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Searching for $\tau \to \mu\gamma$ lepton-flavor-violating decay at super Charm-Tau factory

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Abstract We investigate the possibility of searching the lepton-flavor-violating (LFV) $\tau \to \mu\gamma$ rare decay at the Super Charm-Tau Factory (CTF). By comparing the kinematic distributions of the LFV signal and the standard model background, we develop an optimized event selection criterion which can significantly reduce the background events. It is concluded that the new 2σ upper limit of about 1.9×10^{-9} on $\mathrm{Br}(\tau \to \mu\gamma)$ can be obtained at the CTF, which is beyond the capability of Super-B factory in searching τ lepton rare decay. Within the framework of the scalar leptoquark model, a joint constraint on $\lambda_1\lambda_2$ and M_{LQ} can be derived from the upper bound on $\mathrm{Br}(\tau \to \mu\gamma)$. With $1000\ fb^{-1}$ data expected at the CTF, we get $\lambda_1\lambda_2 < 7.2 \times 10^{-2}$ ($M_{\mathrm{LQ}} = 800\,\mathrm{GeV}$) and $M_{\mathrm{LQ}} > 900\,\mathrm{GeV}$ ($\lambda_1\lambda_2 = 9 \times 10^{-2}$) at 95 % confidence level.

1 Introduction

The standard model (SM) [1,2] of elementary particle physics provides a remarkably successful description of strong, weak and electromagnetic interactions at the energy scale up to $\mathcal{O}(10^2)$ GeV. It is an $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_C \otimes U(1)_{\mathrm{EM}}$ spontaneously broken gauge theory, which also conserves the total baryon number B and the three lepton numbers L_e, L_μ, and L_τ, i.e., the lepton flavors, respectively. However, a number of conceptual and experimental difficulties, such as the hierarchy problem, dark matter, and neutrino oscillations, inspired physicists to consider new mechanisms beyond the SM. Many extensions of the SM, such as the supersymmetric models, left–right symmetric models, the little Higgs model with T parity, and leptoquark models, could bring in lepton-flavor-violating (LFV) terms in natural ways and introduce non-zero neutrino masses and rare decays of the charged lepton. As the heaviest lepton, the τ lepton has more LFV decay modes compared to the μ lepton. The branching ratios of the τ lepton LFV decays are predicted at the level of 10^{-10}–10^{-7} [3–6]. Therefore, searching for τ LFV decays and improving limits on the branching ratio becomes increasingly important of current and future experiments.

All the LFV decays of τ lepton, such as $\tau \to l\gamma$, $\tau \to lll^{(\prime)}$, and $\tau \to lh$, where $l, l' = e$ or μ and h is a hadronic system, are sensitive to new physics beyond the SM. Among these modes, the radiative decays $\tau \to \mu\gamma$ and $\tau \to e\gamma$ are predicted to have the largest probability close to current experimental upper limits in a wide variety of new physics scenarios. So far the most stringent limits are $\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$ and $\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$ at 90 % confidence level (C.L.), with $(963 \pm 7) \times 10^6\ \tau$ decays of data collected by the B factories [7]. To achieve more sensitivity in probing τ LFV decay, high intensive electron–positron beam facilities are desired. One main project is the well-established upgrade of the B factory, i.e., the Super-B factory, running at energies from open charm threshold to above $\Upsilon(5S)$ resonance with intended accumulated luminosity of $75\ ab^{-1}$. The Super-B factory would provide great opportunity in searching τ rare decay, where a 90 % C.L. upper limit on $\mathrm{Br}(\tau \to \mu\gamma)$ is expected as 2.4×10^{-9} [8]. Another proposal, the so-called Super Charm-Tau factory (CTF), is an e^+e^- collider designed to work in the energy region from 2 to 5 GeV with instant luminosity of $10^{35}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$ [9]. Compared to the Super-B factory, τ leptons can be copiously produced in pairs at the CTF with center-of-mass (c.m.s.) energies $E_{e^+e^-}$ not far above the $2m_\tau$ threshold, and the radiative background $e^+e^- \to \tau^+\tau^-\gamma$ is not significant.

In this paper, we investigate the potential of searching the $\tau \to \mu\gamma$ LFV decay at the CTF and demonstrate its better chance than the Super-B factory. The paper is organized as follows: The LFV signal and dominant background at the CTF are discussed. Then a strategy of experimental event selection to improve signal significance is developed, and an

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expected upper limit on Br($\tau \to \mu\gamma$) is presented. Finally, the constraints on the leptoquark model parameters are given as an example of an interpretation of new physics.

2 LFV signal and background at the CTF

At the CTF, the cross section for $e^+e^- \to \tau^+\tau^-$ increases significantly as the increment of the colliding energy from the threshold of τ-pair production ($\sim$3.55 GeV) to the threshold of D meson production ($\sim$3.74 GeV). We set the CTF c.m.s. energy as 3.7 GeV, in order to get τ lepton pair produced copiously.

