Eur. Phys. J. C (2017) 77:129
DOI 10.1140/epjc/s10052-017-4688-x

THE EUROPEAN PHYSICAL JOURNAL C

CrossMark

Regular Article - Theoretical Physics

Excited state mass spectra of doubly heavy Ξ baryons

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Received: 17 December 2016 / Accepted: 8 February 2017 / Published online: 25 February 2017

Abstract In this paper, the mass spectra are obtained for doubly heavy Ξ baryons, namely, Ξ_{cc}^{+}, Ξ_{cc}^{++}, Ξ_{bb}^{-}, Ξ_{bb}^{0}, Ξ_{bc}^{0} and Ξ_{bc}^{+}. These baryons consist of two heavy quarks (cc, bb, and bc) with a light (d or u) quark. The ground, radial, and orbital states are calculated in the framework of the hypercentral constituent quark model with Coulomb plus linear potential. Our results are also compared with other predictions, thus, the average possible range of excited states masses of these Ξ baryons can be determined. The study of the Regge trajectories is performed in (n, M^2) and (J, M^2) planes and their slopes and intercepts are also determined. Lastly, the ground state magnetic moments of these doubly heavy baryons are also calculated.

1 Introduction

Doubly heavy baryons have two families: Ξ and Ω. Ω has a light strange quark, while Ξ has up or down quark(s) with two heavy quarks (c and b). Our previous work [1] exhibited the mass spectra, magnetic moments, and Regge trajectories of doubly heavy Ω baryons, while in the present paper, we established the Ξ baryon family with six members. The only experimental evidence comes for Ξ_{cc}^{+} by the SELEX experiment. One reported a ground state at 3520 MeV containing two charm quarks and a down quark [2–4]. It is yet to be confirmed from the other experiments [5–8]. Recently, the Hamiltonian model [9], Regge phenomenology [10,11], lattice QCD [12–15], QCD sum rules [16,17], the variational approach [18], and many more [19–33] have provided new results in the field of doubly heavy baryons. Many of them have only calculated the ground state masses, while some of them have also shown the excited states.

We have used the QCD inspired hypercentral constituent quark model (hCQM) with Coloumb plus linear potential. The first order correction is also taken into account to the potential and calculation has been performed by solving the six-dimensional hyperradial Schrödinger equation numerically [1,34,35]. We have calculated the mass spectra of radial excited states up to 5S and orbital excited states for 1P–5P, 1D–4D, and 1F–2F states. To the best of our knowledge, all the theoretical approaches have considered the $m_u = m_d$ case so far but the light quark masses are different in our model. Thus, we have obtained the mass spectra with u and d quarks combinations for these baryons. The obtained masses were used in the formation of Regge trajectories in the (n, M^2) and (J, M^2) planes. The determination of the slope and intercept of the Regge trajectories of these baryons is very important as it provides a better understanding of the dynamics of strong interactions in the production of charmed and bottom baryons at high energies.

The paper is organized as follows. We give a brief introduction in Sect. 1 and explain our hypercentral constituent quark model in Sect. 2. We present our mass spectra results of all doubly heavy Ξ baryons in Sect. 3. Regge trajectories and magnetic moments are discussed in Sect. 4. Finally, our conclusion is in Sect. 5.

2 The model

The methodology for the determination of the excited masses follows the same pattern as in our previous work; see [1] and the references therein. Therefore, we discuss the model very briefly in the present paper. We start with the Jacobi coordinates of three quark baryons that are given in terms of mass (m_i) and coordinates ($\mathbf{r_i}$) below [36]. The quark masses are taken in the calculations as $m_u = 0.338$, $m_d = 0.350$, $m_c = 1.275$, and $m_b = 4.67$ (in GeV). The coordinates $\boldsymbol{\rho}$ and $\boldsymbol{\lambda}$ with the respective reduced masses are given by

$$m_\rho = \frac{2m_1 m_2}{m_1 + m_2}, \quad (1)$$

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Table 1 Ground state masses of Ξ baryons are listed and compared. Our work shows the ground state masses of all baryons with both light quark combination as $\Xi_{cc}^{+}/\Xi_{cc}^{++}$, $\Xi_{bb}^{-}/\Xi_{bb}^{0}$ and $\Xi_{bc}^{0}/\Xi_{bc}^{+}$ (in GeV)

Baryons	Ξ_{ccd}/Ξ_{ccu}		Ξ_{bbd}/Ξ_{bbu}		Ξ_{bcd}/Ξ_{bcu}	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our work	3.520/3.511	3.695/3.687	10.317/10.312	10.340/10.335	6.920/6.914	6.986/6.980
Exp. [2]	3.519±0.009		–	–	–	–
Ref. [9]	3.685	3.754	10.314	10.339	–	–
Ref. [10,11]	3.520	3.695	10.199	10.316	–	–
Ref. [12]	3.610(09)(12)	3.694(07)(11)	–	–	–	–
Ref. [13]	3.610	3.692	10.143	10.178	6.943	6.985
Ref. [14]	3.561(22)	3.642(26)	–	–	–	–
Ref. [16,17]	3.720	–	9.960	–	6.720	–
Ref. [18]	3.678	3.752	10.322	10.352	7.014	7.064
Ref. [19]	3.676	3.753	10.340	10.367	7.011	7.074
Ref. [20]	3.547	3.719	10.185	10.216	6.904	6.936
Ref. [21]	3.579	3.656	10.189	10.218	–	–
Ref. [22]	3.620	3.727	10.202	10.237	6.933	6.980
Ref. [23]	3.478	3.610	10.093	10.133	6.820	6.900
Ref.[24]	3.627	3.690	10.162	10.184	6.914	
Ref.[25]	3.519	3.620	9.800	9.890	6.650	6.690
Ref.[26]	3.612	3.706	10.197	10.136	6.919	6.986
Ref.[27]	3.510	3.548	10.130	10.144	6.792	6.827
Ref.[28,29]	3.570	3.610	10.170	10.220	–	–

$$m_\lambda = \frac{2m_3(m_1^2+m_2^2+m_1m_2)}{(m_1+m_2)(m_1+m_2+m_3)}. \tag{2}$$

The Hamiltonian of the three-body baryonic system in the hCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x). \tag{3}$$

