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Analysis of the strong decay $X(5568) \to B_s^0 \pi^+$ with QCD sum rules

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Abstract In this article, we take the $X(5568)$ to be the scalar diquark–antidiquark type tetraquark state, study the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and the disconnected Feynman diagrams; then we calculate the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$ and obtain the value $\Gamma_X = (20.5 \pm 8.1)$ MeV, which is consistent with the experimental data $\Gamma_X = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right)$ MeV from the D0 collaboration.

1 Introduction

Recently, the D0 collaboration observed a narrow structure, $X(5568)$, in the decay $X(5568) \to B_s^0\pi^\pm$ with significance of 5.1σ [1]. The measured mass and width are $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV and $\Gamma_X = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right)$ MeV, respectively. The D0 collaboration fitted the $B_s^0\pi^\pm$ systems with the Breit–Wigner parameters in relative S-wave, the favored quantum numbers are $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to decays $X(5568) \to B_s^*\pi^+ \to B_s^0\pi^+\gamma$, where the low-energy photon is not detected. There have been several possible assignments, such as the scalar-diquark–scalar-antidiquark type tetraquark state [2–7], axialvector-diquark–axialvector-antidiquark type tetraquark state [3,8,9], $B^{(*)}\bar{K}$ hadronic molecule state [10], threshold effect [11].

The calculations based on the QCD sum rules indicate that both the scalar-diquark–scalar-antidiquark type and the axialvector-diquark–axialvector-antidiquark type interpolating currents can give satisfactory mass m_X to reproduce the experimental data [2–4,8]. In Ref. [9], Agaev, Azizi and Sundu choose the axialvector-diquark–axialvector-antidiquark type interpolating current, they calculate the hadronic coupling constant $g_{XB_s\pi}$ with the light-cone QCD sum rules in conjunction with the soft-π approximation and other approximations, and they obtain the partial decay width for the process $X(5568) \to B_s^0\pi^+$. In Ref. [7], Dias et al. choose the scalar-diquark–scalar-antidiquark type interpolating current, calculate the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules in the soft-π limit by taking into account only the connected Feynman diagrams in the leading order approximation, and obtain the partial decay width for the decay $X(5568) \to B_s^0\pi^+$. In previous work [2], we choose the scalar-diquark–scalar-antidiquark type interpolating current to study the mass of the $X(5568)$ with the QCD sum rules. In this article, we extend our previous work to the study of the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension-6 and including both the connected and disconnected Feynman diagrams; then we calculate the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$.

The article is arranged as follows: we derive the QCD sum rule for the hadronic coupling constant $g_{XB_s\pi}$ in Sect. 2; in Sect. 3, we present the numerical results and discussions; and Sect. 4 is reserved for our conclusion.

2 QCD sum rule for the hadronic coupling constant $g_{XB_s\pi}$

We can study the strong decay $X(5568) \to B_s^0\pi^+$ with the three-point correlation function $\Pi(p,q)$,

$$\Pi(p,q) = i^2 \int d^4x d^4y e^{ip\cdot x} e^{iq\cdot y} \times\langle 0|T\left\{J_{B_s}(x)J_\pi(y)J_X(0)\right\}|0\rangle, \quad (1)$$

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where the currents

$$J_{B_s}(x) = \bar{s}(x)i\gamma_5 b(x),$$
$$J_\pi(y) = \bar{u}(y)i\gamma_5 d(y),$$
$$J_X(0) = \epsilon^{ijk}\epsilon^{imn}u^j(0)C\gamma_5 s^k(0)\bar{d}^m(0)\gamma_5 C\bar{b}^n(0), \tag{2}$$

interpolate the mesons B_s, π, and $X(5568)$, respectively, the i, j, k, m, n are color indices, the C is the charge conjugation matrix. In Ref. [7], the axial vector current is used to interpolate the π meson.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_{B_s}(x)$, $J_\pi(y)$, and $J_X(0)$ into the three-point correlation function $\Pi(p,q)$ and isolate the ground state contributions to obtain the following result:

$$\begin{aligned}\Pi(p,q) &= \frac{f_\pi m_\pi^2 f_{B_s} m_{B_s}^2 \lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)} \\ &\quad\times \frac{1}{\left(m_X^2-p'^2\right)\left(m_{B_s}^2-p^2\right)\left(m_\pi^2-q^2\right)} \\ &\quad+\frac{1}{\left(m_X^2-p'^2\right)\left(m_{B_s}^2-p^2\right)}\int_{s_\pi^0}^\infty \mathrm{d}t\,\frac{\rho_{X\pi}(p^2,t,p'^2)}{t-q^2} \\ &\quad+\frac{1}{\left(m_X^2-p'^2\right)\left(m_\pi^2-q^2\right)}\int_{s_{B_s}^0}^\infty \mathrm{d}t\,\frac{\rho_{XB_s}(t,q^2,p'^2)}{t-p^2}+\cdots,\end{aligned} \tag{3}$$

where $p' = p+q$, the f_{B_s}, f_π and λ_X are the decay constants of the mesons B_s, π, and $X(5568)$, respectively, and $g_{XB_s\pi}$ is the hadronic coupling constant.

