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Letter

Meissner effect and a stringlike interaction

Chandrasekhar Chatterjee[1,a], **Ishita Dutta Choudhury**[2,b], **Amitabha Lahiri**[2,c]

[1] Department of Physics at Hiyoshi, and Research and Education Center for Natural Sciences, Keio University, Hiyoshi 4-1-1, Yokohama, Kanagawa 223-8521, Japan
[2] S N Bose National Centre for Basic Sciences, Block JD, Sector III, Kolkata 700106, Salt Lake, India

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Abstract We find that a recently proposed interaction involving the vorticity current of electrons, which radiatively induces a photon mass in $3+1$ dimensions in the low-energy effective theory, corresponds to confining strings (linear potential) between electrons.

The formal theory of superconductivity was developed on the principle of Cooper pair formation [1–5]. According to this principle, the interaction between electrons and phonons generate an effective electron–electron interaction limited to a shell in momentum space around the Fermi surface. When this mutually attractive interaction overcomes the Coulomb repulsion, bound pairs may form. In conventional superconductivity, the bound pair of electrons which possess mutually opposite momenta are in an s-wave state (spin singlet) [1–5]. In this case, the theory has a local order parameter of spin zero and the system undergoes spontaneous symmetry breaking. However, there are superconductors which do not seem to exactly follow the above description. For instance, in the case of topological superconductor, the system may not have any local order parameter like in the Ginzburg–Landau description [6,7]. Furthermore, in an unconventional superconductor the pair states can have non-zero spin. Although a spinorial order parameter was first discovered in superfluid He-3 [8], unconventional superconductivity can be realized in many heavy fermionic compounds. It is well known today that spin interactions, and particularly spin–orbit coupling, play an important role in the physics of topological matter [9]. It has also been shown that long range spin–spin interactions can be induced by collective excitations, like phonons [10].

As is well known, the expulsion of magnetic field from superconductors, or Meissner effect, is described by the London equations,

$$\partial_\mu J_\nu - \partial_\nu J_\mu = \lambda\, F_{\mu\nu}. \tag{1}$$

In conventional superconductors, this is implemented by an underlying model of symmetry breaking, in which a complex Higgs field H of charge $2e$ acquires a vacuum expectation value v. Then the azimuthal part of H, which may be written as $e^{i\phi}$ and is the Nambu–Goldstone mode of the symmetry breaking, is responsible for a current

$$J_\mu = 4i\, ev^2\, e^{-i\phi} D_\mu e^{i\phi}, \quad D_\mu = \partial_\mu - 2ie\, A_\mu. \tag{2}$$

This current is gauge invariant and reduces the London equation to an identity, with $\lambda = 8e^2v^2$.

It is by now well known that there is a 'dual' ansatz for the current which does not require symmetry breaking. Instead of a complex scalar field with a vacuum expectation value, an antisymmetric tensor field $B_{\mu\nu}$ is introduced, with field strength $H_{\mu\nu\lambda} = \partial_\mu B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\lambda B_{\mu\nu}$, [11] and the current is defined to be $J^\mu = \epsilon^{\mu\nu\lambda\rho} H_{\nu\lambda\rho}$. Then the continuity equation $\partial_\mu J^\mu = 0$ becomes an identity, and Eq. (1) can be derived as field equations from the Lagrangian [12,13]

$$S = \int \mathrm{d}^4x\, \left(-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\alpha}H^{\mu\nu\alpha} + \frac{m}{4}\, \epsilon^{\mu\nu\lambda\rho} B_{\mu\nu} F_{\lambda\rho}\right), \tag{3}$$

with $\lambda = 2\,\mathrm{m}^2$. This is usually referred to as a topological mass generation mechanism for the photon because of the $B\wedge F$ interaction term [14], which by itself is an Abelian version of a similar term in many four dimensional topological quantum field theories. This interaction is a four dimensional generalization of the Chern–Simons term in three dimensions [15–17], but unlike the latter it does not break P and T.

