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Final state interactions in $K \to \pi\pi$ decays: $\Delta I = 1/2$ rule vs. ε'/ε

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Abstract Dispersive effects from strong $\pi\pi$ rescattering in the final state interaction (FSI) of weak $K \to \pi\pi$ decays are revisited with the goal to have a global view on their *relative* importance for the $\Delta I = 1/2$ rule and the ratio ε'/ε in the standard model (SM). We point out that this goal cannot be reached within a pure effective (meson) field approach like chiral perturbation theory in which the dominant current–current operators governing the $\Delta I = 1/2$ rule and the dominant density–density (four-quark) operators governing ε'/ε cannot be disentangled from each other. But in the context of a dual QCD approach, which includes both long-distance dynamics and the UV completion, that is, QCD at short-distance scales, such a distinction is possible. We find then that beyond the strict large N limit, N being the number of colours, FSIs are likely to be important for the $\Delta I = 1/2$ rule but much less relevant for ε'/ε. The latter finding diminishes significantly hopes that improved calculations of ε'/ε would bring its SM prediction to agree with the experimental data, opening thereby an arena for important new physics contributions to this ratio.

1 Introduction

Among the most important observables in flavour physics are the ratio of $K \to \pi\pi$ isospin amplitudes $\mathrm{Re}A_0/\mathrm{Re}A_2$ and ε'/ε. The first ratio

$$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} = 22.4, \tag{1}$$

expresses the so-called $\Delta I = 1/2$ rule [1,2] in $K \to \pi\pi$ decays. On the other hand ε'/ε, measured by the NA48 [3] and KTeV [4,5] collaborations to be

$$(\varepsilon'/\varepsilon)_{\mathrm{exp}} = (16.6 \pm 2.3) \times 10^{-4}, \tag{2}$$

expresses CP-violation in $K \to \pi\pi$ decays. In the standard model (SM) the amplitudes $\mathrm{Re}A_{0,2}$ are mostly governed by the $Q_{1,2}$ current–current operators and ε'/ε by the QCD penguin Q_6 and electroweak penguin Q_8 density–density operators. The most recent result for the $\Delta I = 1/2$ rule from the dual approach to QCD reads [6]

$$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)_{\mathrm{dual\ QCD}} = 16.0 \pm 1.5, \tag{3}$$

while the corresponding result from the RBC–UKQCD collaboration is [7]

$$\left(\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2}\right)_{\mathrm{lattice\ QCD}} = 31.0 \pm 11.1. \tag{4}$$

Both results signal that $\mathrm{Re}A_0$ is strongly enhanced over $\mathrm{Re}A_2$ but there is a visible deficit in (3) when compared with (1), while the first lattice QCD result is still rather uncertain.

The present status of ε'/ε in the SM can be summarized as follows. The RBC–UKQCD lattice collaboration calculating the hadronic matrix elements of all operators but not including isospin breaking effects finds [7,8]

$$(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (1.38 \pm 6.90) \times 10^{-4}, \quad (\mathrm{RBC-UKQCD}). \tag{5}$$

Using the hadronic matrix elements of QCD- and EW-penguin $(V-A)\otimes(V+A)$ operators from RBC–UKQCD lattice collaboration but extracting the matrix elements of penguin $(V-A)\otimes(V-A)$ operators from the CP-conserving $K \to \pi\pi$ amplitudes and including isospin breaking effects one finds [9]

$$(\varepsilon'/\varepsilon)_{\mathrm{SM}} = (1.9 \pm 4.5) \times 10^{-4}, \quad (\mathrm{BGJJ}). \tag{6}$$

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A new result in [10],

$$(\varepsilon'/\varepsilon)_{\rm SM} = (1.1 \pm 5.1) \times 10^{-4} \quad ({\rm KNT}), \tag{7}$$

confirms the findings in (5) and (6) that the SM result for ε'/ε is significantly below its experimental value in (2).

