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New effects in the interaction between electromagnetic sources mediated by nonminimal Lorentz violating interactions

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Abstract This paper is dedicated to the study of interactions between external sources for the electromagnetic field in the presence of Lorentz symmetry breaking. We focus on a higher derivative, Lorentz violating interaction that arises from a specific model that was argued to lead to interesting effects in the low energy phenomenology of light pseudoscalars interacting with photons. The kind of higher derivative Lorentz violating interaction we discuss are called nonminimal. They are usually expected to be relevant only at very high energies, but we argue they might also induce relevant effects in low energy phenomena. Indeed, we show that the Lorentz violating background considered by us leads to several phenomena that have no counterpart in Maxwell theory, such as nontrivial torques on isolated electric dipoles, as well as nontrivial forces and torques between line currents and point like charges, as well as among Dirac strings and other electromagnetic sources.

1 Introduction

The standard model (SM) of particle physics describes the fundamental forces as well as the elementary particles that make up all matter, being Lorentz and CPT invariant. However, in high energy scales of the order of the Planck energy $E_P \sim 10^{19}$GeV, it is believed that quantum gravitational effects can not be neglected, and there is the possibility of a spontaneous breaking of Lorentz and CPT symmetries [1], or even a fundamental change in the nature of quantum spacetime and its symmetries [2]. In the last decades, the study of possible Lorentz symmetry violations became an active field of theoretical and experimental research. The motivation is essentially twofold: first, one hopes to learn from eventual positive signs of Lorentz violation (LV) more on the fundamental theory that operates at the Planck scale; second, from each negative measure of LV one obtains a further test of Lorentz symmetry, leading to an extensive set of contemporary, nontrivial tests of relativistic symmetry [3].

An early model to investigate the consequences of an explicit Lorentz symmetry violation was proposed by Carroll, Field and Jackiw [4]. In that work, the Maxwell Lagrangian was augmented by a kind of four dimensional Chern-Simons term $k_\mu \epsilon^{\mu\nu\rho\sigma} A_\nu F_{\rho\sigma}$, where the photon field couples to the Lorentz violating parameter k_μ. A result of this violation is a change in the propagation of electromagnetic waves in the vacuum, which could be detected by experiments, and whose absence induces an experimental constraint on the LV parameter k_μ.

A very systematic approach for the introduction of LV in the SM was developed by Colladay and Kostelecký, in the form of the so-called Standard Model Extension (SME). This model incorporates in the SM structure all the Lorentz and CPT violating terms which respect renormalizability and gauge invariance [5,6]. The SME constitutes a quite general framework that facilitates investigations on the breaking of Lorentz and CPT symmetries. Theoretical aspects of LV have been investigated in Maxwell electrodynamics [7–11], QCD [12], gravity [13,14], noncommutative theories [15], statistical mechanics [16], QED [17–20], supersymmetry [21–24], electromagnetic wave propagation [25–27], to name a few. Experimental tests of LV have been performed in experiments involving photons [28,29], electrons [30,31], muons [32,33], and many others [3].

The SME, understood as an effective field theory, includes renormalizable LV interactions as well as higher derivatives, non renormalizable ones. These later are called nonminimal terms, and by dimensional analysis alone are expected to be subdominant relative to the minimal ones, so in principle they

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could be disregarded except if one deals with extremely high energy phenomena. The systematic study of the nonminimal terms of the SME was started in [34,35], where higher derivatives terms in both photon and fermion sectors were considered. Besides, some studies of different nonminimal LV interactions were carried out in [25,36–41], for example.

In [42], it was shown that nonminimal terms can induce nontrivial effects even in low energy phenomenology, in particular in searches for light pseudoscalars. The QCD axion being its most important representative, these kind of particles are extremely light and weakly interacting, thus constituting a class of WISPs (weakly interacting, slim particles) candidates for dark matter [43,44]. A particular set of nonminimal LV interactions was shown to provide a mechanism for generating the standard, Lorentz invariant (LI), interaction between the photon and light pseudoscalars that is investigated by current experimental efforts. Despite the nonminimal LV interactions being assumedly very small, they might represent a relevant contribution to these phenomenology, since the standard LI interactions involving WISPs are themselves very feeble.

The lesson is that there might exist open windows for the investigation of nonminimal LV effects in low energy physics, and photon physics is a natural place to start looking for this window. A starting point in this direction was the work [45], in which the complete low energy photon effective action for the nomminimal LV interaction studied in [42] was calculated. This result paves the way to investigate possible nonminimal LV effects in electrodynamics. A first result, already discovered in [45], is that the propagation of electromagnetic waves in the vacuum is not affected by the particular LV coupling considered in these articles.

In the present work, we look further for nontrivial effects of nonminimal LV in the classical interaction between electromagnetic sources. Our analysis parallels that of reference [46], which considered a minimal LV term from the non birefringent sector of the SME. As in that paper, we will find essential modifications due to the LV, with some effects that have no counterparts in the standard Maxwell theory. These will be results obtained without the recourse to perturbation theory.

