

Wormholes supported by phantom energy from Shan–Chen cosmological fluids

Deng Wang^{1,a}, Xin-He Meng^{2,3,b}

¹ Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin 300071, People's Republic of China

² Department of Physics, Nankai University, Tianjin 300071, People's Republic of China

³ State key Lab of Theoretical Physics, Institute of Theoretical Physics, CAS, Beijing 100080, People's Republic of China

Received: 26 November 2015 / Accepted: 14 March 2016 / Published online: 28 March 2016

© The Author(s) 2016. This article is published with open access at Springerlink.com

Abstract In the present paper, the exact solutions of spherically symmetrical Einstein field equations describing wormholes supported by phantom energy that violates the null energy condition from Shan–Chen background fluid are obtained. We have considered the important case of the model parameter $\psi \approx 1$, which corresponds to the “saturation effect”, and this regime corresponds to an effective form of “asymptotic freedom” for the fluids, but occurring at cosmological rather than subnuclear scales. Then we investigate the allowed range for the values of the model parameters g and ω when the spacetime metrics describe wormholes and discuss the possible singularities of the solutions, finding that the obtained spacetimes are geodesically complete. Furthermore, we construct two traversable wormholes through matching our obtained interior solutions to the exterior Schwarzschild solutions and analyze the traversabilities of the wormholes. Finally, we consider the case of anisotropic pressure and discover that the transverse pressure also crosses the phantom divide -1 with the growth of the wormhole dimension, and it tends to be the same as the radial pressure with the growth of the wormhole radius.

1 Introduction

Nowadays, all existing observational data are in good agreement with the simplest picture of a six parameter based cosmology, namely, the Λ CDM model, favorably leaving a viable dark-energy component to enrich the cosmological constant effects. According to this model, the universe is described well by a Friedmann–Robertson–Walker (FRW) metric, whose gravity source is a mixture of non-interacting perfect fluids including a cosmological constant with the

main dark matter component. However, no theoretical model able to precisely determine the nature of dark energy is available at present. In recent years, besides modified gravities there have been proposed a variety of models for dark energy, which include, for example, the quintessence model [1,2], scalar field models with nonstandard kinetic terms (k-essence) [3], Chaplygin gas models [4], braneworld models [5,6], dark fluid models [7–12], and cosmological models from scalar–tensor theories of gravity (see, e.g., Refs. [13,14] and references therein). Unfortunately, these models are all described mainly by a similar behavior of the equation of state. Nevertheless, in 1993, a new modified equation of state (EoS) was first proposed by Shan and Chen (SC) in the context of lattice kinetic theory [15] for describing a complex fluid, with the primary intent that repulsion is replaced by a density-dependent attraction. Then 10 years later, Donato Bini et al. have studied the dark-energy properties from cosmological fluids obeying the SC non-ideal equation of state [16]. The main idea is to postulate that the cosmological fluids obey an SC equation with “asymptotic freedom” regimes, namely, ideal gas behaviors at both high and low density regimes, with a liquid–gas coexistence cycle. Through some numerical calculations, they found that in the cosmological Friedmann–Robertson–Walker (FRW) framework a cosmic fluid obeying the SC equation of state naturally evolves toward the present-day universe state with a suitable dark-energy component, with no need of invoking any cosmological constant. Therefore, we are very interested in exploring the SC EoS effects further in the astrophysics scales if the dark-energy phenomenon is universal, especially the possible wormholes supported by this new cosmological fluid, as the phantom dark energy has violated the null energy condition, which is essential for wormhole formation. Another motivation is that two recent studies [17,18] have demonstrated the possible existence of wormholes both in the outer regions of the galactic halo and in the central parts of the halo, respec-

^a e-mail: Cstar@mail.nankai.edu.cn

^b e-mail: xhm@nankai.edu.cn

tively, based on the Navarro–Frenk–White (NFW) density profile and the universal rotation curves (URC) dark matter model simulation and fittings [19,20]. Especially, the second result is an important complement to the earlier results as regards theoretical discussions of possible wormhole properties, thereby inspiring common interests in exploring the possible existence of wormholes in most of the spiral galaxies with a universally distributed dark energy.

