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Tachyon warm-intermediate inflation in the light of Planck data

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Abstract We study the main properties of the warm tachyon inflation model in the framework of the RSII braneworld based on Barrow's solution for the scale factor of the universe. Within this framework we calculate analytically the basic slow-roll parameters for different versions of warm inflation. We test the performance of this inflationary scenario against the latest observational data and we verify that the predicted spectral index and the tensor-to-scalar fluctuation ratio are in excellent agreement with those of *Planck 2015*. Finally, we find that the current predictions are consistent with those of viable inflationary models.

1 Introduction

Standard inflation driven by an inflaton field traces back to early efforts to alleviate the basic problems of the Big-Bang cosmology, namely horizon, flatness, and monopoles [1,2]. The nominal inflationary paradigm contains the slow-roll and the (P)reheating regimes. In the slow-roll phase the kinetic energy (which has the canonical form here) of the scalar field is negligible with respect to the potential energy $V(\phi)$, which implies a de Sitter expansion of the universe. However, after the slow-roll epoch the kinetic energy becomes comparable to the potential energy and thus the inflaton field oscillates around the minimum and progressively the universe is filled by radiation [3,4].

Nevertheless, other theoretical patterns suggested a possible way to treat the physics of the early universe. For example, in the so-called warm-inflationary scenario the radiation production occurs during the slow-roll epoch and the reheating period is avoided [5,6]. The nature of the warm-inflationary scenario is different with respect to that of the standard cold inflation. Warm inflation satisfies the condition $T > H$,

where T is the temperature and H is the Hubble parameter, which implies that the fluctuations of the inflaton field are thermal instead of quantum. An obvious consequence of the above inequality is that in the case of warm inflation, density perturbations arise from thermal fluctuations rather than quantum fluctuations [7–9]. Specifically, thermal fluctuations are produced during the warm-inflationary epoch and they play a central role toward describing the CMB anisotropies and thus providing the initial seeds for the formation of large scale structures. Of course, after this epoch the universe enters in the radiation dominated phase as it should [5,6]. In order to achieve warm inflation one may use a tachyon scalar field for which the kinetic term does not follow the canonical form (k-inflation [10]). It has been found that tachyon fields which are associated with unstable D-branes [11] can be responsible for the cosmic acceleration in early times [10,12,13].

Notice that tachyon potentials have the following two properties: the maximum of the potential occurs when $\phi \to 0$ while the corresponding minimum takes place when $\phi \to \infty$. From the dynamical viewpoint one may obtain the equations of motion using a special Lagrangian [14] which is non-minimally coupled to gravity:

$$L = \sqrt{-g}\left[\frac{R}{16\pi G} - V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}\right]. \qquad (1)$$

Considering a spatially flat Friedmann–Robertson–Walker (hereafter FRW) space-time the stress-energy tensor is given by

$$T_\nu^\mu = \frac{\partial L}{\partial(\partial_\mu\phi)}\partial_\nu\phi - g_\nu^\mu L = \text{diag}(-\rho_\phi, p_\phi, p_\phi, p_\phi) \qquad (2)$$

where ρ_ϕ and p_ϕ are the energy density and pressure of the scalar field. Combining the above set of equations one can derive

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$$\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \tag{3}$$

and

$$P_\phi = -V(\phi)\sqrt{1 - \dot{\phi}^2} \tag{4}$$

where ϕ is the tachyon scalar field in units of the inverse Planck mass M_{pl}^{-1}, and $V(\phi)$ is potential associated with the tachyon field. In the past few years, there was an intense debate among cosmologists and particle physicists regarding those phenomenological models which can be produced in extra dimensions. For example, the reduction of higher-dimensional gravitational scale, down to TeV-scale, could be presented by an extra dimensional scenario [15–17]. In these scenarios, the gravity field propagates in the bulk, while standard models of particles are confined to the lower-dimensional brane. In this framework, the extra dimension induces additional terms in the first Friedmann equation [18–20]. Especially, if we consider a quadratic term in the energy density then we can extract an accelerated expansion of the early universe [21–25]. In the current study we consider the tachyon warm inflation model in the framework of the Randall–Sundrum II braneworld which contains a single, positive tension brane and a non-compact extra dimension.

Following the lines of Ref. [26], we attempt to study the main properties of the warm inflation in which the scale factor evolves as $a(t) \propto \exp(At^f)$, where $0 < f < 1$ ("intermediate inflation"). In this case cosmic expansion evolves faster than the power-law inflation ($a \propto t^p$, $p > 1$) and slower than the standard de Sitter one, $a(t) \propto \exp(H_I t)$ [$H(t) = H_I$ =const.]. More details regarding the cosmic expansion in various inflationary solutions can be found in the paper of Barrow [27].

In the current work, we investigate the possibility of using the intermediate solution in the case of warm tachyon inflation. Specifically, the structure of the article is as follows: in Sect. 2 we briefly discuss the main properties of the warm inflation, while in Sect. 3 we provide the slow-roll parameters. In Sect. 4 we study the performance of our predictions against the *Planck 2015* data. Finally, the main conclusions are presented in Sect. 5.

