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Excited state mass spectra of doubly heavy baryons Ω_{cc}, Ω_{bb}, and Ω_{bc}

Zalak Shah[1], **Kaushal Thakkar**[2], **Ajay Kumar Rai**[1,a]

[1] Department of Applied Physics, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat 395007, India
[2] Department of Applied Sciences and Humanities, GIDC Degree Engineering college, Abrama, Navsari 396406, India

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Abstract We discuss the mass spectrum of Ω baryon with two heavy quarks and one light quark (*ccs, bbs, and bcs*). The main goal of the paper is to calculate the ground state masses and after that, the positive and negative parity excited states masses are also obtained within a hypercentral constituent quark model, using Coulomb plus linear potential framework. We also added a first order correction to the potential. The mass spectra up to 5S for radial excited states and 1P–5P, 1D–4D, and 1F–2F states for orbital excited states are computed for Ω_{cc}, Ω_{bb}, and Ω_{bc} baryons. Our obtained results are compared with other theoretical predictions, which could be a useful complementary tool for the interpretation of experimentally unknown heavy baryon spectra. The Regge trajectory is constructed in both the (n_r, M^2) and the (J, M^2) planes for Ω_{cc}, Ω_{bb}, and Ω_{bc} baryons and their slopes and intercepts are also determined. Magnetic moments of doubly heavy Ω's are also calculated.

1 Introduction

The doubly heavy Ω baryons represent a unique part of three-quark systems because they contain a strange light quark. Experiments and theoretical calculations have been diversely used in studying the heavy hadrons in the last few years. So far singly heavy baryons have been discovered and the quantum numbers of most of the observed states have been assigned. Many experiments, LHCb, BELLE, BABAR, FOCUS, are planning to detect doubly and triply heavy baryons [1,2]. The future project PANDA experiment at GSI is expected to give fruitful results in the heavy baryon sector; especially the charm sector. In fact, Ξ_{cc}^{+} has been discovered [3] but none of the double/triple heavy Ω baryons have been discovered. Predictions for the masses of doubly heavy baryons have been presented by many authors so far. Indeed, this gives additional ground for new theoretical and experimental investigations of doubly as well as triply heavy baryonic properties. Much theoretical work has focused on doubly heavy baryons, like the relativistic three-quark model [4,5], the Salpeter model [6], heavy-quark effective theory [7], QCD sum rule [8–12], semiempirical mass formulas [13], the Hamiltonian model [14], the variational approach [15], the three-body Faddeev method [16], the hypercentral constituent quark model [17,18], lattice QCD [19–24], etc. Recently, Wei et al. used Regge phenomenology, and with the quadratic mass relations, they obtained doubly and triply charmed and bottom baryon masses [25,26].

In this paper we shall study baryons containing two heavy quarks; charm–charm, bottom–bottom, and charm–bottom with a light strange quark. First of all, the mass spectra of these baryons (Ω_{cc}^{+}, Ω_{bb}^{-}, and Ω_{bc}^{0}) are determined in the framework of hypercentral constituent quark model (HCQM) [17,18,27–33]. In this paper we use coloumb plus linear potential and solve six-dimensional hyperradial Schrödinger equation numerically. Second, the first order correction is taken into account in the potential. So, the purpose of our new investigation with an alternative calculation scheme of the baryon mass spectrum are elaborated as compared with the earlier performed investigations through the variational approach in Ref. [17,18]. The quantum numbers of the doubly heavy ground state baryons are as follows:

- Strangeness $S = -1$ and isospin $I = 0$
- Spin-parity $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$
- Quark content *ccs, bbs, bcs*

The calculations have been performed for the radial excited states (up to 5S) and orbital excited states (1P–5P, 1D–4D, and 1F–2F) at ν=1.0[1] for Ω_{cc}, Ω_{bb} and Ω_{bc} systems.

[a] e-mail: raiajayk@gmail.com

[1] The different values of the potential index ν from 0.5 to 2.0 had been taken in the calculations, but convenient results are obtained at 1.0.

Table 1 The quark model parameters

Quark content	$m_s = 0.500$ GeV
	$m_c = 1.275$ GeV
	$m_b = 4.67$ GeV
Model parameter	$\tau = -\frac{2}{3}\alpha_s$
	$\alpha_s = 0.4155$ GeV
	$C_F = \frac{2}{3}$ and $C_A = 3$

To obtain the excited states of these doubly heavy baryons, we follow the same methodology as used in previous calculations for singly charm baryons [27,28]. Ground states were studied by various theoretical approaches for Ω_{cc}, Ω_{bb}, and Ω_{bc} and are listed in Table 1.

The paper is organized as follows. After the introduction, we briefly describe our hypercentral constituent quark model in Sect. 2. In Sect. 3 we present our results and draw Regge trajectories for Ω_{cc}, Ω_{bb}, and Ω_{bc} baryons. We also calculate the magnetic moments for each baryons. In the last section, we draw our conclusions.

2 The HCQM model

The baryons are made of three quarks and they are related to the Jacobi co-ordinates given as [34]

$$\boldsymbol{\rho} = \frac{1}{\sqrt{2}}(\mathbf{r_1} - \mathbf{r_2}), \tag{1}$$

$$\boldsymbol{\lambda} = \frac{m_1\mathbf{r_1} + m_2\mathbf{r_2} - (m_1+m_2)\mathbf{r_3}}{\sqrt{m_1^2 + m_2^2 + (m_1+m_2)^2}}. \tag{2}$$