The signal under discussion is that one τ lepton follows LFV decay into a muon and a photon, while the other follows SM decay into a muon and two neutrinos, i.e., $e^+ + e^- \to \tau^+(\mu^+\nu_\mu\bar{\nu}_\tau) + \tau^-(\mu^-\gamma)$ and $e^+ + e^- \to \tau^+(\mu^+\gamma) + \tau^-(\mu^-\bar{\nu}_\mu\nu_\tau)$. The Feynman diagram for the signal process $e^+e^- \to \tau^+\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\mu^-\gamma$ is depicted in Fig. 1. Due to the fact that $\frac{\Gamma_\tau}{m_\tau} \sim \mathcal{O}(10^{-12})$ is sufficiently small, the naive narrow width approximation (NWA) is adopted when calculating the LFV decay $\tau \to \mu\gamma$, and the muon and photon are assumed isotropic in the rest frame of the τ lepton. However, we do not employ the naive NWA to deal with the SM decay $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ for the LFV signal, and keep the off-shell contribution and spin correlation effect from the potentially resonant intermediate τ lepton. In other words, we treat the LFV signal as 4-body production processes, $e^+e^- \to \tau^{+*}\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-$ and $e^+e^- \to \tau^{-*}\tau^+ \to \mu^-\bar{\nu}_\mu\nu_\tau\tau^+$[1], followed by sequential 2-body LFV decay $\tau \to \mu\gamma$. Then the cross section of the signal process $e^+e^- \to \tau^+\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\mu^-\gamma$ can be factorized as

$$
\begin{aligned}
&\sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\mu^-\gamma) \\
&= \sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\tau^-) \\
&\quad \times \mathrm{Br}(\tau \to \mu\gamma).
\end{aligned}
\tag{2.1}
$$

Given the efficiency of detector resolution, no requirement on the missing energy $\not{E}$ raised by escaping neutrinos is imposed. Thus, the detectable LFV signal at the CTF is comprised of two muons and an isolated photon in the final state as $\mu^+\mu^-\gamma + X$, where X denotes all the undetected neutrinos and one hard muon is from τ LFV decay and the other soft one from the standard τ leptonic decay. Accordingly, the leading background to the LFV signal comes from the $e^+e^- \to \mu^+\mu^-\gamma$ process, which is depicted in Fig. 2 as the leading order (LO) contribution.

However, due to the smallness of the τ LFV decay branching ratio, the effect of the SM background that involves four
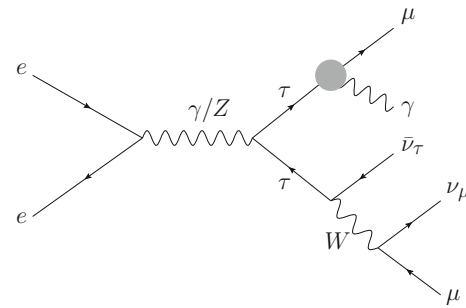
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[1] $\tau^{\pm*}$ might be off-shell depending on the kinematics of the final $\mu^\pm\nu_{\mu(\tau)}\bar{\nu}_{\tau(\mu)}$ system.

Fig. 1 The Feynman diagram for the signal process $e^+e^- \to \tau^+\tau^- \to \mu^+\nu_\mu\bar{\nu}_\tau\mu^-\gamma$

neutrinos in final state cannot be ignored. In our calculation, we mainly consider the resonance contribution from $e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$, where τ^+ and τ^- are treated as on-shell particles. Specifically, the subleading background is that one τ lepton decays into a muon and two neutrinos and the other decays into a muon, two neutrinos, and a photon, i.e., $e^+ + e^- \to \tau^+(\mu^+\nu_\mu\bar{\nu}_\tau) + \tau^-(\mu^-\bar{\nu}_\mu\nu_\tau\gamma)$ and $e^+ + e^- \to \tau^-(\mu^-\bar{\nu}_\mu\nu_\tau) + \tau^+(\mu^+\nu_\mu\bar{\nu}_\tau\gamma)$.

Considering the geometric acceptance of the detector and to avoid the soft divergence induced by low energy infrared radiation, a set of kinematic cuts on the final particles is imposed as baseline,

$$
E_\gamma > 0.5\,\mathrm{GeV}, \quad \eta_\gamma < 5, \quad \eta_\mu < 3, \quad \Delta R(\mu, \gamma) > 0.3,
\tag{2.2}
$$

where E_γ is photon energy, η_γ and η_μ are the pseudorapidities of photon and muon, and $\Delta R(\mu, \gamma) = \sqrt{(\eta_\mu - \eta_\gamma)^2 + (\phi_\mu - \phi_\gamma)^2}$ is the separation on the pseudorapidity-azimuthal-angle plane between muon and photon. After applying the baseline cuts, the cross sections of the LFV signal and the SM background at the $\sqrt{s} = 3.7$ GeV CTF are obtained:

$$
\begin{aligned}
\sigma_S &= \sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma\nu_\mu\bar{\nu}_\tau) \\
&\quad + \sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma\nu_\tau\bar{\nu}_\mu) \\
&= 817.6\,pb \times \mathrm{Br}(\tau \to \mu\gamma), \\
\sigma_B &= \sigma(e^+e^- \to \mu^+\mu^-\gamma) \\
&\quad + \sigma(e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau) \\
&= 968.5\,pb + 0.10\,pb,
\end{aligned}
\tag{2.3}
$$

where in the numerical calculation the SM input parameters are taken as $m_e = 0.511$ MeV, $m_\mu = 105.7$ MeV, $m_\tau = 1.777$ GeV, $\Gamma_\tau = 2.267 \times 10^{-12}$ GeV, $M_W = 80.385$ GeV, $M_Z = 91.1876$ GeV, and $\alpha_{ew} = \alpha(0) = 1/137.036$.

