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Study on decays of $Z_c(4020)$ and $Z_c(3900)$ into $h_c + \pi$

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Abstract At the invariant mass spectrum of $h_c\pi^{\pm}$ a new resonance $Z_c(4020)$ has been observed, however, the previously confirmed $Z_c(3900)$ does not show up at this channel. In this paper we assume that $Z_c(3900)$ and $Z_c(4020)$ are molecular states of $D\bar{D}^*(D^*\bar{D})$ and $D^*\bar{D}^*$, respectively, then we calculate the transition rates of $Z_c(3900) \to h_c + \pi$ and $Z_c(4020) \to h_c + \pi$ in the light-front model. Our results show that the partial width of $Z_c(3900) \to h_c + \pi$ is only three times smaller than that of $Z_c(4020) \to h_c + \pi$. $Z_c(4020)$ seems to be a molecular state, so if $Z_c(3900)$ is also a molecular state it should be observed in the portal $e^+e^- \to h_c\pi^{\pm}$ as long as the database is sufficiently large; to the contrary if the future more precise measurements still cannot find $Z_c(3900)$ at $h_c\pi^{\pm}$ channel, the molecular assignment to $Z_c(3900)$ should be ruled out.

1 Introduction

Since discovery of the exotic XYZ particles as well as the pentaquarks, to determine their inner structure and relevant physics is a challenge to our understanding of the basic principles, especially non-perturbative QCD effects. Gaining knowledge on their inner structure can only be realized through analyzing their production and decays behaviors, absolutely, it is indirect, but efficient. In 2013 the BES collaboration observed a new resonance $Z_c(4020)$ at the $h_c\pi^{\pm}$ invariant mass spectrum by studying the process $e^+e^- \to h_c\pi^+\pi^-$ with the center-of-mass energies from 3.90 to 4.42 GeV [1]. Its mass and width are $(4022.9 \pm 0.8 \pm 2.7)$ MeV and $(7.9\pm2.7\pm2.6)$ MeV. Recently the neutral charmonium-like partner of $Z_c(4020)^0$ has also been experimentally observed [2]. In 2013 $Z_c(3900)$ was measured at the invariant mass spectrum of $J/\psi\pi^{\pm}$ with the mass and width being $(3.899\pm3.6\pm4.9)$ GeV and $(46\pm10\pm10)$ MeV, respectively [3–5]. Since the new resonances $Z_c(4020)$ and $Z_c(3900)$ are charged, they cannot be charmonia, but their masses and decay modes imply that they are hidden charm states, namely they should be exotic states with a $c\bar{c}q\bar{q}'$ structure. The authors of Refs. [6–9] considered that the two resonances should be studied in a unique theoretical framework due to their similarity. It is suggested that the two resonances could be molecular states [9–13], whereas some other authors regard them as tetraquarks [8], mixtures of the two structures [14] or cusp structures [15]. The key point is whether one can use an effective way to confirm their structures. No doubt, it must be done through combining careful theoretical studies and precise measurements in the coming experiments.

Even though the masses of the two resonances are close, their widths are quite apart, especially at present no significant $Z_c(3900)$ signal has been observed at the $h_c\pi^{\pm}$ mass spectrum through the process $e^+e^- \to h_c\pi^+\pi^-$ [1]. Its absence may imply that the two resonances might be different, but do we have evidence to draw a conclusion? If they are of different inner structures, their decay modes should be different, i.e. different structures would lead to different decay rates for the same channel which can be tested by more precise measurements. Theoretically assigning a special structure to any of $Z_c(3900)$ and $Z_c(4020)$, one can predict its decay rate in an appointed channel and then the data would tell if the assignment is valid or should be negated. That is the strategy of this work.

In our earlier paper [16] we explored some strong decays, namely of $Z_c(3900)$ and $Z_c(4020)$, which were assumed to be molecular states of $D\bar{D}^*(D^*\bar{D})$ and $D^*\bar{D}^*$ and the achieved numerical results are satisfactorily consistent with experimental observations. In this paper we are going to study the strong decays $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$ with the same method.

In order to explore the decays of a molecular state [16], we extended the light-front quark model (LFQM), which was thoroughly studied in the literature [17–36]. In this situation the constituents are two mesons instead of a quark

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and an antiquark in the light-front frame. In the case of a covariant form the constituents are off-shell. The effective interactions between the two constituent mesons are adopted following the literature [37–42], where, by fitting relevant processes, the effective coupling constants have been obtained. Using the method given in Ref. [16] we deduce the corresponding form factors and estimate the decay widths of $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$, while both $Z_c(3900)$ and $Z_c(4020)$ are assumed to be molecular states. In fact there exist three degenerate S-wave bound states of $D^*\bar{D}^*$ whose quantum numbers are, respectively, 0^+, 1^+, and 2^+. In our work we evaluate the decay rates of the $D^*\bar{D}^*$ molecules which can be either of the three quantum states.

In this framework, the $q^+ = 0$ condition is applied i.e. $q^2 < 0$, it means that the final mesons are not on-shell, thus the obtained form factors are space-like. Then one needs to extrapolate analytically the form factors from the unphysical space-like region to the time-like region to reach the physical ones. With the form factors we calculate the corresponding decay widths. The numerical results will provide us with information as regards the structures of $Z_c(3900)$ and $Z_c(4020)$.

After the introduction we derive the form factors for transitions $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$ in Sect. 2. Then we numerically evaluate the relevant form factors and decay widths in Sect. 3. In the last section we discuss the numerical results and draw our conclusion. Some details as regards the approach are collected in the appendix.

