}(-x)\gamma_\nu S_q^{b'b}(-x)\right]{\rm Tr}\left[\gamma_5\widetilde{S}_q^{aa'}(x)\gamma_5S_c^{bb'}(x)\right]\Big\},\end{aligned} \tag{5}$$

where

$$\widetilde{S}_{q(b)}^{ab}(x)=CS_{q(b)}^{Tab}(x)C, \tag{6}$$

with $S_q(x)$ and $S_b(x)$ being the q- and b-quark propagators, respectively.

We proceed including into the analysis the well-known expressions of the light and heavy quark propagators. For our aims it is convenient to use the x-space expression of the light quark propagator,

$$\begin{aligned}S_q^{ab}(x)=&\,i\delta_{ab}\frac{\not x}{2\pi^2x^4}-\delta_{ab}\frac{m_q}{4\pi^2x^2}-\delta_{ab}\frac{\langle\overline{q}q\rangle}{12}\\ &+i\delta_{ab}\frac{\not x m_q\langle\overline{q}q\rangle}{48}-\delta_{ab}\frac{x^2}{192}\langle\overline{q}g_s\sigma Gq\rangle\\ &+i\delta_{ab}\frac{x^2\not x m_q}{1152}\langle\overline{q}g_s\sigma Gq\rangle-i\frac{g_sG_{ab}^{\alpha\beta}}{32\pi^2x^2}\left[\not x\sigma_{\alpha\beta}+\sigma_{\alpha\beta}\not x\right]\\ &-i\delta_{ab}\frac{x^2\not x g_s^2\langle\overline{q}q\rangle^2}{7776}-\delta_{ab}\frac{x^4\langle\overline{q}q\rangle\langle g_sq^2G^2\rangle}{27648}+\cdots .\end{aligned} \tag{7}$$

For the heavy $Q=b,\ c$ quarks we utilize the propagator $S_Q^{ab}(x)$ given in the momentum space in Ref. [51]:

$$\begin{aligned}S_Q^{ab}(x)=&\,i\int\frac{d^4k}{(2\pi)^4}e^{-ikx}\Big\{\frac{\delta_{ab}\left(\not k+m_Q\right)}{k^2-m_Q^2}\\ &-\frac{g_sG_{ab}^{\alpha\beta}}{4}\frac{\sigma_{\alpha\beta}\left(\not k+m_Q\right)+\left(\not k+m_Q\right)\sigma_{\alpha\beta}}{(k^2-m_Q^2)^2}\\ &+\frac{g_s^2G^2}{12}\delta_{ab}m_Q\frac{k^2+m_Q\not k}{(k^2-m_Q^2)^4}+\frac{g_s^3G^3}{48}\delta_{ab}\frac{\left(\not k+m_Q\right)}{(k^2-m_Q^2)^6}\\ &\times\left[\not k\left(k^2-3m_Q^2\right)+2m_Q\left(2k^2-m_Q^2\right)\right]\left(\not k+m_Q\right)+\cdots\Big\}.\end{aligned} \tag{8}$$

In the expressions above

$$\begin{aligned}&G_{ab}^{\alpha\beta}=G_A^{\alpha\beta}t_{ab}^A,\quad G^2=G_{\alpha\beta}^AG_{\alpha\beta}^A,\\ &G^3=f^{ABC}G_{\mu\nu}^AG_{\nu\delta}^BG_{\delta\mu}^C,\end{aligned} \tag{9}$$

where $a,\ b=1,2,3$ are color indices and $A,B,C=1,\ 2,\ldots,8$. Here $t^A=\lambda^A/2$, where λ^A are the Gell-Mann matrices, and the gluon field strength tensor is fixed at $x=0$, i.e. $G_{\alpha\beta}^A\equiv G_{\alpha\beta}^A(0)$.

The QCD sum rules can be derived after fixing the Lorentz structures in both the physical and the theoretical expressions of the correlation function and equating the correspondent invariant functions. In the case of the axial-vector particles the Lorentz structures in these expressions behave $\sim g_{\mu\nu}$ and $\sim p_\mu p_\nu$. Because the structures $\sim p_\mu p_\nu$ are contaminated by the scalar states with the same quark contents, we choose $\sim g_{\mu\nu}$ and the invariant function $\Pi^{\rm QCD}(p^2)$ corresponding to this structure. Then on the theoretical side of the sum rule there is only one invariant function $\Pi^{\rm QCD}(p^2)$, which can be represented as the dispersion integral

$$\Pi^{\rm QCD}(p^2) = \int_{\mathcal{M}^2}^{\infty} \frac{\rho^{\rm QCD}(s)}{s-p^2} ds + \cdots , \tag{10}$$

where the lower limit of the integral $\mathcal{M}$ in the case under consideration is equal to $\mathcal{M} = m_b + m_c$. When considering the Z_s state it should be replaced by $\mathcal{M} = m_b + m_c + 2m_s$.

In Eq. (10), $\rho^{\rm QCD}(s)$ is the spectral density calculated as the imaginary part of the correlation function. It is the important component of the sum rule calculations. Because the technical tools necessary for derivation of $\rho^{\rm QCD}(s)$ in the case of the tetraquark states are well known and clearly explained in Refs. [47,52], here we avoid providing details of relevant manipulations, and refrain also from presenting explicit expressions for $\rho^{\rm QCD}(s)$. We want to emphasize only that the spectral density is computed by taking into account vacuum condensates up to dimension eight, and we include the effects of the quark $\langle \overline{q}q \rangle$, gluon $\langle \alpha_s G^2/\pi \rangle$, $\langle g_s^3 G^3 \rangle$, mixed $\langle \overline{q} g_s \sigma G q \rangle$ condensates, and also terms of their products.

Applying the Borel transformation on the variable p^2 to the invariant function $\Pi^{\rm QCD}(p^2)$, equating the obtained expression with $\mathcal{B}_{p^2}\Pi^{\rm Phys}(p)$, and subtracting the contribution of higher resonances and continuum states, one finds the required sum rule. Then the sum rule for the mass of the Z_q state reads

$$m_Z^2 = \frac{\int_{\mathcal{M}^2}^{s_0} ds \rho^{\rm QCD}(s) s e^{-s/M^2}}{\int_{\mathcal{M}^2}^{s_0} ds \rho^{\rm QCD}(s) e^{-s/M^2}}. \tag{11}$$

The meson–current coupling f_Z can be extracted from the sum rule:

$$f_Z^2 m_Z^2 e^{-m_Z^2/M^2} = \int_{\mathcal{M}^2}^{s_0} ds \rho^{\rm QCD}(s) e^{-s/M^2}. \tag{12}$$

In Eqs. (11) and (12) by s_0 we denote the threshold parameter, that separates the ground state's contribution from contributions arising due to higher resonances and continuum.