In the following, we write down the definitions:

$$\langle 0|J_X(0)|X(p')\rangle = \lambda_X,$$
$$\langle 0|J_{B_s}(0)|B_s(p)\rangle = \frac{f_{B_s} m_{B_s}^2}{m_b+m_s},$$
$$\langle 0|J_\pi(0)|\pi(q)\rangle = \frac{f_\pi m_\pi^2}{m_u+m_d}, \tag{4}$$
$$\langle B_s(p)\pi(q)|X(p')\rangle = i g_{XB_s\pi}. \tag{5}$$

The two unknown functions $\rho_{X\pi}(p^2,t,p'^2)$ and $\rho_{XB_s}(t,q^2,p'^2)$ have a complex dependence on the transitions between the ground state $X(5568)$ and the excited states of the π and B_s mesons, respectively. We introduce the parameters $C_{X\pi}$ and C_{XB_s} to parameterize the net effects,

$$C_{X\pi} = \int_{s_\pi^0}^\infty \mathrm{d}t\,\frac{\rho_{X\pi}(p^2,t,p'^2)}{t-q^2},$$
$$C_{XB_s} = \int_{s_{B_s}^0}^\infty \mathrm{d}t\,\frac{\rho_{XB_s}(t,q^2,p'^2)}{t-p^2}, \tag{6}$$

and we rewrite the correlation function $\Pi(p,q)$ into the following form:

$$\begin{aligned}\Pi(p,q) &= \frac{f_\pi m_\pi^2 f_{B_s} m_{B_s}^2 \lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)} \\ &\quad\times \frac{1}{\left(m_X^2-p'^2\right)\left(m_{B_s}^2-p^2\right)\left(m_\pi^2-q^2\right)} \\ &\quad+\frac{C_{X\pi}}{\left(m_X^2-p'^2\right)\left(m_{B_s}^2-p^2\right)} \\ &\quad+\frac{C_{XB_s}}{\left(m_X^2-p'^2\right)\left(m_\pi^2-q^2\right)}+\cdots.\end{aligned} \tag{7}$$

We set $p'^2 = p^2$ and take the double Borel transform with respect to the variable $P^2 = -p^2$ and $Q^2 = -q^2$, respectively, to obtain the QCD sum rule at the left side (LS),

$$\begin{aligned}\mathrm{LS} &= \frac{f_\pi m_\pi^2 f_{B_s} m_{B_s}^2 \lambda_X g_{XB_s\pi}}{(m_u+m_d)(m_b+m_s)}\frac{1}{m_X^2-m_{B_s}^2} \\ &\quad\times\left\{\exp\left(-\frac{m_{B_s}^2}{M_1^2}\right)-\exp\left(-\frac{m_X^2}{M_1^2}\right)\right\}\exp\left(-\frac{m_\pi^2}{M_2^2}\right) \\ &\quad+C_{XB_s}\exp\left(-\frac{m_X^2}{M_1^2}\right)\exp\left(-\frac{m_\pi^2}{M_2^2}\right).\end{aligned} \tag{8}$$

In calculations, we neglect the dependencies of the $C_{X\pi}$ and C_{XB_s} on the variables p^2, p'^2, q^2 therefore the dependencies of the $C_{X\pi}$ and C_{XB_s} on the variables M_1^2 and M_2^2, we take the $C_{X\pi}$ and C_{XB_s} as free parameters, and we choose the suitable values to eliminate the contaminations so as to obtain the stable sum rules with the variations of the Borel parameters [12–14].

Now we carry out the operator product expansion at the large Euclidean space-time region $-p^2 \to \infty$ and $-q^2 \to \infty$, take into account the vacuum condensates up to dimension 6 and neglect the contribution of the three-gluon condensate, as the three-gluon condensate is the vacuum expectation of the operator of the order $\mathcal{O}(\alpha_s^{3/2})$. In other words, we calculate the Feynman diagrams shown in Fig. 1. For example, the first diagram is calculated in the following ways:

$$\begin{aligned}\Pi(p,q) &= -\frac{6}{(2\pi)^8}\int \mathrm{d}^4k\,\mathrm{d}^4l \\ &\quad\times\frac{\mathrm{Tr}\left\{\gamma_5(\not{k}+m_s)\gamma_5(\not{k}+\not{p}+m_b)\gamma_5(\not{l}+\not{q})\gamma_5\not{l}\right\}}{k^2\left[(k+p)^2-m_b^2\right]^2(l+q)^2 l^2} \\ &= -\frac{6}{(2\pi)^8}\frac{(-2\pi i)^2}{2\pi i} \\ &\quad\times\int_{m_b^2}^\infty \mathrm{d}s\,\frac{1}{s-p^2}\int \mathrm{d}^4k\,\delta\left[k^2\right] \\ &\quad\times\delta\left[(k+p)^2-m_b^2\right]\frac{(-2\pi i)^2}{2\pi i} \\ &\quad\times\int_0^\infty \mathrm{d}u\,\frac{1}{u-q^2}\int \mathrm{d}^4l\,\delta\left[l^2\right]\delta\left[(l+q)^2\right]\end{aligned}$$