While these results, based on the hadronic matrix elements from the RBC–UKQCD lattice collaboration, suggest some evidence for the presence of new physics (NP) in hadronic K decays and favour NP models that are able to enhance ε'/ε, the large uncertainties in the hadronic matrix elements in question do not yet preclude that eventually the SM will agree with data. In this context the upper bounds on the matrix elements of the dominant penguin operators from the large N dual QCD approach [11] are important and allow us to derive an upper bound on ε'/ε:

$$(\varepsilon'/\varepsilon)_{\rm SM} \le (8.6 \pm 3.2) \times 10^{-4}, \quad ({\rm BG}). \tag{8}$$

Moreover, taking into account lattice results on the matrix elements of electroweak penguin operators ($B_8^{(3/2)}$) that are better known than those of QCD penguin operators ($B_6^{(1/2)}$) one finds the values of ε'/ε to be significantly below this bound.

While the dual QCD approach allows one to understand the suppression of ε'/ε in (5)–(7) analytically, it does not yet properly include final state interactions (FSIs). The question then arises whether these effects could improve the status of $\Delta I = 1/2$ rule and of ε'/ε bringing the theory in both cases closer to data. In fact the chiral perturbation theory (ChPT) practitioners, already long time ago, put forward the idea that both the amplitude ${\rm Re}A_0$, governed by the current–current operator $Q_2 - Q_1$ and the Q_6 contribution to the ratio ε'/ε could be enhanced significantly through FSI in a correlated manner [12–19]. The goal of this letter is to investigate whether this claim is really justified.

Before entering the details, let us make the following important observation that underlines the main points made in our paper. The QCD penguin operator Q_6, generated by short-distance (SD) evolution from M_W down to scales $\mathcal{O}(1\,{\rm GeV})$ of the *current–current* four-quark operator $(Q_2 - Q_1)$, is unambiguously identified as a *density–density* four-quark operator [6,20]. However, such a distinction between $(Q_2 - Q_1)$ and Q_6 is far from being evident during the further long-distance (LD) evolution below the critical 1 GeV scale of QCD [6,21], though mandatory to consistently identify the strong FSI effects on the corresponding weak hadronic matrix elements.

2 Weak hadronic matrix elements

In the standard ChPT approach based on the power counting in meson momenta, the weak K decay amplitude for the dominant $\Delta I = 1/2$ channel reads [22]

$$A_0 = \langle \pi\pi(I=0)|\, G_8[\partial_\mu U \partial^\mu U^+]_{ds}\, |K\rangle, \quad \text{at} \quad \mathcal{O}(p^2) \tag{9}$$

with $U(\pi)$, a unitary matrix transforming as $(3_L, 3_R^*)$ under global $U(3)_L \otimes U(3)_R$ transformations. Consequently, in this phenomenological approach the four-quark operators $(Q_2 - Q_1)$ and Q_6 contributing to A_0 are somehow merged into a single octet one, at least in the isospin limit [23]. As a result, the corresponding current–current operator cannot be disentangled any more from the density–density operator. In the absence of any UV completion for this effective theory, their respective contributions to the A_0 decay amplitude (9) are encoded in the unique complex coupling G_8. Remarkably, this apparent merging of a priori quite different $\mathcal{O}(p^2)$ operators can be seen at work once fundamental properties of QCD are eventually taken into account.

First of all, in the rather efficient large N limit, N being the number of colours [24–26], both $\Delta S = 1$ bosonized current–current [27] and density–density [28] operators factorize and reduce to form indeed the single octet operator given in (9). Fully exploiting the unitarity of the $U(\pi)$ matrix, one finds, respectively,

$$(Q_2 - Q_1) \propto [\partial_\mu U U^+]_{dq} [\partial^\mu U U^+]_{qs} = -[\partial_\mu U \partial^\mu U^+]_{ds} \quad \text{at} \quad \mathcal{O}(p^2, 0), \tag{10}$$

$$Q_6 \propto \left[U - \frac{1}{\Lambda_\chi^2} \partial_\alpha \partial^\alpha U \right]_{dq} \left[U^+ - \frac{1}{\Lambda_\chi^2} \partial_\beta \partial^\beta U^+ \right]_{qs} = \frac{2}{\Lambda_\chi^2} \left[\partial_\mu U \partial^\mu U^+ \right]_{ds} \quad \text{at} \quad \mathcal{O}(p^2, 0) \tag{11}$$

with Λ_χ a chiral breaking scale fixed by the F_K/F_π ratio of the pseudoscalar decay constants [28,29],

$$\Lambda_\chi^2 = F_\pi \frac{m_K^2 - m_\pi^2}{F_K - F_\pi}. \tag{12}$$

The "0" in $(p^2, 0)$ indicates the strict large N limit: $1/N = 0$.