The sum rules contain the parameters which are necessary for numerical computations: Their numerical values are collected in Table 1. The quark and gluon condensates are well known, therefore we utilize their standard values. Table 1 contains also B_c, ρ and ϕ mesons' masses (see Ref. [53]) and decay constants, which will serve as input parameters when computing the strong couplings and decay widths. It is worth noting that for f_ρ, ϕ and f_{B_c} we use the sum rule estimations from Refs. [54,55].

The sum rules of Eqs. (11) and (12) contain also two parameters s_0 and M^2, choices of which are decisive to extract reliable estimations for the quantities under question. The continuum threshold s_0 determines a boundary that dissects the ground-state contribution from ones due to excited resonances and continuum. It depends on the energy of the first excited state corresponding to the ground-state hadron. The continuum threshold s_0 can also be found from an analysis of the pole to total contribution ratio. The analysis done in the case of the tetraquark Z_q allows us to fix a working interval for s_0 as

$$59\ \text{GeV}^2 \leq s_0 \leq 60\ \text{GeV}^2. \tag{13}$$

The Borel parameter M^2 has also to satisfy well-known requirements. Namely, convergence of OPE and exceeding of the perturbative contribution over the nonperturbative one fixes a lower bound of the allowed values of M^2. The upper limit of the Borel parameter is determined to achieve the largest possible pole contribution to the sum rule. These constraints lead to the following working window for M^2:

$$8.2\ \text{GeV}^2 \leq M^2 \leq 8.4\ \text{GeV}^2. \tag{14}$$

In Figs. 1 and 2 we graphically demonstrate some stages in extracting of the working regions for these parameters. Thus, in Fig. 1 the perturbative and nonperturbative contributions to the sum rule in the chosen regions for s_0 and M^2 are depicted. The convergence of the OPE can be seen by inspecting Fig. 2, where the effects of the operators of the different dimensions are plotted. By varying the parameters s_0 and M^2 within their working ranges we find that the pole contribution to the mass sum rule amounts to $\sim$65% of the result.

Table 1 Input parameters

Parameters	Values
m_{B_c}	(6275.1 ± 1.0) MeV
f_{B_c}	(528 ± 19) MeV
m_ρ	(775.26 ± 0.25) MeV
f_ρ	216 ± 3 MeV
m_ϕ	(1019.461 ± 0.019) MeV
f_ϕ	215 ± 5 MeV
m_b	$4.18^{+0.04}_{-0.03}$ GeV
m_c	(1.27 ± 0.03) GeV
m_s	96^{+8}_{-4} MeV
$\langle \bar{q}q \rangle$	$(-0.24 \pm 0.01)^3$ GeV^3
$\langle \bar{s}s \rangle$	$0.8\ \langle \bar{q}q \rangle$
m_0^2	(0.8 ± 0.1) GeV^2
$\langle \overline{q} g_s \sigma G q \rangle$	$m_0^2 \langle \bar{q}q \rangle$
$\langle \bar{s} g_s \sigma G s \rangle$	$m_0^2 \langle \bar{s}s \rangle$
$\langle \frac{\alpha_s G^2}{\pi} \rangle$	(0.012 ± 0.004) GeV^4
$\langle g_s^3 G^3 \rangle$	(0.57 ± 0.29) GeV^6

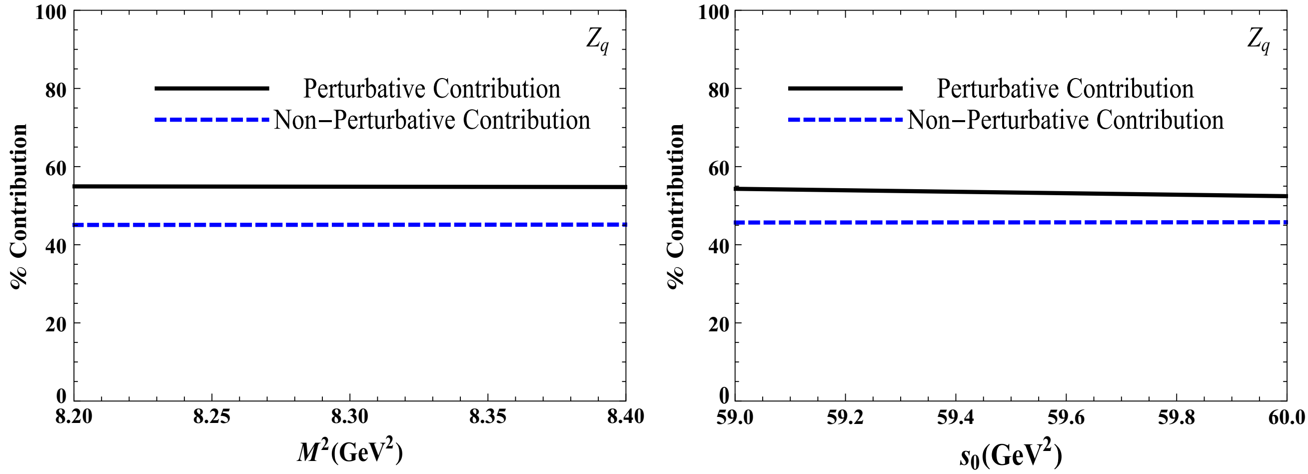

Fig. 1 The perturbative and nonperturbative contributions to the sum rule as functions of M^2 at an average s_0 (*left panel*), and as functions of s_0 at an average M^2 (*right panel*)

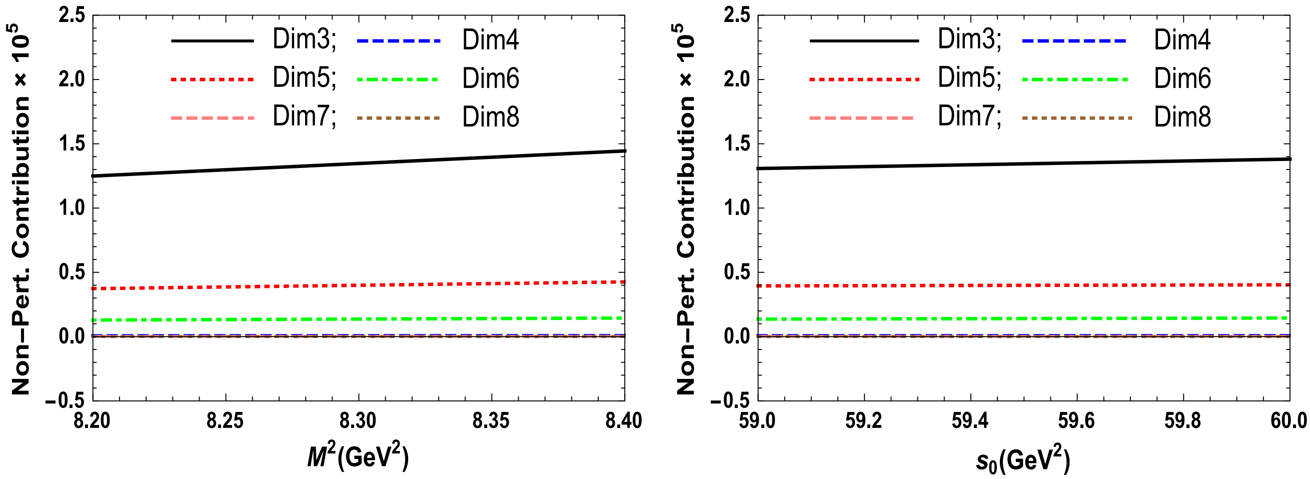

Fig. 2 Contributions to the sum rule arising from the nonperturbative operators of different dimensions are shown as functions of the Borel parameter at an average value of s_0 (*left panel*), and as functions of the threshold parameter s_0 at an average M^2 (*right panel*)