The paper is organized as follows: in Sect. 2 we define the specific model we will study, and calculate the (exact) photon propagator. This result is used to obtain the classical interaction between different electromagnetic sources: point-like stationary charges (Sect. 3), a steady line current and a point-like stationary charge (Sect. 4), and Dirac strings (Sect. 5). Finally, Sect. 6 is dedicated to our final remarks and conclusions. Along the paper we shall deal with models in 3 + 1 dimensional space-time and use Minkowski coordinates with the diagonal metric with signature $(+,-,-,-)$.

2 The model

The explicit model we will consider in this work is defined by the following Lagrangian density,

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}-\frac{1}{2\gamma}\left(\partial_{\mu}A^{\mu}\right)^{2}+\frac{1}{2}\mathrm{d}^{\lambda}\mathrm{d}_{\alpha}\partial_{\mu}F_{\nu\lambda}\partial^{\nu}F^{\mu\alpha}+J^{\mu}A_{\mu}\,, \tag{1}$$

where A^μ is the electromagnetic field, $F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu$ is the field strength, J^μ is the external source, γ is a gauge parameter, and d^λ is a background vector taken to be constant and uniform in the reference frame where the calculations are performed. The parameter d^λ embodies the LV in our model. We restrict ourselves to the case of d^μ being a time-like background vector, namely $\mathrm{d}^2=\mathrm{d}^\mu\mathrm{d}_\mu>0$. The case of a light-like background vector can be obtained from our results by taking the limit $\mathrm{d}^2\to 0$, in which case all effects due to the LV disappear. The case of a space-like background vector is more subtle. Some preliminary results suggest that the interaction energy could exhibit an imaginary part in some circumstances, making the vacuum unstable in the presence of external sources; in addition, we could have tachyonic modes. These facts can be an indication that the model is not consistent for a space-like background vector, so this situation will not be considered in this paper.

Differently from the minimal LV coefficients appearing in the SME, the parameter d^λ is not adimentional but instead has length dimension one. The LV term in (1) is one of the low energy interactions obtained in [45], starting from the basic LV interaction $F_{\mu\nu}\mathrm{d}^\nu\overline{\psi}\gamma^\mu\psi$, between the photon and a very massive fermion field ψ which is integrated out to study the low energy phenomenology of the model.

The propagator $D^{\mu\nu}(x,y)$ for the Lagrangian (1) satisfies the differential equation

$$\left\{\partial^{2}\eta^{\mu\nu}-\left[\left(1-\frac{1}{\gamma}\right)-(\mathrm{d}\cdot\partial)^{2}\right]\partial^{\mu}\partial^{\nu}+\mathrm{d}^{\mu}\mathrm{d}^{\nu}\partial^{4}-\partial^{2}(\mathrm{d}\cdot\partial)\left(\mathrm{d}^{\mu}\partial^{\nu}+\mathrm{d}^{\nu}\partial^{\mu}\right)\right\}D_{\nu}{}^{\beta}(x,y)=\eta^{\mu\beta}\delta^{4}(x-y). \tag{2}$$

Fixing the Feynman gauge $\gamma=1$, one can solve this equation obtaining the exact propagator in the form of the Fourier integral

$$D^{\mu\nu}(x,y)=\int\frac{\mathrm{d}^{4}p}{(2\pi)^{4}}\left\{-\frac{\eta^{\mu\nu}}{p^{2}}+\frac{1}{\left[1-\mathrm{d}^{2}p^{2}+(p\cdot\mathrm{d})^{2}\right]}\times\left[-\mathrm{d}^{\mu}\mathrm{d}^{\nu}-\frac{(p\cdot\mathrm{d})^{2}}{p^{4}}p^{\mu}p^{\nu}+\frac{(p\cdot\mathrm{d})}{p^{2}}\left(p^{\mu}\mathrm{d}^{\nu}+\mathrm{d}^{\mu}p^{\nu}\right)\right]\right\}e^{-ip\cdot(x-y)}\,. \tag{3}$$

This propagator is the basic ingredient we need to obtain several relevant physical quantities of the model. We shall use it to compute the interaction mediated by the electromagnetic field between several different sources in the next sections.

3 Point-like charges

In this section we consider the interaction between two steady point-like charges in the model defined by Eq. (1). This charge configuration is described by the external source

$$J^I_\mu(\mathbf{x}) = q_1\eta^0_\mu\delta^3(\mathbf{x}-\mathbf{a}_1) + q_2\eta^0_\mu\delta^3(\mathbf{x}-\mathbf{a}_2), \tag{4}$$

where the location of the charges are specified by the vectors $\mathbf{a}_1$ and $\mathbf{a}_2$. The parameters q_1 and q_2 are the coupling constants between the vector field and the delta functions, and can be interpreted as electric charges.

The theory is quadratic in the field variables A^μ, so it can be shown that the contribution of the source $J(x)$ to the vacuum energy of the system is given by [47–49]

$$E = \frac{1}{2T}\int d^4y\int d^4x\ J_\mu(x)D^{\mu\nu}(x,y)J_\nu(y), \tag{5}$$

where the integration in y^0 is from $-T/2$ to $T/2$, and the limit $T\to\infty$ is implicit.