2 The strong decays $Z_c(3900) \to h_c + \pi$

In this section we calculate the strong decay rate of $Z_c(3900) \to h_c + \pi$, while assuming $Z_c(3900)$ as a 1^+ $D\bar{D}^*$ molecular state, in the light-front model. Because of the success of applying the method [16] we have reason to believe that this framework also works in this case. The configuration of the $D\bar{D}^*$ molecular state is $\frac{1}{\sqrt{2}}(D\bar{D}^* + \bar{D}D^*)$.

The Feynman diagrams for $Z_c(3900)$ decaying into $h_c\pi$ by exchanging D or D^* mesons are shown in Fig. 1.

Following Ref. [31], the hadronic matrix element corresponding to the diagrams in Fig. 1 is written

$$\mathcal{A}_{11} = i\frac{1}{(2\pi)^4}\int d^4p_1 \frac{[H_{A_{01}}S^{1(a)}_{d\alpha} + H_{A_{10}}S^{1(b)}_{d\alpha}]}{N_1N_1'N_2}\epsilon_1^d\epsilon^\alpha \quad (1)$$

with

$$S^{1(a)}_{d\alpha} = -i\frac{g_{h_cD^*D^*}g_{\pi DD^*}}{\sqrt{2}}g_{\alpha\beta}g^{\mu\beta}g^{\nu\nu'}\varepsilon_{a\mu c\nu}p_1^a(p_1^c - q^c) \times g_{d\nu'}\mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q'),$$
$$S^{1(b)}_{d\alpha} = i\frac{g_{h_cDD^*}g_{\pi D^*D^*}}{\sqrt{2}}g_{\alpha\beta}g^{\mu\beta}g^{\nu\nu'}(p_{1\nu}+q_\nu)P''^{\omega} \times \varepsilon_{\omega d\mu\nu'}\mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q'), \quad (2)$$

$N_1 = p_1^2 - m_1^2 + i\varepsilon$, where $N_1' = q'^2 - m_{q'}^2 + i\varepsilon$ and $N_2 = p_2^2 - m_2^2 + i\varepsilon$. A form factor $\mathcal{F}(m_i, p^2) = \frac{(m_i+\Lambda)^2 - m_i^2}{(m_i+\Lambda)^2 - p^2}$ is introduced to compensate for the off-shell effect caused by the intermediate meson of mass m_i and momentum p. $H_{A_{10}}$ and $H_{A_{01}}$ are vertex functions which include the normalized wavefunctions of the decaying mesons with the assigned quantum numbers and are invariant in the four-dimension. In fact, for the practical computation their exact forms are not necessary, because after integrating over dp_1^- the integral is reduced to a three-dimensional integration, and $H_{A_{10}}$ (or $H_{A_{01}}$) would be replaced by $h_{A_{10}}$ ($h_{A_{01}}$) whose explicit form(s) is (are) calculable. In the light-front frame the momentum p_i is decomposed into its components as $(p_i^-, p_i^+, p_{i\perp})$ and integrating out p_1^- with the methods given in Ref. [29] one has

$$\int d^4p_1 \frac{H_A S_{d\alpha}}{N_1N_1'N_2}\epsilon_1^d\epsilon^\alpha \to -i\pi\int dx_1 d^2p_\perp \frac{h_A\hat{S}_{d\alpha}}{x_2\hat{N}_1\hat{N}_1'}\epsilon_1^d\epsilon^\alpha, \quad (3)$$

with

$$\hat{N}_1 = x_1(M^2 - M_0{}^2),$$
$$\hat{N}_1' = x_2q^2 - x_1M_0{}^2 + x_1M'^2 + 2p_\perp\cdot q_\perp,$$
$$h_A = \sqrt{\frac{x_1x_2}{m_1m_2}}(M^2 - M_0^2)h_A'$$

where M and M' are the masses of initial and finial mesons. The factor $\sqrt{x_1x_2}(M^2 - M_0^2)$ in the expression of h_A was introduced in [31] and an additional normalization factor $\sqrt{\frac{1}{m_1m_2}}$ appears corresponding to the boson constituents in the molecular state. The explicit expressions of the effective form factors h_A' are collected in the appendix.

Since we calculate the transition in the $q^+ = 0$ frame the zero mode contributions, which come from the residues of virtual pair creation processes, are not involved. To include the contributions, $p_{1\mu}$, $p_{1\nu}$, and $p_{1\mu}p_{1\nu}$ in $s^a_{\mu\nu}$ must be replaced by the appropriate expressions as discussed in Ref. [31]. We have

$$p_{1\mu} \to \mathcal{P}_\mu A_1^{(1)} + q_\mu A_2^{(1)},$$
$$p_{1\mu}p_{1\nu} \to g_{\mu\nu}A_1^{(2)} + \mathcal{P}_\mu\mathcal{P}_\nu A_2^{(2)} + (\mathcal{P}_\mu q_\nu + q_\mu\mathcal{P}_\nu)A_3^{(2)} + q_\mu q_\nu A_4^{(2)} \quad (4)$$

where $\mathcal{P} = P' + P''$ and $q = P' - P''$ with P' and P'' denote the momenta of the concerned mesons in the initial and final states, respectively.