Wormholes are a possibly amazing result in the Einstein gravity theory of general relativity (GR), since they can provide an alternative method for rapid interstellar travel with novel properties, which may be defined briefly as tunnels in the spacetime topology connecting different universes or widely separated regions of our universe via a throat [21,22]. Recently, the increasing attention to the investigation of such objects is due in some sense to the discovery that our universe is undergoing an accelerated expansion [23,24], coined the ‘dark-energy phenomenon’. Since the two cases (wormholes and accelerated expansion universe) are both in violation of a null energy condition explicitly (the null energy condition requires the stress-energy tensor $T_{\alpha\beta}k^\alpha k^\beta \geq 0$ for all null vectors), $p + \rho < 0$, and consequently other energy conditions. Therefore, an interesting and implying overlap has appeared between the two seemingly separated subjects. More precisely, the accelerated universe expansion can be described globally by the GR strictly derived Friedmann equation $\ddot{a}/a = -\frac{4\pi}{3}(\rho + 3p)$, i.e., we require $\ddot{a} > 0$. (Here and throughout the article we take units $G = c = 1$.) It is obvious from previous work [25–30] that we know the cosmic acceleration is caused mathematically by a hypothetical negative pressure dark energy with the energy density $\rho > 0$ and $p = \omega\rho$ with the EoS parameter $\omega < -\frac{1}{3}$. An important feature of dark-energy models is the phantom dark energy when the EoS parameter $\omega < -1$, for which the same violation of the null energy condition as for the wormhole case occurs. Thus, we can explore the wormholes supported by phantom energy through studying the situation for $\omega < -1$. Additionally, in the quintessence models the parameter range is $-1 < \omega < -1/3$, and recent astrophysics observations mildly favor a phantom dark-energy scenario. Moreover, the case $\omega = -1$ corresponds to a cosmological constant, and $\omega = -2/3$ is extensively analyzed in the nice work of [31].

A recurring problem in general relativity is to find the exact solutions to the Einstein field equations [32–34]. In this paper, provided that the universe is permeated by a dark-energy fluid, therefore, besides the cosmic scales we should also investigate the astrophysical scale properties of dark energy. So we plan to explore the phantom energy wormholes from SC fluids with considering Shan–Chen’s interesting and intriguing EoS work applying it to the astrophysics scales.

The current work is organized as follows. In the next section, we review some properties of the cosmic wormhole

model and make a brief introduction to the SC equation of state. Moreover, we derive the field equation of the important case for the parameter $\psi = 1$ starting from a general line element and the SC equation of state. In the sub-sections of Sect. 3, we make two special choices for the redshift function: $\Phi = C$ and $\Phi(r) = \frac{1}{2} \ln(r_1/r)$, respectively, and we obtain the corresponding two solutions. We analyze the singularities and construct two traversable wormholes through matching the interior solutions to the exterior Schwarzschild solutions. Moreover, we calculate the total mass of the wormhole when $r \leq a$ or $r \leq b$, and we find that the surface stress-energy σ is zero and the surface tangential pressure \wp is positive when discussing the surface stresses of the solutions. Subsequently, we analyze the traversabilities of the wormholes and take into account the case of an anisotropic pressure. In the final section, we draw some conclusions and present discussions as regards our work and point out possible work in the future.

2 The problem background

The metric of the normal wormhole system can be generally written as

$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\alpha(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where r is the radial coordinate which runs in the range $r_0 \leq r < \infty$; $0 \leq \theta \leq \pi$, and $0 \leq \phi \leq 2\pi$ are the angular coordinates. $\Phi(r)$ is the redshift function, for it is related to the gravitational redshift. The function $\alpha(r)$ has a vertical asymptote at the throat $r = r_0$:

$$\lim_{r \rightarrow (r_0)^+} \alpha(r) = +\infty. \quad (2)$$

Its relationship to the shape function $b(r)$ is

$$e^{2\alpha(r)} = \frac{1}{1 - \frac{b(r)}{r}}. \quad (3)$$

It follows that

$$b(r) = r(1 - e^{-2\alpha(r)}). \quad (4)$$

To describe a wormhole, the metric should obey some conditions [21]. The metric coefficient $e^{2\Phi(r)}$ should be finite and non-vanishing in the vicinity of r_0 , and $b(r)$ should satisfy the following relations:

$$b(r_0) = r_0, \quad (5)$$

$$b'(r_0) < 1, \quad (6)$$

$$b(r) < r, r > r_0. \quad (7)$$

Using the Einstein field equation, $G_{\alpha\beta} = 8\pi T_{\alpha\beta}$, in an orthonormal reference frame, we have the following stress-energy scenario:

$$G_{tt} = 8\pi T_{tt} = 8\pi\rho = \frac{2}{r}e^{-2\alpha(r)}\alpha'(r) + \frac{1}{r^2}(1 - e^{-2\alpha(r)}), \tag{8}$$

$$G_{rr} = 8\pi T_{rr} = 8\pi p_r = \frac{2}{r}e^{-2\alpha(r)}\Phi'(r) - \frac{1}{r^2}(1 - e^{-2\alpha(r)}), \tag{9}$$

$$G_{\theta\theta} = G_{\phi\phi} = 8\pi p_t = e^{-2\alpha(r)} \times \left\{ \Phi''(r) - \Phi'(r)\alpha'(r) + [\Phi'(r)]^2 + \frac{\Phi'(r)}{r} - \frac{\alpha'(r)}{r} \right\}, \tag{10}$$

where p_r denotes the radial pressure and p_t the transverse pressure. Furthermore, it is easy to derive from the conservation law of the stress-energy tensor $T_{\mu;\nu} = 0$ with $\mu = r$ that

$$p'_r = \frac{2}{r}(p_t - p_r) - (\rho + p_r)\Phi'. \tag{11}$$

Then using the SC equation

$$p_r = \omega\rho_{(\text{crit}),0} \left[\frac{\rho}{\rho_{(\text{crit}),0}} + \frac{g}{2}\psi^2 \right], \tag{12}$$

$$\psi = 1 - e^{-\beta\frac{\rho}{\rho_{(\text{crit}),0}}}, \tag{13}$$

where $\rho_{(\text{crit}),0} = 3(H_0)^2/8\pi$ is the present value of the critical density (H_0 denoting the Hubble constant) and the dimensionless quantities $\omega, g \leq 0$, and $\beta \geq 0$ can be regarded as free parameters of the model.