The hyperradius $x = \sqrt{\rho^2+\lambda^2}$ is a collective coordinate and therefore the hypercentral potential contains also the three-body effects. Here $m = \frac{2m_\rho m_\lambda}{m_\rho+m_\lambda}$ is the reduced mass and x is the six-dimensional radial hyper central coordinate of the three-body system. In the present paper, the confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential strings increase linearly with a collective radius r_{3q} as mentioned in [37]. Accordingly the effective two-body interactions can be written as

$$\sum_{i<j} V(r_{ij}) = V(x) + \cdots . \tag{4}$$

In the hypercentral approximation, the potential is only depends on hyper radius(x). More details can be found in the references [37,38]. We consider a reduced hypercentral radial function, $\phi_\gamma(x) = x^{\frac{5}{2}}\Psi_\gamma(x)$ where $\Psi_\gamma(\mathrm{x})$ is the hyper-central wave function and γ is the grand angular quantum number. Thus, the six dimensional hyperradial Schrödinger equation reduces to

$$\left[\frac{-1}{2m}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{\frac{15}{4}+\gamma(\gamma+4)}{2mx^2} + V(x)\right]\phi_\gamma(x) = E\phi_\gamma(x). \tag{5}$$

For the present study we consider the hypercentral potential $V(x)$ as the color Coulomb plus linear potential with first order correction [39–41] given by

$$V(x) = V^0(x) + \left(\frac{1}{m_\rho}+\frac{1}{m_\lambda}\right)V^{(1)}(x) + V_{SD}(x), \tag{6}$$

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x \quad \text{and} \quad V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4x^2}, \tag{7}$$

$$V_{SD}(x) = V_{SS}(x)(\mathbf{S}_{\boldsymbol\rho}.\mathbf{S}_{\boldsymbol\lambda}) + V_{\gamma S}(x)(\boldsymbol\gamma\cdot\mathbf{S}) + V_T(x)\left[S^2 - \frac{3(\mathbf{S}\cdot\mathbf{x})(\mathbf{S}\cdot\mathbf{x})}{x^2}\right]. \tag{8}$$

Here, τ is the hyper-Coulomb strength corresponding to the strong running coupling constant α_s. β is the string tension of the confinement part of potential. C_F and C_A are the Casimir charges of the fundamental and adjoint representation with values $\frac{2}{3}$ and 3. The spin-dependent part,

Table 2 Radial excited states of doubly heavy Ξ baryons(in GeV). Columns 4 and 5 show the masses with light quark d combination whereas Columns 6 and 7 show the masses with light quark u

Particle	State	J^P	A	B	A	B	[9]	[19]	[20]	[21]	[22]	[18]
A→without first order correction, B→with first order correction												
Ξ_{ccd} and Ξ_{ccu}	2S		3.912	3.925	3.905	3.920	4.079	4.029	4.183	3.976	3.910	4.030
	3S	$\frac{1}{2}^+$	4.212	4.233	4.230	4.159	4.206		4.640		4.154	
	4S		4.473	4.502	4.468	4.501						
	5S		4.711	4.748	4.708	4.748						
	2S		3.976	3.988	3.970	3.983	4.114	4.042	4.282	4.025	4.027	4.078
	3S	$\frac{3}{2}^+$	4.244	4.264	4.238	4.261	4.131		4.719			
	4S		4.492	4.520	4.488	4.519						
	5S		4.724	4.759	4.720	4.759						
Ξ_{bbd} and Ξ_{bbu}	2S		10.605	10.612	10.603	10.609	10.571	10.576	10.751	10.482	10.441	10.551
	3S	$\frac{1}{2}^+$	10.851	10.862	10.851	10.862	10.612		11.170		10.630	
	4S		11.073	11.088	11.075	11.090					10.812	
	5S		11.278	11.297	11.282	11.301						
	2S		10.613	10.619	10.611	10.617	10.592	10.578	10.770	10.501	10.482	10.574
	3S	$\frac{3}{2}^+$	10.855	10.855	10.866	10.866	10.593		11.184		10.673	
	4S		11.075	11.090	11.077	11.092					10.856	
	5S		11.280	11.298	11.284	11.302						
Ξ_{bcd} and Ξ_{bcu}	2S		7.236	7.244	7.231	7.240			7.478			7.321
	3S	$\frac{1}{2}^+$	7.495	7.509	7.492	7.507			7.904			
	4S		7.727	7.746	7.726	7.744						
	5S		7.940	7.963	7.940	7.964						
	2S		7.260	7.267	7.256	7.263			7.495			7.353
	3S	$\frac{3}{2}^+$	7.507	7.521	7.505	7.518			7.917			
	4S		7.734	7.752	7.733	7.752						
	5S		7.944	7.968	7.945	7.969						

$V_{SD}(x)$ contains three types of the interaction terms [42]: the spin–spin term $V_{SS}(x)$, the spin–orbit term $V_{\gamma S}(x)$, and the tensor term $V_T(x)$. The details of the terms are given in [34]. We solve the six dimensional Schrodinger equation using Mathematica notebook [43].

3 Mass spectroscopy: Ξ_{cc}, Ξ_{bb}, and Ξ_{bc}

We begin with calculating the ground state masses of doubly heavy Ξ_{cc}, Ξ_{bb} and Ξ_{bc} baryons[1]. The masses are computed for both parities $\frac{1}{2}^+$ and $\frac{3}{2}^+$ mentioned in Table 1. As is well known, the ground state of Ξ_{cc}^+ is found experimentally as $\Xi_{cc}(3520)^+$; but its J^P value is still undefined. Our prediction suggests that it would be $J^P = \frac{1}{2}^+$; a similar suggestion is given by Refs. [10,11,25,27]. The other ground state, with $J^P = \frac{3}{2}^+$, is found as 3.695 GeV by us. The value is closer to other predictions [10,11,24,26] and lattice [12,13] calculations. In the case of Ξ_{bb} baryon, our ground state results (both parities) are matched (with [9,18]) very well. Our predicted ground states of Ξ_{bc} are very close to Refs. [13,22,26]. We have also calculated the ground state spectra of the *ccu*, *bbu*, and *bcu* baryons. They are close to the results of d quark combinations (8, 5, and 6 MeV differences, respectively).