2 Tachyon warm inflation

Let us assume a flat, homogeneous, and isotropic Friedmann–Robertson–Walker (FRW) universe, in which the radiation era is endowed with the scalar field described by the Lagrangian (1) in the context of the Randall–Sundrum II (RSII) brane [28]. Following the notations of [5,6,18–20,26] one may check that the basic cosmological equations are

$$H^2 = \frac{8\pi}{3M_{pl}^2}\rho\left(1 + \frac{\rho}{2\lambda}\right), \tag{5}$$

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\Gamma\dot{\phi}^2, \tag{6}$$

$$\dot{\rho}_\gamma + 3H(\rho_\gamma + p_\gamma) = \Gamma\dot{\phi}^2, \tag{7}$$

where Γ is the dissipation coefficient, in units of M_{pl}^5. Equations (6) and (7) imply the continuity equation, namely $\dot{\rho} + 3H(\rho + p) = 0$. Notice that Eqs. (6) and (7) have been proposed by various authors such as [29–34]. In these studies the quantity $\Gamma\dot{\phi}^2$ is the dissipation term which is introduced phenomenologically in order to describe the nearly thermal radiation bath that is the outcome of the warm-inflationary scenario. It is well known that tachyon inflation in its standard picture (cold inflation) suffers from a serious problem. In particular, reheating and matter creation are both problematic because the tachyon fields in such theories do not oscillate around the minimum of the potential [35]. This problem can be alleviated in the context of warm inflation. In this scenario radiation production occurs during the slow-roll era which implies that reheating is avoided and thus the universe heats up and finally it enters in the radiation era [5] (see Eqs. (6) and (7)).

In the above set of equations, an over-dot denotes derivative with respect to time, $\rho = \rho_\phi + \rho_\gamma$ and $p = p_\phi + p_\gamma$ ($p_\gamma = \rho_\gamma/3$) are the total density and pressure, ρ_ϕ and ρ_γ are the scalar-field and radiation densities, $H = \dot{a}/a$ is the Hubble parameter. Notice that λ is the brane tension, which obeys the following restriction: $\lambda \geq (10\,\mathrm{TeV})^4$ [22,36,37]. Obviously, substituting Eqs. (3) and (4) in Eq. (6) it is easy to derive the modified Klein–Gordon equation which describes the time evolution of the tachyon field. This is

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{V'}{V} = -\frac{\Gamma}{V}\dot{\phi}\sqrt{1 - \dot{\phi}^2}, \tag{8}$$

where $V'(\phi) = \mathrm{d}V/\mathrm{d}\phi$.

The above cosmological equations imply that the model is strongly affected by the quantity Γ. This is due to the fact that radiation is exchanging energy with the tachyon field and this is reflected in the corresponding behavior of dissipation coefficient Γ, which is negligible in the classical inflationary paradigm by definition. Although the precise functional form of Γ is still an open issue, a number of different parametrizations have been proposed in the literature treating the functional form of Γ (see [7,38–40]). In the current work we use the well-known parametrization of

$$\Gamma = \Gamma_c \phi^b T^c, \tag{9}$$

where T is the temperature and Γ_c is constant.

During the warm-inflationary epoch, the energy density of the scalar field dominates the total fluid (stable regime [41]) and thus Eq. (5) becomes

$$H^2 = \frac{8\pi}{3M_{pl}^2} \rho_\phi \left(1 + \frac{\rho_\phi}{2\lambda}\right) \tag{10}$$

or

$$H^2 = \frac{8\pi}{3M_{pl}^2} \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} \left(1 + \frac{V(\phi)}{2\lambda\sqrt{1 - \dot{\phi}^2}}\right). \tag{11}$$

Another important quantity in this kind of studies is the dimensionless dissipation parameter which characterizes the type of inflation and it is defined as (for more details see the appendix)

$$R = \frac{\Gamma}{3H\rho_\phi}. \tag{12}$$

The above definition is presented for warm tachyon inflation in several papers [30,32,33,42–44]. Notice that for the canonical scalar-field model of warm inflation, the corresponding dimensionless ratio is defined as $\frac{\Gamma}{3H}$. In the weak dissipation regime, the ratio R tends to zero ($\Gamma/3H\rho_\phi \ll 1$), however, in the strong dissipation regime, the coefficient Γ guides the damped evolution of the scalar field. Now using Eqs. (8) and (11) in the high-dissipation regime ($\Gamma \gg 3H\rho_\phi$) to prove that

$$\dot{\phi}^2 = -\frac{3M_{pl}^2}{4\pi} \frac{(H\dot{H})}{\Gamma} \left(1 + \frac{3M_{pl}^2 H^2}{4\pi\lambda}\right)^{-\frac{1}{2}}. \tag{13}$$

Owing to the fact that during inflation the parameters H, Γ, and ϕ are slowly varying functions the production of radiation become quasi-stable when $\dot{\rho} \ll 4H\rho_\gamma$, and $\dot{\rho}_\gamma \ll \Gamma\dot{\phi}^2$ [5–7]. Under these conditions, using Eqs. (8) and (13) we write the radiation density as follows:

$$\rho_\gamma = \frac{\Gamma\dot{\phi}^2}{4H} = -\frac{3M_{pl}^2 \dot{H}}{16\pi} \left(1 + \frac{3M_{pl}^2 H^2}{4\pi\lambda}\right)^{-\frac{1}{2}}. \tag{14}$$