Substitution yields

$$\omega\alpha'(r) + \frac{\omega + 1}{2r}(e^{2\alpha(r)} - 1) + 2\pi\omega\rho_{(\text{crit}),0}ge^{2\alpha(r)} \times \left[1 - e^{-\beta\frac{2}{r}e^{-2\alpha(r)}\alpha'(r) + \frac{1}{r^2}(1 - e^{-2\alpha(r)})}{8\pi\rho_{(\text{crit}),0}} \right]^2 = \Phi'(r). \tag{14}$$

We find two solutions when $\rho \gg \rho_*$, $\rho_* = \rho_{(\text{crit}),0}/\beta$ (ρ_* being the typical density above which ψ undergoes a ‘‘saturation effect’’), i.e., $\psi \approx 1$ [16]; this regime corresponds to an effective form of asymptotic freedom. So the SC equation of state (11) becomes

$$p_r = \omega\rho_{(\text{crit}),0} \left[\frac{\rho}{\rho_{(\text{crit}),0}} + \frac{g}{2} \right]. \tag{15}$$

Since the pressure p is negative, $-\rho < \frac{g}{2}\rho_{(\text{crit}),0} < 0$, so $\omega < -\frac{\rho}{\rho + \frac{g}{2}\rho_{(\text{crit}),0}} < -1$ corresponds to a SC version of phantom energy. Moreover, it is not difficult to derive that the

SC fluid corresponds to a quintom-like universe. It follows that

$$\omega\alpha'(r) + \frac{\omega + 1}{2r}(e^{2\alpha(r)} - 1) + 2\pi\omega\rho_{(\text{crit}),0}ge^{2\alpha(r)} = \Phi'(r). \tag{16}$$

This equation tells us the close relationship between $\Phi'(r)$ and $\alpha'(r)$ and hence between $\Phi(r)$ and $\alpha(r)$. It is easy to see when $\beta = 0$ or $g = 0$ that the equation of state will reduce to $p = \omega\rho$.

3 Exact solutions for wormholes

3.1 Special choice for the redshift function: $\Phi = C$

We have the first solution through inserting the redshift function by hand $\Phi'(r) = 0$, resulting in $\Phi = \text{constant} = C$ [35], then Eq. (16) becomes

$$\omega\alpha'(r) + \frac{\omega + 1}{2r}(e^{2\alpha(r)} - 1) + 2\pi\omega\rho_{(\text{crit}),0}ge^{2\alpha(r)} = 0. \tag{17}$$

We have the initial condition $\alpha(r_0) = +\infty$. The solution is

$$e^{2\alpha(r)} = \frac{1}{r^{-1-\frac{1}{\omega}} \left[r^{1+\frac{1}{\omega}} - r_0^{1+\frac{1}{\omega}} + 4\pi g\rho_{(\text{crit}),0} \frac{\omega(r^{2+\frac{1}{\omega}} - r_0^{2+\frac{1}{\omega}})}{1+2\omega} \right]}. \tag{18}$$

So, the line element is

$$ds^2 = -e^{2C}dt^2 + \frac{1}{r^{-1-\frac{1}{\omega}} \left[r^{1+\frac{1}{\omega}} - r_0^{1+\frac{1}{\omega}} + 4\pi g\rho_{(\text{crit}),0} \frac{\omega(r^{2+\frac{1}{\omega}} - r_0^{2+\frac{1}{\omega}})}{1+2\omega} \right]} \times dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{19}$$

Since $\Phi = C$, we have $e^{2\Phi} \neq 0$ and the space-time avoids an event horizon. It is easily checked that $b(r_0) = r_0$, and if we want the shape function to satisfy $b'(r_0) < 1$ and $b(r) < r$, the parameters g and ω must satisfy one relation:

$$\frac{-1 - \frac{1}{\omega}}{4\pi\rho_{(\text{crit}),0}r_0} < g \leq 0. \tag{20}$$

Equation (19) is obtained by the simplest condition that the function $f(r)$ obeys

$$f(r) = r - b(r) = r^{-\frac{1}{\omega}} \times \left[r^{1+\frac{1}{\omega}} - r_0^{1+\frac{1}{\omega}} + 4\pi g\rho_{(\text{crit}),0} \frac{\omega(r^{2+\frac{1}{\omega}} - r_0^{2+\frac{1}{\omega}})}{1+2\omega} \right]. \tag{21}$$

$(f(r_0) = 0)$ is monotonically increasing in the range $r > r_0$ and $b'(r_0) = -\frac{1}{\omega} - 4\pi g\rho_{(\text{crit}),0}r_0 < 1$. (We also observe that if $\omega > -1$ the flare-out condition is no longer satisfied.)

