

# Primordial power spectrum of tensor perturbations in Finsler spacetime

Xin Li<sup>1,2,a</sup>, Sai Wang<sup>2,b</sup>

<sup>1</sup> Department of Physics, Chongqing University, Chongqing 401331, China

<sup>2</sup> State Key Laboratory Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, China

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**Abstract** We first investigate the gravitational wave in the flat Finsler spacetime. In the Finslerian universe, we derive the perturbed gravitational field equation with tensor perturbations. The Finslerian background spacetime breaks rotational symmetry and induces parity violation. Then we obtain the modified primordial power spectrum of the tensor perturbations. The parity violation feature requires that the anisotropic effect contributes to the  $TT$ ,  $TE$ ,  $EE$ ,  $BB$  angular correlation coefficients with  $l' = l + 1$  and  $TB$ ,  $EB$  with  $l' = l$ . The numerical results show that the anisotropic contributions to the angular correlation coefficients depend on  $m$ , and  $TE$  and  $ET$  angular correlation coefficients are different.

## 1 Introduction

Symmetry plays an essential role in studying cosmological physics. Cosmic inflation [1–5], as one of basic ideas of modern cosmology, can be described by a nearly de Sitter (dS) spacetime. The nearly dS spacetime preserves the symmetry of spatial rotations and translations. The primordial power spectrum is scale invariant if the symmetry of time translation of dS spacetime is preserved. The recent astronomical observations on the anisotropy of cosmic microwave background (CMB) [6] show that the exact scale invariance of the scalar perturbation is broken with more than 5 standard deviations. The observations [6] give a stringent limit on the magnitude of deviation from the scale invariance, i.e.,  $\mathcal{O}(10^{-2})$ . It means that the primordial power spectrum for the scalar perturbation is approximately scale invariant and the symmetry of time translation is slightly broken.

Recently, the CMB power asymmetry has been reported [7,8]. One possible physical mechanism that accounts for

CMB power asymmetry is anisotropic inflation models where the rotational symmetry of the nearly dS spacetime is violated. To induce the anisotropy in inflation, the popular approach is to involve a vector field [9–15] that aligned in a preferred direction. In such anisotropic inflation model, the comoving curvature perturbation becomes statistically anisotropic [16–26]. Usually, the background spacetime of the anisotropic inflation model is described by Bianchi spacetime [27].

Instead of choosing the Bianchi spacetime as a background spacetime, we will use Finsler spacetime [28] as a background spacetime to study anisotropic inflation. In general, Finsler spacetime admits less Killing vectors than Riemann spacetime does [29]. Also, there are types of Finsler spacetime that are non-reversible under parity flip,  $x \rightarrow -x$ . A typical non-reversible Finsler spacetime is Randers spacetime [30]. Such a property makes a function in Fourier space  $\phi(-\vec{k})$  different from  $\phi^*(\vec{k})$ . Therefore, in Finsler spacetime, the spatial rotational symmetry and parity symmetry are violated. In Ref. [31], we proposed an anisotropic inflation model in Finsler spacetime. We studied the primordial scalar perturbations and obtained off-diagonal angular correlations for the CMB temperature fluctuation and the E-mode polarization.

In this paper, we apply the Finslerian background spacetime that was used in Ref. [31] to study the possible modulation in the amplitude of tensor perturbations. In the standard model, the  $TB$  and  $EB$  correlations vanish. This is due to the fact that the parity of the CMB temperature fluctuation and E-mode polarization are different from that of the B-mode polarization. However, this is not the case in a Finslerian anisotropic inflation model. In Ref. [31], we have shown that the parity violation feature requires that the anisotropic effect of the primordial power spectrum of scalar perturbations appears in angular correlation coefficients with  $l' = l + 1$ . It means that the anisotropic part of the temperature fluctuations has the same parity as the B-mode polarization. Thus,

<sup>a</sup> e-mail: [lixin1981@cqu.edu.cn](mailto:lixin1981@cqu.edu.cn)

<sup>b</sup> e-mail: [wangsai@itp.ac.cn](mailto:wangsai@itp.ac.cn)

one can expect that the angular correlations  $TB$  and  $EB$  have non-vanishing values.