Moving toward the excited states, the radial excited states are calculated from 2S–5S for $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$. These radial excited states of Ξ_{cc}, Ξ_{bb}, and Ξ_{bc} are mentioned in Table 2. P, D, and F states with their different isospin splittings are computed and the excited state masses from 1P–5P, 1D–4D, and 1F–2F are shown in Tables 3, 4, 5, and 6. One can observe that the B masses are a few MeV higher than the A masses in each case of every baryonic system. The results of different theoretical approaches for all the systems are also compared in the respective tables. Notice that the heavy quark combination (*cc*, *bb*, and *bc*) with light quarks (*u* and *d*) for all three baryons are represented individually in Table 1, 2, 3, 4, 5, and 6.

[1] We compare our *ccd*, *bbd*, and *cbd* baryon combination masses with others for the whole mass spectra discussion.

Table 3 P and D state masses of Ξ_{cc} baryon (in GeV)

State	A Ξ_{cc}^{+}	B	A Ξ_{cc}^{++}	B	[9]	[19]	[21]	[22]	[10]	[23]	[18]	[12]
A→ without correction and B→ with correction												
$(1^2P_{1/2})$	3.853	3.865	3.846	3.861	3.947	3.910	3.880	3.838			4.073	3.892(47)(48)
$(1^2P_{3/2})$	3.834	3.847	3.828	3.842	3.949	3.921		3.959	3.786	3.834	4.079	3.989(58)(58)
$(1^4P_{1/2})$	3.862	3.875	3.856	3.871								
$(1^4P_{3/2})$	3.843	3.856	3.837	3.851								
$(1^4P_{5/2})$	3.818	3.890	3.817	3.888	4.163	4.092		4.155	3.949	4.047	4.089	
$(2^2P_{1/2})$	4.138	4.161	4.134	4.140	4.135	4.074	4.018	4.085				
$(2^2P_{3/2})$	4.121	4.144	4.118	4.140	4.137	4.078	4.197					
$(2^4P_{1/2})$	4.146	4.169	4.143	4.167								
$(2^4P_{3/2})$	4.130	4.152	4.126	4.149								
$(2^4P_{5/2})$	4.108	4.183	4.104	4.181	4.488							
$(3^2P_{1/2})$	4.395	4.426	4.393	4.409	4.149							
$(3^2P_{3/2})$	4.381	4.411	4.379	4.409	4.159							
$(3^4P_{1/2})$	4.402	4.433	4.400	4.432								
$(3^4P_{3/2})$	4.388	4.419	4.386	4.417								
$(3^4P_{5/2})$	4.369	4.399	4.412	4.396	4.534							
$(4^2P_{1/2})$	4.633	4.671	4.633	4.671								
$(4^2P_{3/2})$	4.620	4.658	4.620	4.657								
$(4^4P_{1/2})$	4.640	4.678	4.639	4.678								
$(4^4P_{3/2})$	4.627	4.664	4.626	4.664								
$(4^4P_{5/2})$	4.610	4.646	4.609	4.646								
$(5^2P_{1/2})$	4.857	4.901	4.858	4.902								
$(5^2P_{3/2})$	4.845	4.889	4.846	4.889								
$(5^4P_{1/2})$	4.863	4.908	4.864	4.909								
$(5^4P_{3/2})$	4.851	4.895	4.852	4.896								
$(5^4P_{5/2})$	4.835	4.878	4.835	4.879								
$(1^4D_{1/2})$	4.053	4.077	4.043	4.071								
$(1^2D_{3/2})$	4.026	4.049	4.019	4.044								
$(1^4D_{3/2})$	4.035	4.058	4.027	4.053								
$(1^2D_{5/2})$	4.002	4.024	3.998	4.019	4.043	4.115	4.047		4.391	4.034	4.050	4.388
$(1^4D_{5/2})$	4.011	4.033	4.006	4.029	4.027	4.052						
$(1^4D_{7/2})$	3.982	4.002	3.979	3.998	4.097			4.187	4.089	4.393		
$(2^4D_{1/2})$	4.311	4.345	4.311	4.342								
$(2^2D_{3/2})$	4.289	4.321	4.287	4.318								
$(2^4D_{3/2})$	4.296	4.329	4.295	4.326								
$(2^2D_{5/2})$	4.270	4.299	4.267	4.297	4.164	4.091						
$(2^4D_{5/2})$	4.277	4.307	4.275	4.305								
$(2^4D_{7/2})$	4.253	4.280	4.249	4.278	4.394							
$(3^4D_{1/2})$	4.554	4.592	4.553	4.592								
$(3^2D_{3/2})$	4.534	4.571	4.532	4.570								
$(3^4D_{3/2})$	4.541	4.578	4.539	4.578								
$(3^2D_{5/2})$	4.516	4.552	4.514	4.551	4.348							
$(3^4D_{5/2})$	4.523	4.559	4.521	4.558								
$(3^4D_{7/2})$	4.500	4.535	4.498	4.534								
$(4^4D_{1/2})$	4.780	4.825	4.781	4.826								

Table 3 continued

State	A Ξ_{cc}^{+}	B	A Ξ_{cc}^{++}	B	[9]	[19]	[21]	[22]	[10]	[23]	[18]	[12]
$(4^2D_{3/2})$	4.762	4.806	4.762	4.806								
$(4^4D_{3/2})$	4.768	4.812	4.768	4.813								
$(4^2D_{5/2})$	4.745	4.788	4.745	4.788								
$(4^4D_{5/2})$	4.751	4.795	4.751	4.795								
$(4^4D_{7/2})$	4.731	4.772	4.730	4.772								