Here m_i and $\mathbf{r_i}$ ($i = 1, 2, 3$) denote the mass and coordinate of the ith constituent quark. Quarks masses are shown in Table 1. The respective reduced masses are given by

$$m_\rho = \frac{2m_1m_2}{m_1+m_2}, \tag{3}$$

$$m_\lambda = \frac{2m_3(m_1^2+m_2^2+m_1m_2)}{(m_1+m_2)(m_1+m_2+m_3)}. \tag{4}$$

The hyperradius $x = \sqrt{\rho^2+\lambda^2}$ and hyperangle $\xi = arctan(\frac{\rho}{\lambda})$ are given in terms of the absolute values ρ and λ of the Jacobi coordinates [35–37]. In the center of mass frame ($R_{c.m.} = 0$), the kinetic energy operator can be written as

$$-\frac{\hbar^2}{2m}(\triangle_\rho + \triangle_\lambda) = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{5}{x}\frac{\partial}{\partial x} + \frac{L^2(\Omega)}{x^2}\right) \tag{5}$$

where $L^2(\Omega) = L^2(\Omega_\rho, \Omega_\lambda, \xi)$ is the quadratic Casimir operator of the six-dimensional rotational group O(6) and its eigen functions are the hyperspherical harmonics,

Table 2 Ground state masses of Ω_{cc}^+, Ω_{bb}^-, and Ω_{bc}^0

Baryons	Ω_{cc}^+		Ω_{bb}^-		Ω_{bc}^0	
J^P	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$
Our work	3.650	3.810	10.446	10.467	7.136	7.187
Ref. [4]	3.719	3.746	10.422	10.432	6.999	7.024
Ref. [5]	3.778	3.872	10.359	10.389	7.088	7.130
Ref. [6]	3.648	3.770	10.271	10.289	6.994	7.017
Ref. [8]	4.250	3.810	9.850	10.280	7.020	7.540
Refs. [9,10]	3.710	3.760	10.320	10.380	–	–
Refs. [11,12]	3.730	3.780	9.970	10.500	6.750	7.300
Ref. [14]	3.832	3.883	10.447	10.467	–	–
Ref. [15]	3.815	3.876	10.454	10.486	7.136	7.187
Ref. [16]	3.697	3.769	10.293	10.321	–	–
Ref. [19]	3.747	3.819	–	–	–	–
Ref. [20]	3.713	3.785	–	–	–	–
Ref. [21]	3.738	3.822	10.273	10.308	6.999	7.059
Ref. [23]	3.740	3.779	–	–	–	–
Ref. [22]	3.654	3.724	–	–	–	–
Refs. [25,26]	3.650	3.809	10.320	10.430	–	–
Ref. [40]	3.630	3.710	9.890	9.930	6.750	6.770
Ref. [41]	3.702	3.783	10.260	10.297	6.986	7.046
Ref. [42]	3.667	3758	10.397	10.495	7.103	7.200
Ref. [43]	3.710	3.800	10.208	10.244	6.999	7.063
Ref. [44]	3.590	3.690	10.180	10.200	6.910	6.990
Ref. [45]	3.740	3.820	10.370	10.400	7.045	7.120

$$Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi),$$

satisfying the eigenvalue relation

$$L^2 Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) = -\gamma(\gamma+4)Y_{[\gamma]l_\rho l_\lambda}(\Omega_\rho, \Omega_\lambda, \xi).$$

Here, γ is the grand angular momentum quantum number. In the present paper, the confining three-body potential is chosen within a string-like picture, where the quarks are connected by gluonic strings and the potential strings increase linearly with the collective radius r_{3q} as mentioned in [38]. Accordingly the effective two-body interactions can be written as

$$\sum_{i<j} V(r_{ij}) = V(x) + \cdots \tag{6}$$

In the hypercentral approximation, the potential only depends on the hyperradius (x). More details can be found in Refs. [38,39]. The hyperradius x is a collective coordinate and therefore the hypercentral potential contains also the three-body effects. The Hamiltonian of the three-body baryonic system in the hCQM is then expressed as

$$H = \frac{P_x^2}{2m} + V(x) \tag{7}$$

Table 3 Radial excited states masses of Ω_{cc}, Ω_{bb}, and Ω_{bc} baryons (in GeV)

Baryon	State	J^P	A	B	[14]	[15]	[16]	[6]	[5]
A → without first order correction and B → with first order correction									
	2S	$\frac{1}{2}^+$	4.028	4.041	4.227	4.180	4.112	4.268	4.075
		$\frac{3}{2}^+$	4.085	4.096	4.263	4.188	4.160	4.334	4.174
Ω_{cc}	3S	$\frac{1}{2}^+$	4.317	4.338	4.295			4.714	4.321
		$\frac{3}{2}^+$	4.345	4.365	4.265			4.776	
	4S	$\frac{1}{2}^+$	4.570	4.598					
		$\frac{3}{2}^+$	4.586	4.614					
	5S	$\frac{1}{2}^+$	4.801	4.836					
		$\frac{3}{2}^+$	4.811	4.845					
	2S	$\frac{1}{2}^+$	10.730	10.736	10.707	10.693	10.604	10.830	10.610
		$\frac{3}{2}^+$	10.737	10.743	10.723	10.721	10.622	10.839	10.645
Ω_{bb}	3S	$\frac{1}{2}^+$	10.973	10.983	10.744			11.240	10.806
		$\frac{3}{2}^+$	10.976	10.986	10.730			11.247	10.843
	4S	$\frac{1}{2}^+$	11.191	11.205	10.994				
		$\frac{3}{2}^+$	11.193	11.207	11.031				
	5S	$\frac{1}{2}^+$	11.393	11.411					
		$\frac{3}{2}^+$	11.394	11.412					
	2S	$\frac{1}{2}^+$	7.473	7.480				7.559	
		$\frac{3}{2}^+$	7.490	7.497				7.571	
Ω_{bc}	3S	$\frac{1}{2}^+$	7.753	7.767				7.976	
		$\frac{3}{2}^+$	7.761	7.775				7.985	
	4S	$\frac{1}{2}^+$	8.004	8.023					
		$\frac{3}{2}^+$	8.009	8.028					
	5S	$\frac{1}{2}^+$	8.236	8.260					
		$\frac{3}{2}^+$	8.239	8.263					