3 Results and discussion

Following Eq. (2.3), one can see that the event number of background is about eight orders of magnitude larger than

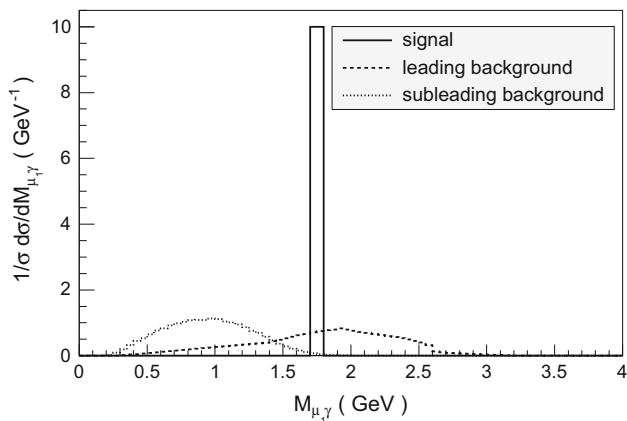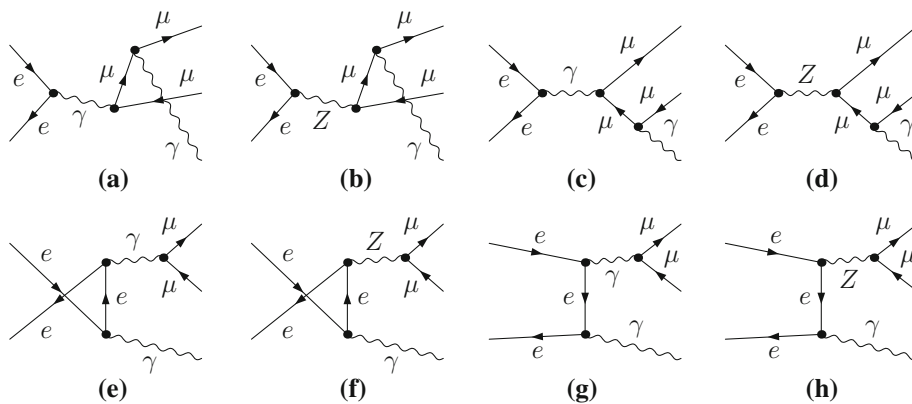**Fig. 2** The LO Feynman diagrams for the background process $e^+e^- \to \mu^+\mu^-\gamma$

Fig. 3 $\mu_1\gamma$ invariant mass distributions for the LFV signal and the SM background after applying baseline cuts
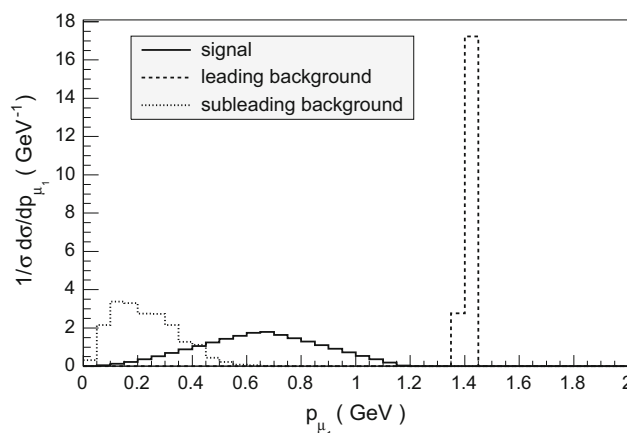
Fig. 4 The trailing muon momentum spectra of the LFV signal and the SM background after applying successive baseline cuts and cut1

that of the LFV signal,

$$\frac{N_S}{N_B} = \frac{\sigma_S}{\sigma_B} \sim \mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}, \qquad (3.1)$$

if only baseline cuts are taken. However, by exploring the distinction between the LFV signal and background kinematic distributions, an optimized event selection algorithm can be developed to suppress the SM background and enhance the sensitivity of searching $\tau \to \mu\gamma$ LFV decay at the CTF.