Substituting (4) into (5), discarding the self-interacting energy of each charge, we have

$$E^I = \frac{q_1q_2}{T}\int d^4y\int d^4x\ d^4y\ D^{00}(x,y) \times\delta^3(\mathbf{x}-\mathbf{a}_1)\,\delta^3(\mathbf{y}-\mathbf{a}_2). \tag{6}$$

By using the explicit form of the propagator in Eq. (3), computing the integrals in the following order: $d^3\mathbf{x}$, $d^3\mathbf{y}$, dx^0, dp^0 and dy^0, using the Fourier representation for the Dirac delta function $\delta(p^0) = \int dx/(2\pi)\exp(-ipx)$, and identifying the time interval as $T = \int_{-\frac{T}{2}}^{\frac{T}{2}} dy^0$, we can write

$$E^I = q_1q_2\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\exp(i\mathbf{p}\cdot\mathbf{a})}{\mathbf{p}^2} - \frac{q_1q_2(d^0)^2}{d^2}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\exp(i\mathbf{p}\cdot\mathbf{a})}{\left(\mathbf{p}^2+\frac{(\mathbf{d}\cdot\mathbf{p})^2}{d^2}\right)+\frac{1}{d^2}}, \tag{7}$$

where $d=\sqrt{d^2}$ and we defined $\mathbf{a}=\mathbf{a}_1-\mathbf{a}_2$, which is the distance between the two electric charges. Remembering that

$$\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\exp(i\mathbf{p}\cdot\mathbf{a})}{\mathbf{p}^2} = \frac{1}{4\pi|\mathbf{a}|}, \tag{8}$$

we can note that the first term in Eq. (7) gives the well-known Coulombian interaction.

In order to calculate the second integral in Eq. (7), we shall perform a change in the integration variables. For this task, we first split the vector $\mathbf{p}$ into two parts,

$$\mathbf{p} = \mathbf{p}_n + \mathbf{p}_p, \tag{9}$$

$\mathbf{p}_p$ being parallel and $\mathbf{p}_n$ normal to the vector $\mathbf{d}$; more explicitly,

$$\mathbf{p}_p = \mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right),\quad \mathbf{p}_n = \mathbf{p}-\mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right). \tag{10}$$

We also define the vector $\mathbf{q}$ as follows,

$$\mathbf{q} = \mathbf{p}_n + \mathbf{p}_p\sqrt{1+\frac{\mathbf{d}^2}{d^2}} \tag{11}$$

$$= \mathbf{p}+\mathbf{d}\left(\frac{\mathbf{d}\cdot\mathbf{p}}{\mathbf{d}^2}\right)\left(\frac{|d^0|}{d}-1\right). \tag{12}$$

With the previous definitions, we can write

$$\mathbf{p}_p = \frac{\mathbf{d}(\mathbf{d}\cdot\mathbf{q})}{\mathbf{d}^2}\frac{d}{|d^0|},\quad \mathbf{p}_n = \mathbf{q}-\frac{\mathbf{d}(\mathbf{d}\cdot\mathbf{q})}{\mathbf{d}^2}, \tag{13}$$

which implies in

$$\mathbf{p} = \mathbf{q}+\frac{(\mathbf{d}\cdot\mathbf{q})\mathbf{d}}{\mathbf{d}^2}\left(\frac{d}{|d^0|}-1\right), \tag{14}$$

and

$$\mathbf{q}^2 = \mathbf{p}^2+\frac{(\mathbf{d}\cdot\mathbf{p})^2}{d^2}. \tag{15}$$

Defining the spatial vector

$$\mathbf{b} = \mathbf{a}+\left(\frac{d}{|d^0|}-1\right)\frac{\mathbf{d}\cdot\mathbf{a}}{\mathbf{d}^2}\mathbf{d}, \tag{16}$$

and using Eq. (13), we can show that

$$\mathbf{p}\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{q}. \tag{17}$$

The Jacobian of the transformation from $\mathbf{p}$ to $\mathbf{q}$ can be calculated from Eq. (13), resulting in

$$\det\left[\frac{\partial\mathbf{p}}{\partial\mathbf{q}}\right] = \frac{1}{\sqrt{1+\frac{\mathbf{d}^2}{d^2}}} = \frac{d}{|d^0|}. \tag{18}$$

Putting all this together, we end up with

$$E^I = \frac{q_1q_2}{4\pi|\mathbf{a}|} - \frac{q_1q_2|d^0|}{d}\int\frac{d^3\mathbf{q}}{(2\pi)^3}\frac{\exp(i\mathbf{b}\cdot\mathbf{q})}{\mathbf{q}^2+\frac{1}{d^2}}. \tag{19}$$