Secondly, in a dual QCD approach going beyond this strict large N factorization limit in a coherent way [6], analytical tools allow us to keep distinguishing $(Q_2 - Q_1)$ from Q_6 operator even at the hadronic level through a matching of the slow SD quark–gluon evolution above 1 GeV [30] with a fast LD meson evolution below 1 GeV [31]. Within such a dual frame based on a consistent $1/N$ expansion in the strong coupling α_s and $1/F_\pi^2$, the hadronic matrix elements of the penguin operator Q_6 in question turn out to lie *below* its large

N value inferred from (11) (and conventionally corresponding to $B_6^{(1/2)} = 1$), namely [11]

$$\langle \pi\pi(I=0)|Q_6|K\rangle_{\text{dual QCD}} \propto B_6^{(1/2)} \\ = 1 - \mathcal{O}(\frac{1}{N}) < 1, \quad \text{at} \quad \mathcal{O}(p^2, 0) + \mathcal{O}(p^0, 1/N). \quad (13)$$

Let us emphasize that the negative sign of the $1/N$ loop correction induced by the zero-derivative operator in (11) is in agreement with the SD evolution of $B_6^{(1/2)}$ parameter analyzed in [32]. This then implies the result in (8), i.e., the 2σ tension when confronted with the measured CP-violating parameter [9,11]. Yet, the question of $1/N$-suppressed strong FSI effects on such a hadronic matrix element may be raised at this point.

3 Strong final state interactions

3.1 Chiral perturbation theory and beyond

In ChPT, strong phase shifts are zero in the leading-order approximation. In this analytical approach a pion loop should be appended to any local weak $K \to \pi\pi$ transition in order to incorporate the strong $\pi\pi \to \pi\pi$ rescattering effects and, in particular, non-zero FSI phase shifts. Following the well-known Cutkosky cutting rule, this effective bubble triggers some ($1/N$-suppressed) absorptive part whenever the two mesons in the loop can be taken on-shell (see Fig. 1). In the field theory, strong phases resulting from these final rescatterings are factorized in the corresponding isospin amplitudes, so that one can use the parametrization

$$A[K \to (\pi\pi)_I] \equiv A_I \exp(i\delta_I) \quad (I = 0, 2). \quad (14)$$

In the limit of CP conservation, the amplitudes A_I are real and positive by definition. They become complex quantities in the presence of CP violation. The measured δ_0 angle being rather large compared to δ_2, here one thus expects non-negligible higher-order dispersive corrections to A_0 since real and imaginary parts resulting from pion loops are necessarily linked by analyticity and unitarity constraints.

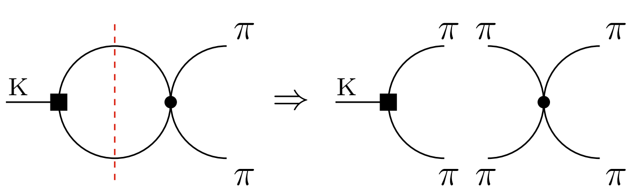

Fig. 1 Strong (•) FSI effect on weak (■) hadronic matrix elements. The Cutkosky cut (- - -) tells us to put internal mesons on the mass-shell to consistently identify any $1/N$-suppressed absorptive part of the Feynman amplitude induced by the $Q_{1,2}$ and $Q_{6,8}$ operators

Going now beyond ChPT (BChPT), one might then advocate [12–19] that an overall dispersive factor $\mathcal{R}_0 \approx \exp(1/N) > 1$ resulting from the all-order resummation of pion loops only should be applied to the weak decay amplitude in (9) and, in particular, to its indistinguishable penguin component. Doing such an exponential rescaling in the strict factorization limit (11) for the Q_6 operator (i.e., for $B_6^{(1/2)} = 1$) to avoid any possible $1/N$ double counting, one would end up this time with a QCD penguin hadronic matrix element well *above* its large N value [12–19],

$$\langle \pi\pi(I=0)|Q_6|K\rangle_{\text{BChPT}} \propto B_6^{(1/2)}\mathcal{R}_0 \\ = 1 + \mathcal{O}(\frac{1}{N}) > 1, \quad \text{at} \quad \mathcal{O}(p^2, 0) + \mathcal{O}(p^2, 1/N). \quad (15)$$