The final results for the mass and meson–current coupling of the Z_q state are drawn in Fig. 3 and collected in Table 2. As is seen from Fig. 3, the quantities extracted from the sum rules demonstrate a mild dependence on M^2, whereas the effects of s_0 on them are sizable. The uncertainties generated by the parameters s_0 and M^2 are main sources of errors, which are an inherent part of sum rule computations and equal to up to 30% of the whole integral.

The mass and meson–current coupling of the Z_s state can be obtained from similar calculations, the difference being only in terms $\sim m_s$ kept in the spectral density, whereas in Z_q calculations we set $m_q = 0$. These modifications and also the replacement $\mathcal{M} \Rightarrow m_b + m_c + 2m_s$ in the integrals result in shifting of the working ranges of the parameters s_0 and M^2 towards slightly larger values, which now read

$$60\ \mathrm{GeV}^2 \leq s_0 \leq 61\ \mathrm{GeV}^2,$$
$$8.4\ \mathrm{GeV}^2 \leq M^2 \leq 8.6\ \mathrm{GeV}^2. \quad (15)$$

Predictions for m_{Z_s} and f_{Z_s} obtained using s_0 and M^2 from Eq. (15) are also written down in Table 2.

3 $Z_q \to B_c\rho$ and $Z_s \to B_c\phi$ decays

In this section we investigate the strong decays of the exotic axial-vector $Z_{q(s)}$ states, and calculate widths of their main decay modes, which, in accordance with results of Sect. 2, are kinematically allowed.

One can see that the quantum numbers, quark content and mass of the Z_q tetraquark make the process $Z_q \to B_c\rho$ its preferable decay mode. The Z_s state may decay to B_c

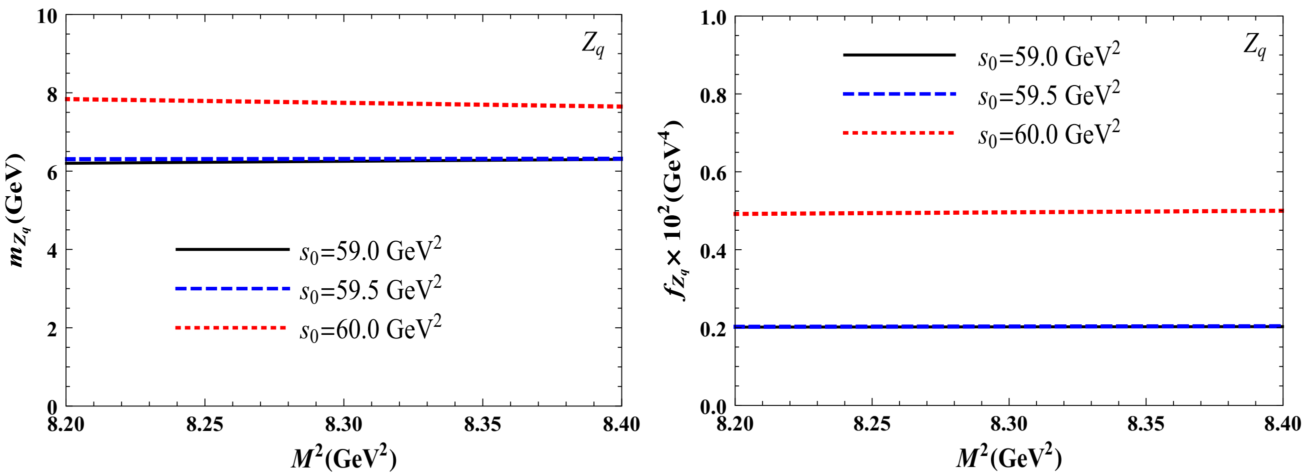

Fig. 3 The mass (*left panel*) and meson–current coupling (*right panel*) as functions of the Borel parameter M^2 at fixed values of the continuum threshold s_0

Table 2 The sum rule results for the masses and meson–current couplings of the axial-vector Z_q and Z_s states

Mass, m.-c. coupling	Results
m_{Z_q}	(7.06 ± 0.74) GeV
f_{Z_q}	$(0.33 \pm 0.11) \times 10^{-2}$ GeV^4
m_{Z_s}	(7.30 ± 0.76) GeV
f_{Z_s}	$(0.63 \pm 0.19) \times 10^{-2}$ GeV^4

and ϕ mesons. It is worth noting that, due to ρ–ω and ω–ϕ mixing, the processes $Z_q \to B_c\omega$ and $Z_s \to B_c\omega$ are also among the kinematically allowed decay channels. But because, for example, the ϕ and ω mesons are almost pure $\bar{s}s$ and $\left(\overline{u}u + \overline{d}d\right)/\sqrt{2}$ states, the $Z_s \to B_c\omega$ process is unessential provided the mass of Z_s allows its decay to ϕ meson. Alternative channels with ω may play an important role in the exploration of the tetraquark states containing an $\bar{s}s$ pair, if their masses are not enough to create a ϕ meson.

We are going to carry out the required analysis and write down all expressions necessary to find the $Z_q \to B_c\rho$ decay's width. After rather trivial replacements in the corresponding formulas and input parameters, the same calculations can easily be repeated for the $Z_s \to B_c\phi$ decay.

As a first step we have to compute the coupling $g_{Z_qB_c\rho}$, which describes the strong interaction in the vertex $Z_qB_c\rho$, and can be extracted from the QCD sum rule. To this end, we explore the correlation function

$$\Pi_\mu(p,q) = i\int d^4x e^{ipx}\left\langle \rho(q)|\mathcal{T}\{J^{B_c}(x)J_\mu^\dagger(0)\}|0\right\rangle, \quad (16)$$

where $J^{B_c}(x)$ is the interpolating current of the B_c meson: It is defined in the form

$$J^{B_c}(x) = i\overline{b}_l(x)\gamma_5 c_l(x). \quad (17)$$

The correlation function in Eq. (16) is introduced in a form which implies usage of the light-cone sum rule method. Indeed, $\Pi_\mu(p,q)$ will be computed employing the QCD sum rule on the light-cone by using the technique of the soft-meson approximation.