Using the fact that, for $\mathrm{d}^2 > 0$ [48],

$$\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3} \frac{\exp(i\mathbf{b}\cdot\mathbf{q})}{\mathbf{q}^2 + \left(\frac{1}{\mathrm{d}^2}\right)} = \frac{1}{4\pi|\mathbf{b}|} \exp\left(-\frac{|\mathbf{b}|}{\mathrm{d}}\right) , \tag{20}$$

and performing some manipulations we arrive at,

$$E^I = \frac{q_1 q_2}{4\pi} \left[\frac{1}{|\mathbf{a}|} - \frac{|\mathrm{d}^0|}{\mathrm{d}} \frac{1}{|\mathbf{b}|} \exp\left(-\frac{|\mathbf{b}|}{\mathrm{d}}\right) \right] , \tag{21}$$

where,

$$|\mathbf{b}| = \sqrt{\mathbf{a}^2 - \frac{(\mathbf{d}\cdot\mathbf{a})^2}{|\mathrm{d}^0|^2}} . \tag{22}$$

It is important to realize that $|\mathbf{b}|$ vanishes only of $\mathbf{a} = 0$. This can be seen by taking a coordinate system where $\mathbf{d}$ lies along the $\hat{z}$ axis. In spherical coordinates, with θ standing for the polar angle for $\mathbf{a}$, Eq. (22) reads

$$|\mathbf{b}| = |\mathbf{a}|\sqrt{1 - \left(\frac{|\mathbf{d}|}{|\mathrm{d}^0|}\cos(\theta)\right)^2} . \tag{23}$$

The restriction $\mathrm{d}^2 > 0$ guarantees that the term inside the square root will always be strictly positive.

Equation (21) gives the interaction energy between two point-like charges mediated by the electromagnetic field with the specific Lorentz violating coupling contained in Eq. (1). The d^μ dependent term in (21) is a correction to the Coulomb interaction due the Lorentz symmetry breaking, leading to an anisotropic interaction between the charges. The LI limit $\mathrm{d}^\mu \to 0$ of this result must be taken with the restriction $(\mathrm{d}^0)^2 > \mathbf{d}^2$: in this case, it can shown that the second term inside the brackets vanishes and we are left with the standard Coulombian interaction between the charges. The same happens for the light-like limit of the vector d^μ, i.e., if $\mathrm{d} \to 0$.

If $\mathbf{d} = 0$, Eq. (21) reduces to the simple form

$$E^I(\mathbf{d} = 0) = \frac{q_1 q_2}{4\pi} \left[\frac{1}{|\mathbf{a}|} - \frac{1}{|\mathbf{a}|} \exp\left(-\frac{|\mathbf{a}|}{|d^0|}\right) \right] , \tag{24}$$

which is the Coulombian interaction corrected by an Yukawa-like interaction, with $1/|d^0|$ as a mass parameter. It is interesting to notice that the same structure for E^I can be found for another (Lorentz invariant) gauge field theory which exhibits higher order derivatives [50], the Podolsky–Lee–Wick electrodynamics [51–55]. As another noteworthy particular case, if the distance vector $\mathbf{a}$ is perpendicular to the background vector $\mathbf{d}$, Eq. (21) becomes

$$E^I(\mathbf{d}\cdot\mathbf{a} = 0) = \frac{q_1 q_2}{4\pi} \left[\frac{1}{|\mathbf{a}|} - \frac{|\mathrm{d}^0|}{\mathrm{d}} \frac{1}{|\mathbf{a}|} \exp\left(-\frac{|\mathbf{a}|}{|\mathrm{d}^0|}\right) \right] . \tag{25}$$

The force between the two charges can be calculated from Eqs. (21) and (22), resulting in

$$\begin{aligned} \mathbf{F}^I &= -\nabla E^I \\ &= \frac{q_1 q_2}{4\pi} \left[\frac{\mathbf{a}}{|\mathbf{a}|^3} - \frac{|d^0|}{d} \frac{1}{|\mathbf{b}|^3} \left(1 + \frac{|\mathbf{b}|}{d}\right) \right. \\ &\quad \left. \times \exp\left(-\frac{|\mathbf{b}|}{d}\right) \left(\mathbf{a} - \frac{(\mathbf{d}\cdot\mathbf{a})\,\mathbf{d}}{|d^0|^2}\right) \right] . \end{aligned} \tag{26}$$

The interaction energy (21) exhibits anisotropy due to the presence of the background vector d^μ. An interesting consequence of this is the emergence of an spontaneous torque on an electric dipole. To see this, we consider a typical dipole composed by two opposite electric charges, $q_1 = -q_2 = q$, placed at the positions $\mathbf{a}_1 = \mathbf{R} + \frac{\mathbf{A}}{2}$ and $\mathbf{a}_2 = \mathbf{R} - \frac{\mathbf{A}}{2}$, $\mathbf{A}$ taken to be a fixed vector. From Eq. (21), we obtain

$$E_{dipole} = -\frac{q^2}{4\pi|\mathbf{A}|} \left[1 - \frac{|\mathrm{d}^0|}{\mathrm{d}} \frac{1}{f(\Theta)} \exp\left(-\frac{|\mathbf{A}|f(\Theta)}{\mathrm{d}}\right) \right] , \tag{27}$$