Table 4 P and D state masses of Ξ_{bb} (in GeV)

State	A Ξ_{bb}^{-}	B	A Ξ_{bb}^{0}	B	[9]	[19]	[21]	[22]	[10]	[18]	Others
A→ without correction and B→ with correction											
$(1^2P_{1/2})$	10.507	10.514	10.504	10.511	10.476	10.493	10.406	10.368		10.691	
$(1^2P_{3/2})$	10.502	10.509	10.499	10.506	10.476	10.495		10.408	10.474	10.692	10.390 [29]
$(1^4P_{1/2})$	10.510	10.517	10.506	10.514							
$(1^4P_{3/2})$	10.505	10.512	10.501	10.509							10.430 [17]
$(1^4P_{5/2})$	10.514	10.521	10.512	10.518	10.759				10.588	10.695	
$(2^2P_{1/2})$	10.758	10.770	10.757	10.770	10.703	10.710	10.612	10.563			
$(2^2P_{3/2})$	10.754	10.766	10.753	10.765	10.704	10.713		10.607			
$(2^4P_{1/2})$	10.760	10.772	10.760	10.772							
$(2^4P_{3/2})$	10.756	10.768	10.755	10.767							
$(2^4P_{5/2})$	10.751	10.763	10.763	10.776	10.973	10.713					
$(3^2P_{1/2})$	10.985	11.001	10.986	11.002	10.740			10.744			
$(3^2P_{3/2})$	10.981	10.997	10.982	10.998	10.742			10.788			
$(3^4P_{1/2})$	10.987	11.003	10.988	11.004							
$(3^4P_{3/2})$	10.983	10.999	10.984	11.000							
$(3^4P_{5/2})$	10.978	10.994	10.991	11.007	11.004						
$(4^2P_{1/2})$	11.194	11.214	11.197	11.217				10.900			
$(4^2P_{1/2})$	11.191	11.210	11.194	11.213							
$(4^2P_{3/2})$	11.196	11.216	11.199	11.219							
$(4^4P_{3/2})$	11.193	11.212	11.796	11.215							
$(4^4P_{5/2})$	11.188	11.208	11.202	11.222							
$(5^2P_{1/2})$	11.390	11.413	11.395	11.418							
$(5^2P_{3/2})$	11.387	11.410	11.392	11.415							
$(5^4P_{1/2})$	11.392	11.415	11.397	11.420							
$(5^4P_{3/2})$	11.389	11.412	11.394	11.417							
$(5^4P_{5/2})$	11.385	11.407	11.399	11.423							
$(1^4D_{1/2})$	10.665	10.677	10.663	10.675							
$(1^2D_{3/2})$	10.658	10.670	10.656	10.668							
$(1^4D_{3/2})$	10.660	10.672	10.659	10.670						11.011	
$(1^2D_{5/2})$	10.652	10.663	10.650	10.661	10.592	10.676			10.742	11.002	
$(1^4D_{5/2})$	10.654	10.666	10.652	10.664							
$(1^4D_{7/2})$	10.647	10.658	10.644	10.656		10.608			10.853	11.011	
$(2^4D_{1/2})$	10.897	10.913	10.897	10.913							
$(2^2D_{3/2})$	10.891	10.907	10.891	10.907							
$(2^4D_{3/2})$	10.893	10.909	10.893	10.909							

Table 4 continued

State	A Ξ_{bb}^{-}	B	A Ξ_{bb}^{0}	B	[9]	[19]	[21]	[22]	[10]	[18]	Others
$(2^4D_{3/2})$	10.886	10.901	10.886	10.901		10.712					
$(2^2D_{5/2})$	10.888	10.903	10.888	10.903	10.613						
$(2^2D_{5/2})$	10.881	10.896	10.881	10.896		11.057					
$(3^4D_{1/2})$	11.109	11.130	11.113	11.133							
$(3^2D_{3/2})$	11.105	11.125	11.108	11.127							
$(3^4D_{3/2})$	11.107	11.126	11.110	11.129							
$(3^4D_{3/2})$	11.101	11.120	11.102	11.122							
$(3^2D_{5/2})$	11.103	11.122	11.105	11.124	10.809						
$(3^2D_{5/2})$	11.097	11.116	11.099	11.118							
$(4^4D_{1/2})$	11.310	11.333	11.314	11.337							
$(4^2D_{3/2})$	11.306	11.328	11.310	11.332							
$(4^4D_{3/2})$	11.307	11.330	11.311	11.334							
$(4^4D_{3/2})$	11.302	11.324	11.305	11.328							
$(4^2D_{5/2})$	11.303	11.325	11.307	11.330							
$(4^2D_{5/2})$	11.298	11.320	11.302	11.324							

The excited states of the doubly heavy Ξ family are unknown experimentally. Starting from radial excited states mentioned in Table 2, we have compared our results with Refs. [9,18–22]. For the 2S state of the system Ξ_{cc} with J^P values $\frac{1}{2}^+(\frac{3}{2}^+)$ we have a lowest prediction of 3.910 (4.027) (by [22]) and a highest of 4.183(4.282) (by [20]). Specifically, our 2S predictions are close to [22]. In a similar way, while analyzing the 2S state of the $\Xi_{bb}(\frac{1}{2}^+)$ and $\Xi_{bb}(\frac{3}{2}^+)$ baryons, the lowest to highest ranges of the masses are found to be (10441–10751) MeV and (10482–10770) MeV, respectively, whereas our model suggested masses that are close to the result of Ref. [19]. The next baryon is Ξ_{bc} and the 2S states with both isospins show more than 100 MeV difference with others ([18,20]). Though Refs. [9,20,22] have computed the 3S state (with $\frac{1}{2}^+$ and $\frac{3}{2}^+$) for Ξ_{cc} and Ξ_{bb} baryons and only [20] has computed for Ξ_{bc} baryon, the mass difference from ours they obtained is large (except [9] for Ξ_{cc}).