The latter formula can be identified with the equation relating ρ_γ with the radiation temperature T. Indeed, under of adiabatic condition we may write

$$\rho_\gamma = C_\gamma T^4 \tag{15}$$

where $C_\gamma = \frac{\pi^2 g_*}{30}$ and g_* is the number of degrees of freedom of the created massless modes [45]. Combining Eqs. (14) and (15) we obtain the temperature

$$T = \left(-\frac{3M_{pl}^2 \dot{H}}{16\pi C_\gamma}\right)^{\frac{1}{4}} \left(1 + \frac{3M_{pl}^2 H^2}{4\pi\lambda}\right)^{-\frac{1}{8}}. \tag{16}$$

Lastly, with the aid of Eqs. (11) and (14) we obtain the potential of the scalar field

$$V = \lambda \left[-1 + \left(1 + \frac{3M_{pl}^2 H^2}{4\pi\lambda}\right)^{\frac{1}{2}}\right]$$
$$\times \left(1 + \frac{3M_{pl}^2}{4\pi} \frac{(H\dot{H})}{\Gamma} \left[1 + \frac{3M_{pl}^2 H^2}{4\pi\lambda}\right]^{-\frac{1}{2}}\right)^{\frac{1}{2}}. \tag{17}$$

3 Slow-roll parameters

Let us present here the main quantities of the tachyonic inflation. In particular, the basic slow-roll parameters are given by

$$\varepsilon = -\frac{\dot{H}}{H^2}, \tag{18}$$

$$\eta = -\frac{\ddot{H}}{2H\dot{H}}. \tag{19}$$

In this context, the number of e-folds is written as

$$N = \int_t^{t_{\text{end}}} H \, dt \tag{20}$$

where t_{end} is the value of the cosmic time at the end of inflation, namely $\varepsilon(\phi_{\text{end}}) \simeq 1$ where $\phi_{\text{end}} = \phi(t_{\text{end}})$.

Also, the power spectrum of the scalar fluctuations is given by [5,6]

$$P_s = \frac{H^2}{\dot{\phi}^2} \delta\phi^2. \tag{21}$$

An important feature of the warm-inflationary model is related with the fact that the origin of $\delta\phi$ is thermal and not quantum as we consider in the nominal inflationary paradigm. In the case of warm inflation it has been found [5–7] that scalar perturbations are written as

$$\delta\phi^2 \simeq \frac{k_F T}{2M_{pl}^4 \pi^2}, \tag{22}$$

where the wave number $k_F = \sqrt{\frac{\Gamma H}{V}} = H\sqrt{\frac{\Gamma}{3HV}} \geq H$ corresponds to the freeze-out scale at the special point where the dissipation damps out to thermally excited fluctuations of inflaton ($\frac{V''}{V'} < \frac{\Gamma H}{V}$) [46]. Notice that Eq. (22) is valid in the high-dissipation regime $R \gg 1$. As we have already mentioned in the previous section we study our model via Eq. (13) in the context of the high-dissipation regime, which means that for scalar perturbations we can utilize Eq. (22).

Inserting the freeze-out wave-number and Eq. (22) into Eq. (21) we find after some simple calculations that the power

spectrum of the tachyonic scalar field is given by

$$P_s \simeq \frac{H^{\frac{5}{2}} \Gamma^{\frac{1}{2}} T}{2\pi^2 M_{pl}^4 V^{\frac{1}{2}} \dot{\phi}^2}. \tag{23}$$

We would like to point out that in the case of canonical scalar fields within the framework of warm inflation one can find other forms of the power spectrum P_s in Refs. [40,47,48]. The corresponding spectral index n_s is defined in terms of the P_s slow-roll parameters, as usual [49], by

$$n_s - 1 = \frac{d \ln P_s}{d \ln k}. \tag{24}$$

On the other hand, it has been found [50] that the power spectrum of the tensor perturbations which are defined on the brane takes the form

$$P_t = \frac{64\pi}{M_{pl}^2} \left(\frac{H}{2\pi}\right)^2 G^2(x) \tag{25}$$

where $x \equiv [\frac{3HM_{pl}^2}{4\pi\lambda}]^{\frac{1}{2}}$ and $G(x) = [\sqrt{1+x^2} - x^2 \sinh^{-1}(\frac{1}{x})]^{-\frac{1}{2}}$ arises from normalization of zero-mod of a graviton [50]. Therefore, using the so-called tensor-to-scalar ratio we arrive at

$$r = 32\pi M_{pl}^2 \frac{V^{\frac{1}{2}} \dot{\phi}^2}{H^{\frac{1}{2}} \Gamma^{\frac{1}{2}} T} G^2(x). \tag{26}$$

In order to proceed with the analysis it would help to know the functional form of the scale factor $a(t)$. Barrow [27] showed that under of specific conditions we can have an intermediate inflation in which the scale factor satisfies the following exponential form:

$$a(t) = a_I \exp(At^f), \tag{27}$$

which provides

$$H(t) = \frac{\dot{a}(t)}{a(t)} = Aft^{f-1}, \tag{28}$$

where f satisfies the restriction $0 < f < 1$. The above expansion evolves faster than the power-law inflation ($a \propto t^p$, $p > 1$) and slower than the standard de Sitter one, $a(t) \propto \exp(H_I t)$ [$H(t) = H_I$ =const.]. Considering the functional form (9), one has to deal in general with the following four parametrizations, which have been considered within different approaches in the literature. Depending on the values of (b, c) we have:

(I) The situation may occur in which the formula is $\Gamma = \Gamma_3 T^3 \phi^{-2}$, $(b, c) = (3, -2)$. The constant parameter Γ_3 corresponds to $0.02h^2 \mathcal{N}_Y$ where there is a generic supersymmetric (SUSY) model with chiral superfields X, Φ, and $Y_i = 1, \ldots \mathcal{N}_Y$. This case is mostly used in the low temperature regime where m_χ (m_χ is the mass of catalyst field) [39,51].