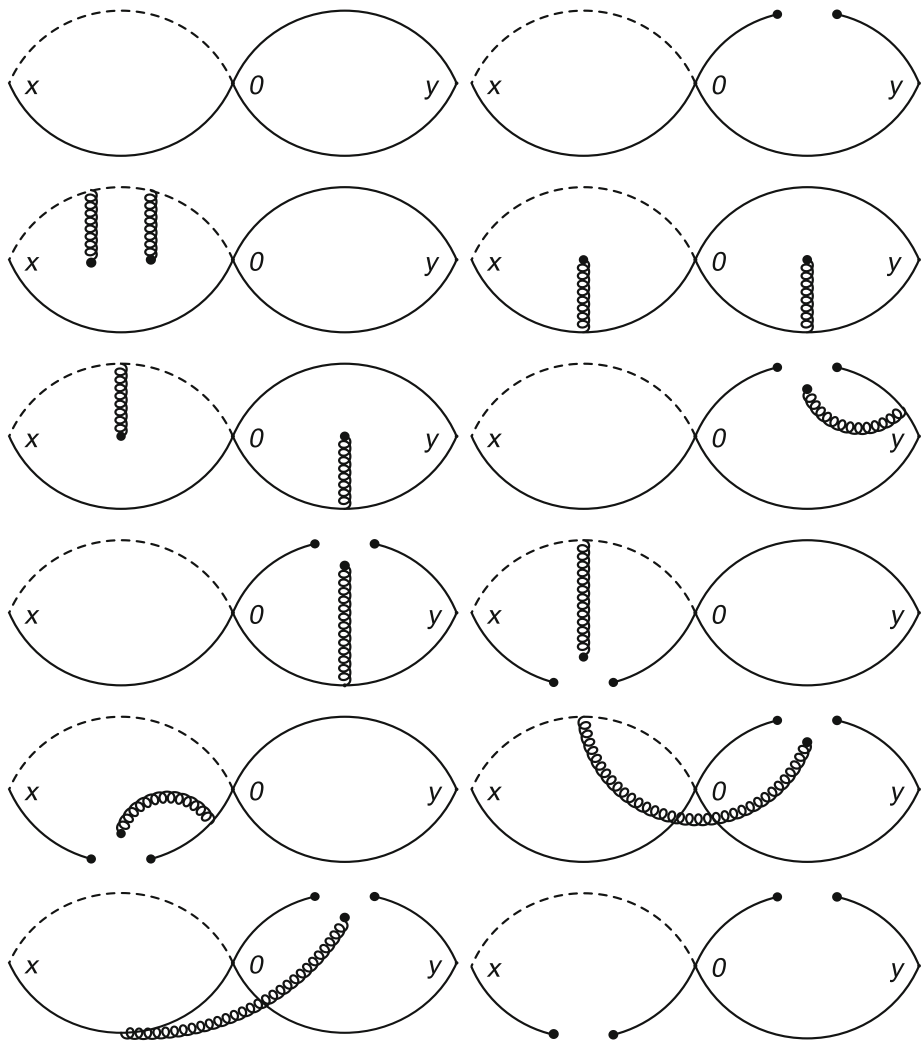

Fig. 1 The Feynman diagrams calculated in this article, where the *solid lines* and *dashed lines* denote the light quarks and heavy quarks, respectively; the *waved lines* denote the gluons. Other diagrams obtained by interchanging of the light quark lines are implied

$$\times \mathrm{Tr}\left\{\gamma_5(\not{k}+m_s)\gamma_5(\not{k}+\not{p}+m_b)\gamma_5(\not{l}+\not{q})\gamma_5 \not{l}\right\}$$
$$= \frac{3}{128\pi^4}\int_{m_b^2}^{\infty} \mathrm{d}s \frac{1}{s-p^2}\frac{(s-m_b^2)^2}{s}\int_0^{\infty}\mathrm{d}u \frac{u}{u-q^2} + \frac{3m_s m_b}{64\pi^4}\int_{m_b^2}^{\infty}\mathrm{d}s \frac{1}{s-p^2}\frac{s-m_b^2}{s}\int_0^{\infty}\mathrm{d}u\frac{u}{u-q^2}. \tag{9}$$

The operator product expansion converges for large $-p^2$ and $-q^2$, it is odd to take the limit $q^2 \to 0$.

Then we set $p'^2 = p^2$, take the quark–hadron duality below the continuum thresholds, and perform the double Borel transform with respect to the variables $P^2 = -p^2$ and $Q^2 = -q^2$, respectively, to obtain the perturbative term,

$$B_{M_1^2,M_2^2}\Pi(p,q) = \frac{3}{128\pi^2}\int_{m_b^2}^{s_0}\mathrm{d}s \times \int_0^{u_0}\mathrm{d}u \frac{(s-m_b^2)^2}{s} u \exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) + \frac{3m_s m_b}{64\pi^2}\int_{m_b^2}^{s_0}\mathrm{d}s \times \int_0^{u_0}\mathrm{d}u\frac{s-m_b^2}{s}u\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right), \tag{10}$$

where s_0 and u_0 are the continuum threshold parameters for the $X(5568)$ and π, respectively.