In the case of Ξ_{cc}, our 1P state $J^P = \frac{1}{2}^-$ shows a difference of 57 MeV (with [19]), 27 MeV (with [21]), 12 MeV (with [22]) and 39 MeV (with [12]), while $J^P = \frac{3}{2}^-$ shows 87 MeV (with [19]) and 0 MeV (with [23]).

For Ξ_{bb}, our 1P states $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ are only 14 and 7 MeV higher than those of Ref. [19] whereas Ref. [9] has masses for $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ that are 31, 26 MeV lower than our prediction. Our 2P state is a few MeV higher than that of [9,19]. Our 1D–2D states have $\approx$35 MeV and $\approx$178 MeV difference with [19]. The P and D states of the Ξ_{bc}^0 baryons are given in Table 5 and it follows the same description as mentioned in [1] and the references therein.

We have compared our results with recent papers [10,11] for 1P and 1D. Their values are higher than ours. The rest of the spectra (2P–5P and 2D–4D) is addressed by us for completeness.

The F state masses for all three doubly heavy baryons are given in Table 6. Apart from our work, Ref. [10,11] has also calculated the 1F state of Ξ_{cc} and Ξ_{bb} for $J^P = \frac{7}{2}^-$ and $\frac{9}{2}^-$. For Ξ_{cc}, their masses are 73 and 280 MeV higher, while for the other system we have values 212 and 328 MeV higher than our predictions. We did not find any other F state calculations for the Ξ_{bc} systems.

4 Regge trajectories and magnetic moments

As discussed in Sect. 3, we have calculated the 1S–5S, 1P–5P, 1D–4D, and 1F–2F state masses for all doubly heavy Ξ baryons. The obtained masses were very much useful in constructing the Regge trajectories in the (n, M^2) and (J, M^2) planes. Here n is the principal quantum number and J is the total spin. The Regge trajectories are presented in Figs. 1, 2, 3, 4, and 5 for the $\Xi_{cc}(ccd)$, $\Xi_{bb}(bbd)$ and $\Xi_{bc}(bcd)$ baryons. Similar trajectories can also be plotted for the rest of the baryons. Straight lines were obtained by the linear fitting in all figures. The ground and radial excited S states ($J^P = \frac{1}{2}^+$) and the orbital excited P ($J^P = \frac{1}{2}^-$), D ($J^P = \frac{5}{2}^+$) and F ($J^P = \frac{7}{2}^-$) states are plotted in Fig 1, 2, and 3 from bottom to top. We use

$$n = \beta M^2 + \beta_0. \quad (9)$$

Table 5 P and D state masses of Ξ_{bc} (in GeV)

State	A	B	A	B	[18]	State	A	B	A	B	[18]
	Ξ_{bc}^0		Ξ_{bc}^+					Ξ_{bc}^0	Ξ_{bc}^+		
A→ without correction and B→ with correction											
$(1^2P_{1/2})$	7.151	7.160	7.146	7.156	7.390	$(1^4D_{1/2})$	7.322	7.336	7.318	7.334	
$(1^2P_{3/2})$	7.140	7.149	7.135	7.144	7.394	$(1^2D_{3/2})$	7.307	7.321	7.303	7.318	
$(1^4P_{1/2})$	7.157	7.166	7.152	7.161	7.399	$(1^4D_{3/2})$	7.312	7.326	7.706	7.308	7.324
$(1^4P_{3/2})$	7.146	7.155	7.141	7.150		$(1^2D_{5/2})$	7.294	7.308	7.290	7.304	
$(1^4P_{5/2})$	7.131	7.175	7.126	7.171		$(1^4D_{5/2})$	7.299	7.313	7.702	7.295	7.309
						$(1^4D_{7/2})$	7.293	7.296	7.708	7.278	7.292
$(2^2P_{1/2})$	7.410	7.425	7.407	7.422		$(2^4D_{1/2})$	7.559	7.425	7.558	7.579	7.579
$(2^2P_{3/2})$	7.401	7.415	7.397	7.412		$(2^2D_{3/2})$	7.547	7.567	7.545	7.565	
$(2^4P_{1/2})$	7.415	7.430	7.411	7.426		$(2^4D_{3/2})$	7.551	7.571	7.549	7.570	
$(2^4P_{3/2})$	7.405	7.420	7.402	7.417		$(2^2D_{5/2})$	7.536	7.555	7.534	7.553	7.538
$(2^4P_{5/2})$	7.393	7.408	7.419	7.434		$(2^4D_{5/2})$	7.540	7.559	7.538	7.558	
						$(2^4D_{7/2})$	7.526	7.545	7.523	7.544	
$(3^2P_{1/2})$	7.643	7.664	7.642	7.662		$(3^4D_{1/2})$	7.779	7.804	7.777	7.804	
$(3^2P_{3/2})$	7.635	7.655	7.634	7.654		$(3^2D_{3/2})$	7.768	7.792	7.7668	7.792	
$(3^4P_{1/2})$	7.647	7.668	7.646	7.666		$(3^4D_{3/2})$	7.772	7.782	7.770	7.796	
$(3^4P_{3/2})$	7.639	7.659	7.638	7.658		$(3^2D_{5/2})$	7.758	7.786	7.757	7.781	
$(3^4P_{5/2})$	7.629	7.648	7.653	7.673		$(3^4D_{5/2})$	7.762	7.772	7.761	7.785	
						$(3^4D_{7/2})$	7.774	7.772	7.749	7.772	
$(4^2P_{1/2})$	7.859	7.884	7.859	8.015		$(4^4D_{1/2})$	7.985	7.801	7.859	7.884	7.797
$(4^2P_{1/2})$	7.852	7.876	7.852	7.877		$(4^2D_{3/2})$	7.975	8.002	7.976	8.004	
$(4^2P_{3/2})$	7.863	7.888	7.863	7.888		$(4^4D_{3/2})$	7.987	8.006	7.979	8.008	
$(4^4P_{3/2})$	7.856	7.880	7.856	7.880		$(4^2D_{5/2})$	7.969	7.993	7.996	7.994	
$(4^4P_{5/2})$	7.846	7.870	7.869	7.895		$(4^4D_{5/2})$	7.958	7.996	7.970	7.998	
						$(4^4D_{7/2})$		7.985	7.958	7.986	
$(5^2P_{1/2})$	8.062	8.091	8.064	8.092							
$(5^2P_{3/2})$	8.05528	8.084	8.057	8.085							
$(5^4P_{1/2})$	8.0652	8.094	8.067	8.096							
$(5^4P_{3/2})$	8.059	8.087	8.060	8.088							
$(5^4P_{5/2})$	8.050	8.078	8.073	8.079							

