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Cosmic constraint on massive neutrinos in viable $f(R)$ gravity with producing ΛCDM background expansion

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Abstract Tensions between several cosmic observations were found recently, such as the inconsistent values of H_0 (or σ_8) were indicated by the different cosmic observations. Introducing the massive neutrinos in ΛCDM could potentially solve the tensions. Viable $f(R)$ gravity producing ΛCDM background expansion with massive neutrinos is investigated in this paper. We fit the current observational data: Planck-2015 CMB, RSD, BAO, and SNIa to constrain the mass of neutrinos in viable $f(R)$ theory. The constraint results at 95% confidence level are: $\Sigma m_\nu < 0.202$ eV for the active-neutrino case, $m^{\rm eff}_{\nu,\rm sterile} < 0.757$ eV with $N_{\rm eff} < 3.22$ for the sterile neutrino case. For the effects due to the mass of the neutrinos, the constraint results on model parameter at 95% confidence level become $f_{R0} \times 10^{-6} > -1.89$ and $f_{R0} \times 10^{-6} > -2.02$ for two cases, respectively. It is also shown that the fitting values of several parameters much depend on the neutrino properties, such as the cold dark matter density, the cosmological quantities at matter–radiation equality, the neutrino density and the fraction of baryonic mass in helium. Finally, the constraint result shows that the tension between direct and CMB measurements of H_0 gets slightly weaker in the viable $f(R)$ model than that in the base ΛCDM model.

1 Introduction

The base 6-parameter Λ–Cold–Dark-Matter (ΛCDM) model is the most popular one to interpret the accelerating expansion of universe. This model is favored by most "observational probes", though the fine-tune problem and the coincidence problem in theory exist. However, some tension was found recently between the cosmic observations when one fitted observational data to this model. For example, the tension is found for estimating the values of H_0: a lower value of $H_0 = 67.3 \pm 1.0$ is provided by Planck-CMB experiment with an indirect estimate on H_0 [1], but the higher value of $H_0 = 74.3 \pm 2.1$ is obtained by SST direct measurements of H_0 [2]; this tension also exists between the Planck-CMB experiment and the rich cluster counts, as they provide the inconsistent value of σ_8 [1,3].

The studies on these tensions are important, since any evidence of a tension may be useful in the search for new physics. One possible interpretation to above tension is that the base 6-parameter ΛCDM model is incorrect or should be extended. Reference [1] shows that introducing $\sum m_\nu$ or introducing $N_{\rm eff}$ solely in the ΛCDM model cannot resolve the above tensions, but the tensions could be solved in the ΛCDM with including both $\sum m_\nu$ and $N_{\rm eff}$ or with including the massive sterile neutrinos $m^{\rm sterile}_{\nu,\rm eff}$. Here $\sum m_\nu$ denotes the total mass of three species of degenerate massive active neutrinos, and $N_{\rm eff}$ denotes the effective number of relativistic degrees of freedom, which relates to the neutrinos and the extra massless species. Combined analysis of cosmic data in other references also indicates the existence of the massive neutrinos, for example, joint analysis from CMB and BAO (baryon acoustic oscillation) [4,5], from solar and atmospheric experiments [6–8], or from the reactor neutrino oscillation anomalies [9,10], etc.

Investigating other scenarios to solve the above tensions and restricting the mass of neutrinos in different scenarios is significant. Reference [11] shows that the possible discovery of a sterile neutrino with mass $m^{\rm eff}_{\nu,\rm sterile} \approx 1.5$ eV, motivated by various anomalies in neutrino oscillation experiments, would favor cosmology based on $f(R)$ gravity rather than the standard ΛCDM. In addition, it is well known that plenty of functions $f(R)$ of the Ricci scalar R [12–28] were presented to modify the Einstein gravity theory, in order to solve puzzles in general relativity. But several forms of $f(R)$ are then found to be nonphysical, since they cannot describe the expansion of universe in matter-dominated time [29,30]. So, studies on observationally viable $f(R)$ theories are necessary. One of

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the viable $f(R)$ theories has been studied in Refs. [31,32], where the $f(R)$ theory can realize the most popular ΛCDM universe at background-dynamics level, while the effects of large scale structure with the cosmological perturbation theory in this $f(R)$ model are different from that in the ΛCDM. In this paper, we investigate the behaviors of massive neutrinos in observationally viable $f(R)$ theories with producing the ΛCDM background expansion history.

2 Viable $f(R)$ gravity theory producing ΛCDM background expansion

The action of $f(R)$ modified gravity theory is written as

$$I=\int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{16\pi G}f(R)+\mathcal{L}_u\right]. \tag{1}$$

$\mathcal{L}_u$ is the Lagrangian density of universal matter including the radiation and the pressureless matter (baryon matter plus cold dark matter). Using the variation principle, one gets

$$f_R R_{\mu\nu}-\frac{1}{2}f(R)g_{\mu\nu}+(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)f_R=8\pi G T_{\mu\nu}. \tag{2}$$

$f_R=\frac{\mathrm{d}f(R)}{\mathrm{d}R}$, $R_{\mu\nu}$ and $T_{\mu\nu}$ denote the Ricci tensor and the energy-momentum tensor of universal matter, respectively. For a universe described by metric $g_{\mu\nu}=\mathrm{diag}(-1,a(t)^2,a(t)^2,a(t)^2)$, the dynamical evolutionary equations of universe in $f(R)$ theory are

$$3f_R H^2=\frac{f_R R-f}{2}-3Hf_{RR}\dot{R}+8\pi G(\rho_m+\rho_r), \tag{3}$$

$$2f_R\dot{H}=H\dot{f}_R-\ddot{f}_R-k^2\left[\rho_m+\frac{4\rho_r}{3}\right]. \tag{4}$$