Other Feynman diagrams are calculated in analogous ways, and finally we obtain the QCD sum rules at the right side (RS),

$$\mathrm{RS} = \frac{3}{128\pi^2}\int_{m_b^2}^{s_0}\mathrm{d}s\int_0^{u_0}\mathrm{d}u\frac{(s-m_b^2)^2}{s}u \times \exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right) + \frac{3m_s m_b}{64\pi^2}\int_{m_b^2}^{s_0}\mathrm{d}s\int_0^{u_0}\mathrm{d}u\frac{s-m_b^2}{s}u$$

$$
\begin{aligned}
&\times \exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\
&+\frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int_{m_b^2}^{s_0} \mathrm{d}s\\
&\times\int_0^{u_0}\mathrm{d}u\left(2-\frac{m_b^2}{s}\right)\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\
&-\frac{m_b\langle\bar{s}s\rangle}{16\pi^2}\int_0^{u_0}\mathrm{d}u u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right)\\
&-\frac{m_s\langle\bar{s}s\rangle}{32\pi^2}\left(1+\frac{m_b^2}{M_1^2}\right)\int_0^{u_0}\mathrm{d}u u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right)\\
&+\frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int_0^{u_0}\mathrm{d}u u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right)\\
&+\frac{1}{128\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int_{m_b^2}^{s_0}\mathrm{d}s\frac{(s-m_b^2)^2}{s}\exp\left(-\frac{s}{M_1^2}\right)\\
&-\frac{m_b\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_0^{u_0}\mathrm{d}u\\
&\times\left(1+\frac{u}{M_1^2}-\frac{um_b^2}{2M_1^4}-\frac{um_sm_b^3}{6M_1^6}\right)\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right).
\end{aligned}
\tag{11}
$$

The terms $\langle\bar{q}q\rangle\langle\bar{s}s\rangle$ disappear after performing the double Borel transform, the last Feynman diagrams in Fig. 1 have no contribution.

In Refs. [14,15], the width of the $Z_c(4200)$ is studied with the three-point QCD sum rules by including both the connected and disconnected Feynman diagrams, which is contrary to Ref. [16], where only the connected Feynman diagrams are taken into account to study the width of the $Z_c(3900)$. In this article, the contributions coming from the connected diagrams can be written as RS_c,

$$
\begin{aligned}
\mathrm{RS}_c=&\frac{1}{192\pi^2}\left\langle\frac{\alpha_s GG}{\pi}\right\rangle\int_{m_b^2}^{s_0}\mathrm{d}s\\
&\times\int_0^{u_0}\mathrm{d}u\left(2-\frac{m_b^2}{s}\right)\exp\left(-\frac{s}{M_1^2}-\frac{u}{M_2^2}\right)\\
&-\frac{m_b\langle\bar{s}g_s\sigma Gs\rangle}{32\pi^2}\int_0^{u_0}\mathrm{d}u\exp\left(-\frac{m_b^2}{M_1^2}-\frac{u}{M_2^2}\right),
\end{aligned}
\tag{12}
$$

which is too small to account for the experimental data [1].

Finally, we obtain the QCD sum rule,

$$
\mathrm{LS}=\mathrm{RS}. \tag{13}
$$

There appear some energy-scale dependence at the hadron side (or LS) of the QCD sum rule according to the factors m_u+m_d and m_b+m_s, we can eliminate the energy-scale dependence by using the currents $\widehat{J}_{B_s}(x)$ and $\widehat{J}_\pi(y)$,

$$
\begin{aligned}
\widehat{J}_{B_s}(x)&=(m_b+m_s)\,\bar{s}(x)i\gamma_5 b(x),\\
\widehat{J}_\pi(y)&=(m_u+m_d)\,\bar{u}(y)i\gamma_5 d(y),
\end{aligned}
\tag{14}
$$

then

$$
\begin{aligned}
\langle 0|\widehat{J}_{B_s}(0)|B_s(p)\rangle&=f_{B_s}m_{B_s}^2,\\
\langle 0|\widehat{J}_\pi(0)|\pi(q)\rangle&=f_\pi m_\pi^2,
\end{aligned}
\tag{15}
$$

and

$$
\begin{aligned}
C_{X\pi}&\to C_{X\pi}\,(m_b+m_s)\,(m_u+m_d)\,,\\
C_{XB_s}&\to C_{XB_s}\,(m_b+m_s)\,(m_u+m_d)\,,
\end{aligned}
\tag{16}
$$

the resulting QCD sum rule at the right side also acquires a factor $(m_b+m_s)\,(m_u+m_d)$, an equivalent QCD sum rule is obtained, the predicted hadronic coupling constant $g_{XB_s\pi}$ is not changed.

We can also study the strong decay $X(5568)\to B_s^0\pi^+$ with the three-point correlation function $\Pi_{\mu\nu}(p,q)$,

$$
\begin{aligned}
\Pi_{\mu\nu}(p,q)=&\,i^2\int \mathrm{d}^4x\mathrm{d}^4y e^{ip\cdot x}e^{iq\cdot y}\\
&\times\left\langle 0|T\left\{\eta_\mu^{\bar{s}b}(x)\eta_\nu^{\bar{u}d}(y)J_X(0)\right\}|0\right\rangle,
\end{aligned}
\tag{17}
$$

where the currents

$$
\begin{aligned}
\eta_\mu^{\bar{s}b}(x)&=\bar{s}(x)\gamma_\mu\gamma_5 b(x),\\
\eta_\nu^{\bar{u}d}(y)&=\bar{u}(y)\gamma_\nu\gamma_5 d(y),
\end{aligned}
\tag{18}
$$