In terms of the physical parameters of the involved particles and coupling $g_{Z_qB_c\rho}$, the function $\Pi_\mu(p,q)$ has a simple form and generates the phenomenological side of the sum rule. Namely,

$$\Pi_\mu^{\text{Phys}}(p,q) = \frac{\langle 0|J^{B_c}|B_c(p)\rangle}{p^2 - m_{B_c}^2}\langle B_c(p)\,\rho(q)|Z_q(p')\rangle \times \frac{\langle Z_q(p')|J_\mu^\dagger|0\rangle}{p'^2 - m_Z^2} + \cdots, \quad (18)$$

where p, q and $p' = p + q$ are the momenta of B_c, ρ and Z_q particles, respectively. The term presented above is the contribution of the ground state: the dots stand for the effects of the higher resonances and continuum states.

We introduce the B_c meson matrix element,

$$\langle 0|J^{B_c}|B_c(p)\rangle = \frac{f_{B_c}m_{B_c}^2}{m_b + m_c}$$

where m_{B_c} and f_{B_c} are the mass and decay constant of the B_c meson, and also the matrix element corresponding to the vertex,

$$\langle B_c(p)\,\rho(q)|Z_q(p')\rangle = g_{Z_qB_c\rho}\left[\left(q\cdot\varepsilon'\right)\left(p'\cdot\varepsilon^*\right) - \left(q\cdot p'\right)\left(\varepsilon^*\cdot\varepsilon'\right)\right]. \quad (19)$$

Then the ground-state term in the correlation function can easily be found:

$$\Pi_{\mu}^{\mathrm{Phys}}(p,q)=\frac{f_{B_c}f_Z m_Z m_{B_c}^2 g_{Z_q B_c \rho}}{\left(p'^2-m_Z^2\right)\left(p^2-m_{B_c}^2\right)(m_b+m_c)} \times\left(\frac{m_Z^2-m_{B_c}^2}{2}\varepsilon_{\mu}^{*}-p'\cdot\varepsilon^{*}q_{\mu}\right)+\cdots. \tag{20}$$

Strong vertices of a tetraquark with two conventional mesons differ from vertices containing only ordinary mesons. The reason here is very simple: the tetraquark Z_q is a state composed of four valence quarks, therefore the expansion of the non-local correlation function $\Pi_{\mu}(p,q)$ leads to the expression, which instead of distribution amplitudes of ρ meson depends on its local matrix elements (of course, same arguments are valid for Z_s, as well). Then the conservation of the four-momentum at the vertex $Z_q B_c \rho$ equals q to zero. In other words, within the light-cone sum rule method the momentum of ρ meson should be equal to zero in our case. In the vertices of ordinary hadrons the four-momenta of all involved particles can take nonzero values. The soft-meson approximation corresponds to a situation when $q=0$. Calculations of the same strong couplings within the full light-cone sum rule method and in the soft-meson approximation demonstrated that the difference between the results extracted using these two approaches is numerically small (for a detailed discussion, see Ref. [46]).

In the soft limit $p=p'$, the only term that survives in Eq. (20) is $\sim\varepsilon_{\mu}^{*}$. The invariant function $\Pi^{\mathrm{Phys}}(p^2)$ corresponding to this structure depends on the variable p^2 and is given as

$$\Pi^{\mathrm{Phys}}(p^2)=\frac{f_{B_c}f_Z m_Z m_{B_c}^2 g_{Z_s B_c \eta}}{2\left(p^2-m^2\right)^2(m_b+m_c)} \times\left(m_Z^2-m_{B_c}^2\right)+\cdots, \tag{21}$$

where $m^2=\left(m_Z^2+m_{B_c}^2\right)/2$.

In the soft-meson approximation we additionally apply the operator

$$\left(1-M^2\frac{\mathrm{d}}{\mathrm{d}M^2}\right)M^2 e^{m^2/M^2}, \tag{22}$$

to both sides of the sum rule. The last operation is required to remove all unsuppressed contributions existing on the physical side of the sum rule in the soft-meson limit (see Ref. [45]).

The second component of the sum rule, i.e. the QCD expression for the correlation function $\Pi_{\mu}^{\mathrm{QCD}}(p,q)$, is calculated employing the quark propagators:

$$\Pi_{\mu}^{\mathrm{QCD}}(p,q)=-i\int \mathrm{d}^4x e^{ipx}\left\{\left[\gamma_5\widetilde{S}_c^{ib}(x)\gamma_5 \times\widetilde{S}_b^{bi}(-x)\gamma_{\mu}\right]_{\alpha\beta}\left\langle\rho(q)|\overline{q}_{\alpha}^{a}q_{\beta}^{a}|0\right\rangle +\left[\gamma_5\widetilde{S}_c^{ib}(x)\gamma_5\widetilde{S}_b^{ai}(-x)\gamma_{\mu}\right]_{\alpha\beta}\langle\rho(q)|\overline{s}_{\alpha}^{a}s_{\beta}^{b}|0\rangle\right\}, \tag{23}$$

with α and β being the spinor indices.

We continue our calculations by employing the expansion

$$\overline{q}_{\alpha}^{a}q_{\beta}^{b}\rightarrow\frac{1}{4}\Gamma_{\beta\alpha}^{j}\left(\overline{q}^{a}\Gamma^{j}q^{b}\right), \tag{24}$$

where $\Gamma^j=1,\ \gamma_5,\ \gamma_{\mu},\ i\gamma_5\gamma_{\mu},\ \sigma_{\mu\nu}/\sqrt{2}$ is the full set of Dirac matrices, and we carry out the color summation.

Prescriptions to perform summation over color indices, as well as procedures to calculate the resulting integrals and extract the imaginary part of the correlation function $\Pi_{\mu}^{\mathrm{QCD}}(p,q)$, were numerously presented in our previous work; see Refs. [47–50]. Therefore, here we skip further details, and we provide the ρ meson local matrix elements that in the soft limit contribute to the spectral density, as well as the final formulas for the spectral density $\rho_c(s)$.