For example, after the replacement $S^{1(a)}_{d\alpha}$ turns into

$$\hat{S}^{1(a)}_{d\alpha} = -i\frac{g_{h_cD^*D^*}g_{\pi DD^*}}{\sqrt{2}}g_{\alpha\beta}g^{\mu\beta}g^{\nu\nu'}\varepsilon_{a\mu c\nu}[g^{ac}A_1^{(2)} + \mathcal{P}^a\mathcal{P}^cA_2^{(2)} + (\mathcal{P}^aq^c + q^a\mathcal{P}^c)A_3^{(2)} + q^aq^cA_4^{(2)} - (\mathcal{P}^aA_1^{(1)} + q^aA_2^{(1)})q^c]g_{d\nu'}$$

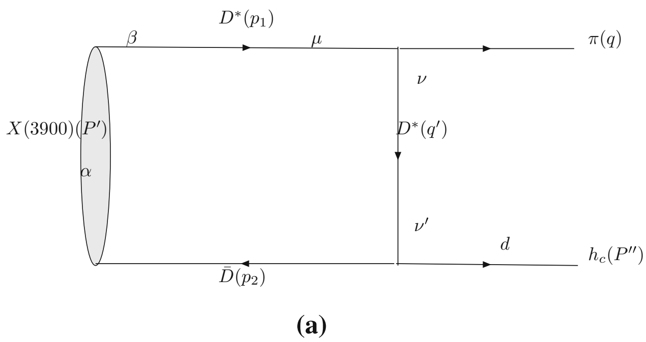

Fig. 1 Strong decays of molecular states (two diagrams where h_c and π in the final states are switched are omitted)

$$
\begin{aligned}
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q')\\
&= i\frac{g_{h_cD^*D^*}g_{\pi DD^*}}{\sqrt{2}}(A_1^{(1)}-A_3^{(2)})\mathcal{P}_a q_c\varepsilon_{acd\alpha}\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q')\\
&= i\frac{g_{h_cD^*D^*}g_{\pi DD^*}}{\sqrt{2}}2(A_1^{(1)}-A_3^{(2)})P'_a q_b\varepsilon_{abd\alpha}\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q'). \qquad (5)
\end{aligned}
$$

Some notations such as $A_i^{(j)}$ and M_0' can be found in Ref. [31]. With the replacement, $h_A\hat{S}_{d\alpha}$ is decomposed into

$$
iF_1P'_a q_b\varepsilon_{abd\alpha}, \qquad (6)
$$

with

$$
\begin{aligned}
F_1 &= \sqrt{2}g_{h_cD^*D^*}g_{\pi DD^*}h_{A01}\left(A_1^{(1)}-A_3^{(2)}\right)\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q')\\
&+\frac{g_{h_cDD}g_{\pi DD^*}}{\sqrt{2}}h_{A10}\left(A_1^{(1)}+A_2^{(1)}+1\right)\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_D,q'). \qquad (7)
\end{aligned}
$$

For convenience of derivation, let us introduce a new form factor, which is defined as follows:

$$
f_1(m_1,m_2)=\frac{1}{16\pi^3}\int \mathrm{d}x_2\mathrm{d}^2p_\perp\frac{F_1}{x_2\hat{N}_1\hat{N}_1'}. \qquad (8)
$$

Then the amplitude is written in terms of $f_1(m_1,m_2)$ as

$$
\mathcal{A}_{11}=if_1(m_1,m_2)P'_a q_b\varepsilon_{abd\alpha}\epsilon_1^d\epsilon^\alpha. \qquad (9)
$$

The contributions from the Feynman diagrams by switching around h_c and π in the final states (in Fig. 1) can be formulated by simply exchanging m_1 and m_2 in the expression $f_1(m_1,m_2)$. Then the total amplitude is

$$
\begin{aligned}
\mathcal{A}_1 &= i[f_1(m_1,m_2)+f_1(m_2,m_1)]P'_a q_b\varepsilon_{abd\alpha}\\
&= ig_1P'_a q_b\varepsilon_{abd\alpha}\epsilon_1^d\epsilon^\alpha, \qquad (10)
\end{aligned}
$$

and the factor g_1 will be numerically evaluated in next section.

3 The strong decay $Z_c(4020)\to h_c+\pi$

Similar to what we have done for $Z_c(3900)$, we calculate the decay rate of $Z_c(4020)\to h_c\pi$ by respectively supposing $Z_c(4020)$ as 0^+, 1^+, and 2^+ $D^*\bar{D}^*$ molecular states. The Feynman diagrams are shown in Fig. 2.

3.1 $Z_c(4020)$ as a 0^+ molecular state

In terms of the vertex function given in the appendix, the hadronic matrix element is

$$
\mathcal{A}_{21}=i\frac{1}{(2\pi)^4}\int \mathrm{d}^4p_1\frac{H_{A_0}}{N_1N_1'N_2}\left(S_d^{2(a)}+S_d^{2(b)}\right)\epsilon_1^d, \qquad (11)
$$

where

$$
\begin{aligned}
S_d^{2(a)} &= ig_{h_cDD^*}g_{\pi DD^*}g_{\mu\nu}g^{\mu\mu'}(2q_{\mu'}-p_{1\mu'})g^{\nu\nu'}g_{\nu'd}\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_D,q')
\end{aligned}
$$

and $S_d^{2(b)} = -ig_{h_cD^*D^*}g_{\pi D^*D^*}g_{\mu\nu}g^{\mu\mu'}g^{\nu\nu'}\varepsilon_{\omega\mu'\rho a}p_1^\omega q'^\rho g^{ac} P''^f\varepsilon_{fdc\nu}\mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q')$. Carrying out the integration and making the required replacements, we have

$$
h_{A_0}\left(\hat{S}_d^{2(a)}+\hat{S}_d^{2(b)}\right)=iF_2q_d, \qquad (12)
$$

with

$$
\begin{aligned}
F_2 &= g_{\psi DD^*}g_{\pi DD^*}h_{A_0}\left(2-A_1^{(1)}-A_2^{(1)}\right)\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_D,q')\\
&-4\,g_{\psi D^*D^*}g_{\pi D^*D^*}h_{A_0}\left(A_1^{(1)}+A_3^{(2)}\right)M''^2\\
&\times \mathcal{F}(m_1,p_1)\mathcal{F}(m_2,p_2)\mathcal{F}^2(m_{D^*},q'). \qquad (13)
\end{aligned}
$$