interpolate the mesons B_s and π, respectively. At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $\eta_\mu^{\bar{s}b}(x)$ and $\eta_\nu^{\bar{u}d}(y)$ into the three-point correlation function $\Pi_{\mu\nu}(p,q)$ and isolate the ground state contributions to obtain the following result:

$$
\begin{aligned}
\Pi_{\mu\nu}(p,q)=&\frac{f_{B_s}f_\pi\lambda_X g_{XB_s\pi}}{\left(m_X^2-p'^2\right)\left(m_{B_s}^2-p^2\right)\left(m_\pi^2-q^2\right)}\left(-p_\mu q_\nu\right)\\
&+\frac{f_{B_{s1}}m_{B_{s1}}f_\pi\lambda_X g_{XB_{s1}\pi}}{\left(m_X^2-p'^2\right)\left(m_{B_{s1}}^2-p^2\right)\left(m_\pi^2-q^2\right)}\\
&\times\left(-q_\mu q_\nu+\frac{p\cdot q}{p^2}p_\mu q_\nu\right)\\
&+\frac{f_{B_{s1}}m_{B_{s1}}f_{a_1}m_{a_1}\lambda_X g_{XB_{s1}a_1}}{\left(m_X^2-p'^2\right)\left(m_{B_{s1}}^2-p^2\right)\left(m_{a_1}^2-q^2\right)}\\
&\times\left(g_{\mu\nu}-\frac{1}{p^2}p_\mu p_\nu-\frac{1}{q^2}q_\mu q_\nu+\frac{p\cdot q}{p^2q^2}p_\mu q_\nu\right)+\cdots,
\end{aligned}
\tag{19}
$$

where $p'=p+q$, the $f_{B_{s1}}$, f_{B_s}, f_{a_1}, and f_π are the decay constants of the mesons $B_{s1}(5830)$, B_s, $a_1(1260)$, and π, respectively, the $g_{XB_{s1}\pi}$ and $g_{XB_{s1}a_1}$ are the hadronic coupling constants.

In the following, we write down the definitions:

$$\begin{aligned}
&\langle 0|\eta^{\bar{s}b}_{\mu}(0)|B_s(p)\rangle = i f_{B_s} p_{\mu},\\
&\langle 0|\eta^{\bar{u}d}_{\nu}(0)|\pi(q)\rangle = i f_{\pi} q_{\nu},\\
&\langle 0|\eta^{\bar{s}b}_{\mu}(0)|B_{s1}(p)\rangle = f_{B_{s1}} m_{B_{s1}} \varepsilon_{\mu},\\
&\langle 0|\eta^{\bar{u}d}_{\nu}(0)|a_1(q)\rangle = f_{a_1} m_{a_1} \epsilon_{\nu}, \qquad (20)\\
&\langle B_{s1}(p)\pi(q)|X(p')\rangle = \varepsilon^* \cdot q\, g_{XB_{s1}\pi},\\
&\langle B_{s1}(p)a_1(q)|X(p')\rangle = i\varepsilon^* \cdot \epsilon^*\, g_{XB_{s1}a_1}, \qquad (21)
\end{aligned}$$

where the ε_{μ} and ϵ_{ν} are polarization vectors of the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$, respectively. From the values $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV [1], $m_{B_{s1}} = (5828.40 \pm 0.04 \pm 0.41)$ MeV, $m_{B_s} = (5366.7 \pm 0.4)$ MeV [17], we can obtain $m_{B_{s1}} - m_{B_s} \approx 462$ MeV and $m_{B_{s1}} - m_X \approx 261$ MeV. If we take the interpolating currents $\eta^{\bar{s}b}_{\mu}(x)$ and $\eta^{\bar{u}d}_{\nu}(y)$, there are contaminations from the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$. We should multiply both sides of Eq.(19) by $p^{\mu}q^{\nu}$ to eliminate the contaminations of the axialvector mesons $B_{s1}(5830)$ and $a_1(1260)$ to find

$$p^{\mu}q^{\nu}\Pi_{\mu\nu}(p,q) = \frac{f_{B_s} f_{\pi} \lambda_X g_{XB_s\pi}}{\left(m_X^2 - p'^2\right)\left(m_{B_s}^2 - p^2\right)\left(m_{\pi}^2 - q^2\right)} \times \left(-p^2q^2\right) + \cdots, \qquad (22)$$

which corresponds to taking the pseudoscalar currents $\widehat{J}_{B_s}(x)$ and $\widehat{J}_{\pi}(y)$ according to the identities

$$\begin{aligned}
&\partial^{\mu}\eta^{\bar{s}b}_{\mu}(x) = (m_b + m_s)\,\bar{s}(x) i\gamma_5 b(x) = \widehat{J}_{B_s}(x),\\
&\partial^{\nu}\eta^{\bar{u}d}_{\nu}(y) = (m_u + m_d)\,\bar{u}(y) i\gamma_5 d(y) = \widehat{J}_{\pi}(y). \qquad (23)
\end{aligned}$$

The axialvector currents $\eta^{\bar{s}b}_{\mu}(x)$ and $\eta^{\bar{u}d}_{\nu}(y)$ can also be chosen to study the strong decay $X(5568) \to B_s^0\pi^+$.