Analysis demonstrates that in the soft limit only the matrix elements

$$\langle 0|\overline{q}\gamma_{\mu}q|\rho^0(p)\rangle=\frac{1}{\sqrt{2}}f_{\rho}m_{\rho}\varepsilon_{\mu} \tag{25}$$

and

$$\langle 0|\overline{q}g\widetilde{G}_{\mu\nu}\gamma_{\nu}\gamma_5 q|\rho^0(p)\rangle=\frac{1}{\sqrt{2}}f_{\rho}m_{\rho}^3\zeta_{4\rho}\varepsilon_{\mu} \tag{26}$$

are involved in the computations, where q denotes one of the u or d quarks. The matrix elements depend on the ρ meson mass m_{ρ} and decay constant f_{ρ}. The twist-4 matrix element in Eq. (26), as a factor, contains also the parameter $\zeta_{4\rho}$. Its numerical value was extracted at the scale $\mu=1$ GeV from the sum rule calculations in Ref. [54] and equals

$$\zeta_{4\rho}=0.07\pm 0.03.$$

The final expression of the spectral density has the form

$$\rho_c(s)=\frac{f_{\rho}m_{\rho}}{24\sqrt{2}}\left[F^{\mathrm{pert.}}(s)+F^{\mathrm{n.-pert.}}(s)\right]. \tag{27}$$

Here $F^{\mathrm{pert.}}(s)$ is the perturbative contribution to $\rho_c(s)$,

$$F^{\mathrm{pert.}}(s)=\frac{1}{\pi^2 s^2}\left\{\left[s^2+s\left(m_b^2+6m_bm_c+m_c^2\right) -2(m_b^2-m_c^2)^2\right]\right\}\sqrt{\left(s+m_b^2-m_c^2\right)^2-4m_b^2 s}, \tag{28}$$

whereas by $F^{\text{n.-pert.}}(s)$ we denote its nonperturbative component. The function $F^{\text{n.-pert.}}(s)$ is the sum of the terms

$$F^{\text{n.-pert.}}(s) = F_G^{\text{n.-pert.}}(s) + \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle \int_0^1 f_{g_s^2 G^2}(z,s)\mathrm{d}z + \left\langle g_s^3 G^3 \right\rangle \int_0^1 f_{g_s^3 G^3}(z,s)\mathrm{d}z + \left\langle \frac{\alpha_s G^2}{\pi} \right\rangle^2 \int_0^1 f_{(g_s^2 G^2)^2}(z,s)\mathrm{d}z. \quad (29)$$

Here $F_G^{\text{n.-pert.}}(s)$ appears from the integration of the perturbative component of one heavy quark propagator with the term $\sim G$ from another one. It can be expressed using the matrix element given by Eq. (26) and has a rather simple form:

$$F_G^{\text{n.-pert.}}(s) = \frac{3m_\rho^2 \zeta_{4\rho}}{2\pi^2 s} \sqrt{\left(s + m_b^2 - m_c^2\right)^2 - 4m_b^2 s}. \quad (30)$$

The nonperturbative factors in front of the integrals and the subscripts of the functions clearly indicate the origin of the remaining terms. In fact, the functions $f_{g_s^2 G^2}$, $f_{g_s^3 G^3}$ are due to products of terms $\sim g_s^2 G^2$ and $\sim g_s^3 G^3$ with the perturbative component of another propagator, whereas $f_{(g_s^2 G^2)^2}$ comes from integrals obtained using $\sim g_s^2 G^2$ components of b and c quarks' propagators. These terms are four, six and eight dimensional nonperturbative contributions to the spectral density $\rho_c(s)$, respectively. Their explicit forms are

$$f_{g_s^2 G^2}(z,s) = \frac{1}{12z^2(z-1)^2} \Big\{ 54(1-z)z^2 \delta(s-\Phi) + \Big[8m_b^2(z-1)^3 + z^2\left(27s(1-z) - 8m_b^2 z\right) + 2m_b m_c \left(4 + 15z + 12z^2\right) \Big] \delta^{(1)}(s-\Phi) - 4s\left[m_b^2(1-z)^3 + m_b m_c z(1-z) - m_c^2 z^3 \right] \delta^{(2)}(s-\Phi) \Big\}, \quad (31)$$

$$f_{g_s^3 G^3}(z,s) = \frac{1}{15 \cdot 2^6 z^5 (z-1)^5} \Big\{ -12z^2(z-1)^2 \Big[3m_b^2(z-1)^5 + 3m_b m_c((1-z)^5 + z^5) + z\Big(-3m_c^2 z^4 + s\left(1 - 8z + 25z^2 - 40z^3 + 33z^4 - 11z^5\right)\Big)\Big] \delta^{(2)}(s-\Phi) + 2z(z-1)\Big[m_b^2(z-1)^5 \left(7m_b^2 - 4m_b m_c - 9sz(2z-1)\right) + 2m_b m_c z^2 \left(2m_c^2 z^3 - 9s(z-1)^2(1-3z+3z^2)\right) + z^3\Big(-7m_c^4 z^2 + 9sm_c^2 z^2(1-3z+2z^2) + 2s^2(z-1)^3(2-7z+7z^2)\Big)\Big] \times \delta^{(3)}(s-\Phi) + \Big[-2m_b^5 m_c(z-1)^5 + 7m_b^4 sz(z-1)^6 - 4m_b^3 m_c s z^2 (z-1)^5 - 6m_b^2 s^2 z^3 (z-1)^6 + 2m_b m_c z^4 \times \left(-3s^2(z-1)^4 + m_c^4 z + 2m_c^2 sz(z-1)^2\right) + s(z-1)z^5 \times \left(s^2(z-1)^4 - 7m_c^4 z + 6m_c^2 sz(z-1)^2\right)\Big] \delta^{(4)}(s-\Phi) \Big\}, \quad (32)$$

$$f_{(g_s^2 G^2)^2}(z,s) = \frac{m_b m_c}{54z^2(z-1)^2} \Big\{ 2\left[m_b m_c - s(1-3z+3z^2) \right] \times \delta^{(4)}(s-\Phi) + s[m_b m_c + s(1-z)z]\delta^{(5)}(s-\Phi) \Big\}, \quad (33)$$

where

$$\delta^{(n)}(s-\Phi) = \frac{\mathrm{d}^n}{\mathrm{d}s^n}\delta(s-\Phi),$$

with Φ being defined as

$$\Phi = \frac{m_b^2(1-z) + m_c^2 z}{z(1-z)}.$$

The final sum rule to evaluate the strong coupling reads

$$g_{Z_q B_c \rho} = \frac{2(m_b + m_c)}{f_{B_c} f_Z m_Z m_{B_c}^2 (m_Z^2 - m_{B_c}^2)} \left(1 - M^2 \frac{\mathrm{d}}{\mathrm{d}M^2}\right) \times M^2 \int_{(m_b+m_c)^2}^{s_0} \mathrm{d}s e^{(m^2-s)/M^2} \rho_c(s). \quad (34)$$

To calculate the width of the decay $Z_q \to B_c \rho$ we use the expression

$$\Gamma\left(Z_q \to B_c \rho\right) = \frac{g_{Z_q B_c \rho}^2 m_\rho^2}{24\pi} \lambda\left(m_Z,\ m_{B_c}, m_\rho\right) \times \left[3 + \frac{2\lambda^2\left(m_{Z_q},\ m_{B_c}, m_\rho\right)}{m_\rho^2}\right], \quad (35)$$

where

$$\lambda(a,\ b,\ c) = \frac{\sqrt{a^4 + b^4 + c^4 - 2\left(a^2 b^2 + a^2 c^2 + b^2 c^2\right)}}{2a}.$$

The parameters necessary for numerical calculations of the strong coupling $g_{Z_q B_c \rho}$ and $\Gamma\left(Z_q \to B_c \rho\right)$ are listed in Table 1.