(II) For $(b, c) = (2, -1)$ we have $\Gamma = \Gamma_2 \phi^2 T^{-1}$. This parametrization has been used for non-SUSY models [52, 53]. (III)- the case where $\Gamma = \Gamma_0$ is a positive constant (hereafter Γ_0-parametrization: see [30,32,33,42–44,54–56]) which implies that the pair (b, c) in Eq. (9) is strictly equal to $(0, 0)$.

(IV) We utilize the so-called high temperature regime (hereafter Γ_1-parametrization) in which we select $(b, c) = (0, 1)$ and thus $\Gamma \propto T$ (see also [57]).

In this paper, we are going to focus on parameterizations (III) and (IV) in order to calculate the slow-roll parameters. Lastly, we remind the reader that in the framework of warm-inflationary model thermal fluctuations dominate over the quantum fluctuations.

Combining the latter argument with the fact that thermal fluctuations are proportional to temperature T while quantum fluctuations are proportional to H, one can easily derive the condition $T > H$. Obviously, if we consider our model in the high temperature regime ($\Gamma \propto T$) then the aforesaid restriction ($T > H$) is satisfied. For more details we refer the reader to Ref. [57].

3.1 Γ_0-parametrization

In this inflationary scenario ($\Gamma = \Gamma_0 =$ const.) with the aid of Eq. (28) we integrate Eq. (13) and we obtain the evolution of the scalar field in terms of the hyper-geometric function [58,59],

$$\phi(t) - \phi_0 = \frac{F(t)}{K} \tag{29}$$

where

$$F(t) = t^{\frac{2f-1}{2}} \\ \times {}_2F_1 \left[\frac{1}{4}, \frac{1-2f}{4(1-f)}, \frac{5-6f}{4(1-f)}, -\frac{3M_{pl}^2 f^2 A^2 t^{2f-2}}{4\pi\lambda} \right], \tag{30}$$

$$K = -\left(\frac{16\pi \Gamma_0 (1-f)}{3M_{pl}^2 f^2 A^2}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{5-6f}{4(1-f)}\right)}{\Gamma\left(\frac{1-2f}{4(1-f)}\right)}, $$

and $\Gamma(n)$ is the normal Gamma-function. Notice that without loss of generality we have set $\phi_0 = 0$.

Now we can derive the Hubble parameter H and the associated potential $V(\phi)$ in the limit of $\dot{\phi}^2 \ll V(\phi)$

$$H(\phi) = fA(F^{-1}[K\phi])^{f-1}$$

$$V(\phi) \simeq \lambda \left(-1 + \sqrt{1 + \frac{3M_{pl}^2 f^2 A^2 [F^{-1}(K\phi)]^{2f-2}}{4\pi\lambda}}\right) \tag{31}$$

where $F^{-1}(\phi)$ is the inverse function of $F(t)$. Clearly, if we substitute Eq. (31) in the slow-roll parameters, then we have

$$\varepsilon = \frac{(1-f)t^{-f}}{fA} = \frac{1-f}{fA[F^{-1}(K\phi)]^f}, \tag{32}$$

$$\eta = \frac{(2-f)t^{-f}}{2fA} = \frac{2-f}{2fA[F^{-1}(K\phi)]^f}. \tag{33}$$

Notice that in order to extract the latter equalities in Eqs. (32) and (33) we used Eqs. (28) and (29). In our warm-intermediate case the condition $\varepsilon = 1$ ensures the beginning of inflation [60,61]. Therefore, utilizing Eq. (20), we can derive the number of e-folds[1]

$$N = \int_{t_{in}}^{t_*} H \mathrm{d}t = A(t_*^f - t_{in}^f)$$
$$= A\left([F^{-1}(K\phi)]^f - [F^{-1}(K\phi_{in})]^f\right). \tag{34}$$

Plugging ϕ_{in} into Eq. (32) and using the constraint $\varepsilon(\phi_{in}) \simeq 1$ we find

$$\phi_{in} = \frac{1}{K}F(y), \quad y = \left(\frac{1-f}{fA}\right)^{\frac{1}{f}}. \tag{35}$$

In order to proceed with the analysis we need to know the values of N and ϕ_{in}. First of all, it is natural to consider that the number of e-folds is 50 or 60. Second, using the condition $\varepsilon(\phi_{in}) = 1$ and Eqs. (34) and (35) we can estimate the slow-roll parameters.