This paper is organized as follows. In Sect. 2, we investigate the gravitational wave in flat Finsler spacetime. The plane-wave solution of gravitational waves is given by imposing three constraints. In Sect. 3, we investigate tensor perturbations for the modified Friedmann–Robertson–Walker (FRW) spacetime in which the spatial part is replaced by a Randers space. In the modified FRW spacetime, we derive the gravitational field equation for the gravitational wave and obtain the primordial power spectrum for gravitational waves. In Sect. 4, the angular correlation coefficients for tensor perturbations are given. We plot the numerical results of the angular correlation coefficients which describe the anisotropic effect. Conclusions and remarks are presented in Sect. 5.

## 2 Gravitational wave in flat Finsler spacetime

Finsler geometry is based on the so-called Finsler structure  $F$  defined on the tangent bundle of a manifold  $M$ , with the property  $F(x, \lambda y) = \lambda F(x, y)$  for all  $\lambda > 0$ , where  $x \in M$  represents the position and  $y$  represents the velocity. The Finslerian metric is given as [28]

$$g_{\mu\nu} \equiv \frac{\partial}{\partial y^\mu} \frac{\partial}{\partial y^\nu} \left( \frac{1}{2} F^2 \right). \quad (1)$$

The Finslerian metric reduces to the Riemannian metric, if  $F^2$  is quadratic in  $y$ . A Finslerian metric is said to be locally Minkowskian if at every point there is a local coordinate system such that  $F = F(y)$  is independent of the position  $x$  [28]. In locally Minkowskian spacetime, the hh-curvature and hv-curvature of the Chern connection vanishes [28]. Also, the vv-curvature of Chern connection vanishes automatically. These facts imply that the curvature two-form of the Chern connection vanishes. In Matsumoto's monograph [32], the locally Minkowskian spacetime implies Riemannian flatness. Thus, the locally Minkowskian spacetime is a flat Finsler spacetime. Throughout this paper, the indices are lowered and raised by  $g_{\mu\nu}$  and its inverse matrix  $g^{\mu\nu}$ .

In Finsler geometry, there is a geometrical invariant quantity, i.e., Ricci scalar. It is of the form [28]

$$\text{Ric} \equiv \frac{1}{F^2} \left( 2 \frac{\partial G^\mu}{\partial x^\mu} - y^\lambda \frac{\partial^2 G^\mu}{\partial x^\lambda \partial y^\mu} + 2G^\lambda \frac{\partial^2 G^\mu}{\partial y^\lambda \partial y^\mu} - \frac{\partial G^\mu}{\partial y^\lambda} \frac{\partial G^\lambda}{\partial y^\mu} \right), \quad (2)$$

where  $G^\mu$  is for the geodesic spray coefficients

$$G^\mu = \frac{1}{4} g^{\mu\nu} \left( \frac{\partial^2 F^2}{\partial x^\lambda \partial y^\nu} y^\lambda - \frac{\partial F^2}{\partial x^\nu} \right). \quad (3)$$

The Ricci scalar only depends on the Finsler structure  $F$ .

There are types of gravitational field equation in Finsler spacetime [33–40]. These gravitational field equations are not equivalent to each other. It is well known that there is only a torsion free connection, i.e., the Christoffel connection in Riemann geometry. However, there are types of connection in Finsler geometry. Therefore, the gravitational field equations that depend on the connection should not be equal to each other. *When one studies self-parallel propagating vectors in the framework of general relativity [41], the norm preservation has to be taken into account which depends on the metric connection (Riemannian counterpart). In Finslerian relativity, the analogous way is successful when the connection is a Cartan connection which is metrical for the h-h parts and v-v parts on the tangent bundle.* In this paper, we construct the gravitational field equation from geometrical invariant quantity in Finsler spacetime. The analogy between geodesic deviation equations in Finsler spacetime and Riemann spacetime gives the vacuum field equation in Finsler gravity [42,43]. It concerns the vanishing of the Ricci scalar. The vanishing of the Ricci scalar implies that the geodesic rays are parallel to each other.