Simultaneously, we have derived the form factor

$$
f_2(m_1,m_2)=\frac{1}{16\pi^3}\int \mathrm{d}x_2\mathrm{d}^2p_\perp\frac{F_2}{x_2\hat{N}_1\hat{N}_1'}. \qquad (14)
$$

With this form factor the transition amplitude is obtained:

$$
\mathcal{A}_{21}=if_2(m_1,m_2)q\cdot\epsilon_1. \qquad (15)
$$

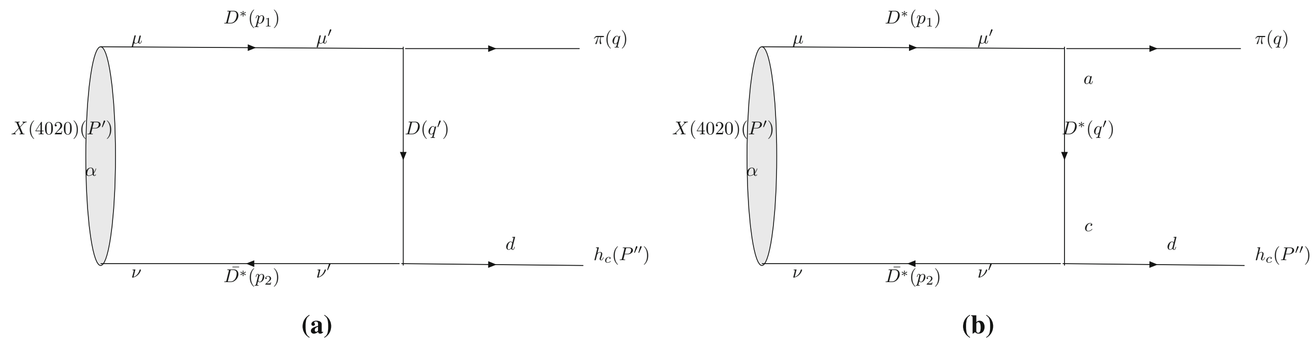

Fig. 2 Strong decays $Z_c(4020) \to h_c\pi$ (the figures with exchanged final states are omitted)

Similarly, the amplitude corresponding the Feynman diagrams where the mesons in the final state are switched around, can easily be obtained by exchanging m_1 and m_2. The total amplitude is

$$\mathcal{A}_2 = i[f_2(m_1, m_2) + f_2(m_2, m_1)]q \cdot \epsilon_1 \\ = ig_2 q \cdot \epsilon_1. \tag{16}$$

3.2 $Z_c(4020)$ as a 1^+ molecular state

For the 1^+ state, the hadronic matrix element would be different from the case where $Z_c(4020)$ is assumed to be a 0^+ meson. Now the hadronic matrix element is written

$$\mathcal{A}_{31} = i\frac{1}{(2\pi)^4}\int \mathrm{d}^4 p_1 \frac{H_{A_1}}{N_1 N_1' N_2}\left(S^{2(a)}_{d\alpha} + S^{2(b)}_{d\alpha}\right)\epsilon_1^d \epsilon^\alpha, \tag{17}$$

where

$$S^{2(a)}_{d\alpha} = ig_{h_c DD^*} g_{\pi DD^*} \varepsilon_{\mu\nu\alpha\beta} g^{\mu\mu'}(2q_{\mu'} - p_{1\mu'})P'^{\beta} g^{\nu\nu'} g_{\nu' d} \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_D, q')$$

and

$$S^{2(b)}_{d\alpha} = -ig_{h_c D^* D^*} g_{\pi D^* D^*} \varepsilon_{\mu\nu\alpha\beta} g^{\mu\mu'} g^{\nu\nu'} P'^{\beta} \varepsilon_{\omega\mu'\rho a} p_1^\omega q'^\rho \\ \times g^{ac} P''^{f} \varepsilon_{fdc\nu} \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q').$$

After integrating over the momentum, we have

$$h_{A_1}(\hat{S}^{2(a)}_{d\alpha} + \hat{S}^{2(b)}_{d\alpha}) = iF_3 P'_a q_b \varepsilon_{abd\alpha}, \tag{18}$$

with

$$F_3 = g_{h_c DD^*} g_{\pi DD^*} h_{A_1}\left(A_2^{(1)} - A_1^{(1)} - 2\right) \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_D, q') \\ + g_{h_c D^* D^*} g_{\pi D^* D^*} h_{A_1}\left(A_1^{(1)} + A_3^{(2)}\right)(M'^2 + M''^2 - q^2) \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q'). \tag{19}$$

The form factor is

$$f_3(m_1, m_2) = \frac{1}{16\pi^3}\int \mathrm{d}x_2 \mathrm{d}^2 p_\perp \frac{F_3}{x_2 \hat{N}_1 \hat{N}_1'}, \tag{20}$$

which will be numerically evaluated. With these form factors the transition amplitude is obtained:

$$\mathcal{A}_{31} = if_3(m_1, m_2)P'_a q_b \varepsilon_{abd\alpha}\epsilon_1^d \epsilon^\alpha. \tag{21}$$

Including the contributions of the Feynman diagrams where we switch around h_c and π in the final states, the amplitude is

$$\mathcal{A}_3 = i[f_3(m_1, m_2) + f_3(m_2, m_1)]P'_a q_b \varepsilon_{abd\alpha} \\ = ig_3 P'_a q_b \varepsilon_{abd\alpha}\epsilon_1^d \epsilon^\alpha. \tag{22}$$

