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Phase space analysis of some interacting Chaplygin gas models

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Abstract In this paper we discuss a phase space analysis of various interacting Chaplygin gas models in general relativity. Linear and nonlinear sign changeable interactions are considered. For each case appropriate late time attractors of field equations are found. The Chaplygin gas is one of the dark fluids actively considered in modern cosmology due to the fact that it is a joint model of dark energy and dark matter.

1 Introduction

One of the active topics of research in modern cosmology is an accelerated expansion of large scale universe [1–8]. There are various approaches to solve the problem. In the case of general relativity it has been suggested to include dark energy [9]. To solve the problem different modifications of general relativity [10] and other interesting ideas [11–14] (to mention a few) have been appeared in Literature. There is an active research toward to all directions. If we keep general relativity as a main theory of gravity, then to describe dark energy we should assume either an explicit form of the EoS or the form of the energy density. It is not excluded that EoS of dark energy can be a solution of an algebraic or a differential equations [15,16]. The last assumption could open a new window toward a fundamental description of darkness of the universe. One of the models of dark fluid actively studied in modern cosmology is the Chaplygin gas [17,18], which has an EoS of the following form:

$$P_c = A\rho_c - \frac{B}{\rho_c^{\alpha}}, \tag{1}$$

where A, B and α are positive constants and ρ_c it is the energy density of gas. This is an example of the fluid which has an explicit EoS. It is well known that this fluid is a joint model of dark energy and dark matter. Therefore, this is one of the main reasons to continue research on this model, despite some criticism [19] related to this issue. From Eq. (1) the case of $A = 0$ recovers the generalized Chaplygin gas EoS, and $A = 0$ together $\alpha = 1$ recovers the original Chaplygin gas EoS. The best fitted parameters are found to be $A = 0.085$ and $\alpha = 1.724$, while Constitution + CMB + BAO and Union + CMB + BAO results are $A = 0.061\pm0.079$, $\alpha = 0.053\pm0.089$, and $A = 0.110\pm0.097$, $\alpha = 0.089\pm0.099$, respectively [20,21]. Observational constraints on the modified Chaplygin gas model using a Markov Chain Monte Carlo approach showed that $A = 0.00189^{+0.00583}_{-0.00756}$, $\alpha = 0.1079^{+0.3397}_{-0.2539}$ at 1σ level and $A = 0.00189^{+0.00660}_{-0.00915}$ with $\alpha = 0.1079^{+0.4678}_{-0.2911}$ at 2σ level [22]. In recent physical literature different modifications of the EoS of the Chaplygin gas could be found (see for instance [23–27] and references therein). On the other hand, it is well known that in dark energy models the cosmological coincidence problem could rise [28], requiring one to explain why

$$r = \frac{\Omega_{\mathrm{DM}}}{\Omega_{\mathrm{DE}}} = r_0, \tag{2}$$

where r_0 is a constant. To solve this problem an interaction between dark components of the universe has been studied. Active research on interacting cosmological models reveals that interaction can change the sign during the evolution of the universe [29,30]. Before, it was thought that it is not possible. In this paper we will consider cosmological models involving various interactions between Chaplygin gas and dark matter and obtain late time attractors of field equations describing the dynamics of such universe. We consider interactions which could be obtained from the forms of interaction/coupling:

$$Q = q^n(3Hb\rho + \gamma\dot{\rho}) \tag{3}$$

and

$$Q = 3Hbq^n\left(\rho + \frac{\rho_i\rho_j}{\rho}\right), \tag{4}$$

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where b, γ, and n are positive constants, H is the Hubble parameter, q is the deceleration parameter, while ρ can be either the energy density of the effective fluid or the energy density of one of the components. ρ_i is the energy density of one of the components (either dark energy or dark matter). We restrict our attention to the models where $n = 0$ and $n = 1$. Interactions described via Eqs. (3) and (4) for $n = 0$ have fixed sign during the whole evolution of the universe. For $n = 1$, interactions Eqs. (3) and (4) represent sign changeable interactions, which is due to the deceleration parameter q. Moreover, interactions given via Eq. (4) are nonlinear with fixed ($n = 0$) and changeable ($n = 1$) signs, respectively. To study the dynamics of the large scale universe, in the physical literature a very highly accurate approximation is used for the effective darkness of the large scale universe, namely, we consider only a mixture of dark energy and dark matter with

$$\rho_{\rm eff} = \rho_{\rm DM} + \rho_{\rm DE} \tag{5}$$

and

$$P_{\rm eff} = P_{\rm DM} + P_{\rm DE}. \tag{6}$$

Therefore, the assumption as regards dark energy and dark matter does allow us to reconstruct the dynamics of the universe. We will present additional assumptions allowing us to construct our cosmological models in Sect. 2.