Before studying the primordial tensor modes in Finslerian inflation model, we investigate the property of gravitational wave in flat Finsler spacetime. We suppose the Finslerian metric is close to the locally Minkowski metric  $\eta_{\mu\nu}(y)$ ,

$$g_{\mu\nu} = \eta_{\mu\nu}(y) + h_{\mu\nu}(x, y), \quad (4)$$

where  $|h_{\mu\nu}| \ll 1$ . To first order in  $h$ , we obtain the Ricci scalar of the metric (4) by making use of Eqs. (2) and (3),

$$F^2 \text{Ric} = \frac{1}{2} \eta^{\mu\nu} y^\alpha y^\beta \left( 2 \frac{\partial^2 h_{\alpha\nu}}{\partial x^\mu \partial x^\beta} - \frac{\partial^2 h_{\alpha\beta}}{\partial x^\mu \partial x^\nu} - \frac{\partial^2 h_{\mu\nu}}{\partial x^\alpha \partial x^\beta} \right) - \frac{1}{4} y^\theta y^\alpha y^\beta \frac{\partial}{\partial y^\mu} \left( 2 \frac{\partial^2 (\eta^{\mu\nu} h_{\alpha\nu})}{\partial x^\theta \partial x^\beta} - \frac{\partial^2 (\eta^{\mu\nu} h_{\alpha\beta})}{\partial x^\nu \partial x^\theta} \right). \quad (5)$$

We consider the gravitational wave to be coming from infinity for simplicity, which means that the gravitational source that produces the gravitational wave can be neglected. The discussion as regards the vacuum field equation in Finsler spacetime requires that such a gravitational wave should satisfy  $\text{Ric} = 0$ . It is rather complicated to solve the equation  $\text{Ric} = 0$  for gravitational wave in Finsler spacetime. We are only interested in a Finslerian plane-wave solution of the equation  $\text{Ric} = 0$  in physics.

In order to get the plane-wave solution of gravitational waves, we suggest three constraints on the gravitational wave. The first one is  $h_\nu^\mu = \eta^{\mu\alpha} h_{\alpha\nu} = h_\nu^\mu(x)$ . The first constraint requires that  $h_\nu^\mu$  is only a function of  $x$ . It means that we choose a special tensor perturbation for flat Finsler spacetime. Such a special perturbation will reduce to standard tensor perturbations in general relativity if the metric of the flat Finsler spacetime  $\eta_{\mu\nu}(y)$  returns to a Minkowski metric. The

second one is the gauge condition

$$\frac{2\partial h^\mu_\nu}{\partial x^\mu} - \frac{\partial h^\mu_\mu}{\partial x^\nu} = 0. \tag{6}$$

Such a gauge condition is the same as the one in general relativity. It can be satisfied in Finsler spacetime, since the Ricci scalar is invariant under a coordinate transformation. The last constraint states that the direction of  $y$  is parallel to  $\frac{\partial}{\partial x}$ . The Finslerian length element  $F$  is constructed on a tangent bundle [28]. Thus, the gravitational field equation should be constructed on the tangent bundle in principle. The last constraint implies that we have restricted the field equation on the base manifold such that the fiber coordinate  $y$  is parallel to  $\frac{\partial}{\partial x}$ . By making use of the three constraints and noticing the relation  $\frac{\partial g^{ij}}{\partial y^i} y_j = 0$ , we find that Eq. (5) reduces to

$$F^2 Ric = -\frac{1}{2} \eta^{\mu\nu}(k) \frac{\partial^2 h_{\alpha\beta}}{\partial x^\mu \partial x^\nu} y^\alpha y^\beta. \tag{7}$$

Since  $\eta^{\mu\nu}$  is a homogeneous function of degree 0 with respect to the variable  $y$  and  $y$  is parallel to the wave vector  $k$  in momentum space, we have replaced the variable  $y$  of  $\eta^{\mu\nu}$  into  $k$  in Eq. (7). Plugging Eq. (7) into the field equation  $Ric = 0$ , we obtain the solution of the field equation

$$h^\mu_\nu = e^\mu_\nu \exp(i\eta_{\alpha\beta}(k)k^\alpha x^\beta) + h.c., \tag{8}$$

where  $e^\mu_\nu$  denotes the polarization tensor of the gravitational wave and the wave vector  $k$  satisfies

$$\eta_{\mu\nu}(k)k^\mu k^\nu = 0. \tag{9}$$

Equation (9) represents that the velocity of gravitational wave depends on the wave vector  $k$ . It means that the Lorentz symmetry is violated, which is a feature of Finsler spacetime [44–46]. The plane-wave solution (8) satisfies the gauge condition (6) if

$$2k_\mu e^\mu_\nu = k_\nu e^\mu_\mu, \tag{10}$$

where  $k_\mu = \eta_{\mu\nu}k^\nu$ . The four relations in (6) imply that the polarization tensor  $e^\mu_\nu$  has six independent components. Following the approach in general relativity [47], one could find that only two components of the polarization tensor are physical.