Also clearly the metric is not asymptotically flat. However, it can be glued to the external Schwarzschild solution

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{22}$$

To match the interior to the exterior, one needs to apply the junction conditions that follow the theory of general relativity. If there is no surface stress-energy term at the surface S , the junction is called a boundary surface. If, on the other hand, surface stress-energy terms are present, the junction is called a thin shell.

A wormhole with finite dimensions, in which the matter distribution extends from the throat, $r = r_0$, to a finite distance $r = a$, obeys the condition that the metric is continuous. Due to the spherical symmetry the components $g_{\theta\theta}$ and $g_{\phi\phi}$ are already continuous, so one needs to impose continuity only on the remaining components at $r = a$:

$$g_{tt(\text{int})}(a) = g_{tt(\text{ext})}(a), \tag{23}$$

$$g_{rr(\text{int})}(a) = g_{rr(\text{ext})}(a). \tag{24}$$

Equations (22) and (23) are for the interior and exterior components, respectively. These requirements, in turn, lead to

$$\Phi_{\text{int}}(a) = \Phi_{\text{ext}}(a), \tag{25}$$

$$b_{\text{int}}(a) = b_{\text{ext}}(a). \tag{26}$$

Particularly,

$$e^{2\alpha(a)} = \frac{1}{1 - \frac{b(a)}{a}} = \frac{1}{1 - \frac{2M}{a}}. \tag{27}$$

So, one can deduce the mass of the wormhole to be given by

$$M = \frac{1}{2}b(a) = \frac{r_0}{2} \left(\frac{r_0}{a}\right)^{\frac{1}{\omega}} - 2\pi g\rho_{(\text{crit}),0} \frac{\omega}{1 + 2\omega} \left[a^2 - (r_0)^2 \left(\frac{r_0}{a}\right)^{1+\frac{1}{\omega}} \right]. \tag{28}$$

Returning to $\Phi = C$, we now have

$$C' = e^{2\Phi} = 1 - \left(\frac{r_0}{a}\right)^{1+\frac{1}{\omega}} + 4\pi g\rho_{(\text{crit}),0} \frac{\omega}{1 + 2\omega} \left[a - r_0 \left(\frac{r_0}{a}\right)^{1+\frac{1}{\omega}} \right]. \tag{29}$$

Thus, the line element becomes

$$ds^2 = -C' dt^2 + \frac{1}{V(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{30}$$

$$V(r) = r^{-1-\frac{1}{\omega}}$$

$$\times \left[r^{1+\frac{1}{\omega}} - r_0^{1+\frac{1}{\omega}} + 4\pi g\rho_{(\text{crit}),0} \frac{\omega(r^{2+\frac{1}{\omega}} - r_0^{2+\frac{1}{\omega}})}{1 + 2\omega} \right]. \tag{31}$$

Our next step is to take into account the surface stresses. Using the Darmois–Israel formalism [36,37], the surface stresses are given by

$$\sigma = -\frac{1}{4\pi a} \left(\sqrt{1 - \frac{2M}{a}} - \sqrt{1 - \frac{b(a)}{a}} \right) \tag{32}$$

and

$$\wp = \frac{1}{8\pi a} \left(\frac{1 - \frac{M}{a}}{\sqrt{1 - \frac{2M}{a}}} - [1 + a\Phi'(a)]\sqrt{1 - \frac{b(a)}{a}} \right). \tag{33}$$

It is thus clear that the surface stress-energy σ is zero and the surface tangential pressure \wp is positive.

Another important consideration is the singularities of the solution. To see this, we can deduce that $V(r) = 0$ when $r = r_0$, but it is not a physical singularity. Then let us consider a free particle with an energy $E = -u_0$ and angular momentum $L = u_\phi$ (in dimensionless units) moving in Eq. (16), u^μ being the 4-velocity. As usual, we choose the plane to be $\theta = \pi/2$, so one can easily obtain from the condition $g_{\mu\nu}u^\mu u^\nu = \varepsilon$ ($\varepsilon = 0$ for massless particles and $\varepsilon = -1$ for massive particles) that

$$(u^r)^2 = V(r) \left(\frac{E^2}{-e^{2C}} - \frac{L^2}{r^2} + \varepsilon \right). \tag{34}$$

Here we just consider radial null geodesics ($L = 0, \varepsilon = 0$), and let $E = 1, \omega = -11/10, e^{2C} = 1, r_0 = 1, g = -7/1000, \rho_{(\text{crit}),0} = 1$. (Here we are just to demonstrate that the proper time is finite at the throat through taking these values which may be not true values for the parameters and quantities; for references, see [38,39].) Hence, the above equation can be rewritten as

$$\left(\frac{dr}{d\tau}\right)^2 = -V(r), \tag{35}$$

τ is the proper time; we solve this equation through some numerical calculations,

$r = \text{InverseFunction}$

$$\times \left[\int_1^m \frac{1}{330 - \frac{330}{n^{11}} + \frac{7\pi}{n^{11}} - 7\pi n} dn \right] \times \left[\pm \frac{\tau}{\sqrt{330}} + D \right], \tag{36}$$

where m, n , and D are the upper limit of integration, the variable of integration, and an integration constant, respectively. Obviously, τ is always finite when $r = r_0 = 1$ is not a

physical singularity. Thus, one can easily see that the metric is geodesically complete.