We also expect to study the strong decay $X(5568) \to B_s^0\pi^+$ with the light-cone QCD sum rules using the two-point correlation function $\overline{\Pi}(p,q)$,

$$\overline{\Pi}(p,q) = i\int d^4x e^{ip\cdot x}\langle \pi(q)|T\left\{J_{B_s}(x)J_X(0)\right\}|0\rangle, \qquad (24)$$

where the $\langle \pi(q)|$ is an external π state.

At the QCD side, we obtain the following result after performing the Wick contraction:

$$\overline{\Pi}(p,q) = i\int d^4x e^{ip\cdot x}\langle \pi(q)|\epsilon^{ijk}\epsilon^{imn}u_j^T(0)C\gamma_5 S_s^{kl}(-x)i\gamma_5 S_b^{ln}(x)\gamma_5 C\bar{d}_m^T(0)|0\rangle, \qquad (25)$$

where the $S_s^{kl}(-x)$ and $S_b^{ln}(x)$ are the full s and b quark propagators, respectively. The u and $\bar{d}$ quarks stay at the same point $x = 0$, the light-cone distribution amplitudes of the π meson are almost useless, the integrals over the π meson's light-cone distribution amplitudes reduce to overall normalization factors. In the light-cone QCD sum rules, such a situation is possible only in the soft pion limit $q \to 0$, and the light-cone expansion reduces to the short-distance expansion [18]. In Ref. [9], Agaev, Azizi and Sundu take the soft pion limit $q \to 0$, and choose the $C\gamma_{\mu} \otimes \gamma^{\mu}C$ type current to interpolate the $X(5568)$, and use the light-cone QCD sum rules to study the strong decay $X(5568) \to B_s^0\pi^+$. The light-cone QCD sum rules are reasonable only in the soft pion approximation.

3 Numerical results and discussions

The hadronic parameters are taken as $m_X = 5.5678$ GeV [1], $\lambda_X = 6.7 \times 10^{-3}\,\text{GeV}^5$, $\sqrt{s_0} = (6.1 \pm 0.1)$ GeV [2], $m_{\pi} = 0.13957$ GeV, $m_{B_s} = 5.3667$ GeV [17], $f_{\pi} = 0.130$ GeV, $\sqrt{u_0} = (0.85 \pm 0.05)$ GeV [19], $f_{B_s} = 0.231$ GeV [20,21], $f_{\pi}m_{\pi}^2/(m_u + m_d) = -2\langle\bar{q}q\rangle/f_{\pi}$ from the Gell-Mann–Oakes–Renner relation, and $M_2^2 = (0.8 - 1.2)\,\text{GeV}^2$ from the QCD sum rules [19]. At the QCD side, the vacuum condensates are taken to be standard values, $\langle\bar{q}q\rangle = -(0.24 \pm 0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle = (0.8 \pm 0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle = m_0^2\langle\bar{s}s\rangle$, $m_0^2 = (0.8 \pm 0.1)\,\text{GeV}^2$ and $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\text{GeV})^4$ at the energy scale $\mu = 1$ GeV [22,23]. The quark condensates and mixed quark condensates evolve with the renormalization group equation, $\langle\bar{q}q\rangle(\mu) = \langle\bar{q}q\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}$ and $\langle\bar{q}g_s\sigma Gq\rangle(\mu) = \langle\bar{q}g_s\sigma Gq\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{27}}$, where $q = u, d, s$.

In this article, we take the $\overline{MS}$ masses $m_b(m_b) = (4.18 \pm 0.03)$ GeV and $m_s(\mu = 2\,\text{GeV}) = (0.095 \pm 0.005)$ GeV from the Particle Data Group [17], and we take into account the energy-scale dependence of the $\overline{MS}$ masses from the renormalization group equation,

$$\begin{aligned}
m_b(\mu) &= m_b(m_b)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_b)}\right]^{\frac{12}{23}},\\
m_s(\mu) &= m_s(2\text{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{4}{9}},\\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1 - \frac{b_1}{b_0^2}\frac{\log t}{t} + \frac{b_1^2(\log^2 t - \log t - 1) + b_0 b_2}{b_0^4 t^2}\right], \qquad (26)
\end{aligned}$$

where $t = \log\frac{\mu^2}{\Lambda^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$, $\Lambda = 213$ MeV, 296 MeV and 339 MeV for the flavors $n_f = 5$, 4, and 3, respectively [17]. Furthermore, we set the u and d quark masses to be zero. In the heavy quark limit, the b-quark can be taken as a static potential well, and unchanged in the decay $X(5568) \to B_s^0\pi^+$. In this article, we take the typical energy scale $\mu = m_b$.