In this paper, the perturbed Finslerian metric  $h(x, y)$  should depend on both  $x$  and  $y$  in Finsler spacetime. Then we have used the three constraints on the gravitational wave to get the plane-wave solution. The last constraint states that the direction of  $y$  is parallel to  $\frac{\partial}{\partial x}$ . The Finslerian length element  $F$  is constructed on a tangent bundle [28]. Thus, the gravitational field equation should be constructed on the tangent bundle in principle. The last constraint implies that we have restricted the field equation on the base manifold. Thus, the solution (8) does not depend on  $y$ , and it only depends on the wave vector  $k$  in momentum space.

### 3 Gravitational wave in Finslerian inflation

In Ref. [31], we proposed a background Finsler spacetime to describe anisotropic inflation. It is of the form

$$F_0^2 = y^t y^t - a^2(t) F_{Ra}^2, \tag{11}$$

where  $F_{Ra}$  is a Randers space [30],

$$F_{Ra} = \sqrt{\delta_{ij} y^i y^j} + \delta_{ij} b^i y^j. \tag{12}$$

Here, we require that the vector  $b^i$  in  $F_{Ra}$  is of the form  $b^i = \{0, 0, b\}$  and  $b$  is a constant. The spatial part of Finsler spacetime (11), i.e.  $F_{Ra}$ , preserves three translation symmetries and one rotational symmetry [29, 31, 43]. It means that  $F_{Ra}$  is only rotational invariant on the plane that is perpendicular to vector  $b^i$ , and the other rotational symmetry of Euclidean space is broken. In this paper, we focus on investigating the tensor perturbations of the background Finsler spacetime (11). The perturbed Finsler structure is of the form

$$F^2 = y^t y^t - a^2(t)(F_{Ra}^2 + h_{ij} y^i y^j). \tag{13}$$

Here, we require the perturbed metric  $h_{ij}$  satisfies the first and third constraint as discussed in Sect. 2. The second constraint, i.e. the gauge condition, is changed into

$$h^i_{,i} = h^i_{j,i} = 0, \tag{14}$$

where the comma denotes the derivative with respect to the spatial coordinate  $\vec{x}$ . By making use of the three constraints, we obtain the Ricci scalar of the perturbed Finsler spacetime (13) in momentum space,

$$F^2 Ric = (a\ddot{a} + 2\dot{a}^2)(F_{Ra}^2 + h_{ij} y^i y^j) - \frac{3\ddot{a}}{a} y^t y^t + \frac{a^2}{2} \left( \ddot{h}_{ij} + \frac{3\dot{a}}{a} \dot{h}_{ij} + \frac{1}{a^2} \eta^{mn} k_m k_n h_{ij} \right) y^i y^j, \tag{15}$$

where the dot denotes the derivative with respect to time and  $\eta^{mn}$  denotes the Finslerian metric of Randers space  $F_{Ra}$ .

In Refs. [43, 48], we have proved that the gravitational field equation in Finsler spacetime

$$G^\mu_\nu = 8\pi G T^\mu_\nu \tag{16}$$

is valid for the modified FRW spacetime (11) and Finslerian Schwarzschild spacetime [43]. Here the modified Einstein tensor in Finsler spacetime is defined as

$$G^\mu_\nu \equiv Ric^\mu_\nu - \frac{1}{2} \delta^\mu_\nu S, \tag{17}$$

and  $T^\mu_\nu$  is the energy-momentum tensor. Here the Ricci tensor is defined as [49]

$$Ric_{\mu\nu} = \frac{\partial^2 (\frac{1}{2} F^2 Ric)}{\partial y^\mu \partial y^\nu}, \tag{18}$$

and the scalar curvature in Finsler spacetime is given as  $S = g^{\mu\nu} \text{Ric}_{\mu\nu}$ . Plugging the equation for the Ricci scalar (15) into the gravitational field equation (16), we obtain the perturbed equation for  $h_j^i$

