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$B^*_{s,d} \to \mu^+\mu^-$ and its impact on $B_{s,d} \to \mu^+\mu^-$

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Abstract This study investigates $B^*_{s,d} \to \mu^+\mu^-$ in the dimuon distributions and the hadronic contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$. The $\mu^+\mu^-$ decay widths of the vector mesons $B^*_{s,d}$ are approximately a factor of 700 larger than the corresponding scalar mesons $B_{s,d}$. The ratio of the branching fractions obtained, $\frac{Br(B^*_{s,d}\to\mu^+\mu^-)}{Br(B_{s,d}\to\mu^+\mu^-)}$, is approximately $\frac{0.3\times\mathrm{eV}}{\Gamma(B^*_{s,d}\to B_{s,d}\gamma)}$. The hadronic contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$ is also estimated. The relative increase in the $B_{s,d} \to \mu^+\mu^-$ amplitude is approximately $(0.01 \pm 0.006)\sqrt{\frac{\Gamma(B^*_{s,d}\to B_{s,d}\gamma)}{100\ \mathrm{eV}}}$. If we select $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) = 2$ eV, then the branching fractions of the vector mesons to the lepton pair are 5.3×10^{-10} and 1.6×10^{-11} for B^*_s and B^*_d, respectively. If we select $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) = 200$ eV, then the updated branching fractions of the scalar mesons to the muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for B_s and B_d, respectively. If we select the recent predicted M1 widths $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) = 313, 1230$ eV (arXiv:1607.02169), then the updated branching fractions are $(3.8 \pm 0.3) \times 10^{-9}$ and $(1.2 \pm 0.1) \times 10^{-10}$ for $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$, respectively. Further studies on $B^*_{s,d}$, including those on dielectron decay and two-body decay with J/ψ, should be conducted.

1 Introduction

The leptonic decays of the $B_{s,d}$ mesons play an important role in the standard model (SM) and the new physics (NP) [1,2]. The leptonic decays are highly suppressed in the SM because flavor-changing neutral current decays are generated through W-box and Z-penguin diagrams. Furthermore, the branching fractions of the leptonic decays of scalar meson go through an additional helicity suppression factor by m^2_μ/M^2_S, where m_μ and M_S denote the masses of the muon lepton and the scalar meson, respectively. The suppression can be removed in several NP models, such as the two-Higgs-doublet models [3], the minimal supersymmetric standard model [4], the next minimal supersymmetric standard model [5], the dark matter [6], the universal extra dimensional model [7], the lepton universality violation model [8], the fourth generation of fermions [9], and so on [10]. The branching fractions of $B_{s,d} \to \mu^+\mu^-$ measured by the CMS and LHCb Collaborations [2], and predicted within the SM [1] with NNLO QCD [11] and NLO EW [12] corrections are presented in Table 1.

On one hand, the experimental branching fractions of $B_{s,d} \to \mu^+\mu^-$ are measured from the dimuon distributions by the CMS and LHCb Collaborations [2]. Thus, the process $B^*_{s,d} \to \mu^+\mu^-$ will enhance the dimuon distributions for mass splitting between $B_{s,d}$ and $B^*_{s,d}$ at approximately 45 MeV. On the other hand, the hadronic contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$ is missing in the theoretical prediction [1]. Therefore, this study focuses on $B^*_{s,d} \to \mu^+\mu^-$ and its impact on $B_{s,d} \to \mu^+\mu^-$ within SM. The $B_s \to B^*_s\gamma \to \mu^+\mu^-\gamma$ process was considered in Ref. [13]. $B^*_{s,d} \to \mu^+\mu^-$ was recently considered in Refs. [14,15]. Moreover, Refs. [16,17]. also investigated the hadronic contribution of charmonium in $B \to K^{(*)}\ell^+\ell^-$ and $B \to X_s\gamma$.