where

$$f(\Theta) = \sqrt{1 - \frac{\mathbf{d}^2\cos^2\Theta}{|\mathrm{d}^0|^2}} , \tag{28}$$

with $\Theta \in [0, 2\pi)$ standing for the angle between $\mathbf{A}$ and the background vector $\mathbf{d}$. This interaction energy leads to an spontaneous torque on the dipole as follows,

$$\begin{aligned} \tau_{dipole} &= -\frac{\partial E_{dipole}}{\partial\Theta} \\ &= \frac{q^2}{8\pi|\mathbf{A}|} \frac{\mathbf{d}^2}{\mathrm{d}|\mathrm{d}^0|} \frac{1}{f^3(\Theta)} \left(1 + \frac{|\mathbf{A}|f(\Theta)}{\mathrm{d}}\right) \\ &\quad \times \sin(2\Theta)\exp\left(-\frac{|\mathbf{A}|f(\Theta)}{d}\right) . \end{aligned} \tag{29}$$

This spontaneous torque on the dipole is an exclusive effect due to the Lorentz violating background. If $d^\mu = 0$, the torque vanishes, as it should, as well as for the specific configurations $\Theta = 0, \pi/2, \pi$. Finally, we note that if $\mathbf{d} = 0$, this effect is also absent.

4 A steady current line and a point-like charge

In this section we study the interaction energy between a steady line current and a point-like stationary charge. This interaction does not occur in Maxwell electrodynamics, but it may emerge in theories with Lorentz violation [46], as well as in LI theories with higher order derivatives [50].

Let us consider a steady line current flowing parallel to the z-axis, along the straight line placed at $\mathbf{A} = (A^1, A^2, 0)$. The electric charge is placed at the position $\mathbf{s}$. The external source for this system is given by

$$J^{II}_{\mu}(\mathbf{x}) = I\eta^3{}_{\mu}\delta^2(\mathbf{x}_\perp - \mathbf{A}) + q\eta^0{}_{\mu}\delta^3(\mathbf{x}-\mathbf{s}), \tag{30}$$

where we defined the vector position perpendicular to the straight line current $\mathbf{x}_\perp = (x^1, x^2, 0)$. The parameters I and q stand for, respectively, the current intensity and the electric charge.

Substituting (30) into (5) and discarding self-interaction terms, we have

$$E^{II} = \frac{qI}{T}\int d^4y \int d^4x\, D^{30}(x,y) \times \delta^2(\mathbf{x}_\perp - \mathbf{A})\,\delta^3(\mathbf{y}-\mathbf{s}), \tag{31}$$

where the integration limits for y^0 are as in the previous section. Substituting the explicit form for the propagator (3) and evaluating the integrals $d^2\mathbf{x}_\perp$, $d^3\mathbf{y}$, dx^3, dp^3, dx^0, dp^0 and dy^0, we obtain

$$E^{II} = -\frac{qId^3d^0}{d^2}\int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2}\frac{\exp(i\mathbf{p}_\perp\cdot\mathbf{a}_\perp)}{\left(\mathbf{p}_\perp^2 + \frac{(\mathbf{d}_\perp\cdot\mathbf{p}_\perp)^2}{d^2}\right) + \frac{1}{d^2}}, \tag{32}$$

where again $\int_{-T/2}^{T/2} dy^0 = T$, and defined the perpendicular momentum $\mathbf{p}_\perp = (p^1, p^2, 0)$ and the distance between the charge and the line current $\mathbf{a}_\perp = (A^1 - s^1, A^2 - s^2, 0)$.

Proceeding as in before, the interaction energy in this case can be written as

$$E^{II} = -\frac{qId^3d^0}{d\sqrt{(d^0)^2-(d^3)^2}}\int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\frac{\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)}{\mathbf{q}_\perp^2+\frac{1}{d^2}}, \tag{33}$$

where we defined

$$\mathbf{r}_\perp = \mathbf{a}_\perp + \left[\frac{d}{\sqrt{(d^0)^2-(d^3)^2}} - 1\right]\frac{\mathbf{d}_\perp\cdot\mathbf{a}_\perp}{\mathbf{d}_\perp^2}\mathbf{d}_\perp. \tag{34}$$

Due to the fact that $d^2 > 0$, we can use that [48]

$$\int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2}\frac{\exp(i\mathbf{q}_\perp\cdot\mathbf{r}_\perp)}{\mathbf{q}_\perp^2+\left(\frac{1}{d}\right)^2} = \frac{1}{2\pi}K_0\left(\frac{|\mathbf{r}_\perp|}{d}\right), \tag{35}$$

thus obtaining

$$E^{II} = -\frac{qI}{2\pi}\frac{d^3d^0}{d\sqrt{(d^0)^2-(d^3)^2}}K_0\left(\frac{|\mathbf{r}_\perp|}{d}\right), \tag{36}$$

where K_0 is a modified Bessel function of the second kind [56], and

$$|\mathbf{r}_\perp| = \sqrt{\mathbf{a}_\perp^2 - \frac{(\mathbf{d}_\perp\cdot\mathbf{a}_\perp)^2}{(d^0)^2-(d^3)^2}}. \tag{37}$$