where $m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}$ is the reduced mass and x is the six-dimensional radial hypercentral coordinate of the three-body system. The hyperradial Schrödinger equation corresponding to the above Hamiltonian can be written

$$\left[\frac{d^2}{dx^2} + \frac{5}{x}\frac{d}{dx} - \frac{\gamma(\gamma+4)}{x^2}\right]\Psi_\gamma(x) = -2m[E - V(x)]\Psi_\gamma(x) \tag{8}$$

where Ψ_γ(x) is the hypercentral wave function and γ is the grand angular quantum number. We consider a reduced hypercentral radial function, $\phi_\gamma(x) = x^{\frac{5}{2}}\Psi_\gamma(x)$. Thus, the six-dimensional hyperradial Schrödinger equation reduces to

$$\left[\frac{-1}{2m}\frac{d^2}{dx^2} + \frac{\frac{15}{4} + \gamma(\gamma+4)}{2mx^2} + V(x)\right]\phi_\gamma(x) = E\phi_\gamma(x). \tag{9}$$

For the present study we consider the hypercentral potential V(x) as the color Coulomb plus linear potential with first order correction [46–51],

$$V(x) = V^0(x) + \left(\frac{1}{m_\rho} + \frac{1}{m_\lambda}\right)V^{(1)}(x) + V_{SD}(x) \tag{10}$$

where $V^0(x)$ is given by

$$V^{(0)}(x) = \frac{\tau}{x} + \beta x \tag{11}$$

and the first order correction is similar to the one given by [46],

$$V^{(1)}(x) = -C_F C_A \frac{\alpha_s^2}{4x^2}. \tag{12}$$

Here, τ is the hyper-Coulomb strength corresponding to the strong running coupling constant α_s [27,28]. β is the

Table 4 Orbitally excited states masses of Ω_{cc} baryons (in GeV)

State	A	B	[14]	[15]	[20]	[5]	[25]	Others
A $\rightarrow$ without first order correction and B $\rightarrow$ with first order correction								
$(1^2P_{1/2})$	3.964	3.989	4.086	4.046	4.061	4.002		4.009 [16]
$(1^2P_{3/2})$	3.948	3.972	4.086	4.052	4.132	4.102	3.910	
$(1^4P_{1/2})$	3.972	3.998						
$(1^4P_{3/2})$	3.956	3.981						3.960 [12]
$(1^4P_{5/2})$	3.935	3.958	4.220	4.152			4.058	
$(2^2P_{1/2})$	4.241	4.273	4.199	4.135		4.251		4.101 [16]
$(2^2P_{3/2})$	4.228	4.259	4.201	4.140		4.345		
$(2^4P_{1/2})$	4.248	4.280						
$(2^4P_{3/2})$	4.234	4.266						
$(2^4P_{5/2})$	4.216	4.247						
$(3^2P_{1/2})$	4.492	4.529						
$(3^2P_{3/2})$	4.479	4.517						
$(3^4P_{1/2})$	4.498	4.536						
$(3^4P_{3/2})$	4.486	4.523						
$(3^4P_{5/2})$	4.469	4.506						
$(4^2P_{1/2})$	4.723	4.767						
$(4^2P_{3/2})$	4.712	4.755						
$(4^4P_{1/2})$	4.728	4.772						
$(4^4P_{3/2})$	4.717	4.761						
$(4^4P_{5/2})$	4.703	4.745						
$(5^2P_{1/2})$	4.939	4.989						
$(5^2P_{3/2})$	4.929	4.978						
$(5^4P_{1/2})$	4.944	4.994						
$(5^4P_{3/2})$	4.934	4.984						
$(5^4P_{5/2})$	4.921	4.969						
$(1^4D_{1/2})$	4.156	4.186						
$(1^2D_{3/2})$	4.133	4.162						
$(1^4D_{3/2})$	4.141	4.170						
$(1^2D_{5/2})$	4.113	4.141	4.264	4.202			4.153	
$(1^4D_{5/2})$	4.121	4.149						
$(1^4D_{7/2})$	4.095	4.122					4.294	
$(2^4D_{1/2})$	4.407	4.446						
$(2^2D_{3/2})$	4.389	4.425						
$(2^4D_{3/2})$	4.395	4.432						
$(2^2D_{5/2})$	4.372	4.407						
$(2^4D_{5/2})$	4.378	4.414	4.299	4.232				
$(2^4D_{7/2})$	4.358	4.391						
$(3^4D_{1/2})$	4.446	4.642						
$(3^2D_{3/2})$	4.425	4.625						
$(3^4D_{3/2})$	4.432	4.631						
$(3^2D_{5/2})$	4.407	4.611						
$(3^4D_{5/2})$	4.414	4.616	4.410					
$(3^4D_{7/2})$	4.391	4.598						
$(4^4D_{1/2})$	4.863	4.911						
$(4^2D_{3/2})$	4.847	4.894						

Table 4 continued

State	A	B	[14]	[15]	[20]	[5]	[25]	Others
$(4^4D_{3/2})$	4.853	4.900						
$(4^2D_{5/2})$	4.833	4.879						
$(4^4D_{5/2})$	4.838	4.885						
$(4^4D_{7/2})$	4.821	4.866						
$(1^4F_{3/2})$	4.313	4.348						
$(1^2F_{5/2})$	4.287	4.321						
$(1^4F_{5/2})$	4.294	4.328						
$(1^4F_{7/2})$	4.271	4.303						
$(1^2F_{7/2})$	4.264	4.296					4.383	
$(1^4F_{9/2})$	4.244	4.274					4.516	
$(1^4F_{3/2})$	4.552	4.593						
$(2^2F_{5/2})$	4.530	4.569						
$(2^4F_{5/2})$	4.536	4.575						
$(2^4F_{7/2})$	4.515	4.553						
$(2^2F_{7/2})$	4.509	4.547						
$(2^4F_{9/2})$	4.490	4.527						

string tension of the confinement part of potential. C_F and C_A are the Casimir charges of the fundamental and adjoint representation. If we compare Eq. (9) with the usual three-dimensional radial Schrödinger equation, the resemblance between angular momentum and hyper angular momentum is given by [52], $l(l+1) \to \frac{15}{4} + \gamma(\gamma+4)$. The spin-dependent part, $V_{SD}(x)$ of Eq. (10) contains three types of interaction terms [53],

$$V_{SD}(x) = V_{SS}(x)(\mathbf{S}_\rho . \mathbf{S}_\lambda) + V_{\gamma S}(x)(\boldsymbol{\gamma} \cdot \mathbf{S}) + V_T(x)\left[S^2 - \frac{3(\mathbf{S}\cdot\mathbf{x})(\mathbf{S}\cdot\mathbf{x})}{x^2}\right]. \quad (13)$$