2 The Decay of $B^*_s(B^*_d) \to \mu^+\mu^-$

An effective Lagrangian related to $b\bar{s} \to \mu^+\mu^-$ within the SM is given in Refs. [18–20]

$$\mathscr{L} = \mathscr{N}\left[C^{\mathrm{eff}}_7(\mu_f)\mathscr{O}^\gamma_7 + C^{\mathrm{eff}}_9(\mu_f)\mathscr{O}^V_9 + C_{10}(\mu_f)\mathscr{O}^A_{10}\right], \tag{1}$$

where $\mathscr{N} = \frac{G_F}{\sqrt{2}}V_{tb}V^*_{ts}\frac{e^2}{4\pi^2}$, and the operators $\mathscr{O}_{7,9,10}$ read as follows:

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Table 1 The branching fractions of $B_{s,d} \to \mu^+\mu^-$ measured by the CMS and LHCb Collaborations [2] and predicted within the SM [1] with NNLO QCD [11] and NLO EW [12] corrections included

	EX [2]	SM [1]	Deviations
$Br(B_d \to \mu^+\mu^-)$	$(3.9^{+1.6}_{-1.4}) \times 10^{-10}$	$(1.06 \pm 0.09) \times 10^{-10}$	2.2σ
$Br(B_s \to \mu^+\mu^-)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$	$(3.66 \pm 0.23) \times 10^{-9}$	1.2σ
$\frac{Br(B_d\to\mu^+\mu^-)}{Br(B_s\to\mu^+\mu^-)}$	$0.14^{+0.08}_{-0.06}$	$0.0295^{+0.0028}_{-0.0025}$	2.3σ

$$\mathscr{O}_7^\gamma = -\frac{2im_b(p_\mu^\nu + p_{\bar\mu}^\nu)}{(p_\mu + p_{\bar\mu})^2}(\bar s\sigma_{\rho\nu}P_R b)(\bar\mu\gamma^\rho\mu), \tag{2}$$

$$\mathscr{O}_9^V = (\bar s\gamma_\rho P_L b)(\bar\mu\gamma^\rho\mu), \tag{3}$$

$$\mathscr{O}_{10}^A = (\bar s\gamma_\rho P_L b)(\bar\mu\gamma^\rho\gamma_5\mu), \tag{4}$$

where $P_L = (1-\gamma_5)/2$, $P_R = (1+\gamma_5)/2$. The Wilson coefficients are $C^{\rm eff}_{7,9,10}(\mu_f) = (-0.316, 4.403-0.47i, -4.493)$ at $\mu_f = m_b = 4.5$ GeV [15]. The superscripts γ, V, and A denote the contributions from photon, vector current, and axial vector current, respectively.

The relationships between the quark level operators and the meson are described as follows:

$$\langle 0|\bar s\gamma^\mu b|B_s^*(q,\varepsilon)\rangle = m_{B_s^*} f_{B_s^*}\,\varepsilon^\mu, \tag{5}$$

$$\langle 0|\bar s\sigma^{\mu\nu} b|B_s^*(q,\varepsilon)\rangle = -i\, f^T_{B_s^*}(q^\mu\varepsilon^\nu - \varepsilon^\mu q^\nu), \tag{6}$$

$$\langle 0|\bar s\gamma^\mu\gamma_5 b|B_s(q)\rangle = if_{B_s}\, q^\mu, \tag{7}$$

where the three decay constants f_{B_s}, $f_{B_s^*}$, and $f^T_{B_s^*}$ depend on the renormalization scale, whose relationships have been investigated in the heavy-quark limit of the Ref. [21]. Ignoring the mass difference between B_s and B_s^* and the high-order QCD corrections, we derive the following expression:

$$f_{B_s^*} = f^T_{B_s^*} = f_{B_s}. \tag{8}$$

Afterward, the $B_s^*(B_s) \to \mu^+\mu^-$ amplitudes are expressed as follows [13]:

$$\mathscr{M}(B_s^* \to \mu^+\mu^-) = f_{B_s^*}\frac{\mathscr{N}}{2}m_{B_s^*}\bar\mu\not\epsilon\Big[C_V^{\rm eff} + C_{10}\gamma_5\Big]\mu,$$
$$\mathscr{M}(B_s \to \mu^+\mu^-) = if_{B_s}\mathscr{N}C_{10}m_\mu\bar\mu\gamma_5\mu, \tag{9}$$

where

$$C_V^{\rm eff} = C_9^{\rm eff} + 2\frac{m_b}{m_{B_s^*}}C_7^{\rm eff}. \tag{10}$$