Here β and β_0 are the slope and intercept, respectively. The fitted slopes and intercepts are given in Table 7. We use the natural parity $J^P = \frac{1}{2}^+, J^P = \frac{3}{2}^-, J^P = \frac{5}{2}^+, J^P = \frac{7}{2}^-$ and the unnatural ($J^P = \frac{3}{2}^+, J^P = \frac{5}{2}^-, J^P = \frac{7}{2}^+, J^P = \frac{9}{2}^-$) parity masses and plot the graphs for the Ξ_{cc} and Ξ_{bb} baryon states (see Figs. 4, 5). For that we use,

$$J = \alpha M^2 + \alpha_0. \tag{10}$$

Here α and α_0 are the slope and intercept, respectively. The fitted slopes and intercepts are given in Table 8. We observe that the square of the calculated masses fit very well to the linear trajectory and are almost parallel and equidistant in S, P, D, and F states. We can determine the possible quantum numbers and prescribe them to a particular Regge trajectory with the help of our obtained results.

To obtain the magnetic moments of the Ξ family, we need to calculate their effective masses first. As the combination of quarks in baryons changes, its binding interaction affects the situation and m_i^{eff} differs. The effective mass for each of the constituting quark m_i^{eff} can be defined as

$$m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i}\right) \tag{11}$$

where $\langle H \rangle$ = E + $\langle V_{\rm spin} \rangle$. Thus, the magnetic moment of baryons with bound quarks are given as [[1] and the references therein]

Table 6 F state masses of Ξ_{cc}, Ξ_{bb}, and Ξ_{bc} baryons (in GeV)

State	Ξ_{ccd}		Ξ_{ccu}		[10]	Ξ_{bbd}		Ξ_{bbu}		[11]	Ξ_{bcd}		Ξ_{bcu}	
	A	B	A	B		A	B	A	B		A	B	A	B
$(1^4F_{3/2})$	4.216	4.247	4.213	4.242		10.805	10.820	10.804	10.819		7.468	7.487	7.466	7.485
$(1^2F_{5/2})$	4.185	4.215	4.182	4.210		10.797	10.812	10.796	10.811		7.451	7.469	7.448	7.467
$(1^4F_{5/2})$	4.158	4.186	4.190	4.219		10.790	10.804	10.798	10.813		7.456	7.474	7.453	7.472
$(1^2F_{7/2})$	4.166	4.194	4.163	4.191		10.799	10.814	10.791	10.806		7.440	7.458	7.437	7.455
$(1^4F_{7/2})$	4.194	4.225	4.154	4.182	4.267	10.792	10.806	10.789	10.803	11.004	7.435	7.453	7.432	7.450
$(1^4F_{9/2})$	4.133	4.159	4.129	4.156	4.413	10.784	10.797	10.783	10.797	11.112	7.421	7.439	7.418	7.436
$(2^4F_{3/2})$	4.462	4.494	4.460	4.497		11.023	11.022	11.024	11.043		7.692	7.715	7.691	7.715
$(2^2F_{5/2})$	4.435	4.468	4.433	4.468		11.016	11.035	11.018	11.036		7.677	7.700	7.676	7.699
$(2^4F_{5/2})$	4.442	4.475	4.440	4.476		11.018	11.036	11.019	11.038		7.681	7.704	7.680	7.703
$(2^2F_{7/2})$	4.410	4.445	4.415	4.450		11.010	11.028	11.013	11.031		7.667	7.690	7.666	7.689
$(2^4F_{7/2})$	4.417	4.452	4.408	4.443		11.012	11.030	11.011	11.029		7.663	7.686	7.662	7.685
$(2^4F_{9/2})$	4.388	4.424	4.386	4.420		11.005	11.022	11.006	11.023		7.651	7.674	7.650	7.672