After matching our interior solution to the exterior Schwarzschild solution, we have constructed a traversable wormhole. So an important consideration affecting the traversability is the proper distance $l(r)$ from the throat to a point away from the throat:

$$l(r) = \int_{r_0}^r \frac{1}{\sqrt{V(r)}} dr, \tag{37}$$

which is finite through some simple numerical calculations.

Another consideration is the time dilation near the throat. Here we let $v = dl/d\tau$ in order that $d\tau = dl/v$ (assuming that the observer who will pass through the wormhole in the spaceship has a non-relativistic speed, i.e. $\gamma = \sqrt{1 - (v/c)^2} \approx 1$). Because $dl = e^{\alpha(r)} dr$ and $d\tau = e^{\Phi(r)} dr$, we have for any coordinate interval Δt

$$\Delta t = \int_{l_a}^{l_b} e^{-\Phi(r)} \frac{dl}{v} = \int_{r_a}^{r_b} \frac{1}{v} e^{-\Phi(r)} e^{\alpha(r)} dr. \tag{38}$$

Going from the throat to r , we get

$$\Delta t = \int_{r_0}^r \frac{1}{v} \frac{1}{\sqrt{C'V(r)}} dr. \tag{39}$$

Through some numerical calculations in which we make some appropriate choices for the parameters ω , r_0 , and g , we find that Δt also behaves well near the throat. For example, we get $l(1.1) = 4.90$ km, $l(1.3) = 9.50$ km and $l(1.5) = 15.17$ km, according to these values of $l(r)$, we also find that $\Delta t = 2.04 \times 10^{-5}$ s, 3.90×10^{-5} s and 6.32×10^{-5} s, respectively (here we still use the above assumed values of parameters, and let $r_0=1$ km, $\Phi = C = 0$, $v = 200$ m/s). Thus, we can discover that the smaller $l(r)$ is, the shorter the time for the comoving observers to pass through the wormhole is, and $l(r)$ increases very fast with the growth of the throat radius r_0 .

3.2 Special choice for the redshift function:

$$\Phi(r) = \frac{1}{2} \ln(r_1/r)$$

Another possibility is $\Phi(r) = \frac{1}{2} \ln(r_1/r)$ [39], for some constant r_1 ; after substitution into Eq. (16) we have

$$\omega\alpha'(r) + \frac{\omega + 1}{2r} (e^{2\alpha(r)} - 1) + 2\pi\omega\rho_{(\text{crit}),0} g e^{2\alpha(r)} = \frac{1}{2r}. \tag{40}$$

Here we still consider the important case $\psi \approx 1$, since the quantity ψ can be interpreted as the density of a chameleon scalar field [40,41], which is used for reconciling large coupling models with local gravity constraints. It follows that

$$e^{2\alpha(r)} = \frac{1}{r^{-1-\frac{2}{\omega}} \left[\frac{(1+\omega)(r^{1+\frac{2}{\omega}} - r_0^{1+\frac{2}{\omega}})}{2+\omega} + 2\pi g \rho_{(\text{crit}),0} (2+\omega) \frac{\omega(r^{2+\frac{2}{\omega}} - r_0^{2+\frac{2}{\omega}})}{(1+\omega)(1+2\omega)} \right]}. \tag{41}$$

So the line element becomes

$$ds^2 = -\frac{r_1}{r} dt^2 + \frac{1}{r^{-1-\frac{2}{\omega}} \left[\frac{(1+\omega)(r^{1+\frac{2}{\omega}} - r_0^{1+\frac{2}{\omega}})}{2+\omega} + 2\pi g \rho_{(\text{crit}),0} (2+\omega) \frac{\omega(r^{2+\frac{2}{\omega}} - r_0^{2+\frac{2}{\omega}})}{(1+\omega)(1+2\omega)} \right]} \times dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{42}$$

Here we have

$$b'(r_0) = -\frac{1}{\omega} - 4\pi g \rho_{(\text{crit}),0} \frac{(2+\omega)r_0}{1+2\omega} < 1, \tag{43}$$

and assuming the simplest condition that

$$f(r) = r - b(r) = r - \frac{1+\omega}{2+\omega} r + \frac{1+\omega}{2+\omega} r^{-\frac{2}{\omega}-1} r_0^{\frac{2}{\omega}+1} - 2\pi g \rho_{(\text{crit}),0} (2+\omega) \frac{2\omega r + 2r^{-\frac{2}{\omega}-1} r_0^{\frac{2}{\omega}+2}}{(1+\omega)(1+2\omega)} \tag{44}$$

is monotonically increasing in the range $r > r_0$ and $f(r_0) = 0$. Hence, we have the relation

$$\frac{-(1+\omega)(1+2\omega)}{4\pi\rho_{(\text{crit}),0}r_0\omega(2+\omega)} < g \leq 0, \omega < -2. \tag{45}$$