Defining $\hat{I}$ as the unit vector along the straight line current and noticing that d^3 is the projection of the vector $\mathbf{d}$ along $\hat{I}$, one can write the energy (36) in the form

$$E^{II} = -\frac{Iq}{2\pi}\frac{(\mathbf{d}\cdot\hat{I})\,d^0}{d\sqrt{(d^0)^2-(\mathbf{d}\cdot\hat{I})^2}}K_0\left(\frac{|\mathbf{r}_\perp|}{d}\right). \tag{38}$$

This interaction energy is an effect due solely to the Lorentz violating background, having no counterpart in Maxwell theory. Clearly, if the background four-vector d^μ is zero, there is no interaction energy. The energy (38) is proportional to the electric charge q as well as to the projection of the Lorentz-symmetry breaking vector $\mathbf{d}$ along the current line. If the current line flows perpendicular to $\mathbf{d}$, there is no interaction; the same happens if $\mathbf{d} = 0$. In the limit of a light-like background vector, $d \to 0$, the energy (38) vanishes.

The force on the point charge can be obtained from Eq. (38) as follows,

$$\begin{aligned}\mathbf{F}^{II} &= -\nabla_{\mathbf{a}_\perp}E^{II}\\ &= -\frac{qI}{2\pi|\mathbf{b}_\perp|}\frac{(\mathbf{d}\cdot\hat{I})\,d^0}{d^2\sqrt{(d^0)^2-(\mathbf{d}\cdot\hat{I})^2}}K_1\left(\frac{|\mathbf{r}_\perp|}{d}\right)\\ &\quad\times\left[\mathbf{a}_\perp - \frac{(\mathbf{d}_\perp\cdot\mathbf{a}_\perp)}{(d^0)^2-(\mathbf{d}\cdot\hat{I})^2}\mathbf{d}_\perp\right].\end{aligned} \tag{39}$$

From Eq. (38), one can also obtain a torque on the line current, due to the interaction with the point charge. Denoting by ϕ the angle between $\mathbf{d}_\perp$ and $\mathbf{a}_\perp$, we have

$$\begin{aligned}\tau^{II} &= -\frac{\partial E^{II}}{\partial\phi}\\ &= -\frac{qI}{4\pi g(\phi)}\frac{d^0\,\mathbf{d}_\perp^2\,(\mathbf{d}\cdot\hat{I})}{d^2\left[(d^0)^2-(\mathbf{d}\cdot\hat{I})^2\right]^{3/2}}\\ &\quad\times K_1\left(\frac{g(\phi)}{d}\right)\sin 2\phi,\end{aligned} \tag{40}$$

where we defined the function

$$g(\phi) = \sqrt{\mathbf{a}_\perp^2 - \frac{\mathbf{d}_\perp^2\mathbf{a}_\perp^2\cos^2\phi}{(d^0)^2-(\mathbf{d}\cdot\hat{I})^2}}. \tag{41}$$

If $\phi = 0, \pi/2, \pi$, $d^\mu = 0$ or $\mathbf{d} = 0$, the torque in Eq. (40) vanishes.

5 Dirac strings

In this section we study the interaction between electromagnetic sources, including Dirac strings. They might be seen as zero width solenoids that connect magnetic monopoles, and their existence is compatible with the standard Maxwell's electrodynamics, where they lead to the Dirac quantization rule for the electric charge. In Maxwell electrodynamics, a Dirac string does not produce any obvious physical effects, because it does not produce electromagnetic field in its exterior region, just a (divergent) magnetic field along the string. It is still relevant to investigate whether a Dirac string can produce observable effects in an extended electrodynamic theory. We will show this is indeed the case, since we will show that non trivial interactions between Dirac strings themselves and other electromagnetic sources will appear, due to the presence of Lorentz symmetry breaking.

We start by considering a system composed by a point-like charge placed at position $\mathbf{a}$ and a Dirac string, both of them stationary. This system is described by the source

$$J^{III}_{\mu}(\mathbf{x}) = J_{(D)\mu}(\mathbf{x}) + q\eta^{0}_{\mu}\delta^{3}(\mathbf{x}-\mathbf{a}), \tag{42}$$

where $J^{\mu}_{(D)}(x)$ stands for the source corresponding to the Dirac string. Choosing a coordinate system where the Dirac string lies along the z-axis with internal magnetic flux Φ, $J^{\mu}_{(D)}(x)$ is given explicitly by [46,57,58]

$$J^{\mu}_{(D)}(\mathbf{x}) = i\Phi(2\pi)^{2}\int \frac{d^{4}p}{(2\pi)^{4}}\delta(p^{0})\delta(p^{3})\varepsilon^{0\mu}_{\ \ \nu 3}\,p^{\nu}e^{-ipx}\ , \tag{43}$$