Now we focus on the power-spectrum formulas. Specifically, inserting the appropriate expressions into Eq. (23) we define the scalar power spectrum

$$P_s = p_1 I(N)^{\frac{3f}{4}} \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}} \right]^{-\frac{1}{2}}$$
$$\times \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{3}{8}} \tag{36}$$

where $p_1 = (\frac{\Gamma_0^6 f^3 A^3}{3^3 M_{pl}^6 \pi^5 (1-f)^3 C_\gamma})^{\frac{1}{4}}$ and $I(N) = [\frac{1+f(N-1)}{fA}]^{\frac{1}{f}}$. Combining the definition of the spectral index n_s (24) and the above equation we obtain

$$n_s - 1 = -\frac{3}{4A}I(N)^{-f} + n_1 + n_2 \tag{37}$$

where in the derivation of the above equality we have used

$$n_1 = \frac{3M_{pl}^4(f-1)fA}{8\pi\lambda}I(N)^{f-2}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{-\frac{1}{2}}$$
$$\times \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}}\right]^{-1}$$
$$n_2 = \frac{9M_{pl}^2(1-f)fA}{16\pi\lambda}I(N)^{f-2}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{-1}. \tag{38}$$

Lastly, based on Eq. (26) we compute the tensor-to-scalar ratio parameter

$$r = r_1 I(N)^{\frac{5f-8}{4}}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{-\frac{3}{8}}$$
$$\times \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} G^2(N) \tag{39}$$

where $r_1 = (\frac{2^{16}\lambda^2 3^3 C_\gamma \pi (fA)^5 (1-f)^3}{M_{pl}^2 \Gamma_0^6})^{\frac{1}{4}}$.

3.2 Γ_1-parametrization

Using the same methodology as in the previous section we provide the basic slow-roll parameters in the case of Γ_1-parametrization, namely $\Gamma = \Gamma_1 T$, where Γ_1 is constant. In particular, from Eqs. (28 and 13) the tachyon field is written as

$$\phi - \phi_0 = \frac{\tilde{F}(t)}{\tilde{K}} \tag{40}$$

where $\tilde{F}(t)$ and $\tilde{K}$ are given by the following expressions:

$$\tilde{F}(t) = t^{\frac{7f-2}{8}}$$
$$\times {}_2F_1\left[\frac{3}{16}, \frac{7f-2}{16(f-1)}, \frac{23f-18}{16(f-1)}, -\frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda}t^{2f-2}\right]$$
$$\tilde{K} = -\left(\frac{2^6 \pi \Gamma_1^4 (1-f)^5}{3M_{pl}^2 C_\gamma f^7 A^7}\right)^{\frac{1}{8}}\frac{\Gamma(\frac{23f-18}{16(f-1)})}{\Gamma(\frac{7f-2}{16(f-1)})}. \tag{41}$$

Also here we have set $\phi_0 = 0$.

Now the Hubble parameter, the potential and the corresponding slow-roll parameters are given by

$$H(\phi) = fA(\tilde{F}^{-1}[\tilde{K}\phi])^{f-1},$$
$$V(\phi) \simeq \lambda\left(-1 + \sqrt{1 + \frac{3M_{pl}^2 f^2 A^2[\tilde{F}^{-1}(\tilde{K}\phi)]^{2f-2}}{4\pi\lambda}}\right), \tag{42}$$

---

[1] In the literature sometimes we replace ϕ by $\phi_\star$, which denotes the value at the horizon crossing.

$$\varepsilon = \frac{1-f}{fA[\tilde{F}^{-1}(\tilde{K}\phi)]^f}, \tag{43}$$

$$\eta = \frac{2-f}{2fA[\tilde{F}^{-1}(\tilde{K}\phi)]^f}, \tag{44}$$

where $\tilde{F}^{-1}(K\phi)$ is inverse function of $\tilde{F}(t)$. In the current case the number of e-folds becomes

$$
\begin{aligned}
N &= \int_{t_{in}}^{t_*} H\mathrm{d}t = A(t_*^f - t_{in}^f) \\
&= A\left([\tilde{F}^{-1}(\tilde{K}\phi)]^f - [\tilde{F}^{-1}(\tilde{K}\phi_{in})]^f\right)
\end{aligned} \tag{45}
$$

and following a standard treatment the end of inflation takes place when

$$\phi_{in} = \frac{1}{\tilde{K}}\tilde{F}(y), \quad y = \left(\frac{1-f}{fA}\right)^{\frac{1}{f}}. \tag{46}$$

The scalar power-spectrum can easily be identified by comparing the current cosmological expressions with Eq. (23), and we find

$$
P_s = p_2 I(N)^{\frac{9f-6}{8f}} \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}
$$

$$
\times \left[1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right]^{\frac{3}{16}} \tag{47}
$$

where $p_2 = \frac{2(\Gamma_1 fA)^{\frac{3}{2}}}{3M_{pl}^2 \lambda^{\frac{1}{2}}}(\frac{16\pi C_\gamma}{3M_{pl}^2(1-f)})^{\frac{3}{8}}$. If we take the aforementioned P_s formula we find the following spectral index:

$$n_s - 1 = -\frac{9f-6}{8fA}I(N)^{-f} + n_1 + n_2 \tag{48}$$

where

$$
n_1 = \frac{3M_{pl}^4(f-1)fA}{8\pi\lambda}I(N)^{f-2}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{-\frac{1}{2}}
$$

$$
\times \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}}\right]^{-1},
$$

$$
n_2 = \frac{9M_{pl}^2(1-f)fA}{32\pi\lambda}I(N)^{f-2}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{-1}.
$$
$$\tag{49}$$

Finally, the tensor-to-scalar ratio [see Eq. (26)] takes the form

$$
r = r_2 I(N)^{\frac{7f-6}{8}}\left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{-3}{16}}
$$

$$
\times \left[-1 + \left(1 + \frac{3M_{pl}^2 f^2 A^2}{4\pi\lambda I(N)^{2-2f}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}} G^2(N) \tag{50}
$$

where $r_2 = \frac{24\lambda^{\frac{1}{2}}(fA)^{\frac{7}{8}}}{\Gamma_1^{\frac{3}{2}}}(\frac{16\pi C_\gamma(1-f)}{3M_{pl}^2})^{\frac{3}{8}}$.