The paper is organized as follows: In Sect. 2 we will give the definition of interacting models in modern cosmology and will present basics on phase space analysis to find late time attractor solutions of the field equations of general relativity. In Sect. 3 a phase space analysis is performed, and late time attractors are found and classified according to their cosmological applicability. To save space we present only attractor solutions. Finally, a discussion of the results obtained is summarized in Sect. 4.

2 Interacting models and autonomous system

It is well known that to describe the dynamics of the large scale flat FRW universe we need the following set of equations obtained from the field equations of general relativity:

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G\rho}{3}, \tag{7}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P). \tag{8}$$

We suppose that the cosmological constant $\Lambda = 0$, the gravitational constant G and c are constants with $c = 8\pi G = 1$. One of the approaches to alleviate the cosmological coincidence problem is to consider interacting dark energy models. In modern cosmology an interaction between effective dark fluid components mathematically means [31–53]

$$\dot{\rho}_m + 3H(\rho_m + P_m) = Q \tag{9}$$

and

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + P_\Lambda) = -Q. \tag{10}$$

The form of Q is determined under phenomenological assumptions, mainly, dimensional analysis is used to construct interactions. It is reasonable to consider interactions which could improve previously known results and at the same time will not make the mathematical treatment of the problems complicated. It is widely believed that a deeper understanding of the nature of dark energy and dark matter could give a fundamental explanation of the phenomenological assumptions as regards interaction. It is well known that the phase space of a dynamical system is a space in which all possible states of the system are represented. There is a huge number of articles presenting a phase space analysis of different cosmological models; see Refs. [55–59] to mention a few. To analyze the dynamical system of an interacting Chaplygin gas we set [54]

$$x = \frac{\rho_c}{3H^2}, \tag{11}$$

$$y = \frac{P_c}{3H^2}, \tag{12}$$

$$z = \frac{\rho_m}{3H^2}, \tag{13}$$

and

$$N = \ln a, \tag{14}$$

where a is the scale factor. To have physically reasonable solutions we should have the constraints $0 \le x \le 1$ and $0 \le z \le 1$. At the same time we should remember that x and z according to Eq. (7) should satisfy the following constraint:

$$x + z = 1. \tag{15}$$

In terms of x and y the EoS parameter of the Chaplygin gas reads

$$\omega_c = \frac{y}{x}, \tag{16}$$

while the EoS parameter of the effective fluid reads

$$\omega_{\rm eff} = \frac{P_c}{\rho_c + \rho_m} = y, \tag{17}$$

because dark matter is considered as a pressureless fluid. It is not hard to show that the deceleration parameter q reads

$$q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2}(1 + 3y). \tag{18}$$

From the next section we will start our study taking into account the general algorithm of finding critical points of an autonomous system x' and y', where $'$ is the derivative with respect to N. Solutions of $x' = 0$ and $y' = 0$ should be found first, then the sign of the determinant and trace of the Jacobian matrix of x' and y' will determine the stability of the critical point. It is well known that if the trace of the Jacobian matrix is negative, while the determinant is positive, then the critical point is stable, because the real parts of the eigenvalues are positive. This is in the case of linear stability. On the other hand a stable critical point is an attractor, which is what we are looking for. Therefore, we need to find the range of the model parameters such that we have physically reasonable stable critical points i.e. $0 \leq x_c \leq 1$ and $0 \leq z_c \leq 1$.

3 Phase space analysis

In this section we will find and discuss late time attractors of the field equations for various interacting Chaplygin gas cosmological scenarios. Four different types of interaction are considered and appropriate late time attractors are found analytically. Combining general experience known from the physical literature we impose the following constraints on the parameters of the models:

$$0 < \alpha \leq 1, \tag{19}$$

$$0 \leq b < 1, \tag{20}$$

$$0 \leq \gamma < 1, \tag{21}$$

and

$$A \geq 0 \quad \text{and} \quad B > 0. \tag{22}$$

In our calculations we used a fact that

$$\rho^{\alpha} = \frac{B}{3H^2(Ax - y)}. \tag{23}$$

3.1 Interaction $Q = 3Hb\rho + \gamma\dot{\rho}$

The interaction considered in this subsection is very well known and has been considered intensively. We are interested in obtaining all late time attractors of field equations, when the interaction between dark energy and dark matter has the following form:

$$Q = 3Hb\rho + \gamma\dot{\rho}, \tag{24}$$

where ρ could be either the energy density of the effective fluid or the energy density one of the components of the effective dark fluid. The general form of the interaction given via Eq. (24) for the two fluid universe reads

$$Q = 3Hb(\rho_m + \rho_c) + \gamma(\dot{\rho}_m + \dot{\rho}_c), \tag{25}$$

which allows us to obtain an explicit form of the autonomous system,

$$x' = \frac{dx}{dN} = -3(b - \gamma - y(-1 + x + y)) \tag{26}$$

Table 1 Critical points corresponding to various interaction terms obtained from general form Eq. (3) for $n = 0$. $r = \sqrt{4Ab(1+\gamma) + (b - A + \gamma)^2}$

Q	$S.P.$	x	y
$3Hb\rho_m + \gamma\dot{\rho}_m$	$E.1.1$	1	-1
$3Hb\rho_c + \gamma\dot{\rho}_c$	$E.1.2$	$1/(1+b)$	-1
$3Hb\rho_c + \gamma\dot{\rho}_m$	$E.1.3$	$1/(1+b)$	-1
$3Hb\rho_m + \gamma\dot{\rho}_c$	$E.1.4$	1	-1
$3Hb\rho_m + \gamma\dot{\rho}_c$	$E.1.5$	$\frac{A-b-\gamma-r}{2A(1+\gamma)}$	$-\frac{b+\gamma-A+r}{2(1+\gamma)}$
$3Hb\rho + \gamma\dot{\rho}$	$E.1.6$	$1-b$	-1
$3Hb(\rho_c + \rho_m)$	$E.1.7$	$1-b$	-1

and

$$y' = \frac{dy}{dN} = 3y(1+y) - 3\frac{(-\alpha y + A(1+\alpha)x)(b + x + y - \gamma(1+y))}{x}. \tag{27}$$

To find stable critical points (attractors), we need to solve $x' = 0$ and $y' = 0$ equations and determine the signs of the eigenvalues of the appropriate Jacobian matrix of x' and y'. Table 1 represents late time attractors for different interactions obtained from Eq. (24). Below we will discuss cosmological consequences for each late time attractor. $E.1.1$ is a stable attractor for α, b, γ, and A satisfying Eqs. (19)–(22). It represents a Chaplygin gas dominated universe, where $\omega_c = -1$, $\omega_{\text{eff}} = -1$ and $q = -1$ i.e. the ΛCDM model is recovered. The stable attractor $E.1.2$ is a scaling solution since

$$r = \frac{\Omega_m}{\Omega_c} = b, \tag{28}$$

when $b \neq 0$. $\omega_c = -1 - b$ means that the phantom phase is in order in this case. In this case the cosmological coincidence problem is solved due to the interaction. Attractor $E.1.3$ is of the same nature as $E.1.2$, while attractor $E.1.4$ has the same nature as $E.1.1$. The late time attractor $E.1.5$ has a completely different nature compared to the previous ones, however, in this case an accelerated expansion is not possible. The late time attractors $E.1.6$ and $E.1.7$ are scaling solutions ($b \neq 0$) because

$$r = \frac{\Omega_m}{\Omega_c} = \frac{b}{1-b}. \tag{29}$$

In this case according to Eq. (18) $q = -1$, while according to Eq. (16)

$$\omega_c = -\frac{1}{1-b}, \tag{30}$$

and the EoS parameter of the effective fluid according to Eq. (17) is equal to -1. In Fig. 1, as an example, we have