3.3 $Z_c(4020)$ as a 2^+ molecular state

Then as we suppose $Z_c(4020)$ is a 2^+ molecule, the hadronic matrix element is

$$\mathcal{A}_{41} = i\frac{1}{(2\pi)^4}\int \mathrm{d}^4 p_1 \frac{H_{A_1}}{N_1 N_1' N_2}\left(S^{2(a)}_{d\mu\nu} + S^{2(b)}_{d\mu\nu}\right)\epsilon_1^d \epsilon^{\mu\nu}, \tag{23}$$

where

$$S^{2(a)}_{d\alpha} = ig_{h_c DD^*} g_{\pi DD^*} g^{\mu\mu'}(2q_{\mu'} - p_{1\mu'}) g^{\nu\nu'} g_{\nu' d} \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_D, q'),$$

and $S^{2(b)}_{d\alpha} = -ig_{h_c D^* D^*} g_{\pi D^* D^*} g^{\mu\mu'} g^{\nu\nu'} \varepsilon_{\omega\mu'\rho a} p_1^\omega q'^\rho g^{ac} P''^{f} \varepsilon_{fdc\nu}\mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q')$. Carrying out the integration, one has

$$h_{A_1}\left(\hat{S}^{2(a)}_{d\alpha} + \hat{S}^{2(b)}_{d\alpha}\right) = i(F_4 q_\mu g_{d\nu} + F_5 q_\nu g_{d\mu} + F_6 q_\nu q_d q_\mu) \tag{24}$$

with

$$F_4 = g_{h_c DD^*} g_{\pi DD^*} h_{A_1}\left(2 + A_1^{(1)} - A_2^{(1)}\right) \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_D, q'),$$

$$F_5 = 2g_{h_c D^* D^*} g_{\pi D^* D^*} h_{A_1}\left(A_1^{(1)} + A_3^{(2)}\right)\frac{(M'^2 + M''^2 - q^2)}{2} \\ \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q'),$$

$$F_6 = 2g_{h_cD^*D^*}g_{\pi D^*D^*}h_{A_1}\left(A_1^{(1)} + A_3^{(2)}\right) \times \mathcal{F}(m_1, p_1)\mathcal{F}(m_2, p_2)\mathcal{F}^2(m_{D^*}, q'). \tag{25}$$

The new form factors are defined as follows:

$$f_a(m_1, m_2) = \frac{1}{16\pi^3}\int \mathrm{d}x_2\mathrm{d}^2p_\perp \frac{F_a}{x_2\hat{N}_1\hat{N}_1'}, \tag{26}$$

where the subscript a denotes 4, 5, and 6. Substituting these form factors into the formulas, the transition amplitude is obtained:

$$\mathcal{A}_{41} = i[f_4(m_1, m_2)q_\mu g_{d\nu} + f_5(m_1, m_2)q_\nu g_{d\mu} + f_6(m_1, m_2)q_\nu q_d q_\mu]\epsilon_1^d\epsilon^{\mu\nu}. \tag{27}$$

Similarly, as all the contributions are incorporated, the total amplitude is

$$\begin{aligned}\mathcal{A}_4 = i\{&[f_4(m_1, m_2) + f_4(m_2, m_1)]q_\mu g_{d\nu} \\ &+ [f_5(m_1, m_2) + f_5(m_2, m_1)]q_\nu g_{d\mu} \\ &+ [f_6(m_1, m_2)q_\nu q_d q_\mu + f_6(m_2, m_1)q_\nu q_d q_\mu]\}\epsilon_1^d\epsilon^{\mu\nu} \\ = i[&g_4 q_\mu g_{d\nu} + g_5 q_\nu g_{d\mu} + g_6 q_\nu q_d q_\mu]\epsilon_1^d\epsilon^{\mu\nu}. \end{aligned} \tag{28}$$

4 Numerical results

In this section we present our predictions on the decay rates of $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$ along with all the input parameters. First we need to calculate the corresponding form factors which we deduced in last section. Those formulas involve some parameters which need to be fixed a priori. We use 3.899 GeV [3] as the mass of $Z_c(3900)$ and the mass of $Z_c(4020)$ is determined to be 4.02 GeV. The masses of the involved mesons are set as $m_{h_c} = 3.525$ GeV, $m_\pi = 0.139$ GeV, $m_D = 1.869$ GeV and $m_{D^*} = 2.007$ GeV according to the data book [43]. The coupling constants $g_{\pi DD^*} = 8.8$ and $g_{\pi D^*D^*} = 9.08$ GeV^{-1} are adopted according to Refs. [37,38]. At present one cannot fix the couplings h_cDD^* and $h_cD^*D^*$ from the data yet. However, there exists a simple but approximate relation, $m_D g_{h_cDD^*} = g_{h_cD^*D^*}$, which is in analogy to the case of the couplings $\psi D^{(*)}D^{(*)}$ [40,41], so only one undetermined parameter remains. Since the values of most coupling constants are of order $O(1)$, we set $g_{h_cD^*D^*} = 1$ as a reasonable choice. If one could fix $g_{h_cD^*D^*}$ later, one just needs to multiply a number to the corresponding form factor and it does not affect our final conclusion. The cutoff parameter Λ in the vertex $\mathcal{F}$ was suggested to be set as 0.88–1.1 GeV [41]. In our calculation we use 0.88 and 1.1 GeV, respectively, to study the effect on the results. The parameter β in the wavefunction is not very certain at present. In Ref. [16] we estimated its value and decided that it is close to or slightly smaller than 0.631 GeV^{-1} [44], and it is the β number for the wavefunction of J/ψ.