where $\varepsilon^{\alpha\beta\mu\nu}$ is the Levi–Civita tensor with $\varepsilon^{0123}=1$. If $\Phi>0$ we have the internal magnetic pointing at the positive direction of $\hat{z}$, whereas for $\Phi<0$, the internal magnetic field points in the opposite direction. In Maxwell electrodynamics, the source (43) produces the vector potential

$$A^{\mu}(x) = \frac{\Phi}{2\pi}\left(0, -\frac{x^{2}}{(x^{1})^{2}+(x^{2})^{2}}, \frac{x^{1}}{(x^{1})^{2}+(x^{2})^{2}}, 0\right)\ , \tag{44}$$

which is, in fact, the vector potential related to a Dirac string, with internal magnetic flux Φ, lying along the z axis.

From now on, the sub-index $\perp$ means the component of a given vector perpendicular to the Dirac string. By following the same steps presented in the previous sections, we obtain for the interaction energy between the Dirac string and the point-like charge the expression

$$E^{III} = -i\frac{q\Phi d^{0}}{d^{2}}\int \frac{d^{2}\mathbf{p}_{\perp}}{(2\pi)^{2}}\frac{\left[\hat{z}\cdot(\mathbf{p}_{\perp}\times\mathbf{d}_{\perp})\right]}{\left(\mathbf{p}_{\perp}^{2}+\frac{(\mathbf{d}_{\perp}\cdot\mathbf{p}_{\perp})^{2}}{d^{2}}\right)+\left(\frac{1}{d}\right)^{2}} \times \exp\left(i\mathbf{p}_{\perp}\cdot\mathbf{a}_{\perp}\right). \tag{45}$$

This integral can be manipulated similarly to Eq. (33). Using (34), we arrive at

$$E^{III} = -\frac{q\Phi d^{0}}{d\sqrt{(d^{0})^{2}-(d^{3})^{2}}}\left[\hat{z}\cdot(\nabla_{\mathbf{r}_{\perp}}\times\mathbf{d}_{\perp})\right] \times\int\frac{d^{2}\mathbf{q}_{\perp}}{(2\pi)^{2}}\frac{\exp\left(i\mathbf{q}_{\perp}\cdot\mathbf{r}_{\perp}\right)}{\mathbf{q}_{\perp}^{2}+\left(\frac{1}{d}\right)^{2}}\ , \tag{46}$$

where $\nabla_{\mathbf{r}_{\perp}} = \left(\frac{\partial}{\partial r^{1}}, \frac{\partial}{\partial r^{2}}, 0\right)$. After some manipulations, and identifying $\hat{z}=\hat{B}_{int}$ as the unit vector pointing along the internal magnetic field, we obtain

$$E^{III} = \frac{q\Phi}{2\pi|\mathbf{r}_{\perp}|}\frac{d^{0}}{d^{2}\sqrt{(d^{0})^{2}-(\mathbf{d}\cdot\hat{B}_{int})^{2}}}K_{1}\left(\frac{|\mathbf{r}_{\perp}|}{d}\right) \times\left[\hat{B}_{int}\cdot(\mathbf{a}_{\perp}\times\mathbf{d}_{\perp})\right]. \tag{47}$$

This interaction energy can be seen to lead to a force between the Dirac string and the charge, as well as to a torque on the Dirac string.

The next example is given by two parallel Dirac strings placed a distance $\mathbf{a}_{\perp}$ apart. We take a coordinate system where the first string lies along the z axis, with internal magnetic flux Φ_{1}, and the second string lies along the line that crosses the xy place at $\mathbf{a}_{\perp}=(a^{1},a^{2},0)$, with internal magnetic flux Φ_{2}. The corresponding source is given by

$$J^{IV}_{\mu}(\mathbf{x}) = J_{\mu(D,1)}(\mathbf{x}) + J_{\mu(D,2)}(\mathbf{x})\ , \tag{48}$$

where $J^{\mu}_{(D,1)}(\mathbf{x})$ is given by the right hand side of Eq. (43), with Φ replaced by Φ_{1}, and

$$J^{\mu}_{(D,2)}(\mathbf{x}) = i\Phi_{2}\int\frac{d^{4}p}{(2\pi)^{2}}\delta(p^{0})\delta(p^{3})\varepsilon^{0\mu}_{\ \ \nu 3}\,p^{\nu}e^{-ipx}e^{-i\mathbf{p}_{\perp}\cdot\mathbf{a}_{\perp}}. \tag{49}$$