First, for an event of $\mu^+\mu^-\gamma + X$ final state, the leading muon μ_1 can be defined as

$$\mu_1 = \begin{cases} \mu^+, & |M_{\mu^+\gamma} - m_\tau| < |M_{\mu^-\gamma} - m_\tau| \\ \mu^-, & |M_{\mu^-\gamma} - m_\tau| < |M_{\mu^+\gamma} - m_\tau| \end{cases}, \qquad (3.2)$$

where $M_{\mu^\pm\gamma}$ is the invariant mass of $\mu^\pm\gamma$ system. The invariant mass distributions of the $\mu_1\gamma$ system are depicted in Fig. 3, for the LFV signal and the SM background, respectively, after applying baseline cuts. The leading muons in signal processes are predominately from the τ lepton LFV radiative decay, and the signal is manifested as a striking peak around m_τ in the $\mu_1\gamma$ mass spectrum. Therefore, a stringent mass window can be imposed to effectively suppress the continuous spectrum of the SM background. Since the τ

lepton decay width is negligible, the spread of the signal mass peak is dominated by the detector resolution. Compared with the BESIII, the CTF puts forward a higher requirement on detector performance, e.g., the energy resolution of electromagnetic calorimeter for photon at the CTF is about 1.5 % at $E_\gamma = 1$ GeV [9], whereas the corresponding resolution of the BESIII is about 2.5 % [10]. The momentum resolutions of the track system for both the CTF and the BESIII are about 0.5 %. Accordingly, a conservative mass window "cut1" on $\mu_1\gamma$ system can be chosen as

$$|M_{\mu_1\gamma} - m_\tau| < 0.06 \text{ GeV}. \qquad (3.3)$$

The second muon other than the leading one is denoted as μ_2. After applying successive baseline cuts and cut1, the trailing μ_2 momentum spectrum of the LFV signal would be well separated from that of the SM leading background, as shown in Fig. 4. For the LFV signal, the trailing muon is from the branch of standard τ 3-body leptonic decay, whose momentum distribution is irrelevant to the $\mu_1\gamma$ invariant mass window imposed around m_τ. But the trailing muon momentum distribution for the SM leading background obviously depends on the $\mu_1\gamma$ invariant mass window and will overlap

that for the LFV signal as the mass window increases to about 0.3 GeV. For the SM subleading background, the trailing muon is certainly from the branch of standard $\tau \to \mu\gamma + 2\nu$ 4-body leptonic decay after cut1 and therefore is relatively soft. As shown in Fig. 4, the trailing muon momentum distribution for the SM subleading background overlaps that for the LFV signal in the low momentum region. Based on the behaviors of these trailing muon momentum distributions, a "cut2" on the trailing muon momentum can be defined to reduce the SM contamination,

$$0.45 \text{ GeV} < p_{\mu_2} < 1.1 \text{ GeV}. \tag{3.4}$$

For the LFV signal, the energy and the momentum of $\mu_1\gamma$ system are strictly limited to $\sqrt{s}/2 = 1.85$ GeV and $\sqrt{s/4 - m_\tau^2} = 0.515$ GeV, respectively. While for the SM subleading background, the energy and the momentum of $\mu_1\gamma$ system are continuously distributed. Therefore, an energy window centered at 1.85 GeV and a momentum window centered at 0.515 GeV on $\mu_1\gamma$ system can greatly reduce the SM subleading background. Taking the energy and the momentum resolutions of the detector mentioned above into consideration, the energy window "cut3" and the momentum window "cut4" on $\mu_1\gamma$ system can be chosen as

$$
\begin{aligned}
|E_{\mu_1\gamma} - 1.85 \text{ GeV}| &< 0.05 \text{ GeV}, \\
|p_{\mu_1\gamma} - 0.515 \text{ GeV}| &< 0.015 \text{ GeV}.
\end{aligned}
\tag{3.5}
$$

Based on the above discussion, an optimized event selection criterion can be proposed:

- Cut1 ($\mu_1\gamma$ invariant mass cut): $|M_{\mu_1\gamma} - m_\tau| < 0.06$ GeV;
- Cut2 (μ_2 momentum cut): 0.45 GeV $< p_{\mu_2} < 1.1$ GeV;
- Cut3 ($\mu_1\gamma$ energy cut): $|E_{\mu_1\gamma} - 1.85$ GeV$| < 0.05$ GeV;
- Cut4 ($\mu_1\gamma$ momentum cut): $|p_{\mu_1\gamma} - 0.515$ GeV$| < 0.015$ GeV.

This four-step cut strategy could theoretically save 79.3 % of signal events, remove all the leading background events, and only retain 1.8×10^{-5} of the subleading background events. Then the significance of the signal over background after the above kinematic cuts applied is given by

$$S = \frac{N_S}{\sqrt{N_B}} = 1.52 \times 10^7 \times \text{Br}(\tau \to \mu\gamma) \cdot \sqrt{\mathcal{L}}, \tag{3.6}$$

where $\mathcal{L}$ is the accumulated luminosity in unit of fb^{-1}, and it is reasonable to presume an annual integrated luminosity of about 1 ab^{-1} at the CTF.