Table 1 The three-parameter form factors with ($\Lambda = 0.88$ GeV, $\beta = 0.631$ GeV^{-1}).

g	$g(0)$	a	b
g_1	−0.253	2.72	4.60
g_2	0.364	2.75	4.70
g_3	−0.129	2.74	3.25
g_4	−0.243	3.24	7.01
g_5	−0.486	2.41	2.42
g_6	−0.0341	2.82	4.88

Since the form factors are derived in the reference frame of $q^+ = 0$ ($q^2 < 0$) i.e. in the space-like region, we need to extend them to the time-like region by means of the normal procedure provided in the literature. In Ref. [31] a three-parameter form factor was suggested:

$$g(q^2) = \frac{g(0)}{\left[1 - a\left(\frac{q^2}{M_{Z_c}^2}\right) + b\left(\frac{q^2}{M_{Z_c}^2}\right)^2\right]}. \tag{29}$$

The resultant form factors are listed in Table 1 and the corresponding decay widths are presented in Table 2. The molecular states of $D^*\bar{D}^*$ can be in three different quantum states, thus the Lorentz structures of their decay amplitudes for $Z_c \to h_c\pi$ are different and the values of the corresponding form factors should also be different. However, we find that the decay widths of all those states are very close to each other, and it is easy to understand because the three states with different spin assignments are degenerate. One can also note that $\Gamma(Z_c(4020) \to h_c\pi)$ is three times larger than $\Gamma(Z_c(3900) \to h_c\pi)$ for different parameter Λ.

In our calculation, we notice that the model parameter β can affect the numerical results to a certain degree. We illustrate the dependence of $\Gamma(Z_c(3900) \to h_c\pi)$ and $\Gamma(Z_c(4020) \to h_c\pi)$ on β in Fig. 3 and depict the dependence of the ratio of $\Gamma(Z_c(4020) \to h_c\pi)/\Gamma(Z_c(3900) \to h_c\pi)$ on β in Fig. 4. Lines A and B in Fig. 3 correspond to $Z_c(3900)$ and $Z_c(4020)$ respectively. It is also noted that the ratio $\Gamma(Z_c(4020) \to h_c\pi)/\Gamma(Z_c(3900) \to h_c\pi) \approx 2 \sim 3$ does not vary much as β changes.

5 Conclusion and discussions

In this work, supposing $Z_c(3900)$ and $Z_c(4020)$ to be $D\bar{D}^*$ and $D^*\bar{D}^*$ molecular states, we calculate the decay rates of $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$, respectively, in the light-front model. It is noted that for the $D^*\bar{D}^*$ system there are three degenerate states whose quantum numbers are 0^+, 1^+, and 2^+ with orbital angular momentum $L = 0$. Thus we calculate the decay rates of the molecular state $D^*\bar{D}^*$ of different quantum numbers in this work. Using the

Table 2 The decay widths of some modes ($\beta = 0.631$ GeV^{-1}).

Decay mode ($\Lambda = 0.88$ GeV)	Width (GeV)	Decay mode ($\Lambda = 1.1$ GeV)	Width (GeV)
$Z_c(3900) \to h_c\pi$	5.85×10^{-5}	$Z_c(3900) \to h_c\pi$	8.91×10^{-5}
$Z_c(4020)(0^+) \to h_c\pi$	1.49×10^{-4}	$Z_c(4020)(0^+) \to h_c\pi$	2.36×10^{-4}
$Z_c(4020)(1^+) \to h_c\pi$	1.51×10^{-4}	$Z_c(4020)(1^+) \to h_c\pi$	2.34×10^{-4}
$Z_c(4020)(2^+) \to h_c\pi$	1.54×10^{-4}	$Z_c(4020)(2^+) \to h_c\pi$	2.38×10^{-4}

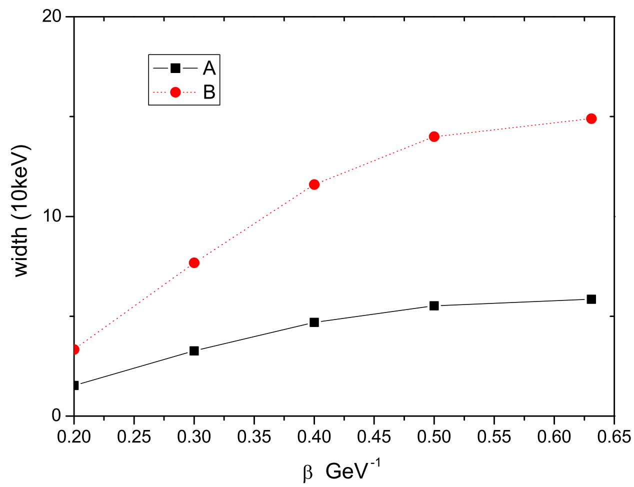

Fig. 3 The dependence of $\Gamma(Z_c(3900) \to h_c\pi)$ (A) and $\Gamma(Z_c(4020) \to h_c\pi)$ (B) on β