After checking, which is easy, we see that the metric satisfies Eqs. (5), (6), and (7) and is also not asymptotically flat. Besides, we have

$$r = \text{InverseFunction} \left[\int_1^m \frac{1}{\sqrt{2000n - 2000n^{\frac{20}{11}} - 2079\pi(n^{\frac{20}{11}} + n^2)}} dn \right] \times \left[\pm \frac{\tau}{60\sqrt{5}} + D' \right] \tag{46}$$

where m , n , and D' are the upper limit of integration, the variable of integration, and an integration constant, respectively. Similarly, here we still consider radial null geodesics ($L = 0$, $\varepsilon = 0$), and let $E = 1$, $\omega = -11/10$, $e^{2C} = 1$, $r_0 = 1$, $g = -7/1000$, $\rho_{(\text{crit}),0} = 1$; we find that the spacetime of the solution is also geodesically complete.

Once again, the metric Eq. (41) can be glued to the Schwarzschild region at $r = b$. Using Eq. (26), we have the mass of the wormhole

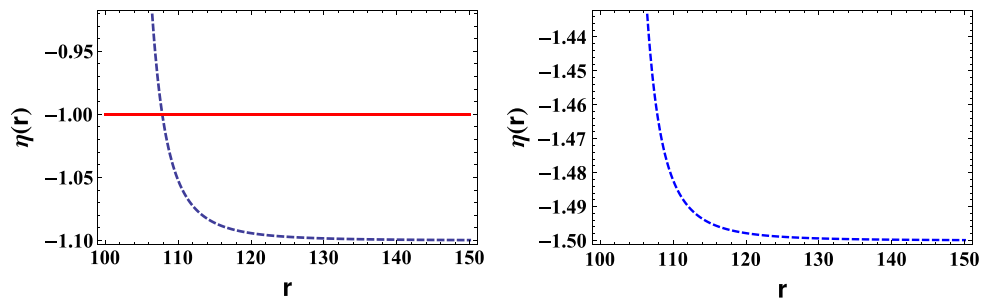


Fig. 1 The relation between the coefficient of the transverse pressure $\eta(r)$ and the radial coordinate r . The dashed (blue) line and the solid (red) line correspond to $\eta(r)$ and the Λ CDM model, respectively. In

the left panel, we take $r_0 = 100$, $g = -1$, $\rho_{(\text{crit}),0} = 6.78 \times 10^{-74}$ and $\omega = -1.1$. In the right panel, we take $\omega = -1.5$

$$M = \frac{b + (1 + \omega)\left(\frac{r_0}{b}\right)^{\frac{2}{\omega}} r_0}{2(2 + \omega)} - \frac{(2 + \omega)\omega \left[b^2 - \left(\frac{r_0}{b}\right)^{\frac{2}{\omega}} r_0^2 \right]}{\pi g \rho_{(\text{crit}),0} (1 + \omega)(1 + 2\omega)}. \tag{47}$$

Returning to $\Phi(r) = \frac{1}{2} \ln(r_1/r)$, we have

$$r_1 = b - 2M = \frac{(1 + \omega) \left[1 - \left(\frac{r_0}{b}\right)^{\frac{2}{\omega}} r_0 \right]}{2 + \omega} + \frac{2\pi g \rho_{(\text{crit}),0} \left[b^2 - \left(\frac{r_0}{b}\right)^{\frac{2}{\omega}} r_0^2 \right]}{(1 + \omega)(1 + 2\omega)}. \tag{48}$$

Here letting $C_1 = r_1/r$, the line element becomes

$$ds^2 = -C_1 dt^2 + \frac{1}{r^{-1-\frac{2}{\omega}} \left[\frac{(1+\omega)(r^{1+\frac{2}{\omega}} - r_0^{1+\frac{2}{\omega}})}{2+\omega} + 2\pi g \rho_{(\text{crit}),0} (2 + \omega) \frac{\omega(r^{2+\frac{2}{\omega}} - r_0^{2+\frac{2}{\omega}})}{(1+\omega)(1+2\omega)} \right]} \times dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \tag{49}$$

Then taking into account the surface stresses in the forms of Eqs. (30)–(31) again, the same conclusion, that the surface stress-energy σ is zero and the surface tangential pressure \wp is positive, is obtained. Similarly, doing the same to the metric as the first, we can also find that the wormhole is traversable and the proper distance $l(r)$ is finite; the coordinate interval Δt , as is clear through some numerical calculations, is well behaved near the throat.