If the LFV signal is not detected with certain accumulated luminosity at the CTF, a new upper bound on $\text{Br}(\tau \to \mu\gamma)$ can be set at a C.L. of 2σ, as shown in Fig. 5. The 2σ upper limits on $\text{Br}(\tau \to \mu\gamma)$ for some typical values of the integrated luminosity are also given in Table 1. We can see that the direct search for the $\tau \to \mu\gamma$ LFV decay at the CTF would give much more stringent limit on $\text{Br}(\tau \to \mu\gamma)$ than

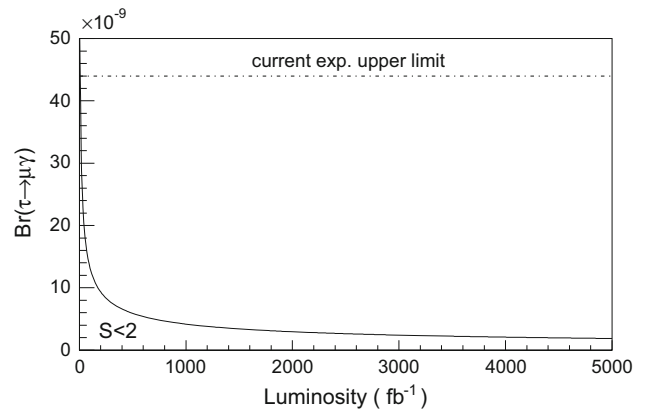**Fig. 5** 2σ upper bound on the branching ratio for $\tau \to \mu\gamma$ as a function of the integrated luminosity at the CTF

the current experimental limit derived at the B factory. For example, with one year data ($\sim 1000 \ fb^{-1}$) taken at the CTF, a new 2σ upper bound $\text{Br}(\tau \to \mu\gamma) < 4.2 \times 10^{-9}$ can be obtained, which is about one order of magnitude smaller than the current experimental upper bound of 4.4×10^{-8}; with three year run, the CTF could surpass the proposed Super-B factory in the sensitivity of searching τ LFV decay.

4 Constraints on new physics

New upper bound on $\text{Br}(\tau \to \mu\gamma)$ expected at the CTF would constrain new physics beyond the SM. Among all the extensions of the SM, the leptoquark (LQ) model is a promising one to interpret LFV decays and has been extensively studied. In addition to the spin and gauge quantum numbers, the leptoquarks carry both lepton number and baryon number, and the spin-0 and spin-1 particles are called scalar and vector leptoquarks, respectively. The renormalizable and $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ invariant interactions between scalar leptoquarks and SM fermions are given by the following Lagrangian [11]:

$$
\begin{aligned}
\mathcal{L}_{\text{LQ}} = &\left[\lambda_0^L \overline{q_L^c} i\tau^2 l_L + \lambda_0^R \overline{u_R^c} e_R\right] S_{0,-\frac{1}{3}}^* + \tilde{\lambda}_0^L \overline{d_R^c} e_R S_{0,-\frac{4}{3}}^* \\
&+ \left[\lambda_{\frac{1}{2}}^R \overline{q_L} i\tau^2 e_R + \lambda_{\frac{1}{2}}^L (\overline{u_R} l_L)^T\right] S_{\frac{1}{2},-\frac{7}{6}}^* \\
&+ \tilde{\lambda}_{\frac{1}{2}}^L (\overline{d_R} l_L)^T S_{\frac{1}{2},-\frac{1}{6}}^* + \lambda_1^L \overline{q_L^c} i\tau^2 \vec{\tau} l_L \cdot \vec{S}_{1,-\frac{1}{3}}^* + \text{h.c.},
\end{aligned}
\tag{4.1}
$$

where q_L and l_L denote the left-handed $SU(2)_L$ doublet quarks and leptons of the SM, and u_R, d_R, and e_R are the right-handed $SU(2)_L$ singlet quarks and charged leptons, respectively. We use $S_{j,\frac{Y}{2}}$ to denote scalar leptoquark, where the subscript j can value 0 and 1/2 indicating $SU(2)_L$ singlet and doublet, respectively, and Y stands for hyper-

Table 1 2σ upper bounds on the branching ratio for $\tau \to \mu\gamma$ at different luminosities

$\mathcal{L}\,[fb^{-1}]$	2σ upper bound on Br($\tau \to \mu\gamma$)	$\mathcal{L}\,[fb^{-1}]$	2σ upper bound on Br($\tau \to \mu\gamma$)
300	7.60×10^{-9}	1500	3.40×10^{-9}
600	5.37×10^{-9}	3000	2.40×10^{-9}
1000	4.16×10^{-9}	5000	1.86×10^{-9}

Fig. 6 The lowest order Feynman diagrams for the LFV decay process $\tau^- \to \mu^-\gamma$ in the scalar leptoquark model

(a)   **(b)**   **(c)**   **(d)**
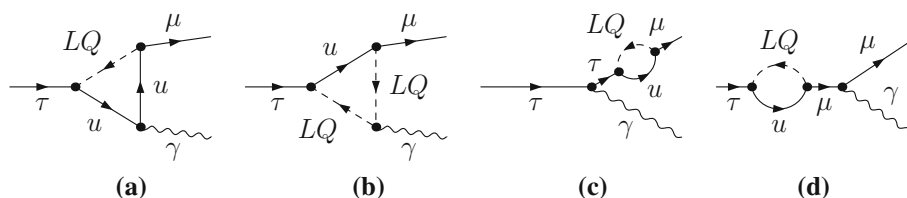
charge. Color and generation indices have been suppressed. τ^i ($i = 1, 2, 3$) are three Pauli matrices.