$$-\frac{1}{2} \left( \ddot{h}_j^i + \frac{3\dot{a}}{a} \dot{h}_j^i + \frac{k_e^2}{a^2} h_j^i \right) = 8\pi G \delta T_j^i, \tag{19}$$

where  $\delta T_j^i = 0$  denotes the first order part of the energy-momentum tensor and the effective wavenumber  $k_e$  is given by

$$k_e^2 = \eta^{ij} k_i k_j = k^2 (1 + b \hat{k} \cdot \hat{n}_z)^2. \tag{20}$$

Here,  $\hat{k}$  denotes the propagational direction of the gravitational wave. The perturbed equation (19) is the same as the one in the standard inflation model, except for replacing the wavenumber  $k$  with the effective wavenumber  $k_e$ .  $k_e$  depends not only on the magnitude of  $k$  but also the preferred direction  $\hat{n}_z$  that induces the rotational symmetry breaking. Then, following the standard quantization process in the inflation model [50], we can obtain the primordial power spectrum of tensor perturbations from the solution of Eq. (19). It is of the form

$$\mathcal{P}^{\pm 2 \pm 2}(\vec{k}) = \mathcal{P}_{\text{iso}}^{\pm 2 \pm 2}(k) \left( \frac{k}{k_e} \right)^3 \simeq \mathcal{P}_{\text{iso}}^{\pm 2 \pm 2}(k) (1 - 3b \hat{k} \cdot \hat{n}_z), \tag{21}$$

where  $\mathcal{P}_{\text{iso}}^{\pm 2 \pm 2}$  is an isotropic power spectrum for tensor perturbations  $h$  which depends only on the magnitude of wavenumber  $k$ . And  $\mathcal{P}_{\text{iso}}^{\pm 2 \pm 2}$  is just the same spectrum with the primordial power spectrum for tensor perturbations that are generated during a de Sitter (inflationary) stage [51]. The term  $3b \hat{k} \cdot \hat{n}_z$  in the primordial power spectrum  $\mathcal{P}^{\pm 2 \pm 2}$  represents the effect of rotational symmetry breaking.

#### 4 The anisotropic effects on angular power spectra

The anisotropic term in Eq. (21) could give an off-diagonal angular correlation for the CMB temperature fluctuation, the E-mode and B-mode polarizations of CMB, and it also contributes to the  $TB$  and  $EB$  spectra, which should vanish in the standard inflation model. The general angular correlation coefficients for the tensor perturbations that describe the anisotropic effect are given by  $C_{XX',ll',mm'}$  [52,53], where  $X = T, E, B$  denotes the CMB temperature fluctuation, the E-mode and B-mode polarizations, respectively. In our anisotropic model, by making use of Eq. (21), we obtain the CMB correlation coefficients for the tensor perturbations as follows:

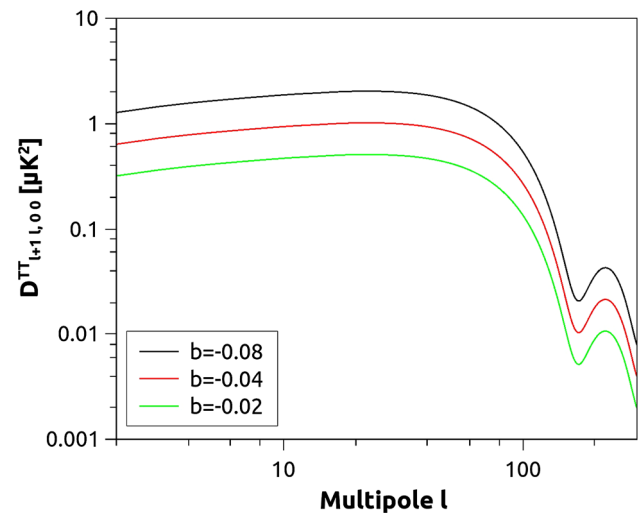
$$C_{XX',ll',mm'}^T = \int \frac{d \ln k}{(2\pi)^3} \Delta_{X,l2}(k) \Delta_{X',l'2}^*(k) P_{ll'mm'}^{\pm 2 \pm 2}, \tag{22}$$