The investigation carried out in accordance with standard requirements of the sum rule calculations allows us to determine the ranges for s_0 and M^2. For example, the pole contribution to the sum rule amounts to ~48 to 60 % of the total result, as is seen from Fig. 4. Other constraints, i.e. convergence of the OPE and prevalence of the perturbative contribution, have been checked, as well. Summing up the performed analysis we fix the interval for the continuum threshold s_0 as in the mass calculations [see Eq. (13)], whereas for the Borel parameter we obtain

$$8\ \text{GeV}^2 \le M^2 \le 9\ \text{GeV}^2, \quad (36)$$

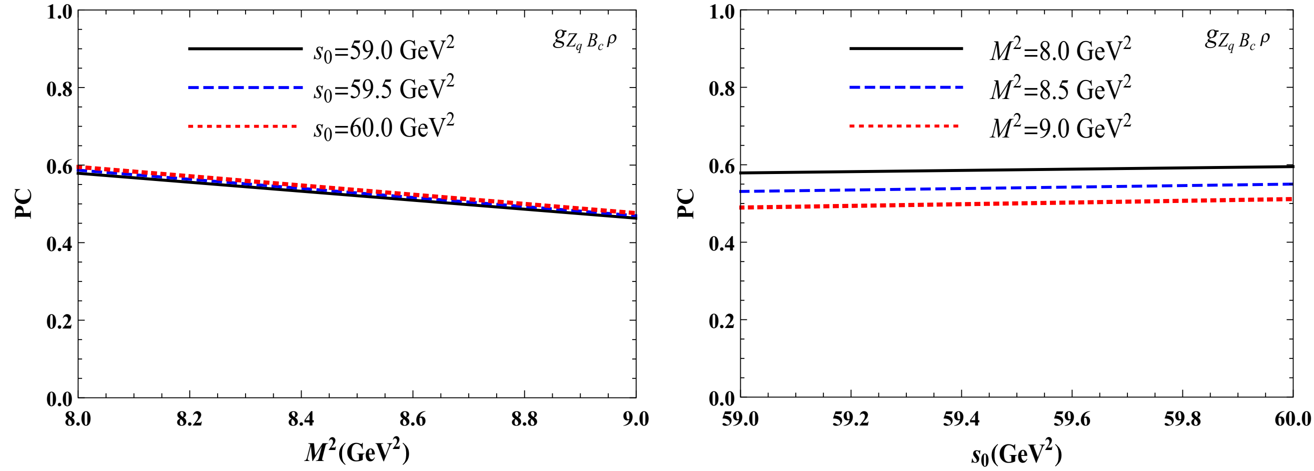

Fig. 4 The pole contribution in the $g_{Z_q B_c \rho}$ coupling sum rule calculations as a function of the Borel parameter M^2 at fixed s_0 (*left panel*), and as a function of the threshold s_0 at fixed values of M^2 (*right panel*)

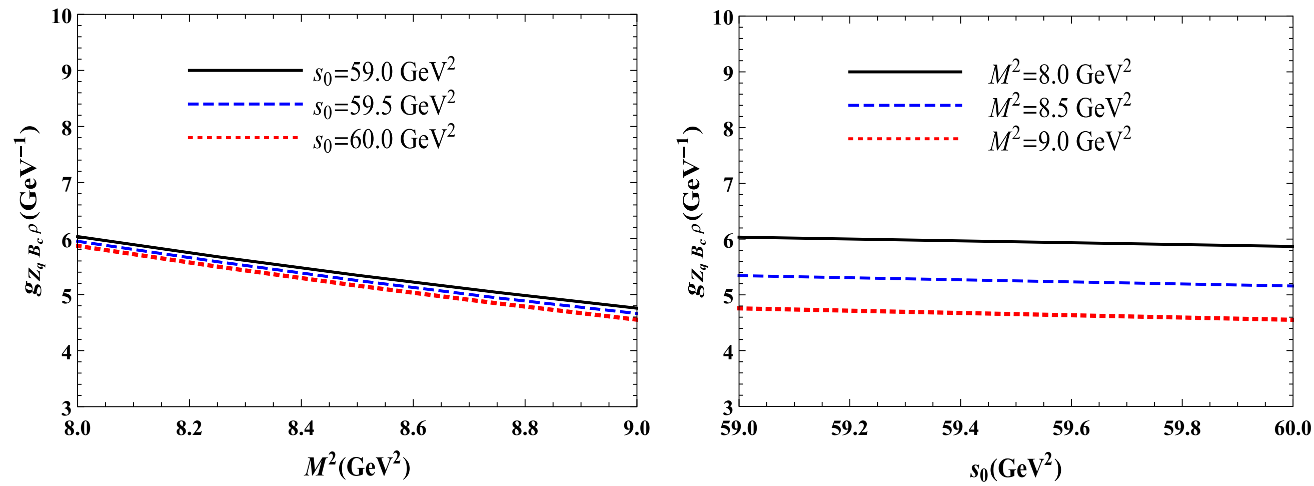

Fig. 5 The strong coupling $g_{Z_q B_c \rho}$ as a function of the Borel parameter M^2 at fixed s_0 (*left panel*), and as a function of the threshold s_0 at fixed values of M^2 (*right panel*)

which is wider than the corresponding window in the mass sum rule.

In Fig. 5 we provide our final results and depict the strong coupling $g_{Z_q B_c \rho}$ as a function of the Borel parameter (at fixed s_0) and as a function of the continuum threshold (at fixed M^2). The dependence of the strong coupling on these parameters has a traditional form, and systematic errors of the calculations are within reasonable limits.

The decay $Z_s \to B_c \phi$ can be considered in analogous manner: One only needs to write down in the relevant expressions the parameters of the ϕ meson. Thus, the matrix elements of the ϕ meson that take part in forming of the spectral density are

$$\langle 0|\overline{s}\gamma_\mu s|\phi(p)\rangle = f_\phi m_\phi \varepsilon_\mu,$$
$$\langle 0|\overline{s}g\widetilde{G}_{\mu\nu}\gamma_\nu\gamma_5 s|\phi(p)\rangle = f_\phi m_\phi^3 \zeta_{4\phi}\varepsilon_\mu,$$

where the twist-4 parameter

$$\zeta_{4\phi} = 0.00 \pm 0.02$$

was estimated and found to be compatible with zero in Ref. [54].

In calculations of the coupling $g_{Z_s B_c \phi}$ the working regions for the Borel parameter and continuum threshold are fixed in the form

$$60\ \text{GeV}^2 \le s_0 \le 61\ \text{GeV}^2,$$
$$8.2\ \text{GeV}^2 \le M^2 \le 9.2\ \text{GeV}^2. \quad (37)$$

Our results for the strong couplings and widths of the decay modes studied in this work are collected in Table 3.