In investigating the rare decay $\tau \to \mu\gamma$, we also require the interactions between the scalar leptoquarks and photon. The photon interactions arise from the following $SU(2)_L \otimes U(1)_Y$ invariant kinematic terms of the scalar leptoquarks:

$$\mathcal{L}_{\text{kinetic}} = (D^\mu S)^\dagger (D_\mu S). \tag{4.2}$$

The $SU(2)_L \otimes U(1)_Y$ covariant derivative D_μ is given by

$$D_\mu = \partial_\mu - ig \sum_{i=1,2,3} W_\mu^i T^i - ig' \frac{Y}{2} B_\mu, \tag{4.3}$$

where W_μ^i ($i = 1, 2, 3$) and B_μ are the $SU(2)_L$ and $U(1)_Y$ gauge fields, respectively, and T^i ($i = 1, 2, 3$) are the generator matrices for the $SU(2)_L$ representation occupied by the scalar leptoquarks. From Eq. (4.2) we obtain the photon interaction for a scalar leptoquark as

$$\mathcal{L}_{LQ,\gamma} = ieQ_{LQ}\left[(\partial_\mu S^\dagger)S - S^\dagger(\partial_\mu S)\right] A^\mu, \tag{4.4}$$

where A_μ is the photon field and Q_{LQ} represents the electric charge of the scalar leptoquark S.

For simplicity, we assume that all the LFV couplings except $\lambda_{\frac{1}{2},ij}^L$ ($i = 1, j = 2, 3$), where i and j are quark and lepton-flavor indices, are zero. Therefore, only τ–u–LQ and μ–u–LQ couplings are non-zero, and the τ lepton can decay into $\mu + \gamma$ via quark–leptoquark involved loops at the LO, as shown in Fig. 6.

The LO decay width for the LFV decay process $\tau \to \mu\gamma$ is given by

$$\Gamma(\tau \to \mu\gamma) = \frac{1}{2}\frac{1}{8\pi}\frac{|\vec{p}_{\mu,\text{cm}}|}{m_\tau^2} \sum_{\text{spin}} |\mathcal{M}_{\tau \to \mu\gamma}|^2, \tag{4.5}$$

where $\mathcal{M}_{\tau \to \mu\gamma}$ is the amplitude for the Feynman diagrams in Fig. 6 and $\vec{p}_{\mu,\text{cm}}$ is the three-momentum of μ in the rest frame of the initial τ lepton. The summation is taken over the

spins of the initial and final state particles and the factor $\frac{1}{2}$ arises from the spin average of the initial state. We compute $\mathcal{M}_{\tau \to \mu\gamma}$ by using the related Feynman rules obtained from Eqs. (4.1) and (4.4), and we adopt the Passarino–Veltman reduction method to convert the one-loop amplitude to scalar integrals. The loop divergence is naturally canceled for these four diagrams with no necessity to introduce any counterterm. The LFV decay branching ratio for $\tau \to \mu\gamma$ can thus be expressed as

$$\text{Br}(\tau \to \mu\gamma) = \frac{9\alpha_{ew}(\lambda_1\lambda_2)^2(m_\tau^2 - m_\mu^2)}{1024\pi^4\Gamma_\tau m_\tau^3}$$
$$\times \left[(|F_1|^2 + |F_2|^2)(m_\tau^2 + m_\mu^2) - 4\text{Re}(F_1 F_2^*)m_\tau m_\mu\right], \tag{4.6}$$

where $\lambda_1 = \lambda_{\frac{1}{2},12}^L$ and $\lambda_2 = \lambda_{\frac{1}{2},13}^L$, denoting the μ–u–LQ and τ–u–LQ coupling strengths, respectively. The form factors $F_{1,2}$ are given in the appendix.

Under the simplicity assumption, there are only three parameters of scalar leptoquark determining the $\tau \to \mu\gamma$ LFV decay, namely the couplings $\lambda_{1,2}$ and the scalar leptoquark mass M_{LQ}. As shown in Eq. (4.6), the τ LFV decay branching ratio is proportional to $(\lambda_1\lambda_2)^2$ but it is in a much complicated way related to M_{LQ}. The dependence of Br($\tau \to \mu\gamma$) as functions of $\lambda_1\lambda_2$ and the leptoquark mass M_{LQ} is presented in Fig. 7.