The spin–spin term $V_{SS}(x)$ gives the spin singlet triplet splittings, the spin–orbit term $V_{\gamma S}(x)$ and tensor term $V_T(x)$ describe the fine structure of the states. The details of the terms are given in [27,28]. We numerically solve the six-dimensional Schrödinger equation using the Mathematica notebook [54]. We have followed the $^{(2S+1)}\gamma_J$ notations for spectra of baryons.

3 Results and discussions

3.1 Mass spectra

The ground and excited states of doubly heavy Ω baryons are still experimentally unknown to us. Therefore, we have calculated the ground as well as excited state masses of doubly heavy baryons Ω_{cc}, Ω_{bb}, and Ω_{bc} (see Tables 2, 3, 4, 5 and 6). Such kind of theoretical study is very much useful to obtain their experimental states (J^P values), masses, and other properties. These mass spectra of doubly heavy baryons are obtained by using Coulomb plus linear potential in the hypercentral constituent quark model. Our computed ground states with $J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$ are compared with different theoretical approaches in Table 2. Our estimated ground state masses of Ω_{cc}^+ have a difference in the range of ≈ 100 MeV with other predictions, whereas the Ω_{bb}^- and Ω_{bc}^0 masses are higher than others.

Lattice QCD calculations for excited states of Ω_{cc} have been performed by Padmanath et al. up to $\frac{7}{2}^+$ and $\frac{7}{2}^-$ parities [55]. They use a variational approach in which they try to write down the eigen states in terms of the operators and determine the energies from the evolution of the correlators of the eigenstates. The energy splittings of the Ω_{cc} states are from the mass of the η_c meson. Reference [56] estimates the ground state mass of Ω_{cc} to be around 3.726 GeV in chiral perturbation theory.

The radial excited state masses for these three baryons are computed from 2S–5S and are compared with Refs. [5, 6,14–16] in Table 3. We can observe that our 2S and 3S states show a smaller difference in MeV, see Ref. [5], than other references for Ω_{cc}^+ and Ω_{bb}^-. Next, in the case of Ω_{bc} only Ref. [6] has calculated radial excited states up to 3S. We noticed that our 2S and 3S state masses (A(B)) with $J^P = \frac{1}{2}^+$ are 76(69) and 225(211), while 80(73) and 224(210) (with $J^P = \frac{3}{2}^+$) are lower, respectively. Note that A are masses without first order correction and B are masses by adding a first order correction in Tables 3, 4, 5, and 6.

The orbital excited states are calculated for 1P–5P, 1D–4D, and 1F–2F mentioned in Tables 4, 5, and 6. Isospin splittings were also considered, which means that we have con-

Table 5 Orbitally excited states masses of Ω_{bb} baryons (in GeV)

State	A	B	[14]	[15]	[5]	[25]	Others
A → without first order correction and B → with first order correction							
$(1^2P_{1/2})$	10.634	10.646	10.607	10.616	10.532		10.519 [16]
$(1^2P_{3/2})$	10.629	10.641	10.608	10.619	10.566	10.593 ± 58	10.520 [9,10]
$(1^4P_{1/2})$	10.636	10.648					
$(1^4P_{3/2})$	10.631	10.643					10.513 [12]
$(1^4P_{5/2})$	10.625	10.637	10.808	10.766	10.798	10.700 ± 60	
$(2^2P_{1/2})$	10.881	10.897	10.796	10.763	10.738		10.683 [16]
$(2^2P_{3/2})$	10.877	10.893	10.797	10.765	10.775		
$(2^4P_{1/2})$	10.883	10.899			10.924		
$(2^4P_{3/2})$	10.879	10.898			10.961		
$(2^4P_{5/2})$	10.874	10.888	11.028				
$(3^2P_{1/2})$	11.104	11.124	10.803		11.083		
$(3^2P_{3/2})$	11.101	11.120	10.805				
$(3^4P_{1/2})$	11.106	11.125					
$(3^4P_{3/2})$	11.103	11.122					
$(3^4P_{5/2})$	11.098	11.177	11.059				
$(4^2P_{1/2})$	11.310	11.332					
$(4^2P_{3/2})$	11.307	11.339					
$(4^4P_{1/2})$	11.312	11.334					
$(4^4P_{3/2})$	11.309	11.331					
$(4^4P_{5/2})$	11.305	11.322					
$(5^2P_{1/2})$	11.503	11.528					
$(5^2P_{3/2})$	11.500	11.525					
$(5^4P_{1/2})$	11.504	11.530					
$(5^4P_{3/2})$	11.502	11.527					
$(5^4P_{5/2})$	11.498	11.523					
$(1^4D_{1/2})$	10.789	10.804					
$(1^2D_{3/2})$	10.783	10.797					
$(1^4D_{3/2})$	10.785	10.800					
$(1^2D_{5/2})$	10.777	10.792	10.729	10.720		10.858 ± 77	
$(1^4D_{5/2})$	10.779	10.794					
$(1^4D_{7/2})$	10.772	10.786				10.964 ± 80	
$(2^4D_{1/2})$	11.017	11.036					
$(2^2D_{3/2})$	11.012	11.030					
$(2^4D_{3/2})$	11.014	11.032					
$(2^2D_{5/2})$	11.008	11.025	10.744	10.734			
$(2^4D_{5/2})$	11.009	11.027					
$(2^4D_{7/2})$	11.004	11.021					
$(3^4D_{1/2})$	11.228	11.249					
$(3^2D_{3/2})$	11.223	11.244					
$(3^4D_{3/2})$	11.225	11.246					
$(3^2D_{5/2})$	11.219	11.240	10.937				
$(3^4D_{5/2})$	11.220	11.241					
$(3^4D_{7/2})$	11.215	11.236					
$(4^4D_{1/2})$	11.424	11.448					
$(4^2D_{3/2})$	11.420	11.444					