Table 3 The strong couplings and decay widths of the Z_q and Z_s tetraquarks

Strong couplings, widths	Predictions
$g_{Z_q B_c \rho}$	(5.31 ± 1.25) GeV^{-1}
$g_{Z_s B_c \phi}$	(6.42 ± 1.52) GeV^{-1}
$\Gamma(Z_q \to B_c \rho)$	(80 ± 32) MeV
$\Gamma(Z_s \to B_c \phi)$	(168 ± 68) MeV

4 Discussion and concluding remarks

In the present work we have calculated the parameters of the open charm-bottom axial-vector tetraquark states Z_q and Z_s within the QCD sum rule method. Their masses and meson–current couplings have been obtained using the two-point sum rule method. In these calculations for Z_q and Z_s we have used the symmetric in color indices interpolating currents by assuming that they are ground states in the corresponding tetraquark multiplets. Indeed, one can anticipate that Z_q and Z_s are the axial-vector components of the $1S$ diquark–antidiquark $[cq][\bar{b}\bar{q}]$ and $[cs][\bar{b}\bar{s}]$ multiplets, respectively.

During last years some progress was achieved in the investigation of the $[cq][\bar{c}\bar{q}]$ and $[cs][\bar{c}\bar{s}]$ multiplets, and classification of the observed hidden-charm tetraquarks as their possible members (see Refs. [56,57]). Thus, within the "type-II" model elaborated in this work, the authors not only identified the multiplet levels with the tetraquarks discovered, but also the estimated masses of the states, which had not yet been observed. This model is founded on some assumptions as regards the nature of inter-quark and inter-diquark interactions, and one considers spin–spin interactions within diquarks as the decisive source of splitting inside of the multiplet.

The information useful for our purposes is accumulated in the axial-vector sector of these multiplets. The axial-vector $J^{PC}=1^{++}$ particle in the ground-state $[cq][\bar{c}\bar{q}]$ multiplet was identified with the well-known $X(3872)$ resonance. A similar analysis carried out for the multiplet of $[cs][\bar{c}\bar{s}]$ states demonstrated that its $J^{PC}=1^{++}$ level may be considered as $X(4140)$. The mass difference of the axial-vector resonances belonging to "q" and "s" hidden-charm multiplets is

$$X(4140) - X(3872) \approx 270 \text{ MeV}. \quad (38)$$

In the present work we have evaluated the masses of the axial-vector states from the $[cq][\bar{b}\bar{q}]$ and $[cs][\bar{b}\bar{s}]$ multiplets. The mass shift between these multiplets,

$$m_{Z_s} - m_{Z_q} \approx 240 \text{ MeV}, \quad (39)$$

is in nice agreement with Eq. (38).

Another question to be addressed here is connected with the masses of the excited states, which in sum rule calculations determine the continuum threshold s_0. We have found that, for the $[cq][\bar{b}\bar{q}]$ and $[cs][\bar{b}\bar{s}]$ multiplets, sum rule calculations fix the lower bounds of the parameter s_0 as $s_0 = 59$ GeV^2 and $s_0 = 60$ GeV^2, respectively. This means that the sum rule has placed a first excited state to the position $\sqrt{s_0}$. In order to estimate the gap between the excited and ground states we invoke $\sqrt{s_0}$ and the central values of Z_q and Z_s masses. Then it is not difficult to see that, for the $[cq][\bar{b}\bar{q}]$ type tetraquarks, it equals

$$\sqrt{s_0}\,\text{GeV} - 7.06\,\text{GeV} \approx 0.62\,\text{GeV}, \quad (40)$$

whereas for the $[cs][\bar{b}\bar{s}]$ one gets

$$\sqrt{s_0}\,\text{GeV} - 7.30\,\text{GeV} \approx 0.45\,\text{GeV}. \quad (41)$$

The masses of the $1S$ and $2S$ states with $J^{PC}=1^{+-}$ from the $[cq][\bar{c}\bar{q}']$ multiplet were calculated by means of the two-point sum rule method in Ref. [58]. The ground-state level $1S$ was identified with the resonance $Z_c(3900)$, whereas the resonance $Z(4430)$ was included into a multiplet of the excited $2S$ states. If this assignment is correct, then the experimental data provides the mass difference between the ground and first radially excited states, which is equal to 530 MeV. The results of the calculations led to the predictions $M_{Z_c(3900)} = 3.91^{+21}_{-17}$ GeV and $M_{Z_c(4430)} = 4.51^{+17}_{-09}$ GeV, and to the mass difference ~ 600 MeV.

The $1S$ and $2S$ multiplets of the $[cs][\bar{c}\bar{s}]$ tetraquarks were explored in the context of the "type-II" model in Ref. [57]. For the axial-vector levels $J^{PC}=1^{++}$, there called X states, the $2S$–$1S$ gap is 4600 MeV − 4140 MeV = 460 MeV, and for the particles $X^{(1)}$ and $X^{(2)}$ with the quantum numbers $J^{PC}=1^{+-}$ one gets 4600 MeV − 4140 MeV = 460 MeV and 4700 MeV − 4274 MeV = 426 MeV, respectively. Comparison of these results with the ones given by Eqs. (40) and (41) can be considered as confirmation of the self-consistent character of the performed analysis.

In the framework of the QCD two-point sum rule approach, the masses of the open charm-bottom diquark–antidiquark states were previously calculated in Ref. [30]. For the masses of the axial-vector tetqaruarks Z_q and Z_s the authors found

$$m_{Z_q} = 7.10 \pm 0.09 \pm 0.06 \pm 0.01 \text{ GeV} \quad (42)$$

and

$$m_{Z_s} = 7.11 \pm 0.08 \pm 0.05 \pm 0.03 \text{ GeV}. \quad (43)$$

These predictions were extracted by using the parameter $s_0 = (55 \pm 2)$ GeV^2 in calculations of m_{Z_q} and m_{Z_s}, and $M^2 = (7.9 - 8.2)$ GeV^2 and $M^2 = (6.7 - 7.9)$ GeV^2 for the "q" and "s" states, respectively. It is seen that the mass differences $m_{Z_s} - m_{Z_q} \approx 10$ MeV and $\sqrt{s}_0 - m_{Z_q} \approx \sqrt{s}_0 - m_{Z_s} \approx 180$ MeV can be neither included into the "q"–"s" mass-hierarchy scheme of the ground-state tetraquarks, nor accepted as giving a correct mass shift between the $1S$ and $2S$ multiplets. Our results for m_{Z_q} and m_{Z_s}, if differences are ignored in the chosen windows for the parameters s_0 and M^2, within theoretical errors may be considered as being in agreement with the predictions of Ref. [30]. But in our case the central value of m_{Z_s} allows the decay process $Z_s \to B_c\phi$, whereas for m_{Z_s} from Eq. (43) it remains among the kinematically forbidden channels.

