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Reply

Reply to "Comment on 'Quantum massive conformal gravity' by F. F. Faria"

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Abstract Recently in (Eur Phys J C 76:341, 2016), Myung has suggested that the renormalizability of massive conformal gravity is meaningless unless the massive ghost states of the theory are stable. Here we show that massive conformal gravity can be renormalizable having unstable ghost states.

Before we address the content of Myung's paper [1], let us take a brief look on the results of Ref. [2]. For this purpose, we consider the massive conformal gravity (MCG) action, which is given by[1] [3]

$$S_{\text{MCG}} = \int d^4x\, \sqrt{-g}\Big[\alpha(\varphi^2 R + 6\partial_\mu\varphi\partial^\mu\varphi) - \frac{1}{m^2}C^{\alpha\beta\mu\nu}C_{\alpha\beta\mu\nu}\Big], \tag{1}$$

where m is a constant with dimensions of mass, α is a dimensionless constant, $C^{\alpha}{}_{\beta\mu\nu}$ is the Weyl tensor, R is the scalar curvature, and φ is a scalar field called dilaton. Using the Lanczos identity, we can write (1) as

$$S_{\text{MCG}} = \int d^4x\, \sqrt{-g}\Big[\alpha(\varphi^2 R + 6\partial_\mu\varphi\partial^\mu\varphi) - \frac{2}{m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{1}{3}R^2\right)\Big], \tag{2}$$

where $R_{\mu\nu}$ is the Ricci tensor.

By performing a perturbative quantization of (2) about the background field expansions

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{3}$$

$$\varphi = \sqrt{\frac{2}{\alpha}}(1+\sigma), \tag{4}$$

it can be shown, after a long but straightforward calculation, that the Feynman propagators for the quantum fields σ and $\Psi_{\mu\nu} = h_{\mu\nu} - \eta_{\mu\nu}h/2$ are given by [2]

$$D_\sigma = i\int \frac{d^4p}{(2\pi)^4}\frac{e^{-ip\cdot(x-y)}}{p^2+m^2-i\chi}, \tag{5}$$

$$D_\Psi^{\mu\nu,\alpha\beta} = -\frac{i}{2}(\eta^{\mu\alpha}\eta^{\nu\beta} + \eta^{\mu\beta}\eta^{\nu\alpha} - \eta^{\mu\nu}\eta^{\alpha\beta}) \times \int \frac{d^4p}{(2\pi)^4}\frac{m^2 e^{-ip\cdot(x-y)}}{(p^2-i\chi)(p^2+m^2-i\chi)}, \tag{6}$$

where χ is an infinitesimal parameter. Since the residues at the poles for both massive terms in (5) and (6) are negative, its corresponding quantum states are taken to have negative energy or negative norm. In either case quantum MCG is supposed unphysical. However, if the states of negative residues (ghost states) are unstable, the theory is unitary.

In his paper [1], Myung argues that the massive ghost states prevents MCG from being treated perturbatively if such ghost states are unstable [4]. In this case, however, we can use a modified perturbation series in which only diagrams without self-energy parts for the unstable ghost states are included, with the bare propagators for these states $D(p^2)$ replaced by the dressed propagators [5]

$$\overline{D}(p^2) = [D^{-1}(p^2) - \Pi(p^2)]^{-1}, \tag{7}$$

where $\Pi(p^2)$ is the sum of all 1PI self-energy parts. Since renormalization concerns only the high-energy behavior of the propagators, the use of dressed propagators does not affect it. So the p^{-4} behavior of the bare propagator (6) at high momenta makes MCG renormalizable regardless of whether the ghost states are unstable or not.

[1] Here we use "absolute units" in which $c = \hbar = 16\pi G = 1$.

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