3.3 Anisotropic pressure

In this section, we will consider the case of anisotropic pressure $p_t = \eta\rho$, which is based on Ref. [39]. For simplicity, we just take into account the specific redshift function $\Phi = C$. Subsequently, replacing Eqs. (8), (15), and (18) in Eq. (11), we can easily obtain the following relationship:

$$\eta(r) = \omega + \frac{1}{16\pi r^2} \{ (r^3 (r_0^{\frac{1}{\omega}+1} r^{\frac{2}{\omega}+2} \times (\omega(4\pi g r_0 \rho_{(\text{crit}),0} + 2) + 1) \times (\omega(96\pi^2 g^2 \rho_{(\text{crit}),0}^2 r^2 \omega^3 + 4\pi g \rho_{(\text{crit}),0} r (2\omega + 1) \times (\omega(2\omega - 7) - 1) - 1))) + 2\omega^2 r_0^{\frac{3}{\omega}+3} \times (\omega(4\pi g r_0 \rho_{(\text{crit}),0} + 2) + 1)^3 - r_0^{\frac{2}{\omega}+2} r^{\frac{1}{\omega}+1} (\omega(4\pi g r_0 \rho_{(\text{crit}),0} + 2) + 1)^2 \times (\omega(8\omega^2 (3\pi g \rho_{(\text{crit}),0} r + 1) - 8\pi g \omega^3 r^{\frac{3}{\omega}+4} \times (4\omega(\pi g \rho_{(\text{crit}),0} r + 6) + 3) + \omega + 1) + 1)) / \times (\omega(r \frac{1}{\omega} + 2(\omega(4\pi g \rho_{(\text{crit}),0} r + 2) + 1) - r r_0^{\frac{1}{\omega}+1} \omega(4\pi g \rho_{(\text{crit}),0} r_0 + 2) + 1))^3 + g\rho / (r_0^{\frac{1}{\omega}+1} (\omega(4\pi g \rho_{(\text{crit}),0} r_0 + 2) + 1) - r_0^{\frac{1}{\omega}+1} (\omega(4\pi g \rho_{(\text{crit}),0} r + 2) + 1))^2 (\omega r_0^{\frac{2}{\omega}+2} \times (\omega(4\pi g \rho_{(\text{crit}),0} r_0 + 2) + 1)^2 - r_0^{\frac{1}{\omega}+1} r^{\frac{1}{\omega}+1} (\omega(4\pi g r_0 \rho_{(\text{crit}),0} + 2) + 1) \times (8\pi g \rho_{(\text{crit}),0} r \omega^2 - 2\omega - 1) + 8\pi g \rho_{(\text{crit}),0} \omega^2 r^{\frac{2}{\omega}+3} \times (2\omega(\pi g \rho_{(\text{crit}),0} r + 1) + 1)) \}. \tag{50}$$

One can find that η is a function of the radial coordinate r in this case, differing from that in [39]. Furthermore, we obtain the most interesting result in this study that the transverse pressure also crosses the phantom divide $\omega = -1$ with the growth of the wormhole dimension, and this tends to be the same as the radial pressure with the growth of the wormhole radius (see Fig. 1). More interestingly, the aforementioned result implies that, if the lateral pressure p_t also violates the NEC, one will get a novel wormhole supported by more exotic matter, which consists of the radial NEC violating SC fluid and the transverse NEC violating perfect fluid. Hence, the dimension of the SC fluid is $r_0 < r < a$ and the dimension of the lateral perfect fluid is $b < r < a$, where a denotes the junction radius and b the location where $\eta(b) = -1$. In addition, it is worth noting that the model parameter g contributes little to the radial pressure and transverse pressure, since the value of the second term in Eq. (15), including $\rho_{(\text{crit}),0}$, is substantially small. At the same time, one can

discover that the SC cosmology is very similar with the viscous ω CDM cosmology, namely, one can reconstruct the SC universe by a viscous ω CDM equation of state.

4 Conclusion and discussions

To summarize, the explanation of the cosmic acceleration expansion requires the introduction of either a cosmological constant, or of a mysterious component so-called dark energy, filling the universe and dominating its expansionary evolution currently. Given that the universe is smoothly permeated by a dark-energy fluid, therefore, we should also investigate the astrophysical scale properties of dark energy. In the first place, we have considered the important case $\psi \approx 1$ in the Shan–Chen cosmological model, which corresponds to the “saturation effect”, and this regime corresponds to an effective form of the “asymptotic freedom” in the fluids, occurring at cosmological rather than subnuclear scales. From this we find two simple solutions of spherically symmetrical Einstein equations with the SC equation of state describing a wormhole. Then we explore the value ranges of the parameters g and ω when the spacetime metrics describe wormholes and discuss the singularities of the solutions, and we find that the spacetimes of the two new solutions are both geodesically complete. Furthermore, we construct two traversable wormholes through matching our interior solutions to the exterior Schwarzschild solutions and calculate the total mass of the wormhole when $r \leq a$ or $r \leq b$, respectively. Subsequently, we take into account the case of the anisotropic pressure, and we find the most interesting result in this study that the transverse pressure also exhibits a quintom-like behavior with the growth of the wormhole dimension, and it tends to approach the radial pressure with the increasing of the wormhole radius. Finally, we see that the surface stress-energy σ is zero and the surface tangential pressure \wp is positive when discussing the surface stresses of the solutions and we analyze the traversabilities of the wormholes.