4 Comparison with observation

The analysis of *Planck* [62] and BICEP2/Keck Array [63] data sets has provided a new constraint on inflationary scenarios [64]. In particular, the comprehensive analysis of *Planck* data [62] indicates that single scalar-field models of slow-roll inflation have a very low tensor-to-scalar fluctuation ratio $r = P_t/P_s \ll 1$, a scalar spectral index $n_s = 0.968 \pm 0.006$ and no appreciable running. The upper bound set by the *Planck* team and the joint analysis of BICEP2/Keck Array/*Planck* [63] on the tensor-to-scalar fluctuation ratio is $r < 0.11$. In this section we attempt to test the performance of the warm-inflationary model against the above observational results.

Let us now concentrate on our results. Notice that in the case of warm inflation the number of degrees of freedom becomes $g_* \simeq 200$ ($C_\gamma \simeq 70$ [65]) Also, for the rest of the paper we have set $\lambda = 10^{-14}$. Concerning the number of e-folds, it is natural to consider that N lies in the interval [50, 60]. Here, we have set it either to 50 or 60. In Fig. 1 (Γ_0 parametrization) and 2 (Γ_1 parametrization) we present the $A-f$ allowed region in which our (n_s, r) results satisfy the above restrictions of *Planck* within 1σ uncertainties. In the case of the Γ_0 model, we observe that for various values of the dissipation coefficient there is a narrow region in the $A-f$ plane which is consistent with the observed values of n_s and r. The absence of $A-f$ pair solutions, and thus of (n_s, r), appears for $\Gamma_0 \leq 10^{-10}$.
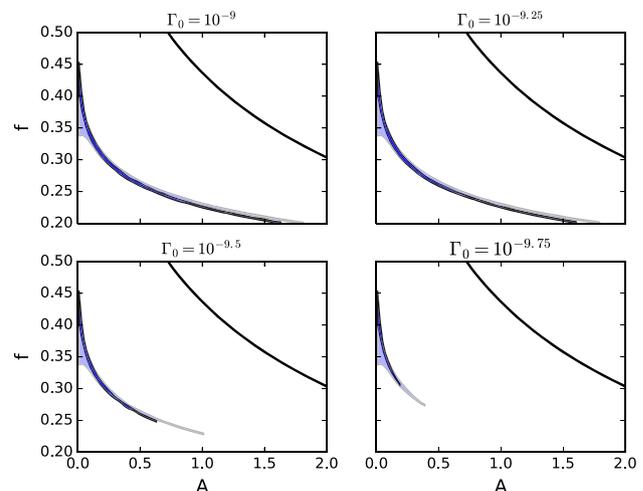

Fig. 1 The $A-f$ diagram which coincides within 1σ confidence level of *Planck* data. The corresponding values of Γ_0 are shown at the *top of panels*. The background transparent (*foreground opaque*) indicates $N = 60$ ($N = 50$). These values of Γ_0 are consistent with $R \gg 1$. The *solid black curve* shows the boundary $T = H$ and the region below the curve is consistent with $T > H$
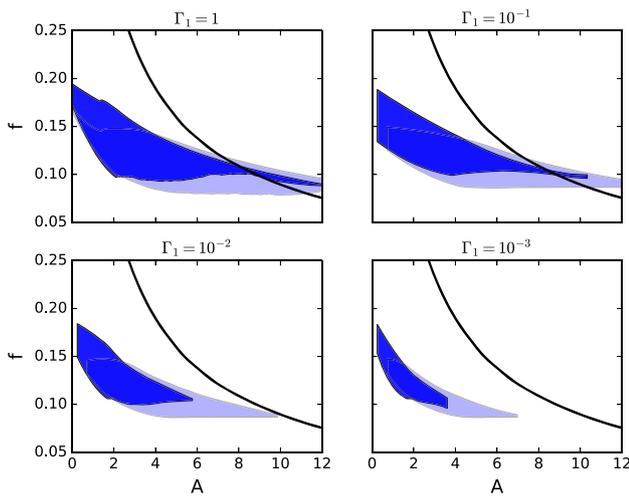
Fig. 2 The A–f region in the case of Γ_1 parametrization. These values of Γ_1 are consistent with $R \gg 1$. The *solid black curve* is the same as Fig. 1