According to the dependence, a joint constraint on $\lambda_1\lambda_2$ and M_{LQ} can be derived from the upper bound on Br($\tau \to \mu\gamma$) expected at the CTF, as shown in Fig. 8. One can estimate from the plot the upper bound on $\lambda_1\lambda_2$ and the lower bound on M_{LQ} for given M_{LQ} and $\lambda_1\lambda_2$, respectively. For example, with 1000 fb^{-1} data expected at the CTF, one can get

$$\lambda_1\lambda_2 < 7.2 \times 10^{-2}, \quad (M_{LQ} = 800\,\text{GeV}, \ 95\,\%\ \text{C.L.}),$$
$$M_{LQ} > 900\,\text{GeV}, \quad (\lambda_1\lambda_2 = 9 \times 10^{-2}, \ 95\,\%\ \text{C.L.}).$$
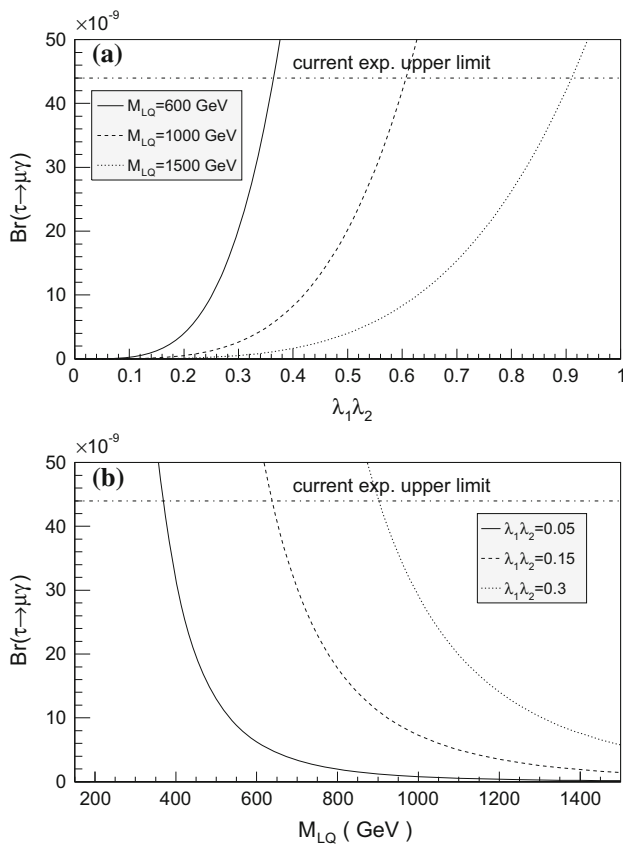$$\tag{4.7}$$

Fig. 7 The branching ratio for $\tau \to \mu\gamma$ as functions of **a** $\lambda_1\lambda_2$ and **b** M_{LQ}
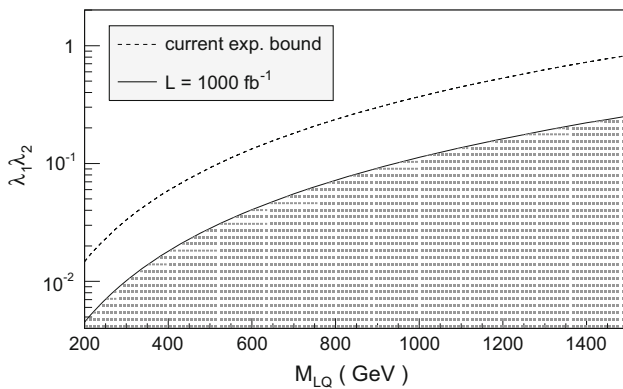

Fig. 8 The joint constraint on $\lambda_1\lambda_2$ and M_{LQ} with 1000 fb^{-1} integrated luminosity at 95 % C.L.

Besides the above specified leptoquark interpretation, the τ LFV decay can be expressed in a model-independent formalism. An effective vertex τ–μ–γ can be introduced in the form of $\frac{i}{m_\tau}\sigma^{\mu\nu}p_\nu(\sigma_L P_L + \sigma_R P_R)$, where $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, $P_{L,R} = (1 \mp \gamma^5)/2$, and p_ν is the momentum of the photon [12]. Then the branching ratio for $\tau \to \mu\gamma$ can be simply
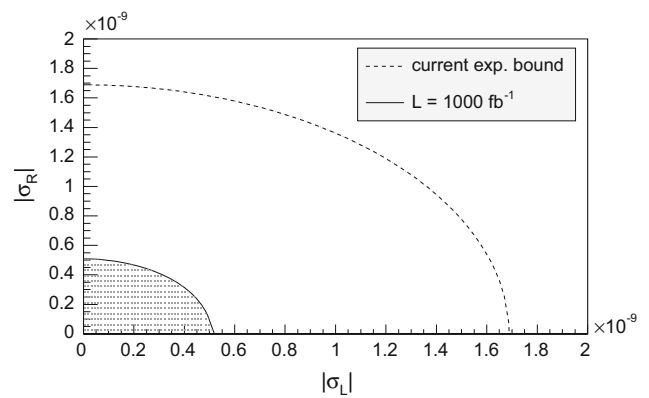

Fig. 9 The joint upper bound on $|\sigma_L|$ and $|\sigma_R|$ with 1000 fb^{-1} integrated luminosity at 95 % C.L.