As shown in Ref. [32], the viable $f(R)$ theory which realizes the popular ΛCDM universe at background-dynamics level does not have an analytical expression of $f(R)$ to describe a physical universe from the radiation-dominated epoch to the late-time acceleration, but it really has the analytical solutions of $f(R)$ in different evolutionary epochs of the universe. Concretely, Ref. [32] gives the forms of $f(R)$ in two cases: one describes the evolution of the ΛCDM background from the radiation-dominated epoch to the matter-dominated epoch, and the other one represents the evolution of the ΛCDM background from the matter-dominated era to the future expansion. In this paper we focus on studying the $f(R)$ function producing a ΛCDM background expansion from the matter-dominated epoch to the late-time acceleration,[1] which has a form as follows [31,32]:

$$f(R)=R-2\Lambda-\varpi\left(\frac{\Lambda}{R-4\Lambda}\right)^{p_+-1}\times{}_2F_1\left[q_+,p_+-1;r_+;-\frac{\Lambda}{R-4\Lambda}\right], \tag{5}$$

where $\varpi=\frac{3H_0^2\Omega_\Lambda D}{p_+-1}(\frac{\Omega_m}{\Omega_\Lambda})^{p_+}$, ${}_2F_1[a,b;c;z]$ is the Gaussian hypergeometric function with $q_+=\frac{1+\sqrt{73}}{12}$, $r_+=1+\frac{\sqrt{73}}{6}$, and $p_+=\frac{5+\sqrt{73}}{12}$. D is the model parameter in this $f(R)$ modified gravity, which can relate to the current value f_{R0} and the current value of the Compton wavelength B_0 by

$$f_{R0}=1+D\times{}_2F_1\left[q_+,p_+;r_+;-\frac{\Omega_\Lambda}{\Omega_m}\right], \tag{6}$$

$$B_0=\frac{2Dp_+}{\Omega_m^2\left\{1+D{}_2F_1\left[q_+,p_+;r_+;-\frac{\Omega_\Lambda}{\Omega_m}\right]\right\}}\times\left\{\Omega_\Lambda\frac{q_+}{r_+}{}_2F_1\left[q_++1,p_++1;r_++1;-\frac{\Omega_\Lambda}{\Omega_m}\right]-\Omega_m{}_2F_1\left[q_+,p_+;r_+;-\frac{\Omega_\Lambda}{\Omega_m}\right]\right\}, \tag{7}$$

where the Compton wavelength is derived by $B=\frac{f_{RR}}{f_R}\frac{\mathrm{d}R}{\mathrm{d}\ln a}\frac{H}{\mathrm{d}H/\mathrm{d}\ln a}=\frac{\partial f_R/\partial\ln a}{f_R}\frac{H}{\partial H/\partial\ln a}$.

Obviously, Eq. (5) can partly realize the background expansion as that of the ΛCDM universe, while the cosmological perturbation behaviors in this $f(R)$ model are different from that in the ΛCDM model. Given that it is not natural by using two $f(R)$ functions to mimic one total ΛCDM universe, in this paper we consider our universe including two stages: the early universe $a<0.02$ (including the radiation-dominated epoch and the early stage of the matter-dominated era) is described by the ΛCDM, and the universe $a\geq 0.02$ (including the deep matter-dominated epoch and the late-time acceleration) is depicted by the above viable $f(R)$ model.

3 Cosmological perturbations in viable $f(R)$ gravity theory producing ΛCDM background expansion

The line element with the perturbation reads

$$\mathrm{d}s^2=a^2[-(1+2\psi Y^{(s)})\mathrm{d}\tau^2+2BY_i^{(s)}\mathrm{d}\tau\mathrm{d}x^i+(1+2\phi Y^{(s)})\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j+\varepsilon Y_{ij}^{(s)}\mathrm{d}x^i\mathrm{d}x^j], \tag{8}$$

[1] An accelerating cosmological model can be used to interpret the current observations. For the ΛCDM background expansion from the matter-dominated epoch to the late-time acceleration, R can be written by $R=3\Omega_m a^{-3}+12\Omega_\Lambda=3\Omega_m a^{-3}+4\Lambda$.

where γ_{ij} is the three-dimensional spatial metric in the spherical coordinate

$$[\gamma_{ij}] = \begin{pmatrix} \frac{1}{1-Kr^2} & 0 & 0 \\ 0 & r^2 & 0 \\ 0 & 0 & r^2\sin^2\theta \end{pmatrix}, \quad (9)$$

$(\triangle + k^2)Y^{(s)} = 0$, $Y_j^{(s)} \equiv -\frac{1}{k}Y_{|j}^{(s)}$ and $Y_{ij}^{(s)} \equiv \frac{1}{k^2}Y_{|ij}^{((s)} + \frac{1}{3}\gamma_{ij}Y^{((s)}$ are the scalar harmonic functions. Considering the synchronous gauge, we have $\psi = 0$, $B = 0$, $h_L = 6\phi$ and $\eta_T = -(\phi + \varepsilon/6)$, where $\eta_T = \frac{\delta R^{(3)}}{-4k^2+12K} = \frac{6\delta K}{-4k^2+12K}$ denotes the conformal 3-space curvature perturbation. The perturbed modified Einstein equations in $f(R)$ theory can be derived as follows [33]:

$$\left(f_R\mathcal{H} + \frac{1}{2}f_R^{'}\right)k\mathcal{Z} = \frac{\kappa^2}{2}a^2\delta\rho + f_Rk^2\eta_T\beta_2 - \frac{3}{2}\mathcal{H}\delta f_R^{'} - \frac{1}{2}\delta f_Rk^2 + \frac{3}{2}\mathcal{H}^{'}\delta f_R, \quad (10)$$

$$\frac{k^2}{3}f_R(\beta_2\sigma - \mathcal{Z}) = \frac{\kappa}{2}a^2q + \frac{1}{2}k\delta f_R^{'} - \frac{1}{2}k\mathcal{H}\delta f_R, \quad (11)$$

$$\sigma^{'} + 2\mathcal{H}\sigma + \frac{f_R^{'}}{f_R}\sigma = k\eta_T - \kappa^2a^2\frac{p\Pi}{f_Rk} - k\frac{\delta f_R}{f_R}, \quad (12)$$

$$\mathcal{Z}^{'} + \left(\frac{1}{2}\frac{f_R^{'}}{f_R} + \mathcal{H}\right)\mathcal{Z} = \left(-k\beta_2 + \frac{k}{2} + \frac{3\mathcal{H}^2}{k}\right)\frac{\delta f_R}{f_R} - \frac{\kappa^2a^2}{2kf_R}(\delta\rho + 3\delta p) - \frac{3}{2}\frac{\delta f_R^{''}}{kf_R}, \quad (13)$$

where $q = (\rho + p)v$, $\beta_2 = \frac{k^2-3K}{k^2}$, $f_R = 1 + Da^{3p_+} \times_2 F_1[q_+, p_+; r_+; -a^3\frac{\Omega_\Lambda}{\Omega_m}]$, $\mathcal{H} = a^{'}/a$ is the conformal Hubble parameter, and the superscript $'$ denotes the derivative with respect to conformal time. In addition, in the CAMB code, the curvature perturbations are characterized by $\mathcal{Z} = \frac{h_L^{'}}{2k}$ and $\sigma = \frac{(h_L+6\eta_T)^{'}}{2k}$ with $\eta_T^{'} = \frac{k}{3}(\sigma - \mathcal{Z})$. The evolutionary equation of the perturbed field δf_R reads

$$\delta f_R^{''} + 2\mathcal{H}\delta f_R^{'} + a^2\left(\frac{k^2}{a^2} + m_{f_R}^2\right)\delta f_R = \frac{\kappa^2a^2}{3}(\delta\rho - 3\delta p) - kf_R^{'}\mathcal{Z}. \quad (14)$$

The source term of the CMB temperature anisotropy is described by

$$S_T(\tau, k) = e^{-\varepsilon}(\alpha^{''} + \eta_T^{'}) + g\left(\triangle_{T0} + 2\alpha^{'} + \frac{v_b^{'}}{k} + \frac{\zeta}{12\sqrt{\beta_2}} + \frac{\zeta^{''}}{4k^2\sqrt{\beta_2}}\right) + g^{'}\left(\alpha + \frac{v_b}{k} + \frac{\zeta^{'}}{2k^2\sqrt{\beta_2}}\right) + \frac{1}{4}\frac{g^{''}\zeta}{k^2\sqrt{\beta_2}}$$

$$= e^{-\varepsilon}\left(\frac{\sigma^{''}}{k} + \frac{k\sigma}{3} - \frac{k\mathcal{Z}}{3}\right) + g\left(\triangle_{T0} + 2\frac{\sigma^{'}}{k} + \frac{v_b^{'}}{k} + \frac{\zeta}{12\sqrt{\beta_2}} + \frac{\zeta^{''}}{4k^2\sqrt{\beta_2}}\right) + g^{'}\left(\frac{\sigma}{k} + \frac{v_b}{k} + \frac{\zeta^{'}}{2k^2\sqrt{\beta_2}}\right) + \frac{1}{4}\frac{g^{''}\zeta}{k^2\sqrt{\beta_2}} \quad (15)$$

where $g = -\dot{\varepsilon}e^{-\varepsilon} = an_e\sigma_Te^{-\varepsilon}$ is the visibility function and ε is the optical depth. ζ is given by $\zeta = (\frac{3}{4}I_2 + \frac{9}{2}E_2)$, where I_2, E_2 indicate the quadrupole of the photon intensity and the E-like polarization, respectively.

4 Data fitting and results

4.1 Used data

In this section, we apply the cosmic data to constrain the above viable $f(R)$ model. The used data are as follows.