[a] e-mails: chandra@phys-h.keio.ac.jp; chandra.chttrj@gmail.com
[b] e-mail: ishitadutta.choudhury@bose.res.in
[c] e-mail: amitabha@bose.res.in

Chern–Simons theory finds application in three dimensional condensed matter physics, in particular in describing fractional quantum Hall effect [18]. The action corresponding to Eq. (3) is a mixed Chern–Simons theory in three dimensions, and it has been used in quantum hall systems and in superconductors with a gap in the single particle spectrum [6]. This description relies on the duality between the BF action of Eq. (3) and the Abelian Higgs model, albeit with a frozen Higgs degree of freedom. In four dimensions however, there is a subtle distinction between the two dual theories, because $B_{\mu\nu}$ can in principle have couplings which have no analogue in the Higgs picture. It has been argued in [7] that this implies that superconductors described by BF theory are different from the usual kind. Indeed, the four dimensional theory of Eq. (3) has proven very difficult to implement in realistic systems. This difficulty is based on the lack of a sensible interaction between fermions and the $B_{\mu\nu}$ field, namely, one that is invariant under the vector gauge transformation $B_{\mu\nu} \to B_{\mu\nu} + \partial_\mu \chi_\nu - \partial_\nu \chi_\mu$.

It was shown recently that such an interaction could be constructed, provided $B_{\mu\nu}$ was coupled to a non-local fermion current. Such a coupling induces the $B \wedge F$ term at one loop level even if it is not present in the original Lagrangian [19], and the pseudovector charge density of the non-local current can be interpreted as the vorticity field of the fermion. In other words, a mass for the photon is induced by the non-local interactions, producing an alternative low-energy effective theory of superconductivity, valid for energy scales well below an ultraviolet cut-off Λ. The induced photon mass is cut-off dependent, being proportional to $m \log \frac{\Lambda^2}{m^2}$, with m the fermion mass.

The form of the coupling between fermions and the $B_{\mu\nu}$ field raises an interesting question. Just as the ordinary gauge potential couples to worldlines of charged particles, the antisymmetric tensor gauge field $B_{\mu\nu}$ couples naturally to worldsheets swept out by strings. Interactions between strings can be thought of as being mediated by the B field, just as interactions between charged particles can be thought of as being mediated by vector gauge fields, as has been known from the early days of string theory. Now that we have a coupling between the B field and fermions, a coupling which is not localized at a point, the question naturally arises as to whether the system contains stringlike objects, and what such strings would be made of.

In this letter we derive the remarkable result that the static potential between a pair of fermions in this theory has a component that is linear and attractive, independent of the charge of the fermions. For a pair of electrons, this is like a Cooper pair connected by a confining string.

The result is fairly easy to obtain. We start with the partition function

$$Z = \int \mathscr{D}B\, \mathscr{D}A\, \mathscr{D}\bar\psi\, \mathscr{D}\psi\, e^{iS} \tag{4}$$

for the action [19]

$$S = \int d^4x \Big[-\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\lambda}H^{\mu\nu\lambda} + \bar\psi(i\gamma_\mu\partial^\mu - m)\psi + g\, m\, \epsilon^{\mu\nu\lambda\sigma} B_{\mu\nu} \frac{\partial_\lambda}{\Box} \bar\psi\gamma_\sigma\psi + eA_\mu\bar\psi\gamma^\mu\psi \Big]. \tag{5}$$

Here $\psi(x)$ is a charged fermion of mass m interacting with electromagnetic gauge field A_μ with coupling e, and also with an anti-symmetric tensor field $B_{\mu\nu}$ with coupling g. This action is invariant under the aforesaid vector gauge symmetry in addition to the usual $U(1)$ gauge symmetry. Therefore, in order to perform the integrals over $B_{\mu\nu}$ and A_μ, we add the gauge fixing terms $-\frac{1}{2\zeta}(\partial_\mu B^{\mu\nu})^2 - \frac{1}{2\eta}(\partial_\mu A^\mu)^2$ to the Lagrangian.