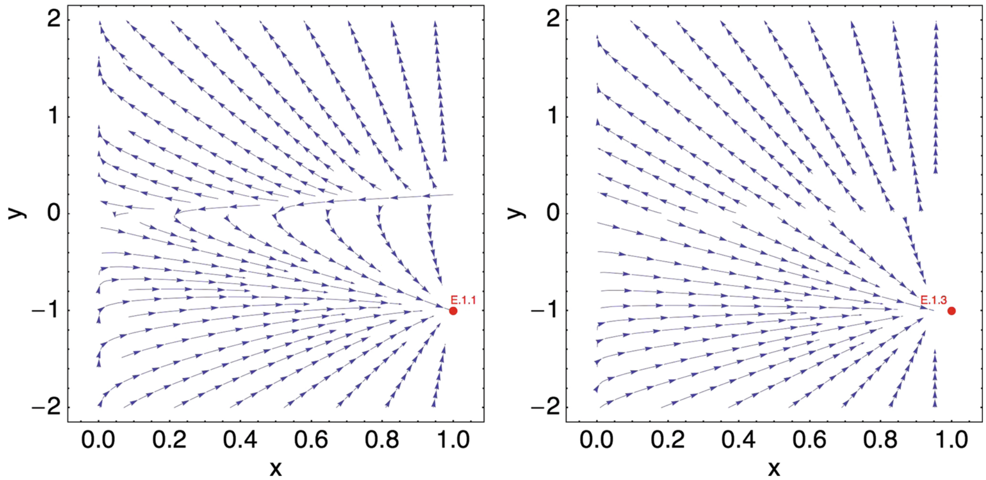

Fig. 1 Phase space portraits for the models with $Q = 3Hb\rho_m + \gamma\dot{\rho}_m$ and $Q = 3Hb\rho_c + \gamma\dot{\rho}_m$

illustrated the phase space portraits corresponding to $E.1.1$ and $E.1.3$.

3.2 Interaction $Q = q(3Hb\rho + \gamma\dot{\rho})$

The cosmological models considered in this subsection contain interactions between dark energy and dark matter, which are particular examples of a sign changeable interaction of the following form (two fluid case):

$$Q = q(3Hb(\rho_c + \rho_m) + \gamma(\dot{\rho}_c + \dot{\rho}_c)). \tag{31}$$

Sign changeable interactions of this form use the deceleration parameter q to have the desirable effect. For the general case given via Eq. (31), the critical points are solutions of the following two equations:

$$x' = \frac{3}{2}(-b(1+3y) + \gamma + y(-2 + 2x + 4\gamma + 3\gamma y)) \tag{32}$$

and

$$y' = 3y(1+y) + \frac{3}{2x}(-\alpha y + A(1+\alpha)x)(b(1+3y) + 2(x+y) - (1+y)(1+3y)\gamma). \tag{33}$$

Late time attractors for different interactions obtained from Eq. (31) are collected in Table 2. Our analysis shows that late time attractors possibly are obtained if $b = 0$ (Fig. 2). This means that only one type of late time attractor is possible so as to obtain i.e. $x = 1$ and $y = -1$. Such an attractor describes a Chaplygin gas dominated universe.

Table 2 Critical points corresponding to various interaction terms obtained from general form Eq. (3) for $n = 1$

Q	$S.P.$	x	y
$q(3Hb\rho_c + \gamma\dot{\rho}_c)$	$E.2.1$	$1/(1-b)$	-1
$\gamma q(\dot{\rho}_c + \dot{\rho}_m)$	$E.2.2$	1	-1
$bq(3H(\rho_c + \rho_m) + \dot{\rho}_m + \dot{\rho}_c)$	$E.2.3$	$1+b$	-1
$3Hbq(\rho_m + \rho_c)$	$E.2.4$	$1+b$	-1
$3Hbq\rho_c$	$E.2.5$	$1/(1-b)$	-1
$3Hbq\rho_m$	$E.2.6$	1	-1

3.3 Interaction $Q = 3Hb\left(\rho + \frac{\rho_i\rho_j}{\rho}\right)$

In the physical literature there is an active discussion on nonlinear interactions (see for instance [60,61]). Nonlinear interactions considered in this paper have a phenomenological origin and are based on dimensional analysis. The general form of the nonlinear interaction considered by us has the following form (for two fluid models):

$$Q = 3Hb\left(\rho_c + \rho_m + \frac{\rho_{ij}}{\rho_c + \rho_m}\right), \tag{34}$$

where ρ_{ij} is the product of ρ_c and ρ_m and if $i = j = m$ we have ρ_m^2 etc. In Table 3 we summarize late time attractors when interaction between dark energy and dark matter can be described via Eq. (4) for $n = 0$. We have considered the cases, when critical points are possibly obtained analytically. $E.3.1$ is a scaling late time attractor for $0 < \alpha \leq 1$, $0 < b < 1$ and $A \geq 0$, because in this case

$$\frac{\Omega_m}{\Omega_c} = \frac{1}{2}\left(b - 1 + \sqrt{1 + b(6+b)}\right). \tag{35}$$

It is not hard to see that $\omega_{\text{eff}} = -1$, $q = -1$, and

$$\omega_c = \frac{2b}{1 + b - \sqrt{1 + b(6+b)}}. \tag{36}$$

Another scaling late time attractor when $0 < b < 1/2$, $0 < \alpha \leq 1$, and $A \geq 0$, might be obtained, if we consider the following interaction:

$$Q = 3Hb\left(\rho_c + \rho_m + \frac{\rho_m^2}{\rho_c + \rho_m}\right). \tag{37}$$