(1) The CMB temperature and polarization information released by Planck 2015 [1]: the high-l C_l^{TT} likelihood (PlikTT), the high-l C_l^{EE} likelihood (PlikEE), the high-l C_l^{TE} likelihood (PlikTE), the low-l data and the lensing data.
(2) The 10 datapoints of the redshift space distortion (RSD): the RSD measurements from 6dFGS ($f\sigma_8(z = 0.067) = 0.42 \pm 0.06$) [34], 2dFGRS ($f\sigma_8(z = 0.17) = 0.51 \pm 0.06$) [35], WiggleZ ($f\sigma_8(z = 0.22) = 0.42 \pm 0.07$, $f\sigma_8(z = 0.41) = 0.45 \pm 0.04$, $f\sigma_8(z = 0.60) = 0.43 \pm 0.04$, $f\sigma_8(z = 0.78) = 0.38 \pm 0.04$) [36], SDSS LRG DR7 ($f\sigma_8(z = 0.25) = 0.39 \pm 0.05$, $f\sigma_8(z = 0.37) = 0.43 \pm 0.04$) [37], BOSS CMASS DR11 ($f\sigma_8(z = 0.57) = 0.43 \pm 0.03$) [38], and VIPERS ($f\sigma_8(z = 0.80) = 0.47 \pm 0.08$) [39]. Here $f = \frac{d\ln D}{d\ln a}$, D is the linear growth rate of matter fluctuations, σ_8 is the RMS matter fluctuations in linear theory. RSD reflects the coherent motions of galaxies, so it provides information as regards the formation of large-scale structure [40–42].
(3) The BAO data: the 6dFGS [43], the SDSS-MGS [44], the BOSSLOWZ BAO measurements of $D_V = r_{\rm drag}$ [44] and the CMASS-DR11 anisotropic BAO measurements [44]. Since the WiggleZ volume partially overlaps that of the BOSSCMASS sample, we do not use the WiggleZ results in this paper. 6dFGS denotes the six-degree-Field Galaxy survey (6dFGS) at $z_{\rm eff} = 0.106$ [43], SDSS-MGS denotes the SDSS Main Galaxy Sample (MGS) at $z_{\rm eff} = 0.15$ [44], BOSSLOWZ denotes the Baryon Oscillation Spectroscopic Survey (BOSS) "LOWZ" at $z_{\rm eff} = 0.32$ [44], and CMASS-DR11 denotes the BOSS CMASS at $z_{\rm eff} = 0.57$ [44]. The recent analysis of the latter two

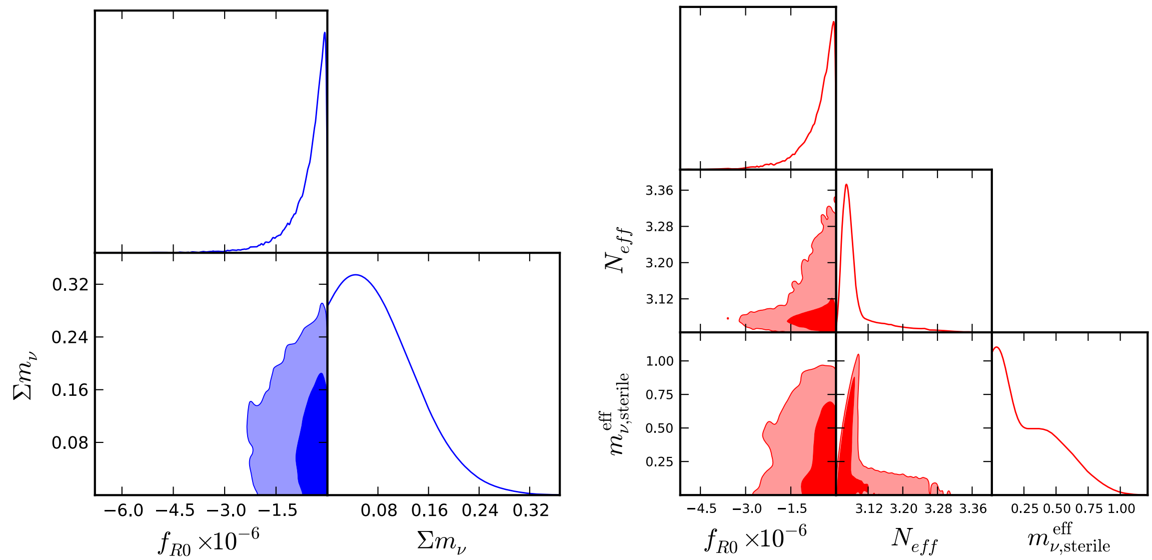

Fig. 1 The contours of model parameters in viable $f(R)$ theory with massive neutrino by fitting the Planck TT, TE, EE+lowP and the low-redshift data: Planck lesning+RSD+BAO+JLA

BAO data use peculiar velocity field reconstructions to sharpen the BAO feature and reduce the errors on $D_V = r_{\rm drag}$. The point labeled BOSS CMASS at $z_{\rm eff} = 0.57$ shows $D_V = r_{\rm drag}$ from the analysis of [45], updating the BOSS-DR9 analysis.

(4) The supernova Ia (SN Ia) data from SDSS-II/SNLS3 joint light-curve analysis (JLA) [46,47].

The prior value of the Hubble constant $H_0 = 100h$ km s^{-1} Mpc^{-1} is usually taken in cosmic analysis, though there are hundreds of measurement value of H_0 and lots of them are mutually inconsistent.[2] Reference [57] points out that the prior value of the H_0 affects cosmological parameter estimation, but not very significantly. Here we take the HST prior, $H_0 = 73.8 \pm 2.4\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [58].

Table 1 The 95% confidence level of basic parameters in viable $f(R)$ model with the massive neutrino by fitting the Planck TT, TE, EE+lowP and the low-redshift data: Planck lesning+RSD+BAO+JLA

Parameters	Active	Sterile
Σm_ν	<0.202	–
$m^{\rm eff}_{\nu,\rm sterile}$	–	<0.757
$N_{\rm eff}$	–	<3.22
$f_{R0} \times 10^{-6}$	>−1.89	>−2.02
$\Omega_b h^2$	$0.02233^{+0.00028}_{-0.00028}$	$0.02228^{+0.00031}_{-0.00029}$
$\Omega_c h^2$	$0.1178^{+0.0022}_{-0.0022}$	$0.1147^{+0.0063}_{-0.0068}$
$100\theta_{MC}$	$1.04090^{+0.00060}_{-0.00059}$	$1.04096^{+0.00063}_{-0.00065}$
τ	$0.053^{+0.026}_{-0.027}$	$0.060^{+0.028}_{-0.028}$
$\ln(10^{10}A_s)$	$3.035^{+0.048}_{-0.050}$	$3.049^{+0.052}_{-0.053}$
n_s	$0.9683^{+0.0080}_{-0.0079}$	$0.9713^{+0.0097}_{-0.0087}$