where

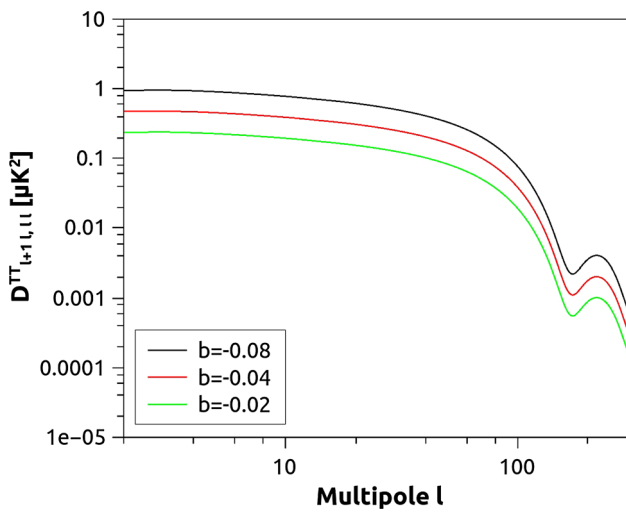
$$\begin{aligned} P_{ll'mm'}^{\pm 2 \pm 2} &= \int d\Omega \mathcal{P}^{\pm 2 \pm 2}(\vec{k}) (-_2 Y_{lm}^* -_2 Y_{l'm'} \pm +_2 Y_{lm}^* +_2 Y_{l'm'}) \\ &= \mathcal{P}_{\text{iso}}^{\pm 2 \pm 2} \sqrt{\frac{2l'+1}{2l+1}} \left( C_{00lm}^{l'm'} (C_{00l2}^{l'2} \pm C_{00l-2}^{l'-2}) \right. \\ &\quad \left. - 3b C_{10lm}^{l'm'} (C_{10l2}^{l'2} \pm C_{10l-2}^{l'-2}) \right), \end{aligned} \tag{23}$$

$\Delta_{X,l2}(k)$  denote the transfer functions for tensor modes and  $C_{LMlm}^{l'm'}$  are the Clebsch–Gordan coefficients. Here, the  ${}_s Y_{lm}$  in Eq. (23) are for the spin- $s$  spherical harmonic function [54]. In Eq. (23), the ‘+’ of ‘ $\pm$ ’ in the brackets corresponds to the  $TT, TE, EE, BB$  correlations and the ‘-’ of ‘ $\pm$ ’ corresponds to the  $TB, EB$  correlations. By making use of the symmetry of the Clebsch–Gordan coefficients  $C_{LMlm}^{l'm'} = (-1)^{L+l-l'} C_{L-Ml-m}^{l'-m'}$ , one can find from Eq. (22) that  $TT, TE, EE, BB$  correlations have non-zero values for  $l' = l, l + 1$  and  $TB, EB$  correlations have non-zero values for  $l' = l$ . The anisotropic term  $3b \hat{k} \cdot \hat{n}_z$  in the primordial power spectrum  $\mathcal{P}^{\pm 2 \pm 2}$  that describes the deviation from statistical isotropy violates the parity symmetry. Thus, it contributes to the  $TT, TE, EE, BB$  correlations if  $l' = l + 1$  and the  $TB, EB$  correlations if  $l' = l$ .

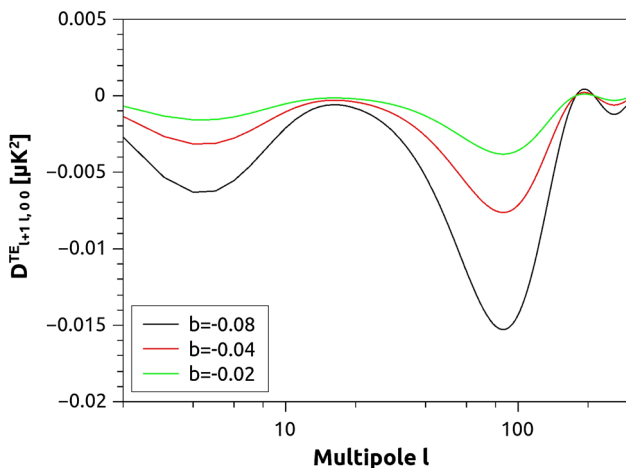
Here, by making use of the formula of the angular correlation coefficients (22) and the Planck 2015 data [55], we plot numerical results for the anisotropic contribution to  $C_{ll'} \equiv C_{XX',ll',mm'}$ . The anisotropic part of the  $C_{ll'}$  has three properties that differ from the isotropic part. The first one is that the  $TE$  and  $ET$  correlation coefficients are different. The second one shows that the anisotropic part of the  $C_{ll'}$  depends on  $m$ . The last one shows that  $C_{ll}^{TB}$  and  $C_{ll}^{EB}$  have non-vanishing values. These properties are obvi-