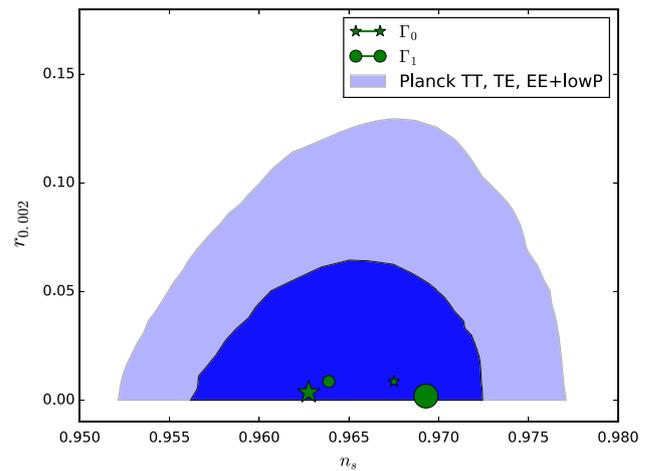**Fig. 3** 1σ and 2σ confidence regions borrowed from *Planck* [62]. *Stars* (*squares*) indicate warm inflation with Γ_0 (Γ_1) parametrization. *Big and small points* correspond to $N = 60$ and $N = 50$, respectively. For Γ_0 we set $f = 0.28$, $A = 0.35$, and $\Gamma_0 = 10^{-9}$. For Γ_1 we set $f = 0.13$, $A = 3.$, and $\Gamma_1 = 10^{-3}$

For the Γ_1 parametrization the situation is slightly different. Figure 2 shows broader A–f regions with respect to those of the Γ_0 parametrization. Also in this case we verify that, for $\Gamma_1 \leq 10^{-10}$, there are no A–f pairs which satisfy the observational criteria. The theoretical curves of cold intermediate inflation model in Einstein General Relativity in the n_s–r plane are well outside of the 95 % C.L. region. Our aim here is to test the viability of warm inflation, involving the latest *Planck2015* data. In Fig. 3 we present the confidence contours in the (n_s, r) plane. On top of Fig. 3 we provide the solid stars for the individual sets of (n_s, r) which are based on the Γ_0 parametrization, whereas in the same figure we display the corresponding solid points in the case of Γ_1 parametrization. From the comparison it becomes clear that our (n_s, r) results are in excellent agreement with those of *Planck 2015*. Indeed, we find:

(a) Γ_0 parametrization: if we use $N = 50$ then we find $n_s = 0.9675$ and $r = 0.0086$, whereas for $N = 60$ we have $n_s = 0.9627$ and $r = 0.0036$.

(b) Γ_1 parametrization: in the case of $N = 50$ we obtain $n_s = 0.9638$ and $r = 0.00857$ and for $N = 60$ we have $n_s = 0.9692$ and 0.00187.

Below we compare the current predictions with those of viable literature potentials. This can help us to understand the variants of the warm-inflationary model from the observationally viable inflationary scenarios.

– Chaotic inflation [66]: In this inflationary model the potential is $V(\phi) \propto \phi^k$. Therefore, the basic slow-roll parameters are written as $\varepsilon = k/4N$, $\eta = (k-1)/2N$ which implies $n_s = 1 - (k+2)/2N$ and $r = 4k/N$. It has been found that monomial potentials with $k \geq 2$ cannot

accommodate the Planck priors [62]. For example, using $k = 2$ and $N = 50$ we obtain $n_s \simeq 0.96$ and $r \simeq 0.16$. For $N = 60$ we have $n_s \simeq 0.967$ and $r \simeq 0.133$. It is interesting to mention that chaotic inflation also corresponds to the slow-roll regime of intermediate inflation [60,61,67,68] with Hubble rate during inflation given by $H \propto t^{k/(4-k)}$ with $n_s = 1 - (k+2)r/8k$ and $k = -2$ giving $n_s = 1$ exactly to first order.

– R^2 inflation [69]: In Starobinsky inflation the asymptotic behavior of the effective potential becomes $V(\phi) \propto [1 - 2e^{-B\phi/M_{pl}} + \mathcal{O}(e^{-2B\phi/M_{pl}})]$, which provides the following slow-roll predictions [70,71]: $n_s \approx 1 - 2/N$ and $r \approx 8/B^2N^2$, where $B^2 = 2/3$. Therefore, if we select $N = 50$ then we obtain $(n_s, r) \approx (0.96, 0.0048)$. For $N = 60$ we find $(n_s, r) \approx (0.967, 0.0033)$. It has been found that the Planck data [62] favors the Starobinsky inflation. Obviously, our results (see Fig. 3) are consistent with those of R^2 inflation.

– Hyperbolic inflation [72]: In hyperbolic inflation the potential is given by $V(\phi) \propto \sinh^b(\phi/f_1)$. Initially, Rubano and Barrow [73] proposed this potential in the context of dark energy. Recently, Basilakos and Barrow [72] investigated the properties of this scalar-field potential back in the inflationary epoch. Specifically, the slow-roll parameters are written as

$$\varepsilon = \frac{b^2 M_{pl}^2}{2f_1^2} \coth^2(\phi/f_1),$$

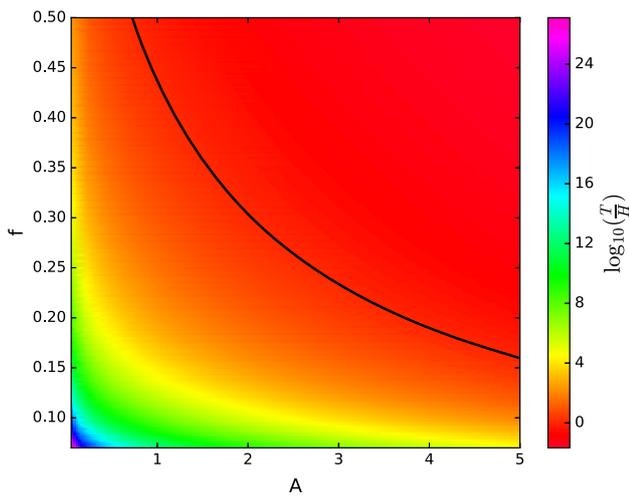$$\eta = \frac{b M_{pl}^2}{f_1^2} \left[(b-1)\coth^2(\phi/f_1) + 1 \right],$$