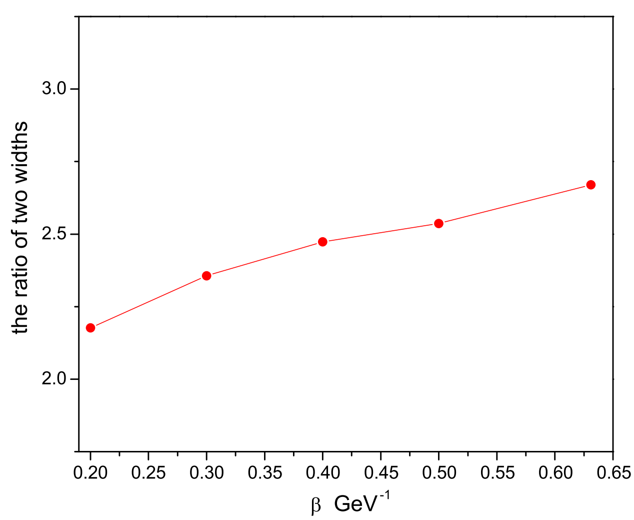

Fig. 4 The dependence of the ratio $\Gamma(Z_c(4020) \to h_c\pi)/\Gamma(Z_c(3900) \to h_c\pi)$ on β

effective interactions we calculate the corresponding form factors for the decays $Z_c(3900) \to h_c\pi$ and $Z_c(4020) \to h_c\pi$. Our numerical results show $\Gamma(Z_c(4020)(0^+) \to h_c\pi)$, $\Gamma(Z_c(4020)(1^+) \to h_c\pi)$, and $\Gamma(Z_c(4020)(2^+) \to h_c\pi)$ are indeed close to each other. By the results one would think that $Z_c(4020)$ behaves as a molecular state.

It is noticed that the resultant $\Gamma(Z_c(3900) \to h_c\pi)$ is only three times smaller than $\Gamma(Z_c(4020) \to h_c\pi)$ for various values of Λ and β.

Considering the total width, even though the branching ratio of $\Gamma(Z_c(3900) \to h_c\pi)$ is slightly small, we still have a remarkable opportunity to observe $Z_c(3900)$ in this channel. If $Z_c(3900)$ and $Z_c(4020)$ are $D\bar{D}^*$ and $D^*\bar{D}^*$ molecular states, we should observe the $Z_c(3900)$ peak at the invariant mass spectrum of $e^+e^- \to h_c\pi$. No doubt, since this portal has not been "seen" at BES III so far, the reason may be attributed to the relatively small database at present. Thus with more data accumulating to a reasonable stack, the experimental exploration of $Z_c(3900) \to h_c\pi$ will eventually reach a conclusion, namely that a peak at 3900 MeV does or does not show up or. Namely, if it does appear, one can celebrate the assumption that $Z_c(3900)$ is indeed a valid molecular state of $D\bar{D}^*(D^*\bar{D})$, or at least it possesses a large fraction of the molecular state. On the contrary, if there is still no signal of $Z_c(3900)$ to be observed at the $h_c\pi$ invariant mass spectrum, the proposal that $Z_c(3900)$ were a $D\bar{D}^*$ molecular state would not be favored or ruled out.

Even though in our calculation the coupling constant $g_{h_cD^*D^*}$ is not well determined, so that the estimated widths are not precise. However, the ratio $\Gamma(Z_c(3900) \to h_c\pi)/\Gamma(Z_c(4020) \to h_c\pi)$ does not depend on the coupling. Therefore, our scheme for judging whether $Z_c(3900)$ is a molecular state is still working. A relevant question arises: what is the inner structure of $Z_c(3900)$ if it is not a molecule? In Ref. [45] the authors study some strong decays of $Z_c(3900)$ by assuming it to be a tetraquark with the QCD sum rules, but unfortunately the channel of $Z_c(3900) \to h_c\pi$ was not discussed in their work. In our next work we will explore some strong decays of $Z_c(3900)$ as a tetraquark, especially including $Z_c(3900) \to h_c\pi$ in the light-front model, and will show the partial width of this channel should indeed be small.

Since $Z_c(3900)$ was found from the final state $J/\psi\pi$, it is natural to suggest that one should detect if $Z_c(4020)$ shows up in the invariant mass spectrum of $J/\psi\pi$. Postulating both $Z_c(3900)$ and $Z_c(4020)$ to be molecular states we find $\Gamma(Z_c(4020) \to J/\psi\pi)$ is five times larger than $\Gamma(Z_c(3900) \to J/\psi\pi)$ [16]. Thus we suggest our experimental colleagues to adjust the center-of-mass-energy to produce a larger database for $Z_c(4020)$ to measure the corresponding decay rate. It will be an ideal scheme to determine the identities of both $Z_c(3900)$ and $Z_c(4020)$.

Moreover, at the invariant mass spectrum of $D^*\bar{D}^*$, another resonance, $Z_c(4025)$, was measured with a mass of $(4026.3\pm2.6\pm3.7)$ MeV and width $(24.8\pm5.6\pm7.7)$ MeV [46]. Its peak heavily overlaps with that of $Z_c(4020)$, and the deviation is within 1.5σ, therefore it seems that $Z_c(4020)$ and $Z_c(4025)$ might be degenerate, even more, that they are the same state, but the measurement errors cause a misidentification. Thus in future work it is our task to identify them as either two different resonances whose masses are close, or just degenerate states or the same one.