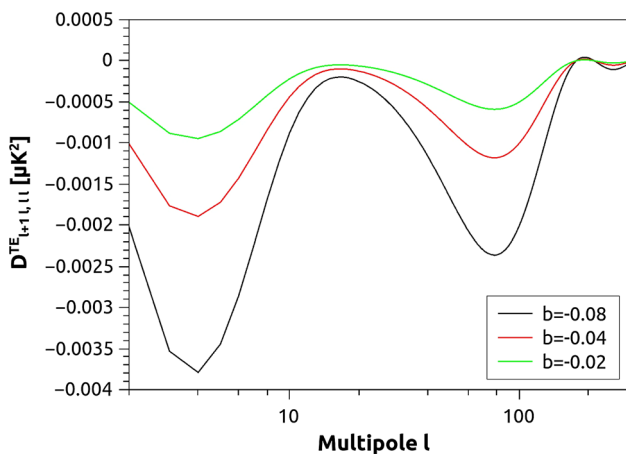
**Fig. 1** The anisotropic part of the  $TT$  correlation coefficients with  $m = 0$ . The black, red, and green curves correspond to a different Finslerian parameter, i.e.  $b = -0.08, -0.04, -0.02$ , respectively. The following figures are given by the same convention



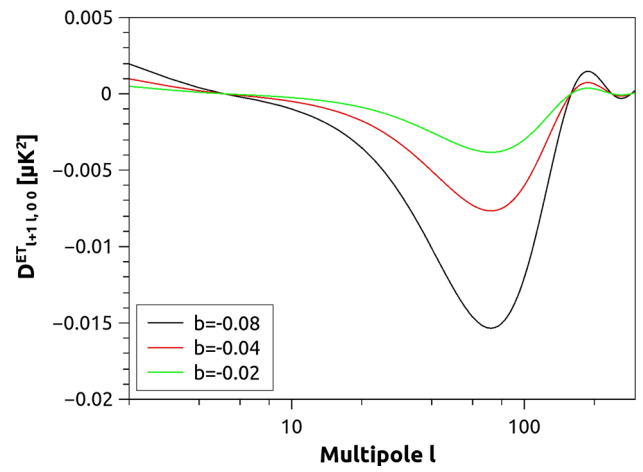
**Fig. 2** The anisotropic part of the  $TT$  correlation coefficients with  $m = l$ . It demonstrates that the anisotropic part of the correlation coefficients  $C_{ll}$  depend on  $m$



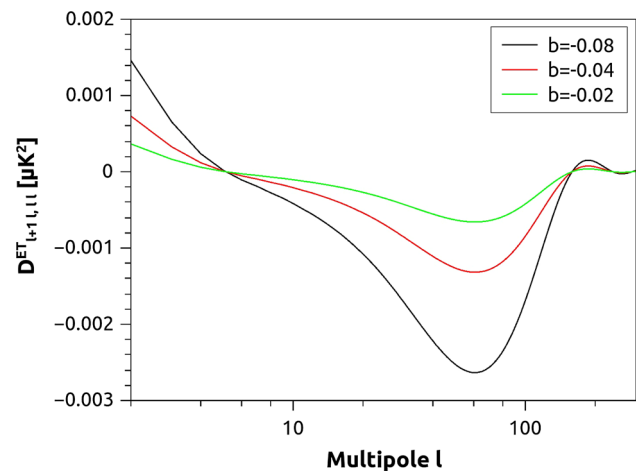
**Fig. 3** The anisotropic part of the  $TE$  correlation coefficients with  $m = 0$