Table 5 continued

State	A	B	[14]	[15]	[5]	[26]	Others
$(4^4 D_{3/2})$	11.421	11.445					
$(4^2 D_{5/2})$	11.416	11.440					
$(4^4 D_{5/2})$	11.418	11.441					
$(4^4 D_{7/2})$	11.413	11.437					
$(1^4 F_{3/2})$	10.927	10.943					
$(1^2 F_{5/2})$	10.920	10.936					
$(1^4 F_{5/2})$	10.922	10.938					
$(1^4 F_{7/2})$	10.915	10.932					
$(1^2 F_{7/2})$	10.913	10.930				11.118 ± 96	
$(1^4 F_{9/2})$	10.907	10.924				11.221 ± 99	
$(2^4 F_{3/2})$	11.142	11.162					
$(2^2 F_{5/2})$	11.136	11.155					
$(2^4 F_{5/2})$	11.137	11.157					
$(2^4 F_{7/2})$	11.132	11.151					
$(2^2 F_{7/2})$	11.130	11.149					
$(2^4 F_{9/2})$	11.125	11.144					

sidered all possible combinations of total spin S and total angular momentum J to obtain orbital states mass spectra. We can observe that the total combinations for P, D, and F states number 5, 6, and 6, respectively. Other theoretical approaches have also calculated these orbital excited states but they have not considered all (S, J) combinations. We have compared our outcomes with other models in Tables 4, 5, and 6.

Our obtained orbital excited masses (A) are compared and discussed with other predictions in the following paragraph. For Ω_{cc}, our 1P state $J^P = \frac{1}{2}^-$ shows 38 MeV (with [5]), $J^P = \frac{3}{2}^-$ shows 38 MeV and $J^P = \frac{5}{2}^-$ show 123 MeV (with [25]) difference. Our 2P state $J^P = \frac{1}{2}^-$ shows 10 MeV (with [5]), $J^P = \frac{3}{2}^-$ shows 27 MeV (with [14]) difference. Our 1D state $J^P = \frac{5}{2}^+$ shows 40 MeV and $J^P = \frac{3}{2}^-$ shows 199 MeV difference with [25]. For the 2D–3D states the difference is 79 and 4 MeV for $J^P = \frac{5}{2}^+$ with Ref. [14]. Our 1F state values are $119(J^P = \frac{7}{2}^-)$ and $272(J^P = \frac{9}{2}^-)$ MeV lower than Ref. [25].

For Ω_{bb}, our 1P state $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ are 18 and 10 MeV higher than Ref. [15]. Reference [14] has masses for $J^P = \frac{1}{2}^-$, $J^P = \frac{3}{2}^-$, and $J^P = \frac{5}{2}^-$ which are 85, 80, and 154 MeV lower than our prediction. Our 3P state shows 39 MeV difference with [14]. Our 1D state $J^P = \frac{5}{2}^+$ shows 48 and 57 MeV difference with [14,15], respectively. We have also compared our results with the recent paper [26] for 1P, 1D, and 1F states. Their values are higher than ours. The orbital mass spectra of the third doubly heavy baryon, Ω_{bc}, is not calculated in any approach. References [5,44] stated that excited levels are not possible for Ω_{bc} because the excited

Table 6 Orbitally excited states masses of Ω_{bc} baryons (in GeV)

State	A	B
A → without first order correction and B → with first order correction		
$(1^2 P_{1/2})$	7.375	7.386
$(1^2 P_{3/2})$	7.363	7.373
$(1^4 P_{1/2})$	7.381	7.392
$(1^4 P_{3/2})$	7.369	7.379
$(1^4 P_{5/2})$	7.353	7.363
$(2^2 P_{1/2})$	7.657	7.674
$(2^2 P_{3/2})$	7.647	7.664
$(2^4 P_{1/2})$	7.662	7.679
$(2^4 P_{3/2})$	7.652	7.669
$(2^4 P_{5/2})$	7.639	7.655
$(3^2 P_{1/2})$	7.912	7.935
$(3^2 P_{3/2})$	7.903	7.925
$(3^4 P_{1/2})$	7.916	7.939
$(3^4 P_{3/2})$	7.908	7.930
$(3^4 P_{5/2})$	7.896	7.918
$(4^2 P_{1/2})$	8.147	8.175
$(4^2 P_{3/2})$	8.140	8.167
$(4^4 P_{1/2})$	8.151	8.179
$(4^4 P_{3/2})$	8.143	8.171
$(4^4 P_{3/2})$	8.133	8.160
$(5^2 P_{1/2})$	8.368	8.400
$(5^2 P_{3/2})$	8.361	8.393
$(5^4 P_{1/2})$	8.372	8.404
$(5^4 P_{3/2})$	8.365	8.396