Fig. 4 The value of $\frac{T}{H}$ at beginning of inflation in the A–f plane. The *solid black curve* shows the boundary $T = H$

and

$$\phi = f_1 \cosh^{-1}\left[e^{NbM_{pl}^2/f^2}\cosh(\phi_{\text{end}}/f_1)\right].$$

where $\phi_{\text{end}} \simeq \frac{f}{2}\ln(\frac{\theta+1}{\theta-1})$. Comparing this model with the data Basilakos and Barrow [72] found $n_s \simeq 0.968$, $r \simeq 0.075$, $1 < b \leq 1.5$, and $f_1 \geq 11.7M_{pl}$.

– Other inflationary models: The origin of brane [74,75] and exponential [76,77] inflationary models are motivated by the physics of extra dimensions and supergravity, respectively. It has been found that these models are in agreement with the Planck data although the R^2 inflation is the winner from the comparison [62].

At this point we would like to mention that in the high-dissipation regime $R \gg 1$, there is always a region in the A–f plane, which is consistent with the warm inflation condition $T > H$. To clarify this issue we plot in Fig. 4 the diagram of $\log_{10}\frac{T}{H}$ in the A–f plane. The solid line corresponds to the boundary limit $T = H$. Clearly, based on the condition $T > H$ we can reduce the parameter space and thus produce one of the strongest existing constraints (to the best of our knowledge) on A and f. Note that in order to produce the above diagram we have fixed the initial values of T and H to those at the beginning of inflation. After the triggering of inflation the inflaton/photon interaction takes place, which leads to radiation production and thus it guarantees that the above condition holds during the inflationary era.

Finally, we investigate the possibility to treat λ as a free parameter. In fact there are three main conditions which we need to use in order to provide a viable limit on the λ. These are: (a) the high-dissipation regime $R \gg 1$, (b) the warm inflation condition $T > H$, and (c) to recover the *Planck2015* (n_s, r) observational constraints. Our investigation shows

that λ is correlated with the (A, f) pair. For example for $(A, f) = (0.5, 0.25)$ we find $\lambda > 6 \times 10^{-16}$, which is consistent with the above conditions, while for $(A, f) = (0.4, 0.2)$ we obtain $\lambda > 1.5 \times 10^{-20}$. In general we verify that it is not possible to find a lower value of λ for all pairs of (A, f).

5 Conclusions

In this article we investigate the warm inflation for the Friedmann–Robertson–Walker spatially flat cosmological model in which the scale factor of the universe satisfies the form of Barrow [27], namely $a(t) = a_I \exp(At^f)$ $(0 < f < 1)$. Within this context, we estimate analytically the slow-roll parameters and we compare our predictions with those of other inflationary models as well as we test the performance of warm inflation against the observational data. We find that currently the warm-inflationary model is consistent with the results given by *Planck 2015* within 1σ uncertainties.

6 Appendix

In this paper we have studied our model in natural unit ($\frac{h}{2\pi} = c = 1$) therefore we have ([mass] = M, [time] = T, and [length] = L where $[A]$ means dimension of "A")

$$[c] = LT^{-1} = 1[h] = ML^2T^{-1}$$
$$\Rightarrow T = L = M^{-1}. \tag{51}$$

Using Eq. (5) we have

$$[H^2] = \left[\frac{8\pi}{M^2}\rho_\phi\left(1 + \frac{\rho_\phi}{2\lambda}\right)\right]$$
$$\Rightarrow \frac{[a^2]}{[a^2]T^2} = \frac{[\rho_\phi]}{[M^2]} \Rightarrow [\rho_\phi] = [T_\mu^\nu] = [P_s] = M^4 \tag{52}$$

where ρ_ϕ is the scalar-field energy density with dimension M^4. From Eq. (3) we have

$$[\dot{\phi}] = 1 \Rightarrow [\phi] = M^{-1}. \tag{53}$$

It appears that the tachyon scalar field has dimensions of M^{-1}. In Eq. (6) the r.h.s. and the l.h.s. have dimension M^4,

$$[\dot{\rho_\phi}] + [3H\rho_\phi] + [3HP_\phi] = [\Gamma\dot{\phi}^2]$$
$$\Rightarrow \frac{[\rho_\phi]}{T} + \frac{[\rho_\phi]}{T} + \frac{[P_\phi]}{T} = [\Gamma]$$
$$\Rightarrow [\Gamma] = M^5. \tag{54}$$

Now based on Eq. (12) we find

$$[R] = \frac{[\Gamma]}{[H][\rho_\phi]} = \frac{M^5}{M M^4} = 1. \tag{55}$$

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