The helicity suppression factor m_μ^2/m_M^2 in the decay width is removed in the vector meson decay. Then the $B_s^*(B_s) \to \mu^+\mu^-$ decay widths are obtained:

$$\Gamma(B_s^* \to \mu^+\mu^-) = \frac{G_f^2\alpha_{em}^2}{96\pi^3}\left|V_{tb}V_{ts}^*\right|^2\left(|C_{10}|^2 + \left|C_V^{\rm eff}\right|^2\right) \times m^3_{B_s^*} f^2_{B_s^*}\left(1 + \mathscr{O}(m_\mu^2/m_{B_s}^2)\right),$$
$$\Gamma(B_s \to \mu^+\mu^-) = \frac{G_f^2\alpha_{em}^2}{16\pi^3}\left|V_{tb}V_{ts}^*\right|^2|C_{10}|^2 \times m_\mu^2 m_{B_s} f^2_{B_s}\left(1 + \mathscr{O}(m_\mu^2/m_{B_s}^2)\right),$$
$$\frac{\Gamma(B_s^* \to \mu^+\mu^-)}{\Gamma(B_s \to \mu^+\mu^-)} = \frac{\left(|C_{10}|^2 + \left|C_V^{\rm eff}\right|^2\right)m^3_{B_s^*}f^2_{B_s^*}}{6\,|C_{10}|^2\,m_\mu^2 m_{B_s} f^2_{B_s}} \times\left(1 + \mathscr{O}(m_\mu^2/m_{B_s}^2)\right). \tag{11}$$

The decay width ratio is approximately 700 for $B_s^{(*)}$ and $B_d^{(*)}$ both.

3 The impact of $B_s^*(B_d^*) \to \mu^+\mu^-$ on $B_s(B_d) \to B_s^*(B_d^*)\gamma \to \mu^+\mu^-$

Furthermore, $B^*_{s,d}$ will impact on the $B_{s,d}$ leptonic decay through the loop contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$. The Feynman diagrams are shown in Fig. 1. This calculation is a part of EM corrections to $B_{s,d} \to \mu^+\mu^-$. The NLO EW corrections of $B_{s,d} \to \mu^+\mu^-$ within the SM have been calculated in Refs. [1,12]. The hadronic contribution of $B_{s,d} \to B^*_{s,d}\gamma_{\rm soft} \to \mu^+\mu^- + \gamma_{\rm soft}$ has been calculated in Ref. [13]. However, the contribution of $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$ is missing in the previous calculation. This calculation is incomplete. For instance, there is double counting between the NLO EW corrections $B_s \to b+s+\gamma \to \mu^+\mu^-$ [12] and this calculation $B_s \to B_s^*\gamma \to \mu^+\mu^-$. We considered that the contribution of $B_s \to B_s^*\gamma \to \mu^+\mu^-$ will be retained only when the B_s^* is nearly on-shell. If B_s^* is far away from

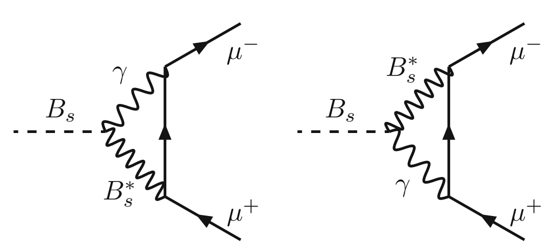

Fig. 1 Feynman diagrams of $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$

mass shell, $B_s \to b+s+\gamma \to \mu^+\mu^-$ is dominant. As is well known, the propagator of hadron will be modified duo to the off-shell of hadron [22], and the Wilson coefficients $C^{\rm eff}_{7,9,10}$ will be modified too [17,19]. Therefore, this treatment may be regarded as a crude estimate, and the error may be large in this treatment.

The vertex of $B_{s,d} \to B^*_{s,d}\gamma$ is expressed as the following operator [23,24]:

$$\mathscr{M}_{B_sB^*_s\gamma} = \sum_{q=s,b} < B^*_s\gamma|iee_q\bar{q}(p_{\bar{q}})\gamma_\mu q(p_q)|B_s > \\ = \sum_{q=s,b} \varepsilon^\mu_\gamma p^\nu_\gamma < B^*_s|iee_q\bar{q}(p_{\bar{q}})\frac{i\sigma_{\mu\nu}}{2m_q}q(p_q)|B_s > .$$