Integrating over the A and B fields, we get

$$Z = N \int \mathcal{D}\bar\psi\, \mathcal{D}\psi\, e^{iS[\psi,\bar\psi]}, \tag{6}$$

where

$$\begin{aligned} S[\psi,\bar\psi] &= S_D[\psi,m] + \frac{1}{2}\int \frac{d^4k}{(2\pi)^4} \Big[J^\sigma(-k)\frac{e^2}{k^2} J_\sigma(k) - g^2 J_{\mu\nu}(-k) \frac{g^{\mu[\rho}g^{\lambda]\nu}}{k^2} J_{\rho\lambda}(k) \Big] \\ &= S_D[\psi,m] + S_A[\psi,\bar\psi] + S_B[\psi,\bar\psi]. \end{aligned} \tag{7}$$

$S_D[\psi,m]$ consists of the kinetic and mass terms of the fermion, and in writing the other two terms we have used the fact that both J_μ and $J_{\mu\nu}$ are conserved currents.

The antisymmetric tensor current $J_{\mu\nu}$ is defined to be what couples to $B_{\mu\nu}$ in Eq. (5), and it is easy to see that it is related to the fermion current J_μ by

$$J_{\mu\nu}(k) = -m\,\epsilon_{\alpha\mu\nu\sigma} \frac{ik^\alpha}{k^2} J^\sigma(k), \tag{8}$$

which when inserted into the last term of Eq. (7) gives

$$S_B[\psi,\bar\psi] = 2\int \frac{d^4k}{(2\pi)^4} J^\sigma(-k) \frac{g^2m^2}{k^4} J_\sigma(k). \tag{9}$$

To obtain the form of the potential in non-relativistic limit, we expand the fermion fields in terms of annihilation and creation operators $a_r(\mathbf{p})$ and $a_r^\dagger(\mathbf{p})$ etc. We note that in the non-relativistic limit, only the J^0 component will contribute to the effective action of Eq. (7) because J^i is made of the lower components of Dirac spinors and thus can be neglected for energies much lower than their mass. Therefore, the leading contribution from this term is

$$\int \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \frac{d^3 q'}{(2\pi)^3} \sum_{r,s} a_r^\dagger(\mathbf{q}') a_r(\mathbf{q}) \times \left[\frac{e^2}{(p-p')^2} + 2\frac{g^2 m^2}{(p-p')^4} \right] a_s^\dagger(\mathbf{p}') a_s(\mathbf{p}). \quad (10)$$

In the non-relativistic limit, we keep terms only to the lowest order in the 3-momenta so that we can write $p = (m, \mathbf{p}),\ p' = (m, \mathbf{p}')$, and

$$(p-p')^2 \approx -|\mathbf{p} - \mathbf{p}'|^2. \quad (11)$$

What we have calculated in Eq. (10) is an effective scattering amplitude for electron–electron interaction. The static potential has an extra factor of (-1) compared to this [20], which means that in three dimensional momentum space the static potential has the form

$$V(\mathbf{k}) = \frac{e^2}{|\mathbf{k}|^2} - 2\frac{g^2 m^2}{|\mathbf{k}|^4}, \quad (12)$$

where $\mathbf{k} = \mathbf{p} - \mathbf{p}'$.

The Fourier transform of $V(\mathbf{k})$ gives the expression for the static potential in three dimensional coordinate space,

$$V(r) = \frac{e^2}{4\pi r} + \frac{g^2 m^2 r}{2\pi}. \quad (13)$$

The force resulting from the static potential given in Eq. (13) has the form

$$-\nabla V(r) = \left(\frac{e^2}{4\pi r^2} - \frac{g^2 m^2}{2\pi} \right) \hat{\mathbf{r}}. \quad (14)$$

The first term in Eq. (14) is the Coulomb force which is repulsive for the electron–electron interaction. But the second term, which comes from a confining linear potential and is independent of the electric charges of the fermions, is attractive for the electron–electron interaction. Following the idea of "Cooper instability" [1,2], we can say that the attractive interaction generates a kind of pairing between electrons. For chargeless fermions, this pairing would be responsible for gauge-invariant mass generation of $B_{\mu\nu}$ similar to Schwinger mechanism [21,22], by shifting the pole of the $B_{\mu\nu}$ field. When the fermions couple also to A_μ, it generates an effective $B \wedge F$ interaction. This shows that although the fermions are confined by linear confining potential, the interacting field $B_{\mu\nu}$ is not confined, rather it would behave as a short-ranged force field.