Proceeding as in the previous cases, and identifying the length of the Dirac string as $L=\int dx^{3}$, we can show that the interaction energy between the two Dirac strings is given by

$$E^{IV} = \Phi_{1}\Phi_{2}L\Bigg[-\int\frac{d^{2}\mathbf{p}_{\perp}}{(2\pi)^{2}}e^{i\mathbf{p}_{\perp}\cdot\mathbf{a}_{\perp}} + \frac{1}{d\sqrt{(d^{0})^{2}-(\mathbf{d}\cdot\hat{B}_{int})^{2}}} \times\int\frac{d^{2}\mathbf{q}_{\perp}}{(2\pi)^{2}}\frac{\left[(\mathbf{d}_{\perp}\cdot\mathbf{q}_{\perp})^{2}-\mathbf{q}_{\perp}^{2}\mathbf{d}_{\perp}^{2}\right]}{\mathbf{q}_{\perp}^{2}+\left(\frac{1}{d}\right)^{2}}e^{i\mathbf{q}_{\perp}\cdot\mathbf{r}_{\perp}}\Bigg]. \tag{50}$$

Provided that $\mathbf{a}_\perp$ is non-zero, the first term inside the brackets in this result vanishes. The remaining integral can be calculated with the procedure outlined in the previous sections. The interaction energy between the two parallel Dirac strings per unit length $\mathcal{E}^{IV}$ ends up given by,

$$\mathcal{E}^{IV}=\frac{E^{IV}}{L}=\frac{\Phi_1\Phi_2}{4\pi}\frac{1}{\mathrm{d}^3\sqrt{(\mathrm{d}^0)^2-(\mathbf{d}\cdot\hat{B}_{int})^2}}\times\left\{\mathbf{d}_\perp^2K_0\left(\frac{|\mathbf{r}_\perp|}{d}\right)+\frac{1}{\mathbf{r}_\perp^2}K_2\left(\frac{|\mathbf{r}_\perp|}{\mathrm{d}}\right)\left[\mathbf{d}_\perp^2\mathbf{a}_\perp^2-\left(2-\frac{\mathbf{d}_\perp^2}{(\mathrm{d}^0)^2-(\mathbf{d}\cdot\hat{B}_{int})^2}\right)(\mathbf{d}_\perp\cdot\mathbf{a}_\perp)^2\right]\right\}. \quad (51)$$

The last example we consider is given by a Dirac string alongside a steady line current, both parallel to each other. The corresponding external source is

$$J_\mu^V(\mathbf{x})=I\eta^3{}_\mu\delta^2(\mathbf{x}_\perp-\mathbf{a}_\perp)+J^\mu_{(D)}(\mathbf{x})\ , \quad (52)$$

where $J^\mu_{(D)}(\mathbf{x})$ is given by (43). Proceeding as before, we obtain the result

$$\mathcal{E}^V=\frac{E^V}{L}=\frac{I\Phi}{2\pi|\mathbf{r}_\perp|}\frac{\mathbf{d}\cdot\hat{B}_{int}}{\mathrm{d}^2\sqrt{(\mathrm{d}^0)^2-(\mathbf{d}\cdot\hat{B}_{int})^2}}\times K_1\left(\frac{|\mathbf{r}_\perp|}{\mathrm{d}}\right)\left[\hat{B}_{int}\cdot(\mathbf{a}_\perp\times\mathbf{d}_\perp)\right]\ , \quad (53)$$

for the energy line density.

The results of this section are all exclusive effects of the LV, having no counterpart in Maxwell theory, in which the interaction energy vanishes in all cases considered here. In the limit $\mathrm{d}^\mu\to0$, all these effects disappears, as they should. The same happens in the limit of a light-like background vector, $\mathrm{d}\to0$.

6 Conclusions and perspectives

In this paper we investigated the interaction between sources for the electromagnetic field in the presence of the Lorentz violating higher derivative interaction $\mathrm{d}^\lambda\mathrm{d}_\alpha\partial_\mu F_{\nu\lambda}\partial^\nu F^{\mu\alpha}$. This interaction is induced by a specific setting of nonminimal LV which was shown to lead to low energy effects relevant for the physics of light pseudoscalars interacting with photons [42,45]. We obtained results with no resource to perturbation theory in the background vector for the specific case where $\mathrm{d}^\mu\mathrm{d}_\mu=\mathrm{d}^2>0$ (time-like interval), which provided us with different physical effects with no counterpart in Maxwell theory. The case of a light-like background vector, $\mathrm{d}\to0$, can be obtained from our results. In this situation, the interaction between two point-like charges becomes the Coulombian one, and all other nontrivial interactions obtained in this paper vanish. On the other hand, a space-like background vector was not considered here. In this situation the calculations are much more difficult and some preliminary results suggest that it could lead to inconsistencies.

We have shown the emergence of an spontaneous torque on a classical electromagnetic dipole, as well as a nontrivial interaction between a steady straight line current and a point-like charge. We also investigated some phenomena due to the presence of Dirac strings. We showed that a Dirac string have a nontrivial interaction with a point charge, with a straight line steady current, as well as with another Dirac string. All these phenomena are effects due to the Lorentz-violation background. The nontrivial LV effects uncovered in this paper represent another instance where nominimal LV terms may induce low energy phenomenology, and might open up a window to look for experimental limits on these LV coefficients.

As a final remark, we point out that this paper all field sources are spinless. An interesting extension of this work would be the investigation of spin effects in the interactions between field sources.

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