Table 6 continued

State	A	B
$(5^4P_{5/2})$	8.355	8.386
$(1^2D_{1/2})$	7.562	7.577
$(1^2D_{3/2})$	7.545	7.561
$(1^4D_{3/2})$	7.551	7.566
$(1^2D_{5/2})$	7.531	7.547
$(1^4D_{5/2})$	7.536	7.552
$(1^4D_{7/2})$	7.518	7.534
$(2^2D_{1/2})$	7.821	7.843
$(2^2D_{3/2})$	7.807	7.829
$(2^4D_{3/2})$	7.812	7.834
$(2^2D_{5/2})$	7.795	7.816
$(2^4D_{5/2})$	7.799	7.821
$(2^4D_{7/2})$	7.784	7.805
$(3^2D_{1/2})$	8.060	8.088
$(3^2D_{3/2})$	8.048	8.075
$(3^4D_{3/2})$	8.052	8.079
$(3^2D_{5/2})$	8.037	8.063
$(3^4D_{5/2})$	8.041	8.068
$(3^4D_{7/2})$	8.028	8.054
$(4^2D_{1/2})$	8.285	8.317
$(4^2D_{3/2})$	8.274	8.305
$(4^4D_{3/2})$	8.277	8.309
$(4^2D_{5/2})$	8.263	8.294
$(4^4D_{5/2})$	8.267	8.298
$(4^4D_{7/2})$	8.254	8.285
$(1^4F_{3/2})$	7.721	7.742
$(1^2F_{5/2})$	7.702	7.723
$(1^4F_{5/2})$	7.708	7.728
$(1^4F_{7/2})$	7.690	7.711
$(1^2F_{7/2})$	7.685	7.705
$(1^4F_{9/2})$	7.670	7.690
$(2^4F_{3/2})$	7.721	7.965
$(2^2F_{5/2})$	7.702	7.949
$(2^4F_{5/2})$	7.708	7.953
$(2^4F_{7/2})$	7.690	7.938
$(2^2F_{7/2})$	7.685	7.934
$(2^4F_{9/2})$	7.670	7.921

states of diquarks $\{bc\}$ are not stable due to the emission of soft gluons. We have not considered the diquark mechanism in our approach, so that we calculated orbital mass spectra for the Ω_{bc} baryon. We guess that we are the first to calculate these spectra.

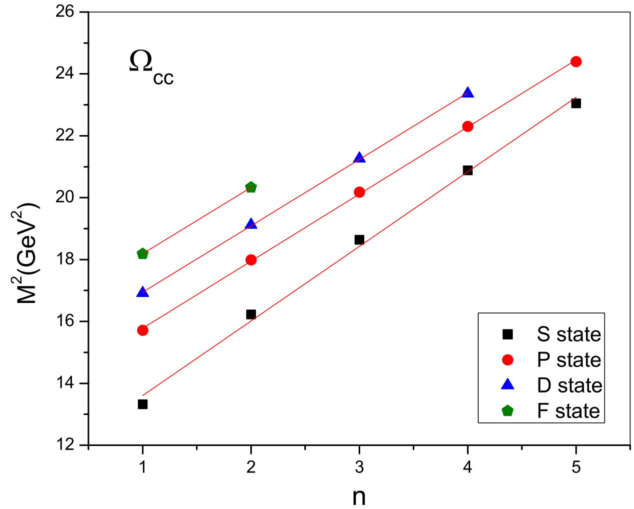

Fig. 1 Regge trajectory ($M^2 \to$ n) for Ω_{cc} baryon

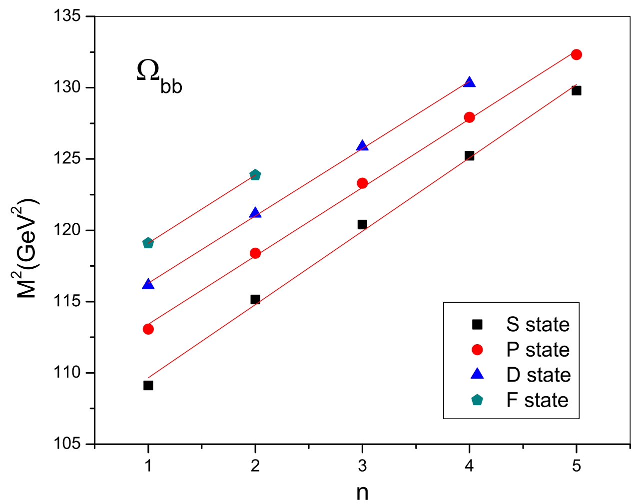

Fig. 2 Regge trajectory ($M^2 \to$ n) for Ω_{bb} baryon

3.2 Regge trajectory

We calculated both radial and orbital excited states masses up to L = 3. Using them we are able to construct Regge trajectories in the (n, M^2) and (J, M^2) planes. n is the principal quantum number and J is the total spin. The Regge trajectories are presented in Figs. 1, 2, 3, 4, and 5. Straight lines were obtained by the linear fitting in all figures. The ground and radial excited states S ($J^P = \frac{1}{2}^+$) and the orbital excited state P ($J^P = \frac{1}{2}^-$), D ($J^P = \frac{5}{2}^+$), and F ($J^P = \frac{7}{2}^-$) are plotted in Figs. 1, 2, and 3 from bottom to top. We use

$$n_r = \beta M^2 + \beta_0 \tag{14}$$

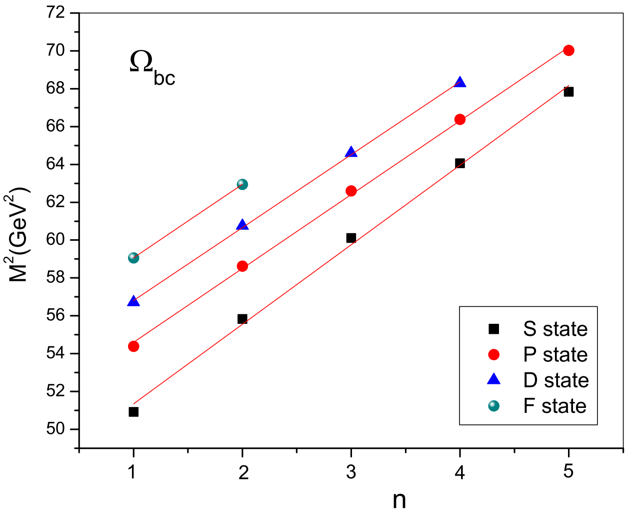

Fig. 3 Regge trajectory ($M^2 \rightarrow$ n) for Ω_{bc} baryon

where β and β_0 are the slope and intercept, respectively, and n_r= n-1. The fitted slopes and intercepts are given in Table 7.