This would imply a better agreement with the measured value of ε'/ε in (2) whenever the $\Delta I = 1/2$ rule (1) is assumed to begin with. However, resumming only part of the higher order ChPT corrections into a simple dispersive factor is well known to be dangerous. Moreover, one should keep in mind that ε' in itself is proportional to the imaginary part of ratio A_2/A_0:

$$\varepsilon' = \frac{i}{\sqrt{2}}\text{Im}\left(\frac{A_2}{A_0}\right)\exp(i(\delta_2 - \delta_0)). \quad (16)$$

Taken as such without using (1), any overall increase of A_0 (and decrease of A_2) as proposed in [12–19] would then imply a decrease of ε'/ε.

But which bound on the $B_6^{(1/2)}$ should one trust, the upper one (13) from dual QCD or the lower one (15) from chiral perturbation supplemented by a large N limit?

3.2 Dual QCD approach

In the dual QCD approach [6], the $\mathcal{O}(p^2, 1/N)$ bubble correction generated by the current–current operators $Q_{1,2}$ does also require some FSI dispersive rescaling. Indeed, the associated $\mathcal{O}(p^2, 0)$ on-shell tree-level amplitude corresponding to the right diagram in Fig. 1 and computed from (10) is proportional to the SU(3)-breaking factor $(m_K^2 - m_\pi^2)$ and thus non-vanishing. However, the common $\mathcal{O}(p^2, 0)$ result in (10) and (11) of a strict large N factorization does not necessarily imply that the $1/N$-suppressed FSI effects on $(Q_2 - Q_1)$ and Q_6 matrix elements are identical.

In fact, the (density–density) operator Q_6 can also generate the chiral octet operator $[\partial_\mu U \partial^\mu U^+]_{ds}$ through its zero-derivative term $[U]_{dq}[U^+]_{qs}$ in (11). In general, the factorizable $1/N$ corrections to this term should exactly cancel the non-factorizable ones in order to preserve the unitarity of the $U(\pi)$ matrix. Yet, in our dual QCD approach these factorizable $1/N$ corrections to the bosonized $\bar{q}q$ densities are already included in the running of quark masses since the QCD mass terms $\bar{q}_L m q_R + h.c.$ are scale independent. As a consequence, a non-zero $1/N$ contribution survives even after contracting the q and q' flavour indices in the following non-factorizable LD evolution [11] from scale $\Lambda = \mathcal{O}(1\,\mathrm{GeV})$ to scale $M = \mathcal{O}(m_K)$:

$$U^{dq}U^{\dagger q's}(\Lambda) \to U^{dq}U^{\dagger q's}(M) - \frac{\ln(\Lambda^2/M^2)}{(4\pi F_\pi)^2}(\partial^\mu U \partial_\mu U^\dagger)^{ds}\delta^{qq'}. \quad (17)$$

This is the genuine $\mathcal{O}(p^0, 1/N)$ one-loop correction to the Q_6 hadronic matrix element [11]

$$B_6^{(1/2)} = 1 - \frac{3}{2}\left(\frac{\Lambda_\chi^2}{(4\pi F_\pi)^2}\right)\ln\left(\frac{\Lambda^2}{M^2}\right), \quad (18)$$

with $\Lambda_\chi^2 (\approx 1\,\mathrm{GeV}^2)$, given in (12), a sizeable momentum-independent substitute for $p^2 (\approx m_K^2)$ as already outlined in (13) but obviously missing in (15). Evidently this $\mathcal{O}(p^0, 1/N)$ contribution is absent in the matrix element of the two-derivative operator $(Q_2 - Q_1)$ in (10) implying that $1/N$-suppressed loop effects on Q_6 and $(Q_2 - Q_1)$ matrix elements are not identical, in contrast to the claim made in [12–19].

In any analytical approach relying on some (truncated) expansion, what is called *FSI effects* might be a misnomer with respect to the well-defined Watson factorization theorem (14) in field theory. In this context one should carefully distinguish between *dispersive* and *absorptive* contributions from the $1/N$-suppressed loop diagrams in Fig. 1.