4.2 Constraint on neutrino mass and the base parameters in viable $f(R)$ model producing ΛCDM expansion

Constraints on the neutrino mass in ΛCDM model or in dynamical dark energy models or in $f(R)$ theory have been discussed in some references [1,59–64]. Given that the constraints on Σm_ν (or $m^{\rm sterile}_{\nu,\rm eff}$) are model-dependent, we fit the cosmic data to limit the mass of neutrinos in the above viable $f(R)$ model by using the MCMC method [65–70]. Obviously, extra parameters f_{R0} and Σm_ν (or $m^{\rm sterile}_{\nu,\rm eff}$ with the required $N_{\rm eff}$) are added, relative to the base ΛCDM model. Table 1 and Fig. 1 show the 95% limits of basic parameters in $f(R)$ model. One can see the upper bound on the mass of active neutrino $\Sigma m_\nu < 0.202$, which is comparable with other results. For example, adding a single free parameter Σm_ν to the base ΛCDM model, fitting different data gives $\Sigma m_\nu < 0.177$ eV [59], $\Sigma m_\nu < 0.17$ eV [1] or $\Sigma m_\nu < 0.254$ eV [60]; adding Σm_ν to the dynamical DE model, fitting cosmic data gives $\Sigma m_\nu < 0.304$ eV in wCDM model [59] or $\Sigma m_\nu < 0.113$ eV in the holographic DE model [59]; and $\Sigma m_\nu < 0.451$ eV and $\Sigma m_\nu < 0.214$ eV are given in the Starobinsky $f(R)$ and exponential $f(R)$

[2] Measurements provide some different results on H_0, which are almost in the region (60–80) km s^{-1} Mpc^{-1}, such as the higher values: $H_0 = 74.3 \pm 2.6\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [2], the lower values: $H_0 = 63.7 \pm 2.3\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [48] and the concordance value: $H_0 = 69.6 \pm 0.7\,{\rm km\,s^{-1}\,Mpc^{-1}}$ [49], etc. For other measurement values of H_0, one can see Refs. [50–56].

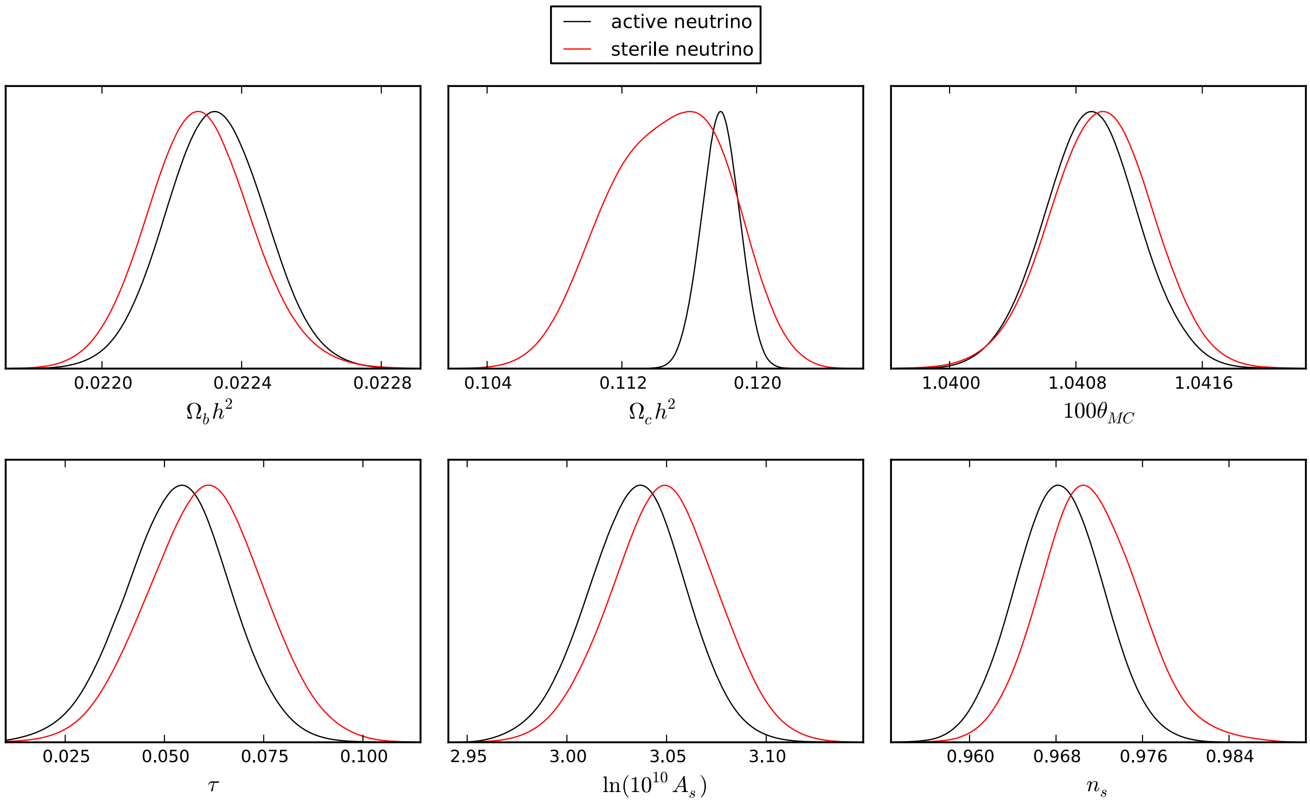

Fig. 2 The 1-D distributions of basic cosmological parameters in viable $f(R)$ model with massive neutrino