We use the natural ($J^P = \frac{1}{2}^+$, $J^P = \frac{3}{2}^-$, $J^P = \frac{5}{2}^+$, $J^P = \frac{7}{2}^-$) and unnatural ($J^P = \frac{3}{2}^+$, $J^P = \frac{5}{2}^-$, $J^P = \frac{7}{2}^+$, $J^P = \frac{9}{2}^-$) parity masses and plotted graphs for Ω_{cc} and Ω_{bb} (see Figs. 4, 5). For that reason we use

$$J = \alpha M^2 + \alpha_0 \tag{15}$$

where α and α_0 are the slope and intercept, respectively. The fitted slopes and intercepts for both natural and unnatural parities are given in Table 8. We observe that the squares of the calculated masses fit very well to the linear trajectory and yield almost parallel states, equidistant in S, P, D, and F. We can determine the possible quantum numbers and pre-

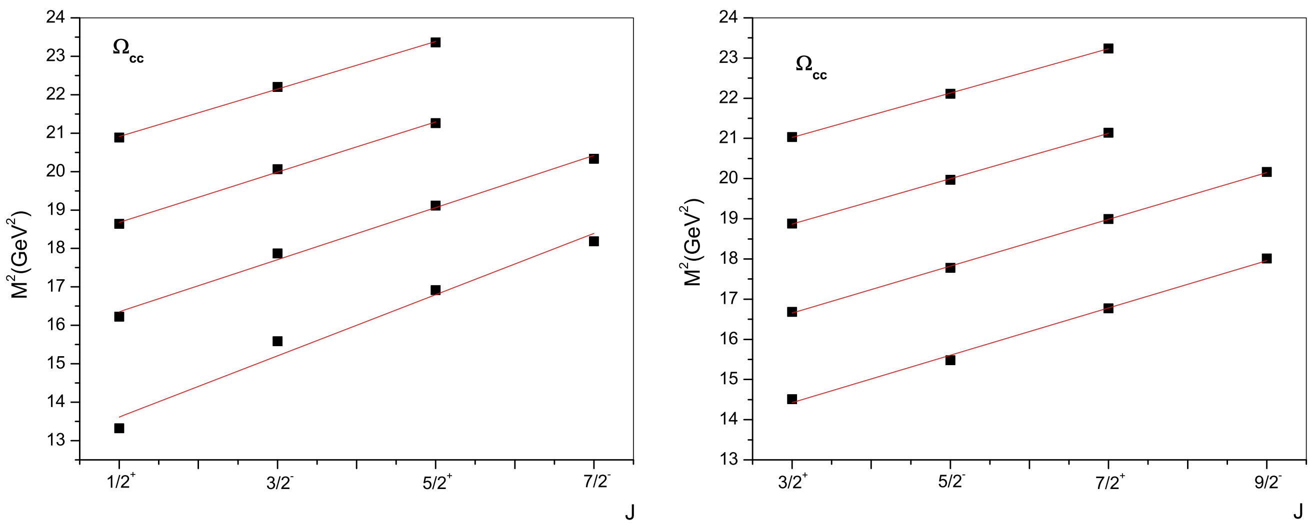

Fig. 4 Regge trajectory ($M^2 \rightarrow$ J) for Ω_{cc} baryon

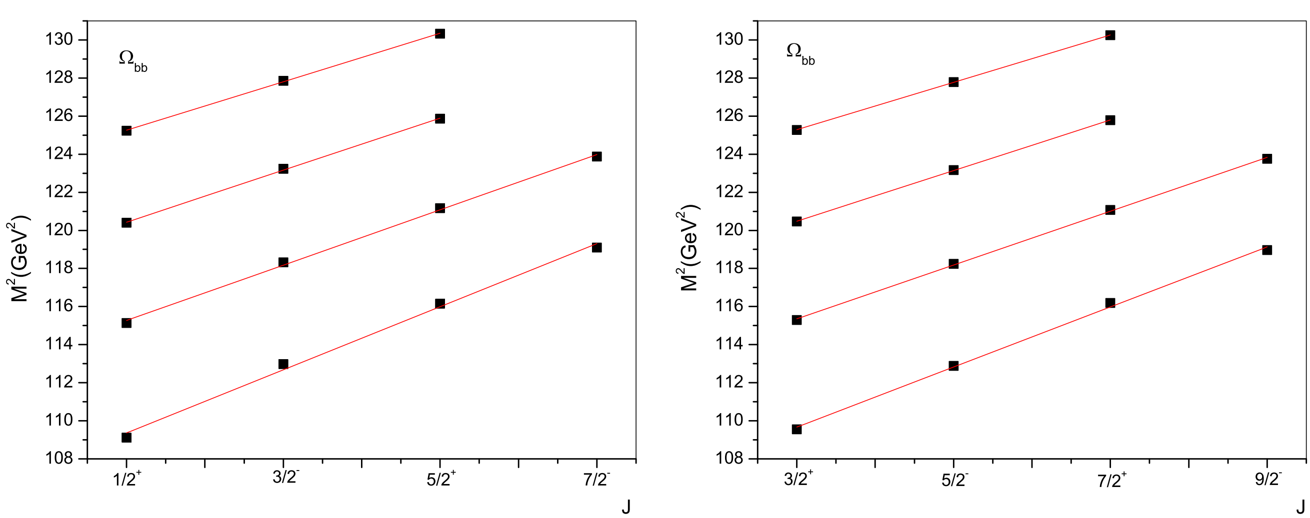

Fig. 5 Regge trajectory ($M^2 \rightarrow$ J) for Ω_{bb} baryon

Table 7 Fitted slopes and intercepts of the Regge trajectories in the (n, M^2) plane