The centennially celebrated GR is still full of puzzling implications, especially when confronting the dark matter and dark-energy mysteries in various scales if the dark sectors are universal. Wormholes are theoretically objects in the universe which now appear to attract more observational astrophysics interests and may provide a new window for new physics. Further work could be to explore the wormhole dynamics relations with the analogous studies of black holes by considering an obvious sensible relation between a transverse pressure and the energy density and to make some special choices for the shape function $b(r)$ as well. At the same time, it is worth to analyze the geodesic motion in the wormhole spacetime from SC cosmological fluids in detail. In addition, one can also try to reconstruct viscous cosmology by the SC cosmological fluid.

Acknowledgments For helpful discussions and comments, we thank Prof. Saibal Ray and Jingling Chen, Qixiang Zou, Guang Yang, Shengsen Lu, and Liwei Yu. This work is partly supported by the National Science Foundation of China.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. M.S. Turner, [arXiv:astro-ph/0108103](https://arxiv.org/abs/astro-ph/0108103)
2. R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003)
3. C. Armendariz-Picon, V. Mukhanov, P.J. Steinhardt, *Phys. Rev. Lett.* **85**, 4438 (2000)
4. M.C. Bento, O. Bertlami, A.A. Sen, *Phys. Rev. D* **66**, 043507 (2002)
5. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999)
6. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 4690 (1999)
7. X.-H. Meng, J. Ren, M.G. Hu, *Commun. Theor. Phys.* **47**, 379 (2007). [arXiv:astro-ph/0509250](https://arxiv.org/abs/astro-ph/0509250)
8. J. Ren, X.-H. Meng, *Phys. Lett. B* **636**, 5 (2006)
9. J. Ren, X.-H. Meng, *Phys. Lett. B* **633**, 1 (2006)
10. M.G. Hu, X.-H. Meng, *Phys. Lett. B* **635**, 186 (2006)
11. X.-H. Meng, X. Dou, *Commun. Theor. Phys.* **52**, 377 (2009)
12. Xu Dou, X.-H. Meng, *Adv. Astron.* **2011**, 829340 (2011)
13. V. Sahni, A.A. Starobinsky, *Int. J. Mod. Phys. D* **15**, 2105 (2006)
14. P. Ruiz-Lapuente, *Class. Quantum Grav.* **24**, R91 (2007)
15. X. Shan, H. Chen, *Phys. Rev. E* **47**, 1815 (1993)
16. D. Bini et al., *Phys. Rev. D* **88**, 063007 (2013)
17. F. Rahaman, P.K.F. Kuhfittig, S. Ray, N. Islam, *Eur. Phys. J. C* **74**, 2750 (2014)
18. P.K.F. Kuhfittig, *Eur. Phys. J. C* **462**, 2818 (2014)
19. F. Rahaman et al., *Ann. Phys.* **350**, 561–567 (2014)
20. J.F. Navarro, C.S. Frenk, S.D.M. White, *Astrophys. J.* **462**, 563 (1996)
21. M.S. Moris, K.S. Thorne, *Am. J. Phys.* **56**, 395 (1988)
22. M. Visser, *Lorentzian Wormholes: From Einstein to Hawking* (Springer, Berlin, 1997)
23. A.G. Riess et al., *Astro. J.* **116**, 1009 (1998)
24. S.J. Perlmutter et al., *Astro. J.* **517**, 565 (1999)
25. X.-H. Meng, P. Wang, *Class. Quantum Grav.* **20**, 4949 (2003)
26. X.-H. Meng, P. Wang, *Class. Quantum Grav.* **21**, 951 (2004)
27. X.-H. Meng, P. Wang, *Phys. Lett. B* **584**, 1 (2004)
28. X.-H. Meng, Z.Y. Ma, *Eur. Phys. J. C* **72**, 2012 (2012)
29. P. Wang, X.-H. Meng, *Class. Quantum Grav.* **22**, 283 (2005)
30. P.K.F. Kuhfittig, *Class. Quantum Grav.* **23**, 5853 (2006)
31. P.F. Gonzales-Diaz, *Phys. Rev. D* **65**, 104035 (2002)
32. F.S.N. Lobo, *Phys. Rev. D* **71**, 124022 (2005)
33. F. Canfora, H. Maeda, *Phys. Rev. D* **87**, 084049 (2013)
34. F. Canfora, *Phys. Rev. D* **88**, 065028 (2013)
35. F.S.N. Lobo, *Phys. Rev. D* **71**, 084011 (2005)
36. N. Sen, *Ann. Phys. (Leipzig)* **73**, 365 (1924)
37. K. Lanczos, *Ann. Phys. (Leipzig)* **74**, 518 (1924)
38. R. Shankar Kuniyal et al., [arXiv:1509.05131v1](https://arxiv.org/abs/1509.05131v1)
39. O.B. Zaslavskii, *Phys. Rev. D* **72**, 061303(R) (2005)
40. J. Khoury, A. Weltman, *Phys. Rev. Lett.* **93**, 171104 (2004)
41. J. Khoury, A. Weltman, *Phys. Rev. D* **69**, 044026 (2004)