models [61], respectively.[3] Table 1 also exhibits the constraint result $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}} < 0.757$ with $N_{\mathrm{eff}} < 3.22$ for the sterile-neutrino case in viable $f(R)$ model. One can compare these results with other ones. For example, fitting the different cosmic data gives $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}} < 0.52$ eV with $N_{\mathrm{eff}} < 3.7$ [1], $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}} < 0.479$ eV with $\Delta N_{\mathrm{eff}} = <0.98$ [60], or $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}} < 0.43$ with $N_{\mathrm{eff}} < 3.96$ in ΛCDM model [62], and $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}} < 0.61$ with $N_{\mathrm{eff}} < 3.95$ in the $f(R)$ model [62]. Obviously, a higher upper limit on $m_{\nu,\mathrm{eff}}^{\mathrm{sterile}}$ and a lower limit on N_{eff} are obtained in our study. Some inconsistent results on the sterile-neutrino mass can also be found, for example, the sterile neutrino mass $0.47\,\mathrm{eV} < m_{\nu,\mathrm{sterile}}^{\mathrm{eff}} < 1\,\mathrm{eV}$ (2σ) is given in a $f(R)$ model and $0.45\,\mathrm{eV} < m_{\nu,\mathrm{sterile}}^{\mathrm{eff}} < 0.92\,\mathrm{eV}$ is given in the ΛCDM model [63], or the active-neutrino mass $\sum m_\nu = 0.35 \pm 0.10$ is presented in the ΛCDM model [4]. The constraint results on the model parameter in a viable $f(R)$ theory are $f_{R0} \times 10^{-6} > -1.89$ for the active-neutrino case and $f_{R0} \times 10^{-6} > -2.02$ for the sterile-neutrino case at the %95 limit. Though the fitting results on f_{R0} are affected by the additional parameters Σm_ν (or $m_{\nu,\mathrm{sterile}}^{\mathrm{eff}}$ with N_{eff}), for using the Planck 2015 data in this paper it has a more stringent constraint than the result given by Ref. [71]: $f_{R0} \times 10^{-6} = -2.58^{+2.14}_{-0.58}$ in the 1σ region.

Table 1 also lists the values of six basic cosmological parameters. $\Omega_b h^2$ is the current baryon density, $\Omega_c h^2$ is the cold dark matter density at present, θ_{MC} denotes the approximation to r_*/D_A, τ represents the Thomson scattering optical depth due to reionization, $\ln(10^{10}A_s)$ is the Log power of the primordial curvature perturbations, and n_s is the scalar spectrum power-law index. From Table 1 and Fig. 2, it can be seen that the neutrino properties to a much higher extent affect the fitting value of the cold dark matter density than the fitting values of the other parameters. These results could be interpreted as follows. Since the massive neutrinos are considered as one kind of dark matter in the universe, the mass of the neutrino (active or sterile) would directly affect the dimensionless energy density of dark matter. According to the constraint results on $\Omega_c h^2$ and $\Omega_\nu h^2$, one can see that the larger uncertainty of $\Omega_c h^2$ value is caused by the looser constraint on the dimensionless energy density of sterile neutrino $\Omega_\nu h^2$, which maybe reflects that there is less information on sterile neutrino from cosmic observations. However, the constraint on $\Omega_c h^2$ is stricter for the active-neutrino case, since the constraint

[3] Constraints on the total mass of the active neutrino are also investigated including an additional free parameter N_{eff} in the theoretical model, for example, $\Sigma m_\nu < 0.826$ with $N_{\mathrm{eff}} = 3.49^{+0.71}_{-0.73}$ [64] and $\Sigma m_\nu = 0.533^{+0.254}_{-0.411}$ with $N_{\mathrm{eff}} = 3.78^{+0.84}_{-0.64}$ [61] are given in the $f(R)$ models.

Table 2 The 95% confidence level of derived parameters in viable $f(R)$ model with the massive neutrino by fitting the Planck TT, TE, EE+lowP and the low-redshift data: Planck lesning+RSD+BAO+JLA