- The operator $[U]_{dq}[U^+]_{qs}$ contributes to the left loop diagram with *off-shell* intermediate mesons and leads to the non-zero $\mathcal{O}(p^0, 1/N)$ *dispersive* term in (18) calculated in [11]. This term competes with the $\mathcal{O}(p^2, 0)$ tree-level value of the $B_6^{(1/2)}$ parameter normalized to one as possibly foreseen from a simultaneous expansion in $p^2 = \mathcal{O}(\delta)$ and $1/N = \mathcal{O}(\delta)$, the joint chiral and colour counting already invoked elsewhere [33] for strong interaction physics. Being of the same order in δ as the leading term but having opposite sign, it is the main origin of the suppression of $B_6^{(1/2)}$ and thus of ε'/ε.
- Most importantly, following the Cutkosky cutting rule, the operator $[U]_{dq}[U^+]_{qs}$ does not imply any *absorptive* part since, once again, the associated tree-level amplitude with on-shell pions in the right loop diagram of Fig. 1 identically vanishes due to the unitarity property of the $U(\pi)$ matrix for the light pseudoscalars,

$$\langle \pi\pi(I=0)|[U]_{dq}[U^+]_{qs}|K\rangle_{\text{tree-level}} = 0\,. \quad (19)$$

  Consequently, the leading pion-loop contribution to the Q_6 matrix elements is purely dispersive such that $B_6^{(1/2)}$ is under control. In contrast, the leading pion-loop contribution to the $Q_{1,2}$ matrix elements is *both* dispersive *and* absorptive.

This disparity between the pion FSI effects on the matrix elements of $(Q_2 - Q_1)$ and Q_6 operators is the main result of our paper, which cannot be highlighted within an effective (meson) field approach like chiral perturbation theory where these two operators are indistinguishable from the beginning.

The absence of on-shell rescattering impact on $B_6^{(1/2)}$ at $\mathcal{O}(p^0, 1/N)$ gives us the confidence in the bound in (13), which is crucial for the suppression of ε'/ε below the data. This absence has been checked explicitly in [34] through a full one-loop calculation of both factorizable and non-factorizable LD contributions that are generated by the density–density penguin operator Q_6. In our dual QCD picture, the absorptive part of the former cannot be included in the running of the quark masses, while the absorptive part of the latter cannot be matched with SD evolution. So, they have to cancel each other, supporting in that manner the leading upper bound (13) at the expense of the subleading lower bound (15).

In fact, any attempt to include the first impact of strong FSI on $B_6^{(1/2)}$ would require an expansion beyond the consistent $\mathcal{O}(\delta)$ bound (13). Unfortunately a full $\mathcal{O}(\delta^2)$ estimate of the Q_6 matrix element, with further $\mathcal{O}(p^2, 1/N)$ as well as genuine $\mathcal{O}(p^4, 0)$ and $\mathcal{O}(p^0, 1/N^2)$ corrections in (15), is a task beyond the authors present skills. At best, we can quote the following *partial* results:

$$\delta B_6^{(1/2)}(p^4, 0) \supset \frac{(m_K^2 + m_\pi^2)}{2\Lambda_\chi^2} \approx +0.15 \quad (20)$$

from (11) alone and

$$\delta B_6^{(1/2)}(p^0, 1/N^2) \supset -\frac{4}{N}\left(\frac{m_0}{4\pi F_\pi}\right)^2 \approx -0.35 \quad (21)$$

from the anomalous effective Lagrangian that solves the so-called $U(1)_A$ problem [11]. These versatile numbers encourage us to stick to a consistent $\mathcal{O}(\delta)$ calculation for $B_6^{(1/2)}$ rather than to venture in an unreliable $\mathcal{O}(\delta^2)$ estimate of this hadronic parameter. In other words, our upper bound (13) on $B_6^{(1/2)}$ follows from the above δ expansion under the assumption

$$\mathcal{O}(\delta^2) < \mathcal{O}(\delta), \quad (22)$$

while the lower bound (15) would require large $\mathcal{O}(\delta^2)$ corrections to be true. After all, the pseudoscalar mass spectrum is reproduced within 15% on the sole basis of the effective Lagrangian for strong interactions at $\mathcal{O}(\delta)$ [35], with the axial

$U(1)$ breaking scale $m_0 = \mathcal{O}(0.85\,\text{GeV})$ associated to a large η' mass, its $\mathcal{O}(p^0, 1/N)$ component.