The unknown parameter is chosen as $C_{XB_s} = -0.00059$ GeV^8. There appears a platform in the region $M_1^2 = (4.5 -$

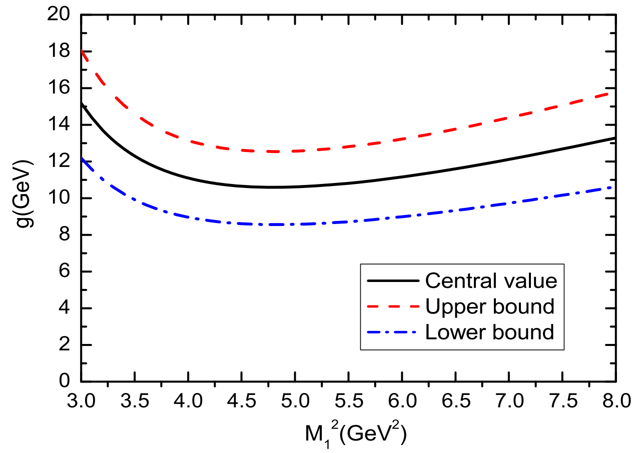

Fig. 2 The hadronic coupling constant $g_{XB_s\pi}$ with variation of the Borel parameter M_1^2

5.5) GeV^2. Now we take into account the uncertainties of the input parameters and obtain the value of the hadronic coupling constant $g_{XB_s\pi}$, which is shown explicitly in Fig. 2,

$$g_{XB_s\pi} = (10.6 \pm 2.1)\ \text{GeV}. \tag{27}$$

Now we obtain the partial decay width,

$$\begin{aligned}\Gamma\left(X(5568) \to B_s^0\pi^+\right) &= \frac{g^2_{XB_s\pi}}{16\pi M_X^3}\sqrt{\left[m_X^2-(m_{B_s}+m_\pi)^2\right]\left[m_X^2-(m_{B_s}-m_\pi)^2\right]} \\ &= (20.5 \pm 8.1)\ \text{MeV}.\end{aligned} \tag{28}$$

The decays $X(5568) \to B^+\bar{K}^0$ are kinematically forbidden, so the width Γ_X can be saturated by the partial decay width $\Gamma\left(X(5568) \to B_s^0\pi^+\right)$, which is consistent with the experimental value $\Gamma_X = 21.9 \pm 6.4^{+5.0}_{-2.5}$ MeV from the D0 collaboration [1]. The present work favors assigning the $X(5568)$ to be the scalar diquark–antidiquark type tetraquark state.

In the following, we perform a Fierz re-arrangement to the current J_X both in the color and Dirac-spinor spaces to obtain the result

$$\begin{aligned}J_X = \frac{1}{4}\Big\{ &-\bar{b}s\,\bar{d}u + \bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u - \bar{b}\gamma^\mu s\,\bar{d}\gamma_\mu u \\ &- \bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u + \frac{1}{2}\bar{b}\sigma_{\mu\nu}s\,\bar{d}\sigma^{\mu\nu}u \\ &+ \bar{b}u\,\bar{d}s - \bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s + \bar{b}\gamma^\mu u\,\bar{d}\gamma_\mu s \\ &+ \bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s \\ &- \frac{1}{2}\bar{b}\sigma_{\mu\nu}u\,\bar{d}\sigma^{\mu\nu}s \Big\};\end{aligned} \tag{29}$$

the components $\bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u$ and $\bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u$ couple potentially to the meson pair $B_s\pi^+$, while the components $\bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s$ and $\bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s$ couple potentially to the meson pair $B^+\bar{K}^0$. The strong decays

$$X(5568) \to B_s\pi^+ \tag{30}$$

are Okubo–Zweig–Iizuka super-allowed, while the decays

$$X(5568) \to B^+\bar{K}^0 \tag{31}$$

are kinematically forbidden, which is consistent with the observation of the D0 collaboration [1]. In previous work, we observed that the $C\gamma_5 \otimes \gamma_5 C$ type hidden-charm tetraquark states have slightly smaller masses than the $C\gamma_\mu \otimes \gamma^\mu C$ type hidden-charm tetraquark states; the predicted lowest masses are $m_{C\gamma_5\otimes\gamma_5 C} = \left(3.82^{+0.08}_{-0.08}\right)$ GeV and $m_{C\gamma_\mu\otimes\gamma^\mu C} = \left(3.85^{+0.15}_{-0.09}\right)$ GeV [24,25]. We expect that a $C\gamma_\mu \otimes \gamma^\mu C$ type current can also reproduce the experimental value $m_X = \left(5567.8 \pm 2.9^{+0.9}_{-1.9}\right)$ MeV approximately [3,8].

Now we construct the current η_X and perform a Fierz re-arrangement both in the color and Dirac-spinor spaces to obtain the following result:

$$\begin{aligned}\eta_X &= \epsilon^{ijk}\epsilon^{imn}u^j C\gamma_\mu s^k \bar{d}^m \gamma^\mu C\bar{b}^n, \\ &= \bar{b}s\,\bar{d}u + \bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u + \frac{1}{2}\bar{b}\gamma_\mu s\,\bar{d}\gamma^\mu u \\ &\quad - \frac{1}{2}\bar{b}\gamma_\mu\gamma_5 s\,\bar{d}\gamma^\mu\gamma_5 u \\ &\quad + \bar{b}u\,\bar{d}s + \bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s + \frac{1}{2}\bar{b}\gamma_\mu u\,\bar{d}\gamma^\mu s \\ &\quad - \frac{1}{2}\bar{b}\gamma_\mu\gamma_5 u\,\bar{d}\gamma^\mu\gamma_5 s,\end{aligned} \tag{32}$$

the components $\bar{b}i\gamma_5 s\,\bar{d}i\gamma_5 u$ and $\bar{b}\gamma^\mu\gamma_5 s\,\bar{d}\gamma_\mu\gamma_5 u$ couple potentially to the meson pair $B_s\pi^+$, while the components $\bar{b}i\gamma_5 u\,\bar{d}i\gamma_5 s$ and $\bar{b}\gamma^\mu\gamma_5 u\,\bar{d}\gamma_\mu\gamma_5 s$ couple potentially to the meson pair $B^+\bar{K}^0$, which is analogous to the current J_X. It is also sensible to assign the $X(5568)$ to be an axialvector-diquark–axialvector-antidiquark type tetraquark state or the $X(5568)$ has some axialvector-diquark–axialvector-antidiquark type tetraquark components.