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Appendix A: The vertex function of a molecular state

Supposing $Z_c(3900)$ and $Z_c(4030)$ are molecular states which consists of D and $\bar{D}^*$ and D^* and $\bar{D}^*$ respectively. The wavefunction of a molecular state with total spin J and momentum P is [16]

$$|X(P,J,J_z)\rangle = \int \{d^3\tilde{p}_1\}\{d^3\tilde{p}_2\}\,2(2\pi)^3\delta^3(\tilde{P}-\tilde{p}_1-\tilde{p}_2) \times \sum_{\lambda_1}\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) \times \mathcal{F}\,|\,D^{(*)}(p_1,\lambda_1)\bar{D}^*(p_2,\lambda_2)\rangle. \quad (A1)$$

For the 0^+ molecular state of $D^*\bar{D}^*$

$$\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) = C_0\varphi(x,p_\perp)\epsilon_1(\lambda_1)\cdot\epsilon_2(\lambda_2) = h'_{C_0}\epsilon_1(\lambda_1)\cdot\epsilon_2(\lambda_2), \quad (A2)$$

for the 1^+ molecular state of $D^*\bar{D}^*$

$$\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) = C_1\varphi(x,p_\perp)\varepsilon^{\mu\nu\alpha\beta}\epsilon_{1\mu}(\lambda_1) \times \epsilon_{2\nu}(\lambda_2)\epsilon_\alpha(J_z)P_\beta = h'_{C_1}\varepsilon^{\mu\nu\alpha\beta}\epsilon_{1\mu}(\lambda_1)\epsilon_{2\nu}(\lambda_2)\epsilon_\alpha(J_z)P_\beta, \quad (A3)$$

for the 2^+ molecular state of $D^*\bar{D}^*$

$$\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) = C_2\varphi(x,p_\perp)\epsilon_{1\mu}(\lambda_1)\epsilon_{2\nu}(\lambda_2)\epsilon^{\mu\nu}(J_z) = h'_{C_2}\epsilon_{1\mu}(\lambda_1)\epsilon_{2\nu}(\lambda_2)\epsilon^{\mu\nu}(J_z), \quad (A4)$$

and for the 1^+ molecular state of $D\bar{D}^*$

$$\Psi^{SS_z}(\tilde{p}_1,\tilde{p}_2,\lambda_1,\lambda_2) = C_{01(10)}\varphi(x,p_\perp)\epsilon_{1\mu}(\lambda_1)\cdot\epsilon_\alpha(J_z) = h'_{C_{01(10)}}\epsilon_{1\mu}(\lambda_1)\cdot\epsilon_\alpha(J_z), \quad (A5)$$

where C_{01}, C_{10}, C_0, C_1, and C_2 are the normalization constants, which can be fixed by normalizing the state [31]

$$\langle X(P',J',J'_z)|X(P,J,J_z)\rangle = 2(2\pi)^3P^+\delta^3(\tilde{P}'-\tilde{P})\delta_{JJ'}\delta_{J_ZJ_{Z'}}, \quad (A6)$$

and we let the normalization $\int\frac{dxd^2p_\perp}{2(2\pi)^3}\varphi'^*_{L',L'_Z}(x,p_\perp)\varphi_{L,L_Z}(x,p_\perp)=\delta_{L,L'}\delta_{L_Z,L'_Z}$ hold.

For example C_0 is fixed by calculating Eq. (A6) with the 0^+ state,

$$\int\frac{dxd^2p_\perp}{2(2\pi)^3}C_0^2\epsilon_1^*(\lambda_1)\cdot\epsilon_2^*(\lambda_2)\epsilon_1(\lambda_1) \cdot\epsilon_2(\lambda_2)\varphi^*(x,p_\perp)\varphi(x,p_\perp)=1, \quad (A7)$$

then $C_0=\frac{2m_1m_2}{\sqrt{M_0{}^4-2M_0{}^2(m_1{}^2+m_2{}^2)+m_1{}^4+10m_1{}^2m_2{}^2+m_2{}^4}}$. It is noted that $P^2=M_0^2$, $p1\cdot P=e_1M_0$, and $p2\cdot P=e_2M_0$ are used as discussed in Ref. [31].

Similarly one can obtain

$$C_{01}=\frac{\sqrt{3}m_1}{\sqrt{e_1^2+2m_1^2}},\quad C_{10}=\frac{\sqrt{3}m_2}{\sqrt{e_2^2+2m_2^2}},$$

$$C_1=\frac{2\sqrt{3}m_1m_2}{\sqrt{M^2[4e_1{}^2m_2{}^2-4e_1e_2(-M_0{}^2+m_1{}^2+m_2{}^2)+4e_2{}^2m_1{}^2+10m_1{}^2m_2{}^2-C_A]}},$$

$$C_2=\frac{\sqrt{120}m_1m_2}{\sqrt{4e_1{}^2(4e_2{}^2+7m_2{}^2)+4e_1e_2(-M_0{}^2+m_1{}^2+m_2{}^2)+28e_2{}^2m_1{}^2+54m_1{}^2m_2{}^2+C_A}},$$

$$C_A=M_0{}^4-2M_0{}^2(m_1{}^2+m_2{}^2)+m_1{}^4+m_2{}^4,$$

and $\varphi = 4(\frac{\pi}{\beta^2})^{3/4}\frac{e_1 e_2}{x_1 x_2 M_0}\exp(\frac{-\mathbf{p}^2}{2\beta^2})$.

All other notations can be found in Refs. [22–24].

Appendix B: The effective vertices

The effective vertices can be found in [37–41],

$$\mathcal{L}_{\pi DD^*} = ig_{\pi DD^*}(D^{*\mu}\partial_\mu \pi \bar{D} - \partial^\mu D\pi \bar{D}^*_\mu + h.c.), \quad \text{(B1)}$$

$$\mathcal{L}_{\pi D^*D^*} = -g_{\pi D^*D^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu \bar{D}^*_\nu \pi \partial_\alpha D^*_\beta, \quad \text{(B2)}$$

$$\mathcal{L}_{h_c D^*D^*} = -ig_{h_c D^*D^*}\varepsilon^{\mu\nu\alpha\beta}\partial_\mu h_{c\nu} D^*_\alpha \bar{D}^*_\beta, \quad \text{(B3)}$$

$$\mathcal{L}_{h_c DD^*} = g_{h_c DD^*} h_c{}^\nu D\bar{D}^*_\nu. \quad \text{(B4)}$$

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