Baryon	J^P	State	β	β_0
Ω_{cc}	$\frac{1}{2}^+$	S	0.413 ± 0.0144	-5.614 ± 0.270
	$\frac{1}{2}^-$	P	0.461 ± 0.004	-7.273 ± 0.09
	$\frac{3}{2}^-$	P	0.460 ± 0.05	-7.195 ± 0.091
	$\frac{5}{2}^+$	D	0.466 ± 0.003	-7.890 ± 0.007
Ω_{bb}	$\frac{1}{2}^+$	S	0.194 ± 0.006	-21.242 ± 0.752
	$\frac{1}{2}^-$	P	0.208 ± 0.004	-23.583 ± 0.560
	$\frac{3}{2}^-$	P	0.208 ± 0.05	-23.520 ± 0.056
	$\frac{5}{2}^+$	D	0.212 ± 0.004	-24.613 ± 0.490
Ω_{bc}	$\frac{1}{2}^+$	S	0.194 ± 0.006	-21.242 ± 0.752
	$\frac{1}{2}^-$	P	0.255 ± 0.004	-13.885 ± 0.282
	$\frac{3}{2}^-$	P	0.208 ± 0.05	-23.520 ± 0.056
	$\frac{5}{2}^+$	D	0.259 ± 0.004	-14.731 ± 0.238

scribe them to particular Regge trajectory with the help of our obtained results.

3.3 Magnetic moments

The magnetic moment of baryons are obtained in terms of the spin, charge, and effective mass of the bound quarks as [18,29–33]

$$\mu_B = \sum_i \langle \phi_{sf} | \mu_{iz} | \phi_{sf} \rangle)$$

where

$$\mu_i = \frac{e_i \sigma_i}{2 m_i^{eff}} \tag{16}$$

e_i is the charge and σ_i is the spin of the respective constituent quark corresponding to the spin flavor wave function of the baryonic state. The effective mass for each of the constituting quark m_i^{eff} can be defined as

$$m_i^{eff} = m_i \left(1 + \frac{\langle H \rangle}{\sum_i m_i}\right) \tag{17}$$

where $\langle H \rangle$ = E + $\langle V_{spin} \rangle$. Using these equations, we calculated the magnetic moments of the Ω_{cc}^+, Ω_{bb}^-, and Ω_{bc}^- baryons. The spin flavor wave function [57] and magnetic moments are given in Table 9. Our obtained ground state magnetic moments are also compared with others and the results are reasonably close.

4 Conclusions

The ground state as well as excited state masses are obtained for doubly heavy Ω baryons in hCQM; they are tabulated in Tables 2, 3, 4, 5, and 6. We have compared our results with other approaches and they are not in mutual agreement. Moreover, all are unknown experimentally. Due to that fact, we cannot single out any model. It is very important to compare results with lattice QCD but for most of them have calculated only ground state masses of these baryons [21–23].

Table 8 Fitted parameters α and α_0 are slope and Intercept of parent and daughter Regge trajectories. Columns 3, 4 are for natural parities and 5, 6 are for unnatural parities

Baryon	Trajectory	α	α_0	α	α_0
Ω_{cc}	Parent	0.614 ± 0.0653	-7.332 ± 1.051	0.846 ± 0.0358	-11.202 ± 0.582
	1 Daughter	0.733 ± 0.039	-10.983 ± 0.731	0.858 ± 0.0125	-13.291 ± 0.231
	2 Daughter	0.762 ± 0.039	-13.225 ± 0.783	0.884 ± 0.017	-15.688 ± 0.341
	3 Daughter	0.807 ± 0.0295	-15.897 ± 0.655	0.906 ± 0.0108	-180.055 ± 0.241
Ω_{bb}	Parent	0.301 ± 0.013	-31.881 ± 1.523	0.316 ± 0.009	-33.699 ± 1.081
	1 Daughter	0.344 ± 0.009	-38.61 ± 1.052	0.354 ± 0.005	-39.804 ± 0.649
	2 Daughter	0.366 ± 0.008	-43.065 ± 0.955	0.376 ± 0.004	-44.354 ± 0.437
	3 Daughter	0.393 ± 0.006	-48.172 ± 0.794	0.402 ± 0.003	-49.381 ± 0.341

Table 9 Magnetic moment (in nuclear magnetons) of J^P: $\frac{1}{2}^+$ and $\frac{3}{2}^+$ doubly heavy baryons

Baryons	Wave-function	Our	[58]	[18]	[59]	[60,61]	[62]	[63]
Ω_{cc}^+	$\frac{4}{3}\mu_c - \frac{1}{3}\mu_s$	0.692	0.668	0.785	0.635	–	0.639	0.66
Ω_{cc}^{+*}	$\mu_c + \mu_s$	0.285	0.332	0.121	0.139	0.210	–	
Ω_{bb}^-	$\frac{4}{3}\mu_b - \frac{1}{3}\mu_s$	0.108	0.120	0.109	0.101	0.111	0.100	
Ω_{bb}^{-*}	$2\mu_b + \mu_s$	−1.239	−0.730	−0.711	−0.662	−0.703	–	
Ω_{bc}^0	$\frac{2}{3}\mu_b + \frac{2}{3}\mu_c - \frac{1}{3}\mu_s$	0.439	0.034	0.397	0.368	0.399	–	
Ω_{bc}^{0*}	$\mu_b + \mu_c + \mu_s$	−0.181	−0.111	−0.317	−0.261	−0.274	–	

We have also constructed the Regge trajectories from the masses obtained. This study will help future experiments to identify the baryonic states from resonances. In addition, we have calculated the magnetic moments and they are fairly in agreement with other calculations. We have successfully employed this model to study doubly heavy Ω baryons. Now, we would like to calculate the properties of Ξ baryons in the future.

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