expressed in terms of the form factors σ_L and σ_R as

$$\mathrm{Br}(\tau \to \mu\gamma) = \frac{(m_\tau^2 - m_\mu^2)^3 \left(|\sigma_L|^2 + |\sigma_R|^2\right)}{16\pi \, \Gamma_\tau m_\tau^5}. \quad (4.8)$$

Similarly, a joint upper bound on $|\sigma_L|$ and $|\sigma_R|$ can be deduced from the upper bound on $\mathrm{Br}(\tau \to \mu\gamma)$. As shown in Fig. 9, a more stringent upper bound of $\sqrt{|\sigma_L|^2 + |\sigma_R|^2} < 5.2 \times 10^{-10}$, much smaller than the current experimental limit, could be derived, if the $\tau \to \mu\gamma$ LFV decay is not detected with 1000 fb^{-1} integrated luminosity expected at the CTF.

5 Summary

In this paper, we investigate the potential of searching $\tau \to \mu\gamma$ LFV decay at the CTF. With a center-of-mass energy of 3.7 GeV of electron–positron collisions, τ leptons can be copiously produced in pairs at the CTF. The LFV signal processes $e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma \nu_{\mu(\tau)}\bar{\nu}_{\tau(\mu)}$ are featured by a detectable final state of $\mu_1\mu_2\gamma + X$, namely one hard leading μ_1 along with a hard photon from τ radiative LFV decay and one soft trailing μ_2 from the standard τ leptonic decay, and leaving the missing energy from escaping neutrinos unmeasured. To improve the significance of the $\tau \to \mu\gamma$ LFV decay at the CTF, we propose a four-step event selection strategy: an invariant mass window on $\mu_1\gamma$ system around m_τ and a momentum cut on μ_2 are imposed to eliminate the dominant $e^+e^- \to \mu^+\mu^-\gamma$ SM background, and then an energy window and a momentum window on $\mu_1\gamma$ system are successively applied to significantly suppress the $e^+e^- \to \tau^+\tau^- \to \mu^+\mu^-\gamma \nu_\mu \bar{\nu}_\mu \nu_\tau \bar{\nu}_\tau$ subleading SM background. It can be predicted with a couple years of CTF running, new sensitivities on $\mathrm{Br}(\tau \to \mu\gamma)$, which could surpass current experimental upper bound and those expected at the Super-B factory, can be achieved. The new upper limit on $\mathrm{Br}(\tau \to \mu\gamma)$ expected at the CTF would cer-

tainly constrain the parameter space of new physics beyond the SM, either in specific theories as leptoquark theory or in a model-independent effective formalism.

Appendix

The form factors F_1 and F_2 are expressed as

$$
\begin{aligned}
F_1 = Q_u \Bigg[& 2C_{00}^{(1)} + m_\tau^2 \left(C_1^{(1)} + C_{11}^{(1)} + C_{12}^{(1)} \right) \\
& + m_\mu^2 \left(C_2^{(1)} + C_{22}^{(1)} + C_{12}^{(1)} \right) - m_u^2 C_0^{(1)} - \frac{1}{2} \Bigg] \\
& - 2Q_{\mathrm{LQ}} C_{00}^{(2)} - \frac{m_\mu^2}{m_\tau^2 - m_\mu^2} \left(B_0^{(1)} + B_1^{(1)} \right) \\
& + \frac{m_\tau^2}{m_\tau^2 - m_\mu^2} \left(B_0^{(2)} + B_1^{(2)} \right),
\end{aligned}
$$

$$
\begin{aligned}
F_2 = & - Q_u m_\tau m_\mu \left(C_0^{(1)} + C_1^{(1)} + C_2^{(1)} \right) \\
& - \frac{m_\tau m_\mu}{m_\tau^2 - m_\mu^2} \left(B_0^{(1)} + B_1^{(1)} - B_0^{(2)} - B_1^{(2)} \right),
\end{aligned}
\tag{5.1}
$$

where

$$
B_i^{(1)} = B_i(m_\mu^2, M_{\mathrm{LQ}}^2, m_u^2),
$$

$$
B_i^{(2)} = B_i(m_\tau^2, M_{\mathrm{LQ}}^2, m_u^2),
$$

$$
C_{\{j,jk\}}^{(1)} = C_{\{j,jk\}}(m_\tau^2, 0, m_\mu^2, M_{\mathrm{LQ}}^2, m_u^2, m_u^2),
$$

$$
C_{\{j,jk\}}^{(2)} = C_{\{j,jk\}}(m_\tau^2, m_\mu^2, 0, M_{\mathrm{LQ}}^2, m_u^2, M_{\mathrm{LQ}}^2),
\tag{5.2}
$$

and M_{LQ} is the mass of scalar leptoquark. The definitions of one-loop 2- and 3-point functions B_i ($i = 0, 1$) and $C_{\{j,jk\}}$ ($j, k = 0, 1, 2$) are given in Ref. [13].

Note added After submitting this paper we found another calculation of this LFV signal at the HIEPA [14].

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