We have also calculated the widths of the $Z_q \to B_c\rho$ and $Z_s \to B_c\phi$ decays, which are new results of this work. The obtained predictions for $\Gamma(Z_q \to B_c\rho)$ and $\Gamma(Z_s \to B_c\phi)$ show that Z_q may be considered as a narrow resonance, whereas Z_s belongs to a class of wide tetraquark states.

Investigation of the open charm-bottom axial-vector tetraquarks performed in the present work within the diquark–antidiquark picture led to quite interesting predictions. Theoretical explorations of other members of the $[cq][\bar{b}\bar{q}]$ and $[cs][\bar{b}\bar{s}]$ tetraquark multiplets, as well as their experimental studies, may shed light on the nature of multi-quark hadrons.

Acknowledgements The work of K. A. was financed by TUBITAK under the Grant No. 115F183.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (http://creativecommons.org/licenses/by/4.0/), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made.
Funded by SCOAP³.

References

1. H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Phys. Rep. **639**, 1 (2016)
2. H.X. Chen, W. Chen, X. Liu, Y.R. Liu, S.L. Zhu. arXiv:1609.08928 [hep-ph]
3. A. Esposito, A.L. Guerrieri, F. Piccinini, A. Pilloni, A.D. Polosa, Int. J. Mod. Phys. A **30**, 1530002 (2015)
4. C.A. Meyer, E.S. Swanson, Prog. Part. Nucl. Phys. **82**, 21 (2015)
5. S.-K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003)
6. D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004)
7. V.M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004)
8. B. Aubert et al. [BaBar Collaboration], Phys. Rev. D **71**, 071103 (2005)
9. K. Abe et al. [Belle Collaboration], BELLE-CONF-0540. arXiv:hep-ex/0505037
10. B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. **102**, 132001 (2009)
11. T. Barnes, S. Godfrey, E.S. Swanson, Phys. Rev. D **72**, 054026 (2005)
12. I.V. Danilkin, Y.A. Simonov, Phys. Rev. Lett. **105**, 102002 (2010)
13. F.E. Close, P.R. Page, Phys. Lett. B **578**, 119 (2004)
14. N.A. Tornqvist, Phys. Lett. B **590**, 209 (2004)
15. C.M. Zanetti, M. Nielsen, R.D. Matheus, Phys. Lett. B **702**, 359 (2011)
16. F.K. Guo, C. Hanhart, Y.S. Kalashnikova, U.G. Meißner, A.V. Nefediev, Phys. Lett. B **742**, 394 (2015)
17. L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D **71**, 014028 (2005)
18. L. Maiani, A.D. Polosa, V. Riquer, Phys. Rev. Lett. **99**, 182003 (2007)
19. F.S. Navarra, M. Nielsen, Phys. Lett. B **639**, 272 (2006)
20. S. Dubnicka, A.Z. Dubnickova, M.A. Ivanov, J.G. Korner, Phys. Rev. D **81**, 114007 (2010)
21. Z.G. Wang, T. Huang, Phys. Rev. D **89**, 054019 (2014)
22. B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. **90**, 242001 (2003)
23. D. Besson et al. [CLEO Collaboration], Phys. Rev. D **68**, 032002 (2003). [Erratum: Phys. Rev. D **75** , 119908 (2007)]
24. V.M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. **117**, 022003 (2016)
25. The D0 Collaboration, D0 Note 6488-CONF (2016)
26. R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. **117**, 152003 (2016)
27. The CMS Collaboration, CMS PAS BPH-16-002 (2016)
28. J.R. Zhang, M.Q. Huang, Phys. Rev. D **80**, 056004 (2009)
29. J.R. Zhang, M.Q. Huang, Commun. Theor. Phys. **54**, 1075 (2010)
30. W. Chen, T.G. Steele, S.L. Zhu, Phys. Rev. D **89**, 054037 (2014)
31. S. Zouzou, B. Silvestre-Brac, C. Gignoux, J.M. Richard, Z. Phys, C **30**, 457 (1986)
32. B. Silvestre-Brac, C. Semay, Z. Phys, C **59**, 457 (1993)
33. D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, Phys. Rev. D **76**, 114015 (2007)
34. Z.F. Sun, X. Liu, M. Nielsen, S.L. Zhu, Phys. Rev. D **85**, 094008 (2012)
35. R.M. Albuquerque, X. Liu, M. Nielsen, Phys. Lett. B **718**, 492 (2012)
36. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **95**, 034008 (2017)
37. M.A. Shifman, A.I. Vainshtein, V.I. Zhakharov, Nucl. Phys. B **147**, 385 (1979)
38. V.M. Braun, A.V. Kolesnichenko, Phys. Lett. B **175**, 485 (1986)
39. V.M. Braun, Y.M. Shabelski, Sov. J. Nucl. Phys. **50**, 306 (1989). [Yad. Fiz. **50**, 493 (1989)]
40. I.I. Balitsky, D. Diakonov, A.V. Yung, Phys. Lett. B **112**, 71 (1982)
41. I.I. Balitsky, D. Diakonov, A.V. Yung, Z. Phys, C **33**, 265 (1986)
42. J. Govaerts, L.J. Reinders, H.R. Rubinstein, J. Weyers, Nucl. Phys. B **258**, 215 (1985)
43. J. Govaerts, L.J. Reinders, J. Weyers, Nucl. Phys. B **262**, 575 (1985)
44. I.I. Balitsky, V.M. Braun, A.V. Kolesnichenko, Nucl. Phys. B **312**, 509 (1989)
45. B.L. Ioffe, A.V. Smilga, Nucl. Phys. B **232**, 109 (1984)
46. V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Rückl, Phys. Rev. D **51**, 6177 (1995)
47. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **93**, 074002 (2016)
48. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **93**, 114007 (2016)
49. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **93**, 094006 (2016)
50. S.S. Agaev, K. Azizi, H. Sundu, Eur. Phys. J. Plus **131**, 351 (2016)
51. L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. **127**, 1 (1985)
52. S.S. Agaev, K. Azizi, H. Sundu, Phys. Rev. D **93**, 074024 (2016)
53. C. Patrignani, Chin. Phys. C **40**, 100001 (2016)
54. P. Ball, V.M. Braun, A. Lenz, JHEP **0708**, 090 (2007)
55. M.J. Baker, J. Bordes, C.A. Dominguez, J. Penarrocha, K. Schilcher, JHEP **1407**, 032 (2014)
56. L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, Phys. Rev. D **89**, 114010 (2014)
57. L. Maiani, A.D. Polosa, V. Riquer, Phys. Rev. D **94**, 054026 (2016)
58. Z.G. Wang, Commun. Theor. Phys. **63**, 325 (2015)