The $C \otimes C$ type current $\widetilde{J}_X$ and $C\gamma_\mu\gamma_5 \otimes \gamma_5\gamma^\mu C$ type current $\widetilde{\eta}_X$ are expected to couple potentially to the scalar tetraquark with much larger masses,

$$\begin{aligned}\widetilde{J}_X &= \epsilon^{ijk}\epsilon^{imn}u^j C s^k \bar{d}^m C\bar{b}^n, \\ \widetilde{\eta}_X &= \epsilon^{ijk}\epsilon^{imn}u^j C\gamma_\mu\gamma_5 s^k \bar{d}^m \gamma_5\gamma^\mu C\bar{b}^n,\end{aligned} \tag{33}$$

as the favored configurations are the scalar diquarks ($C\gamma_5$-type) and axialvector diquarks ($C\gamma_\mu$-type) from the QCD sum rules [26–28].

4 Conclusion

In this article, we take $X(5568)$ to be the scalar diquark–antidiquark type tetraquark state, we study the hadronic coupling constant $g_{XB_s\pi}$ with the three-point QCD sum rules, then we calculate the partial decay width of the strong decay $X(5568) \to B_s^0\pi^+$ and obtain the value $\Gamma_X = (20.5 \pm 8.1)$ MeV, which is consistent with the experimental data, $\Gamma_X = \left(21.9 \pm 6.4^{+5.0}_{-2.5}\right)$ MeV, from the D0 collaboration. In the calculation, we carry out the operator product expansion up to the vacuum condensates of dimension-6, and we take into account both the connected and disconnected Feynman diagrams. The present prediction favors assigning the $X(5568)$ to the diquark–antidiquark type tetraquark state with $J^P = 0^+$. However, the quantum numbers $J^P = 1^+$ cannot be excluded according to the decays $X(5568) \to B_s^*\pi^+ \to B_s^0\pi^+\gamma$, where the low-energy photon is not detected.

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References

1. V.M. Abazov et al., arXiv:1602.07588
2. Z.G. Wang, arXiv:1602.08711
3. W. Chen, H.X. Chen, X. Liu, T.G. Steele, S.-L. Zhu, arXiv:1602.08916
4. C.M. Zanetti, M. Nielsen, K.P. Khemchandani, arXiv:1602.09041
5. W. Wang, R. Zhu, arXiv:1602.08806
6. Y.R. Liu, X. Liu, S.L. Zhu, arXiv:1603.01131
7. J.M. Dias, K.P. Khemchandani, A.M. Torres, M. Nielsen, C.M. Zanetti, arXiv:1603.02249
8. S.S. Agaev, K. Azizi, H. Sundu, arXiv:1602.08642
9. S.S. Agaev, K. Azizi, H. Sundu, arXiv:1603.00290
10. C.J. Xiao, D.Y. Chen, arXiv:1603.00228
11. X.H. Liu, G. Li, arXiv:1603.00708
12. B.L. Ioffe, A.V. Smilga, Nucl. Phys. B **232**, 109 (1984)
13. Z.G. Wang, W.M. Yang, S.L. Wan, Phys. Rev. D **72**, 034012 (2005)
14. Z.G. Wang, Int. J. Mod. Phys. A **30**, 1550168 (2015)
15. W. Chen, T.G. Steele, H.X. Chen, S.L. Zhu, Eur. Phys. J. C **75**, 358 (2015)
16. J.M. Dias, F.S. Navarra, M. Nielsen, C.M. Zanetti, Phys. Rev. D **88**, 016004 (2013)
17. K.A. Olive et al., Chin. Phys. C **38**, 090001 (2014)
18. V.M. Belyaev, V.M. Braun, A. Khodjamirian, R. Ruckl, Phys. Rev. D **51**, 6177 (1995)
19. P. Colangelo, A. Khodjamirian, arXiv:hep-ph/0010175
20. Z.G. Wang, JHEP **1310**, 208 (2013)
21. Z.G. Wang, Eur. Phys. J. C **75**, 427 (2015)
22. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B **147**(385), 448 (1979)
23. L.J. Reinders, H. Rubinstein, S. Yazaki, Phys. Rep. **127**, 1 (1985)
24. Z.G. Wang, Mod. Phys. Lett. A **29**, 1450207 (2014)
25. Z.G. Wang, Commun. Theor. Phys. **63**, 466 (2015)
26. Z.G. Wang, Eur. Phys. J. C **71**, 1524 (2011)
27. R.T. Kleiv, T.G. Steele, A. Zhang, Phys. Rev. D **87**, 125018 (2013)
28. Z.G. Wang, Commun. Theor. Phys. **59**, 451 (2013)