We can simplify the matrix element $< B^*_s|\bar{q}(p)\sigma_{\mu\nu}q(p)|B_s >$ with the procedure in Refs. [25,26],[1]

$$\mathscr{M}_{B_sB^*_s\gamma} = \sum_{q=s,b} \frac{-ee_q}{2m_q}\varepsilon^\mu_\gamma p^\nu_\gamma < B^*_s|\bar{q}(p)\sigma_{\mu\nu}q(p)|B_s > \\ = i\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu_\gamma p^\nu_\gamma \varepsilon^\alpha_{B^*_s} p^\beta_{B^*_s} \sum_{q=s,b}\left(\frac{ee_q}{m_q}\right)\mathscr{I}. \tag{12}$$

$\mathscr{I}$ is related to the wave functions of B^*_s and B_s [25], which is $\mathscr{I} =< B_s|j_0(p_\gamma r)|B^*_s >\sim 1$ [27,28]. We can rewrite Eq. (12) as follows:

$$\mathscr{M}_{B_sB^*_s\gamma} = i\frac{g_{B_sB^*_s\gamma}}{m_{B^*_s}}\epsilon_{\mu\nu\alpha\beta}\varepsilon^\mu_\gamma p^\nu_\gamma \varepsilon^\alpha_{B^*_s} p^\beta_{B^*_s}. \tag{13}$$

Here the dimensionless vector–scalar–photon coupling constant $g_{B_sB^*_s\gamma}$ is related to the magnetic moments of b and s quarks; and the phase factor i is consistent with the amplitude of $\gamma^* \to VP$ in Ref. [29].

Ultraviolet (UV) logarithmic divergences are observed in the evaluation of loop integrals. In the NLO EW corrections of $B_{s,d} \to \mu^+\mu^-$ [12], the UV divergences are canceled by the renormalization of C_{10}. Just as $R-$ value of hadron production in e^+e^- annihilation, the hadronic contributions will return the quark contributions if the B^*_s is far away from the mass shell. So that the UV part of loop integral will be suppressed due to avoid the double counting. We introduce a cutoff regularization scheme for the following UV divergence integral:

[1] The decay constants defined of vector meson in Eq. (5) is different from Eq. (44) in Ref. [25] with an additional i.

$$\int\frac{{\rm d}^4q}{(2\pi)^4}\frac{1}{\left(q_i^2-m_i^2\right)\left(q_j^2-m_j^2\right)} \\ \to \int\frac{{\rm d}^4q}{(2\pi)^4}\left[\frac{1}{\left(q_i^2-m_i^2\right)\left(q_j^2-m_j^2\right)}\right. \\ \left. -\frac{1}{\left(q_i^2-(m_i+\Lambda)^2\right)\left(q_j^2-(m_j+\Lambda)^2\right)}\right], \tag{14}$$

where $i, j = B^*_s,\ \gamma$ or μ, and q_i corresponds to the i momentum in the loop. $\Lambda \ll M_W$ for the amplitude is UV finite when W boson is involved. The hadronic contribution will be suppressed when $\sqrt{q_j^2}-m_j \gg \Lambda_{QCD}$, where Λ is approximately several Λ_{QCD}. The cutoff regularization scheme is similar to Pauli–Villars regularization scheme; however, the cutoff regularization scheme acts on two propagators. The Pauli–Villars regularization scheme of the UV divergence integral is the same as the form factor $\mathscr{F}$ introduced in the $B_sB^*_s\gamma$ vertex in Ref. [30],

$$\mathscr{F} = \left(\frac{\Lambda^2-m^2_{B^*_s}}{\Lambda^2-q^2_{B^*_s}}\right), \tag{15}$$

for

$$\frac{1}{q^2_{B^*_s}-m^2_{B^*_s}}\mathscr{F} = \frac{1}{q^2_{B^*_s}-m^2_{B^*_s}} - \frac{1}{q^2_{B^*_s}-\Lambda^2}. \tag{16}$$

However, the cutoff regularization scheme acts on the UV divergence term, as well as the two propagators. Then the soft contribution will be maintained in our calculation.

The amplitude from $B_s \to B^*_s\gamma \to \mu^+\mu^-$ can be written as follows:

$$\mathscr{M}(B_s \to B^*_s\gamma \to \mu^+\mu^-) \\ = ie\mathscr{N} g_{B_sB^*_s\gamma}R(\Lambda)C^{\rm eff}_V f_{B^*_s}m_\mu\bar{\mu}\gamma_5\mu, \tag{17}$$

where $C^{\rm eff}_V = C^{\rm eff}_9 + 2m_b/m_{B^*_s}C^{\rm eff}_7$ and considered as a constant. m_μ reappears in the amplitude of the leptonic decay of the scalar mesons. The $R(\Lambda)$ factor serves as a function of the high energy cut, as shown in Fig. 2. Detailed information on the $R(\Lambda)$ factor is provided in the appendix.