Parameters	Active	Sterile	Parameters	Active	Sterile
H_0	$67.9^{+1.1}_{-1.1}$	$68.4^{+1.1}_{-0.99}$	σ_8	$0.813^{+0.023}_{-0.023}$	$0.811^{+0.023}_{-0.022}$
Ω_m	$0.306^{+0.014}_{-0.013}$	$0.301^{+0.012}_{-0.013}$	$\Omega_m h^2$	$0.1411^{+0.0019}_{-0.0019}$	$0.1410^{+0.0032}_{-0.0027}$
$\Omega_m h^3$	$0.09579^{+0.00085}_{-0.00096}$	$0.0964^{+0.0026}_{-0.0013}$	$\sigma_8\Omega_m^{0.5}$	$0.450^{+0.012}_{-0.012}$	$0.445^{+0.014}_{-0.014}$
$\sigma_8\Omega_m^{0.25}$	$0.605^{+0.015}_{-0.015}$	$0.601^{+0.017}_{-0.017}$	$z_{\rm re}$	$7.5^{+2.6}_{-2.8}$	$8.2^{+2.6}_{-3.0}$
$10^9 A_s$	$2.08^{+0.10}_{-0.10}$	$2.11^{+0.11}_{-0.11}$	$10^9 A_s e^{-2\tau}$	$1.870^{+0.023}_{-0.022}$	$1.869^{+0.025}_{-0.023}$
t_0(Gyr)	$13.803^{+0.068}_{-0.061}$	$13.762^{+0.074}_{-0.12}$	z_*	$1089.79^{+0.47}_{-0.46}$	$1089.89^{+0.49}_{-0.48}$
r_*	$145.02^{+0.51}_{-0.49}$	$144.87^{+0.88}_{-1.4}$	$100\theta_*$	$1.04111^{+0.00060}_{-0.00059}$	$1.04113^{+0.00064}_{-0.00069}$
$z_{\rm drag}$	$1059.68^{+0.60}_{-0.58}$	$1059.61^{+0.78}_{-0.72}$	$r_{\rm drag}$	$147.71^{+0.52}_{-0.50}$	$147.57^{+0.91}_{-1.5}$
$k_{\rm D}$	$0.14018^{+0.00058}_{-0.00060}$	$0.1401^{+0.0012}_{-0.00092}$	$z_{\rm eq}$	3350^{+49}_{-50}	3252^{+120}_{-150}
$k_{\rm eq}$	$0.01022^{+0.00015}_{-0.00015}$	$0.01001^{+0.00035}_{-0.00038}$	$100\theta_{\rm s,eq}$	$0.4543^{+0.0050}_{-0.0048}$	$0.465^{+0.017}_{-0.013}$
$\log_{10}(B_0)$	$-5.74^{+0.92}_{-1.00}$	$-5.69^{+0.90}_{-1.00}$	$\Omega_\nu h^2$	$0.00094^{+0.0013}_{-0.00097}$	$0.0039^{+0.0048}_{-0.0034}$
Y_P	$0.24537^{+0.00012}_{-0.00013}$	$0.2460^{+0.0018}_{-0.00071}$			

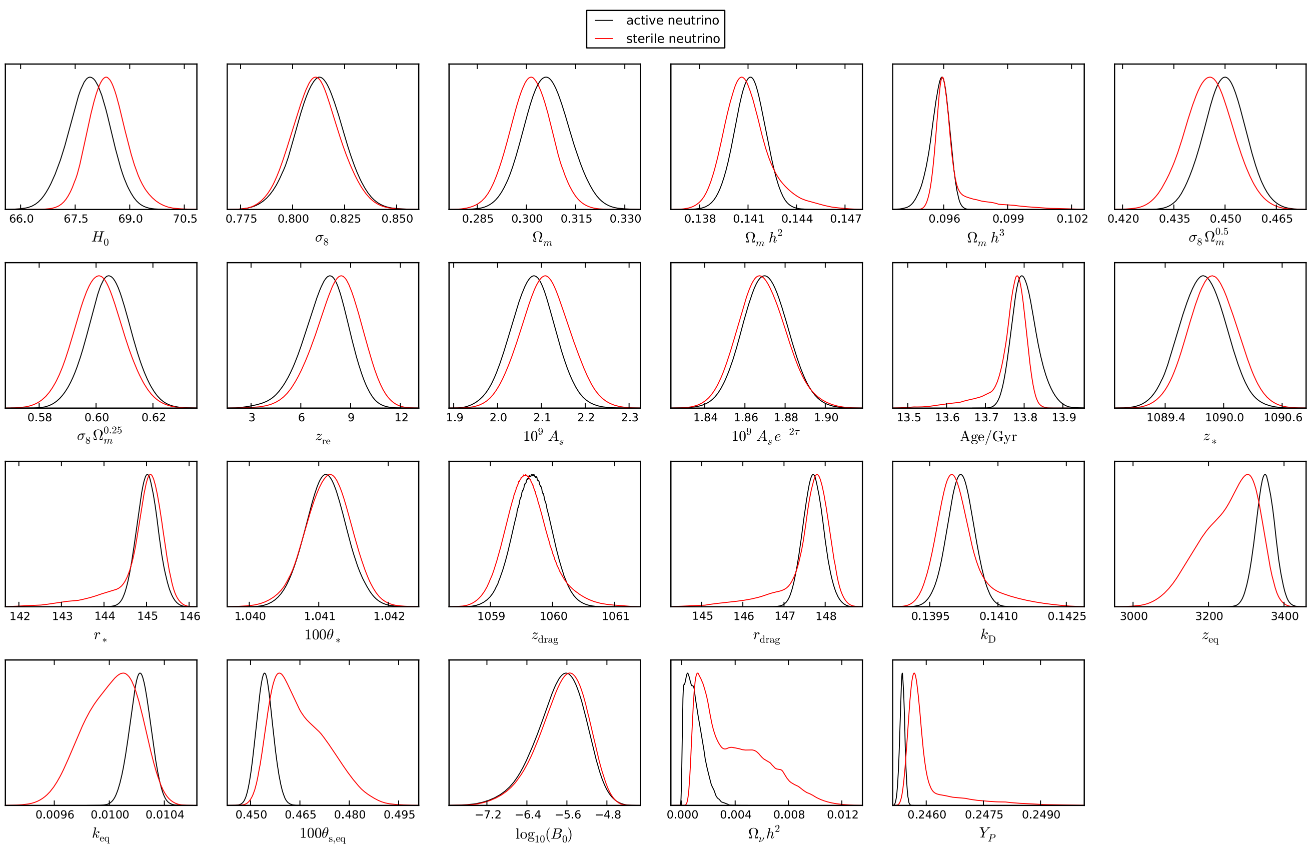

Fig. 3 The 1-D distributions of derived cosmological parameters in viable $f(R)$ model with massive neutrino

on the dimensionless energy density of the active neutrino $\Omega_\nu h^2$ is tighter than the case of the sterile neutrino, which maybe reflects that there is more information on the active neutrino from cosmic observations. Except for $\Omega_c h^2$, the other basic parameters in Table 1 are not directly related to the neutrino density, so the effects on the fitting values of the other basic parameters from the neutrino characters are smaller.