The action in Eq. (5) we started with describes low-energy effective interactions of the system. So the exact description of the interactions at very short distance scale may not be possible. However, we may make an estimation of the scale below which photons become short ranged. This can be thought of as the separation of the pair, or a 'correlation length' r_{cor}, which can be estimated by setting the net force to zero,

$$\frac{1}{r^2{}_{\text{cor}}} = \frac{2g^2 m^2}{e^2}. \quad (15)$$

At r_{cor}, the potential has the value $\frac{egm}{2\sqrt{2}\pi}$.

We may think of the potential of Eq. (13) as describing a system of two spins (electrons) connected by a string of fixed length. If the length of the string exceeds the value given in Eq. (15), the attractive linear potential dominates, and the spins are attracted to each other. If they come closer than r_{cor}, the Coulomb repulsion dominates, leading to the spins moving away from each other. We can also expect that the effect of interactions with other nearby spins could modify the structure of the string. As is clear from Eq. (10) the attractive interaction is independent of spin, so the Cooper pair wave function can have degenerate spin combinations

$$C_1 \mid \uparrow, \uparrow\rangle + C_{01} \mid \uparrow, \downarrow\rangle + C_{02} \mid \downarrow, \uparrow\rangle + C_{-1} \mid \downarrow, \downarrow\rangle. \quad (16)$$

Here the coefficients in general are functions of space time coordinates and orbital angular momentum. This indicates the system can be in a spin one or spin zero superconducting phase.

How does our work relate to, or differ from, earlier work? It is obvious that the confining potential of Eq. (13) must appear when the interacting charged particles are connected by a flux tube or string. The Kalb–Ramond field $B_{\mu\nu}$ also arises naturally in such situations. For example, in the context of confining strings in gauge theories [23–25], or in a field theoretic description of topological matter [26,27], the antisymmetric tensor field $B_{\mu\nu}$ originates from the condensation of topological charge in compact U(1) gauge theory. In these cases one finds flux tubes carrying electric (or magnetic, or both electric and magnetic) flux. The model we have considered differs from these in both respects. The B field, and its non-local coupling with fermions which was our starting point in this investigation, must arise from some underlying theory in terms of an effective action, but the details of that process is not clear. More importantly, it is not clear what corresponds to flux tubes in this model, since electric charge is unconfined, and what couples to the B field does not appear to be magnetic charge. Further, the B interaction does not appear to give rise semi-classically to topological defects. While the current which couples to $B_{\mu\nu}$ corresponds to the vorticity of the fermion field in a semi-classical picture, this does not lead to a winding number for a given pair of fermions. The effective potential of Eq. (13) comes from a purely quantum calculation, and in that regard our results are perhaps closest in spirit to the ideas of [28].

To summarize, in this letter we have described a model which exhibits an unconventional pairing of fermions, and at the same time produces Meissner effect at low energy by a radiatively induced effective $B \wedge F$ interaction. This process does not involve any spontaneous symmetry breaking by any local order parameter [6,29]. It could be useful as a description of unconventional superconductors which expels magnetic field from the bulk but pairs fermions by flux tubes or something analogous. The main result of our paper is that the effective potential between two pairing electrons has a part which is linear and attractive, which means that the theory is in a confining phase. According to the 't Hooft–Mandelstam [30,31] description of confinement, the system must be in a disordered state [32]. So the non-existence of local order parameter is natural here, although following order–disorder duality, it may be possible to describe the confining system by a dual order parameter. In the present case we have not found such a dual order parameter yet, but Eq. (16) shows that pair formation due to the confinement mechanism would create a kind of ordering.

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