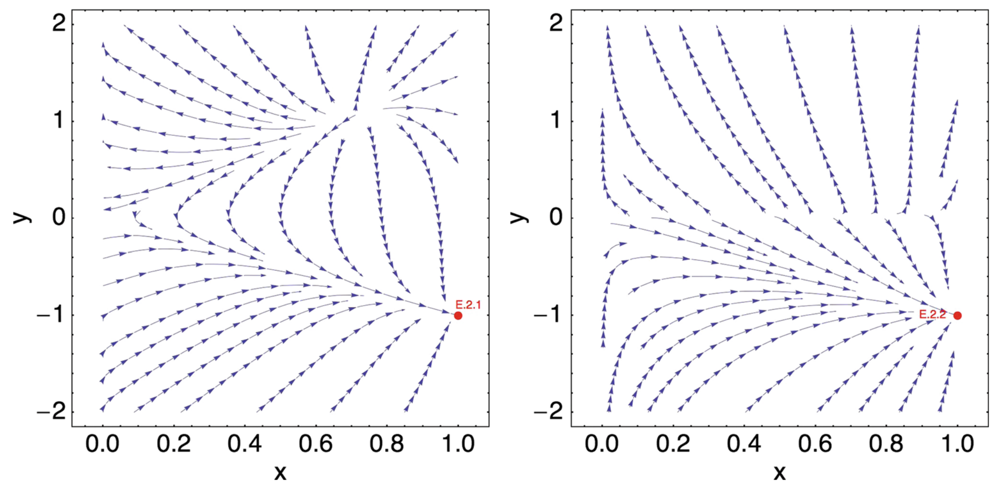

Fig. 2 Phase space portraits for the models with $Q = q(3Hb\rho_c + \gamma\dot{\rho}_c)$ and $Q = \gamma q(\dot{\rho}_c + \dot{\rho}_m)$

Table 3 Critical points corresponding to various interaction terms obtained from general form Eq. (4) for $n = 0$, where $r_1 = \sqrt{1+b(6+b)}$, $r_2 = \sqrt{1-4b^2}$ and $r_3 = \sqrt{1-4(b-1)b}$

Q	$S.P.$	x	y
$3Hb(\rho_c + \rho_c^2/(\rho_c+\rho_m))$	$E.3.1$	$(-1-b+r_1)/2b$	-1
$3Hb(\rho_c + \rho_m + \rho_m^2/(\rho_c+\rho_m))$	$E.3.2$	$(-1+2b+r_2)/2b$	-1
$3Hb(\rho_c + \rho_m + \rho_c^2/(\rho_c+\rho_m))$	$E.3.3$	$(r_3-1)/3b$	-1
$3Hb(\rho_m + \rho_m^2/(\rho_c+\rho_m))$	$E.3.4$	1	-1

In Table 3 it corresponds to $E.3.2$, for which

$$\frac{\Omega_m}{\Omega_c} = -\frac{1}{2} + \frac{\sqrt{1-4b^2}}{2(1-2b)}. \tag{38}$$

It is not hard to see that

$$\omega_c = -\frac{2b}{2b-1+\sqrt{1-4b^2}}. \tag{39}$$

Another scaling late time attractor is $E.3.3$, when $0 < \alpha \leq 1$, $0 < b < 1$, and $A \geq 0$ with

$$\frac{\Omega_m}{\Omega_c} = \frac{2b-1+\sqrt{1-4(b-1)b}}{2(b-1)}, \tag{40}$$

$$\omega_c = -\frac{2b}{\sqrt{1-4(b-1)b}-1}, \tag{41}$$

and $q = -1$, $\omega_{\text{eff}} = -1$. Finally, if we consider an interaction of the following form:

$$Q = 3Hb\left(\rho_m + \frac{\rho_m^2}{\rho_c+\rho_m}\right), \tag{42}$$

then the late time attractor $E.3.4$ could be found, which describes a Chaplygin gas dominated universe with $\omega_c = -1$. In all cases an accelerated expansion is on face. Consideration of nonlinear interactions gives qualitatively four different scaling solutions (for considered interactions). Therefore, future consideration of additional possibilities for nonlinear interactions in modern cosmology will enrich our experience.

In Fig. 3, as an example, we have illustrated phase space portraits corresponding to $E.3.1$ and $E.3.3$.