Compared with Eq. (9), the previous amplitude added the factor F,

$$F(B^*_s) = \frac{\mathscr{M}(B_s \to B^*_s\gamma \to \mu^+\mu^-)}{\mathscr{M}(B_s \to \mu^+\mu^-)} \\ = \frac{C^{\rm eff}_V f_{B^*_s}}{C_{10}f_{B_s}}eg_{B_sB^*_s\gamma}R(\Lambda). \tag{18}$$

We can estimate $g_{B_sB^*_s\gamma}$ in several ways, including the heavy-quark and chiral effective theories [31,32] with the radiative and pion transition widths of D^{*+}, the light cone QCD sum rules [33,34], and the radiative M1 decay widths of $B^*_s \to B_s\gamma$ from the potential model [27,35].

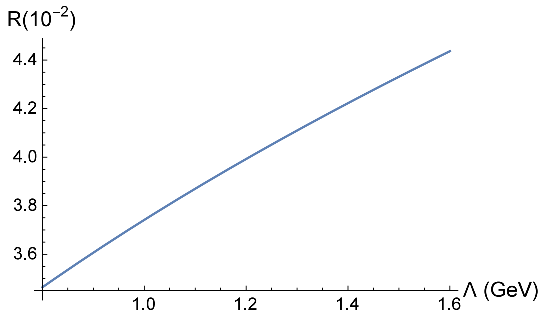

Fig. 2 $R(\Lambda)$ of $B_s \to \mu^+\mu^-$ defined in Eq. (17) as a function of the cut off energy

The heavy-quark and chiral effective theories yield the following expression [13–15]:

$$g_{B_d B_d^* \gamma} = -1.7 \pm 0.2,$$
$$g_{B_s B_s^* \gamma} = -1.2 \pm 0.2. \qquad (19)$$

The light cone QCD sum rules yield the following expression [33,34]:

$$g_{B_d B_d^* \gamma} = -2.3 \pm 0.3,$$
$$g_{B_s B_s^* \gamma} = -1.5 \pm 0.2. \qquad (20)$$

The radiative M1 decay width of $B_s^* \to B_s\gamma$ is derived as follows:

$$g_{B_s B_s^* \gamma} = -m_{B_s^*}\left(\frac{12\pi}{E_\gamma^3}\Gamma(B_s^* \to B_s\gamma)\right)^{1/2}. \qquad (21)$$

The predicted M1 widths are 0.15–400 eV and 10–300 eV for $B_s^* \to B_s\gamma$ and $B_d^* \to B_d\gamma$, respectively [24,26,27,35,36]. Recently new predicted M1 widths were given in Ref. [28]:

$$\Gamma_{B_s^* \to B_s\gamma} = 0.313 \text{ KeV},$$
$$\Gamma_{B_d^* \to B_d\gamma} = 1.23 \text{ KeV}. \qquad (22)$$

Then we can get the following value:

$$g_{B_d B_d^* \gamma} = -3.8,$$
$$g_{B_s B_s^* \gamma} = -2.0. \qquad (23)$$

4 Numerical result

The parameters in the numerical calculation are selected as follows [37]:

$$\Lambda = 1.2 \text{ GeV},$$
$$m_b = 4.2 \text{ GeV},$$
$$\alpha_{em} = 1/137. \qquad (24)$$

In the branch fraction of $B_{s,d}^*$, the weak decay is less than the M1 decay, $\Gamma_{\text{tot}}(B_{s,d}^*) \approx \Gamma(B_{s,d}^* \to B_{s,d}\gamma)$. The following ratio is obtained:

$$\frac{Br(B_s^* \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} = (0.34 \pm 0.03) \times \frac{\text{e}V}{\Gamma(B_s^* \to B_s\gamma)},$$
$$\frac{Br(B_d^* \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)} = (0.33 \pm 0.03) \times \frac{\text{e}V}{\Gamma(B_d^* \to B_d\gamma)}. \qquad (25)$$

The main uncertainty is derived from the $f_{B_{s,d}}^*$ value. The dimuon invariant mass distribution measured by the CMS and LHCb Collaborations in Ref. [2] should include the $B_{s,d}^* \to \mu^+\mu^-$ contributions. If $\Gamma(B_s^* \to B_s\gamma) = 1$ eV [27], we can get $\frac{Br(B_s^* \to \ell^+\ell^-)}{Br(B_s \to \mu^+\mu^-)} = 0.34$ for $\ell = e, \mu$. Afterward, $B_s^* \to e^+e^-$ may be searched by the CMS and LHCb experiments with larger data samples.