4 Comments and conclusion

The dispersive rescaling factors $R_0 \approx 1.55$ and $R_2 \approx 0.92$, corresponding, respectively, to $\delta_0 \gg 0$ and $\delta_2 < 0$, have been extracted from an all-order resummation of the 1/N-suppressed FSI in [16,19]. Naively applied to the hadronic matrix elements of the free $|\Delta S| = 1$ weak Hamiltonian to avoid, once again, any possible double counting, they would imply the following $\Delta I = 1/2$ enhancement:

$$\frac{\text{Re}A_0}{\text{Re}A_2} = \sqrt{2} \times \frac{\mathcal{R}_0}{\mathcal{R}_2} \approx 2.4. \tag{23}$$

In this rather peculiar large N limit indeed, only the Q_2 operator with its two charged currents survives such that the $K^0 \rightarrow \pi^0\pi^0$ neutral channel is purely induced by $\pi^+\pi^- \rightarrow \pi^0\pi^0$ rescattering. Further $\mathcal{O}(1/N)$ corrections from strong interactions, namely LD and SD evolutions [6], are obviously required to understand the measured value in (1). Whatever the approach adopted these corrections must also be large, even if formally $\mathcal{O}(p^2, 1/N)$, and properly combined with the FSI LD one given in (23).

Similarly, the FSI rescaling factors $\mathcal{R}_{0,2}$ applied to a strict large N value of the QCD and electroweak penguin hadronic matrix elements (i.e., the one obtained for $B_6^{(1/2)} = 1$ and $B_8^{(3/2)} = 1$), namely

$$B_6^{(1/2)} = 1 \times \mathcal{R}_0 \approx 1.55, \quad B_8^{(3/2)} = 1 \times \mathcal{R}_2 \approx 0.92 \tag{24}$$

would also miss strong $\mathcal{O}(p^0)$ and mild $\mathcal{O}(p^2)$ $1/N$ contributions, respectively. Again, such a disparity between FSI effects on $B_6^{(1/2)}$ and $B_8^{(3/2)}$ is due to the fact that

$$\langle \pi\pi(I=2)|[U]_{dq} e_q [U^+]_{qs}|K\rangle_{\text{tree-level}} \neq 0 \tag{25}$$

instead of (19) when the quark electric charges e_q are introduced.

Relying now more specifically on a simultaneous expansion in $p^2 = \mathcal{O}(\delta)$ and $1/N = \mathcal{O}(\delta)$ in the dual QCD approach involving both SD and LD operator evolutions at the one-loop level, we draw the following conclusions.

- The all-order resummation of FSI effects from the $Q_{1,2}$ current–current operators would definitely help filling the persistent gap of about 30% between theory and experiment for the $\Delta I = 1/2$ rule [6], though some $\mathcal{O}(p^2, 1/N)$ double counting at the LD level seems difficult to avoid within present analytical techniques relying on some expansion. Here non-perturbative approaches like lattice QCD could turn out to be more successful. In lattice computations, the strong phases are determined using the Luscher relation between the two-pion energies in a finite volume and the phase shifts [36,37]. The moduli are fully calculated [36–40] and the amplitudes are then given by (14).
- The first FSI effects induced by the Q_6 density–density operator being subleading in either p^2 or $1/N$ within an appropriate chiral/color expansion, they do not really relax the tension recently highlighted in [9,11] for the CP-violating parameter ε'/ε.

In other words, the FSI rescaling factors $\mathcal{R}_I$ extracted from dispersive treatments beyond one-loop [12–19] are relevant for the $\Delta I = 1/2$ rule in [6], enhancing the expectations that the $\Delta I = 1/2$ rule is fully governed by SM dynamics.

On the other hand our findings imply that FSIs are much less relevant for ε'/ε and diminish significantly hopes that improved calculations of ε'/ε would bring it within the SM to agree with the experimental data, opening thereby an arena for important new physics contributions to this ratio. For the latest analyses of such contributions see [41–48].

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