3.4 Interaction $Q = 3Hbq\left(\rho + \frac{\rho_i\rho_j}{\rho}\right)$

Consideration of nonlinear sign changeable interactions is another phenomenological assumption of this work. Mainly motivated by the results obtained in Sect. 3.3, we decided to consider the following possibility:

$$Q = 3Hbq\left(\rho_c + \rho_m + \frac{\rho_{ij}}{\rho_c+\rho_m}\right). \tag{43}$$

We followed the recipes of the previous subsections to find all late time attractors for this case. We have considered two particular forms of interaction,

$$Q = 3Hbq\left(\rho_c + \frac{\rho_c^2}{\rho_c+\rho_m}\right) \tag{44}$$

and

$$Q = 3Hbq\left(\rho_m + \frac{\rho_m^2}{\rho_c+\rho_m}\right), \tag{45}$$

and we found that late time attractors do not exist in these cosmological models. We are not closing our study of this type of sign changeable interactions and we will come back to them in our next work.

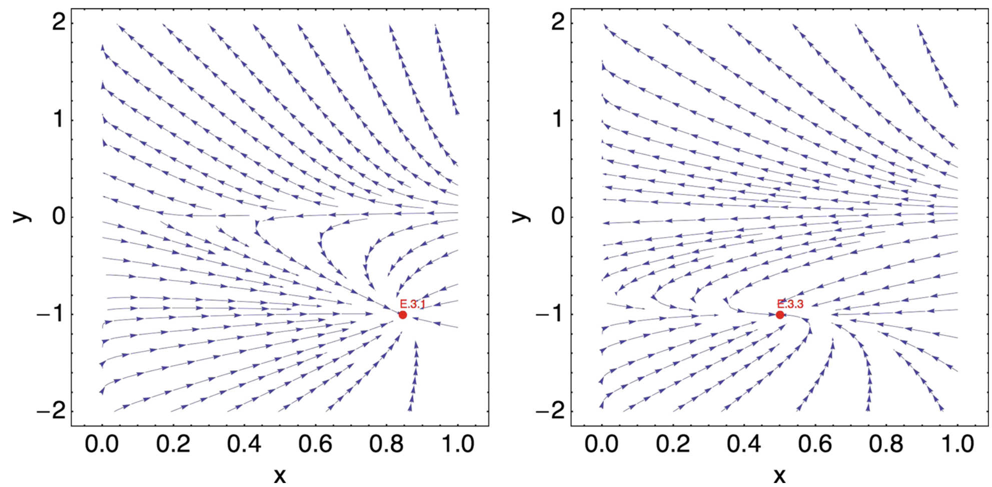

Fig. 3 Phase space portraits for the models with interactions $3Hb(\rho_c + \rho_c^2/(\rho_c + \rho_m))$ and $3Hb(\rho_c + \rho_m + \rho_c^2/(\rho_c + \rho_m))$

4 Discussion

The subject of our interest has been a phase space analysis of some interacting Chaplygin gas cosmological models, where dark matter is a pressureless fluid. In this paper we have another goal as well—we have constructed some nonlinear and sign changeable nonlinear interactions and studied appropriate cosmological models from this perspective. The new nonlinear interactions considered have a phenomenological origin. To perform the phase space analysis we have adopted a new set of parameters allowing us to obtain an appropriate autonomous system. We have restricted ourselves to the cases when critical points can be found analytically. To find physically reasonable solutions, restrictions from cosmological and astrophysical studies known in the physical literature have been imposed as well. Our study showed that in the case of some sign changeable nonlinear interactions, constructed in this paper, stable critical points and late time attractors are missing. However, a study of the models when $Q = q(3Hb\rho + \gamma\dot{\rho})$ reveals that only one type late time attractor is possible. It has been found that it is a late time attractor with $x = 1$ and $y = -1$ describing Chaplygin gas dominated phase with $\omega_c = -1$ and $q = -1$. On the other hand, the situation is completely different if we consider fixed sign interactions. In the case of nonlinear interactions (among the considered ones) four different types of late time scaling attractors were found, while for the models with $Q = 3Hb\rho + \gamma\dot{\rho}$ only two late time scaling solutions of a different physical nature were found. One of them describes a large scale universe in phantom regime. General cases of interacting models (particularly nonlinear interacting models) which cannot be analyzed analytically could be studied numerically to investigate new possibilities. It is not excluded that some interesting results different from the ones presented could appear. We hope to report some new results on this issue in our next articles.

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