Moreover, we observe that the amplitude of $B_{s,d} \to \mu^+\mu^-$ is modified by the contributions of $B_{s,d}^*$ with a factor

$$F(B_{s,d}^*) = (0.011 \pm 0.006)\sqrt{\frac{\Gamma(B_{s,d}^* \to B_{s,d}\gamma)}{100\text{eV}}}, \qquad (26)$$

if $\Gamma(B_{s,d}^* \to B_{s,d}\gamma) \sim 100$ eV. The main uncertainty is caused by the Λ value. The new predictions of $\Gamma(B_{s,d} \to \mu^+\mu^-)$ are provided as follows:

$$Br(B_s \to \mu^+\mu^-) = (36.6 \pm 2.3) \times$$
$$\times \left(1 + (0.023 \pm 0.012)\sqrt{\frac{\Gamma_{\text{tot}}(B_s^*)}{100\text{eV}}}\right) \times 10^{-10},$$
$$Br(B_d \to \mu^+\mu^-) = (10.6 \pm 0.9) \times$$
$$\times \left(1 + (0.023 \pm 0.012)\sqrt{\frac{\Gamma_{\text{tot}}(B_d^*)}{100\text{eV}}}\right) \times 10^{-11}.$$

If $\Gamma(B_{s,d}^* \to B_{s,d}\gamma) = 200$ eV, then this factor will increase the $\Gamma(B_{s,d} \to \mu^+\mu^-)$ decay width by a factor of $(3.3 \pm 1.7)\%$, which is approximately a factor 10 times larger than the neglect NLO EW correction factor 0.3% at the decay width in Ref. [1]. In addition, the corresponding $g_{B_{s,d}B_{s,d}^*\gamma} = -1.5$ about a factor 15 times larger than $e_q e = -1/3\sqrt{4\pi\alpha_{em}} = -0.10$. $\Gamma(B_{s,d}^* \to B_{s,d}\gamma)$ may be measured through two-body decay $B_{s,d}^* \to J/\psi + M$ by CMS and LHCb.

If $\Gamma(B_{s,d}^* \to B_{s,d}\gamma) = 313, 1230$ eV [28], then we can get

$$\frac{Br(B_s^* \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} = (1.1 \pm 0.1) \times 10^{-3},$$
$$\frac{Br(B_d^* \to \mu^+\mu^-)}{Br(B_d \to \mu^+\mu^-)} = (2.7 \pm 0.3) \times 10^{-4}. \qquad (27)$$

New predictions of $\Gamma(B_{s,d} \to \mu^+\mu^-)$ are provided as follows:

Table 2 The branching fractions of $B_{s,d} \to \mu^+\mu^-$ measured by the CMS and LHCb Collaborations [2] and updated SM prediction with $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) =$ 313, 1230 eV [28]

	EX [2]	SM [1]	Updated SM
$Br(B_d \to \mu^+\mu^-)$	$(3.9^{+1.6}_{-1.4}) \times 10^{-10}$	$(1.06 \pm 0.09) \times 10^{-10}$	$(1.2 \pm 0.1) \times 10^{-10}$
$Br(B_s \to \mu^+\mu^-)$	$(2.8^{+0.7}_{-0.6}) \times 10^{-9}$	$(3.66 \pm 0.23) \times 10^{-9}$	$(3.8 \pm 0.3) \times 10^{-9}$
$\frac{Br(B_d \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)}$	$0.14^{+0.08}_{-0.06}$	$0.0295^{+0.0028}_{-0.0025}$	0.031 ± 0.0036

$$\begin{aligned} &Br(B_s \to \mu^+\mu^-) = (3.8 \pm 0.3) \times 10^{-9}, \\ &Br(B_d \to \mu^+\mu^-) = (1.2 \pm 0.1) \times 10^{-10}, \\ &\frac{Br(B_d \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} = 0.031 \pm 0.0036. \end{aligned} \tag{28}$$

The numerical results are shown in Table 2 too.