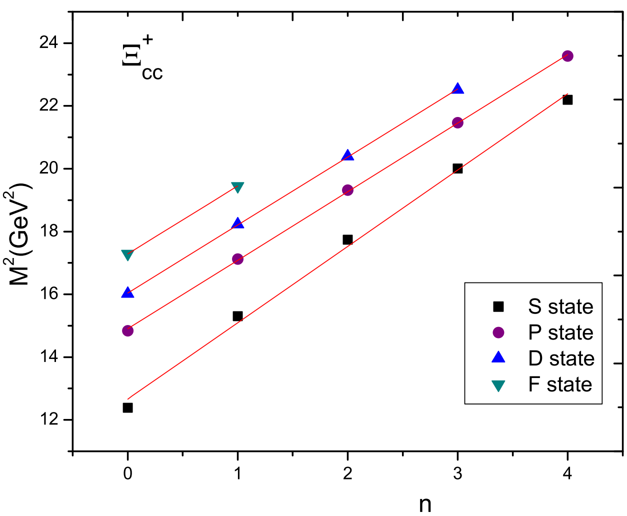

Fig. 1 Regge trajectory ($M^2 \to n$) for Ξ_{cc} baryon

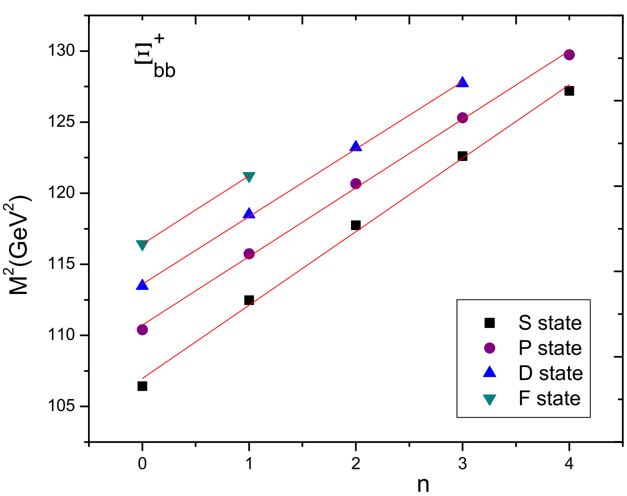

Fig. 2 Regge trajectory ($M^2 \to n$) for Ξ_{bb} baryon

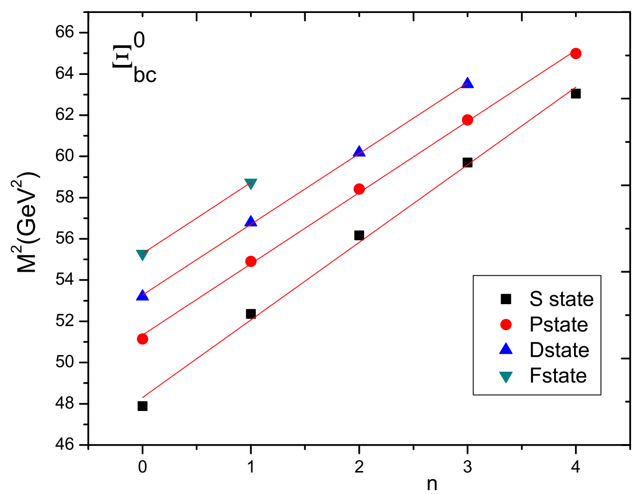

Fig. 3 Regge trajectory ($M^2 \to n$) for Ξ_{bc} baryon

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle) \tag{12}$$

where

$$\mu_i = \frac{e_i \sigma_i}{2m_i^{eff}}. \tag{13}$$

e_i is a charge and σ_i is the spin of the respective constituent quark corresponding to the spin flavor wave function of the baryonic state. Using these equations and our obtained ground state masses (mentioned in Table 1), we calculated the magnetic moments of all six Ξ baryons. The spin flavor wave function and magnetic moments are given in Table 9. Our obtained ground state magnetic moments are also compared with others, as shown in Table 9.