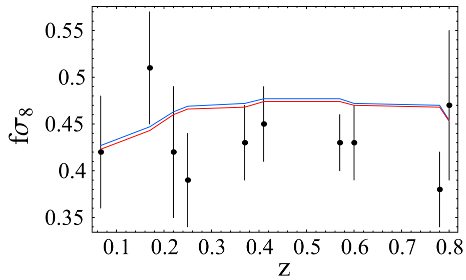

Fig. 4 The values of $f\sigma_8$ calculated in the viable $f(R)$ model with the massive neutrino, where *blue line* denotes the active-neutrino case, and *red line* denotes the sterile-neutrino case. *Ten dots* with *error bar* denote the observational datapoints from Refs. [34–39]

4.3 Constraint on derived parameters in viable $f(R)$ model producing ΛCDM expansion

The values of the derived parameters of interest are calculated and listed in Table 2. It includes 20 parameters listed in table 4 of Ref. [1] and three other parameters $[\log_{10}(B_0), \Omega_\nu h^2, Y_p]$. Concretely, Ω_m is the current dimensionless matter density, z_{re} is the redshift at which universe is half reionized, t_0 denotes the age of the universe today (in Gyr), z_* denotes the redshift for which the optical depth equals unity, r_* denotes the comoving size of the sound horizon at $z = z_*$, θ_* denotes the angular size of sound horizon at $z = z_*(r_*/D_A)$, $z_{\rm drag}$ denotes the redshift at which the baryon-drag optical depth equals unity, $r_{\rm drag}$ denotes the comoving size of the sound horizon at $z = z_{\rm drag}$, k_D denotes the characteristic damping comoving wavenumber (Mpc^{-1}), $z_{\rm eq}$ denotes the redshift of the matter–radiation equality, $\Omega_\nu h^2$ is the neutrino density, Y_p denotes the fraction of baryonic mass in helium. Obviously, from Table 2 we can see that the constraint results on H_0 and σ_8 are: $H_0 = 67.9^{+1.1}_{-1.1}$ and $\sigma_8 = 0.813^{+0.023}_{-0.023}$ for the active-neutrino case, and $H_0 = 68.4^{+1.1}_{-0.99}$ and $\sigma_8 = 0.811^{+0.023}_{-0.022}$ for the sterile-neutrino case, which are compatible with results given by [59]. For these constraint results on H_0, it is also shown that the tension between direct and CMB measurements of H_0 gets slightly weaker in our considered model than that in the base ΛCDM model, where $H_0 = 67.6 \pm 0.6$ is given by Ref. [1]. In addition, it is found from Fig. 3 that the neutrino properties much affect the fitting value of parameters: $z_{\rm eq}$, $k_{\rm eq}$, $100\theta_{s,\rm eq}$, $\Omega_\nu h^2$, and Y_p, which could be partly explained by the dependence of the parameters on the cold dark matter density and might be useful for testing the neutrino properties in experiments. The values of σ_8 in a viable $f(R)$ model are almost the same for the cases of different-species neutrinos, and the same result is also suitable for the parameters $f\sigma_8$, $A_s e^{-2\tau}$, and θ_*, as exhibited in Figs. 3 and 4.

5 Conclusion

Tension between several observations was found recently. The studies of tensions are important, since they are useful to search for new physics. The massive neutrinos are introduced in cosmological models to solve the tensions concerning the inconsistent values of H_0 (or σ_8). Investigating other scenarios to solve these tensions and restricting the mass of neutrinos in different scenarios are significant. Given that several forms of $f(R)$ are found to be nonphysical, we study the viable $f(R)$ gravity with the massive neutrinos in this paper. We fit the current observational data: Planck-2015 CMB, RSD, BAO, and SNIa to constrain the mass of neutrinos in viable $f(R)$ theory. The constraint results at 95% confidence level are $\Sigma m_\nu < 0.202$ eV for the active-neutrino case and $m^{\rm eff}_{\nu,\rm sterile} < 0.757$ eV with $N_{\rm eff} < 3.22$ for the sterile-neutrino case, which are comparable with some other results. For the effects by the mass of the neutrinos, the constraint results on model parameter become $f_{R0} \times 10^{-6} > -1.89$ and $f_{R0} \times 10^{-6} > -2.02$ for the two cases, respectively. It is also shown that the fitting values of several parameters strongly depend on the neutrino properties, such as the cold dark matter density $\Omega_c h^2$, the cosmological quantities at matter–radiation equality: $z_{\rm eq}$, $k_{\rm eq}$, and $100\theta_{s,\rm eq}$, the neutrino density $\Omega_\nu h^2$, and the fraction of baryonic mass in helium Y_p. Finally, the constraint result shows that the tension between direct and CMB measurements of H_0 gets slightly weaker in the viable $f(R)$ model than that in the base ΛCDM model.

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