5 Summary

In summary, this study investigated $B^*_{s,d} \to \mu^+\mu^-$ in the dimuon distributions and the hadronic contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$. The $\mu^+\mu^-$ decay widths of the vector mesons $B^*_{s,d}$ are approximately a factor of 700 larger than the corresponding scalar mesons $B_{s,d}$. The obtained ratio of the branching fractions $\frac{Br(B^*_{s,d} \to \mu^+\mu^-)}{Br(B_{s,d} \to \mu^+\mu^-)}$ is approximately $\frac{0.3 \times \mathrm{eV}}{\Gamma(B^*_{s,d} \to B_{s,d}\gamma)}$. The hadronic contribution $B_{s,d} \to B^*_{s,d}\gamma \to \mu^+\mu^-$ is also estimated. The relative increase in the $B_{s,d} \to \mu^+\mu^-$ amplitude is approximately $(0.01 \pm 0.006)\sqrt{\frac{\Gamma(B^*_{s,d} \to B_{s,d}\gamma)}{100\ \mathrm{eV}}}$. If we select $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) =$ 2 eV, then the branching fractions of the vector mesons to the lepton pair are 5.3×10^{-10} and 1.6×10^{-11} for B^*_s and B^*_d, respectively. If we select $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) = 200$ eV, then the updated branching fractions of the scalar mesons to the muon pair are $(3.78 \pm 0.25) \times 10^{-9}$ and $(1.09 \pm 0.09) \times 10^{-10}$ for B_s and B_d, respectively. If we select the recent predicted M1 widths $\Gamma(B^*_{s,d} \to B_{s,d}\gamma) = 313, 1230$ eV [28], then the updated branching fractions are $(3.8 \pm 0.3) \times 10^{-9}$ and $(1.2 \pm 0.1) \times 10^{-10}$ for $B_s \to \mu^+\mu^-$ and $B_d \to \mu^+\mu^-$, respectively. Further studies on $B^*_{s,d}$, including those on dielectron decay and two-body decay with J/ψ should be conducted.

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Appendix A: Appendix $R(\Lambda)$

$R(\Lambda)$ of $B_s \to B^*_s\gamma \to \mu^+\mu^-$ defined in Eq. (17) is obtained as follows:

$$\begin{aligned} R(\Lambda) = &\frac{1}{32\pi^2 {m_{B_s}}^2 m_\mu^2} \Big\{ 3{m_{B_s}}^2 m_\mu^2 - 2m_\mu^2 \left({m_{B_s}}^2 - {m_{B_s^*}}^2\right)^2 \\ &\times C_0\left({m_{B_s}}^2, m_\mu^2, m_\mu^2, {m_{B_s^*}}^2, 0, m_\mu^2\right) \\ &+ {m_{B_s}}^2 (m_\mu^2 - {m_{B_s^*}}^2)\left(B_0\left(0, m_\mu^2, {m_{B_s^*}}^2\right)\right. \\ &\left.- B_0\left(0, (\Lambda + m_\mu)^2, (\Lambda + m_{B_s^*})^2\right)\right) \\ &+ \left(m_{B_s}^2\left(2m_\mu^2 + {m_{B_s^*}}^2\right) + 2m_\mu^2 {m_{B_s^*}}^2\right) \\ &\left(B_0\left(m_\mu^2, m_\mu^2, {m_{B_s^*}}^2\right)\right. \\ &\left.- B_0\left(m_\mu^2, (\Lambda + m_\mu)^2, (\Lambda + m_{B_s^*})^2\right)\right) \\ &+ m_\mu^2\left(3{m_{B_s}}^2 - 2{m_{B_s^*}}^2\right)\left(B_0\left(m_\mu^2, 0, m_\mu^2\right)\right. \\ &\left.- B_0\left(m_\mu^2, \Lambda^2, (\Lambda + m_\mu)^2\right)\right)\Big\}. \end{aligned} \tag{A.1}$$

The scalar functions B_0 and C_0 are given in Refs. [38–40]. As a numerical fit between 0.5 − 2 GeV, $R(\Lambda)$ is obtained as follows:

$$R(\Lambda) = 0.022 + 0.062 \times ln\left(\frac{\Lambda + m_{B_s}}{m_{B_s}}\right). \tag{A.2}$$

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