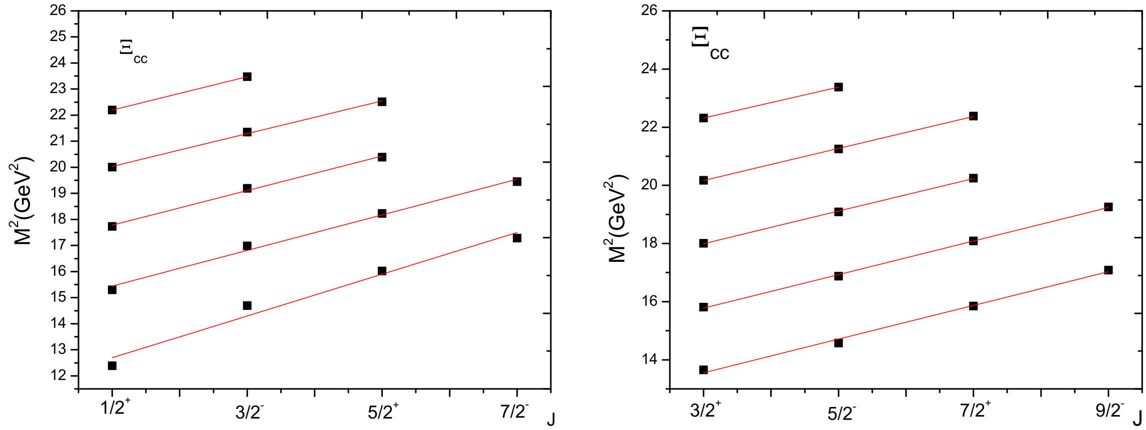

Fig. 4 Regge trajectory ($M^2 \to J$) for Ξ_{cc} baryon

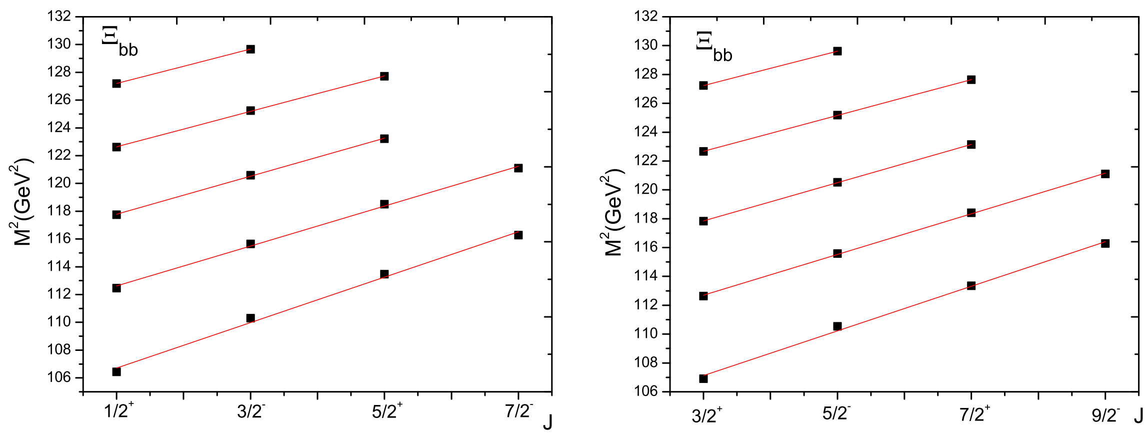

Fig. 5 Regge trajectory ($M^2 \to J$) for Ξ_{bb} baryon

Table 7 Fitted slopes and intercepts of the Regge trajectories in (n, M^2) plane

Baryon	J^P	State	β	β_0
Ξ_{cc}	$\frac{1}{2}^+$	S	0.409 ± 0.0138	-5.181 ± 0.247
	$\frac{1}{2}^-$	P	0.457 ± 0.004	-6.819 ± 0.073
	$\frac{3}{2}^-$	P	0.456 ± 0.003	-6.729 ± 0.075
	$\frac{5}{2}^+$	D	0.462 ± 0.003	-7.408 ± 0.058
Ξ_{bb}	$\frac{1}{2}^+$	S	0.193 ± 0.006	-20.627 ± 0.721
	$\frac{1}{2}^-$	P	0.206 ± 0.004	-22.914 ± 0.540
	$\frac{3}{2}^-$	P	0.207 ± 0.045	-22.850 ± 0.054
	$\frac{5}{2}^+$	D	0.210 ± 0.004	-23.900 ± 0.461
Ξ_{bc}	$\frac{1}{2}^+$	S	0.265 ± 0.009	-12.777 ± 0.501
	$\frac{1}{2}^-$	P	0.289 ± 0.005	-14.832 ± 0.298
	$\frac{3}{2}^-$	P	0.288 ± 0.005	-14.738 ± 0.305
	$\frac{5}{2}^+$	D	0.291 ± 0.004	-15.522 ± 0.215

5 Conclusion

The hypercentral constituent quark model is used to construct the mass spectra of doubly heavy Ξ baryons. Ground states as well as excited state masses are obtained successfully. The mass difference between the light quarks (u and d) is 12 MeV in our model. So, it is obvious that when we move toward the calculations of the excited states the baryon masses would also have a very small mass difference. For the sake of completeness we calculated whole mass spectrum for all six doubly heavy baryon and noticed that it hardly differs less than ≈10 MeV, which can be observed in Tables 1, 2, 3, 4, 5, and 6. The ground state of Ξ_{cc} is experimentally known and while comparing our ground states of the Ξ_{cc}^{++}, Ξ_{cc}^{+} baryons we define the state with parity $J^P = \frac{1}{2}^+$.

We successfully plotted Regge trajectories of present work in both (n, M^2) and (J,M^2) planes and fortunately assigned the quantum number for each cases of six Ξ baryons. The magnetic moments of the ground states are also calculated

Table 8 Fitted parameters α and α_0 are slope and intercept of parent and daughter Regge trajectories

Baryon	Trajectory	α	α_0	α	α_0
Ξ_{ccd}	Parent	0.609 ± 0.0607	-6.694 ± 1.022	0.859 ± 0.042	-10.650 ± 0.648
	1 Daughter	0.726 ± 0.042	-10.197 ± 0.737	0.865 ± 0.016	-12.649 ± 0.288
	2 Daughter	0.751 ± 0.041	-12.351 ± 0.799	0.893 ± 0.019	-15.069 ± 0.368
	3 Daughter	0.795 ± 0.0319	-14.935 ± 0.680	0.907 ± 0.014	-17.292 ± 0.298
Ξ_{bbd}	Parent	0.304 ± 0.0156	-31.433 ± 1.733	0.322 ± 0.013	-33.518 ± 1.481
	1 Daughter	0.347 ± 0.0112	-38.094 ± 1.312	0.354 ± 0.005	-38.947 ± 0.561
	2 Daughter	0.365 ± 0.007	-41.989 ± 0.945	0.377 ± 0.003	-43.419 ± 0.344
	3 Daughter	0.391 ± 0.006	-46.983 ± 0.772	0.402 ± 0.003	-48.306 ± 0.326

Table 9 Magnetic moment (in nuclear magnetons) of J^P $\frac{1}{2}^+$ and $\frac{3}{2}^+$ doubly heavy baryons

Baryons	Magnetic moment	Our	[31]	[44]	[45,46]	[47]
Ξ_{cc}^{+}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_d$	0.784	0.859	0.722	0.80	0.785
Ξ_{cc}^{++}	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_u$	0.031	−0.133	0.114	−0.12	−0.208
Ξ_{bb}^{-}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_d$	0.196	0.190	0.086	0.215	0.251
Ξ_{bb}^{0}	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_u$	−0.663	−0.656	−0.432	−0.630	−0.742
Ξ_{bc}^{0}	$\frac{2}{3}\mu_b+\frac{2}{3}\mu_c - \frac{1}{3}\mu_d$	0.354	0.476	0.068	0.480	0.518
Ξ_{bc}^{+}	$\frac{2}{3}\mu_b+\frac{2}{3}\mu_c - \frac{1}{3}\mu_u$	−0.204	−0.400	−0.236	−0.369	−0.475
Ξ_{cc}^{+*}	$2\mu_c+\mu_d$	0.068	−0.168	0.163	0.035	−0.311
Ξ_{cc}^{++*}	$2\mu_c+\mu_u$	2.218	2.749	2.001	–	2.670
Ξ_{bb}^{-*}	$2\mu_b+\mu_d$	−1.737	−0.951	−0.652	−1.029	−0.522
Ξ_{bb}^{0*}	$2\mu_b+\mu_u$	−1.607	1.576	0.916	1.507	1.870
Ξ_{bc}^{0*}	$\mu_b+\mu_c+\mu_d$	−0.372	−0.567	−0.257	−0.508	−0.712
Ξ_{bc}^{+*}	$\mu_b+\mu_c+\mu_u$	1.562	2.052	1.414	2.022	2.270

using obtained masses. We can observe that our obtained results are close to other predictions (except Ξ_{cc}^{++}, Ξ_{cc}^{+*}, Ξ_{bb}^{0*} baryons).

This study will definitely help future experiments and other theoretical models to identify the baryonic states from resonances. We would like to extend this model to calculate the mass spectra and other properties of triply heavy baryons.

Acknowledgements A. K. Rai acknowledges the financial support extended by DST, India, under SERB fast track scheme SR/FTP/PS-152/2012. We are very much thankful to Prof. P. C. Vinodkumar and Dr. Kaushal Thakkar for their valuable suggestions throughout the work.

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