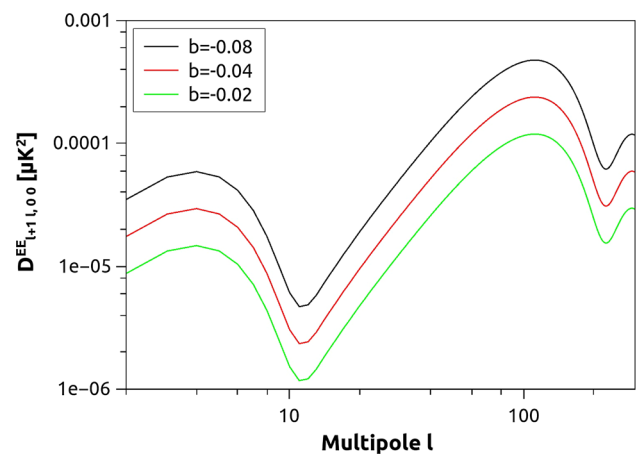
**Fig. 4** The anisotropic part of the  $TE$  correlation coefficients with  $m = l$



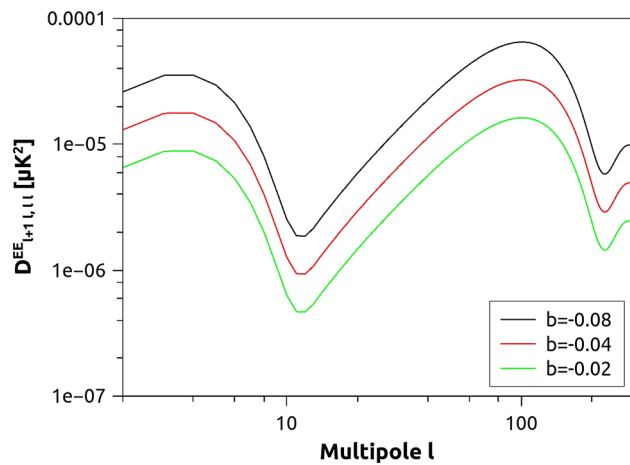
**Fig. 5** The anisotropic part of the  $ET$  correlation coefficients with  $m = 0$



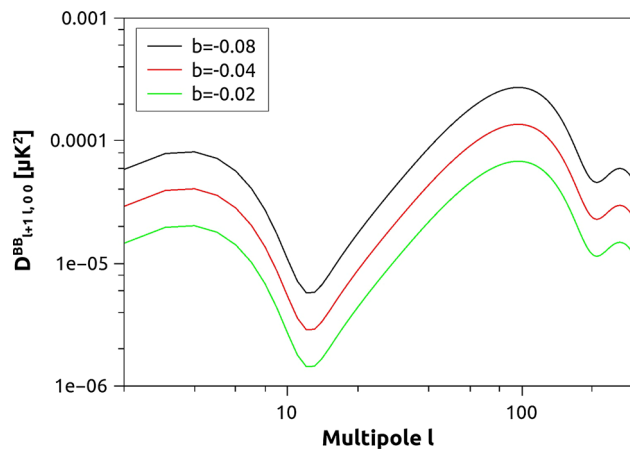
**Fig. 6** The anisotropic part of the  $ET$  correlation coefficients with  $m = l$



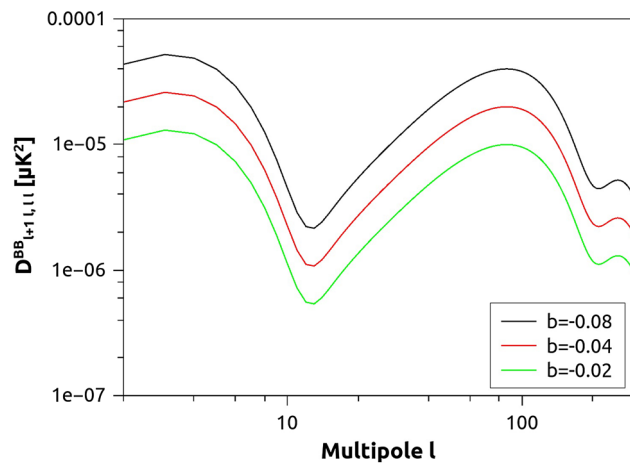
**Fig. 7** The anisotropic part of the  $EE$  correlation coefficients with  $m = 0$



**Fig. 8** The anisotropic part of the  $EE$  correlation coefficients with  $m = l$

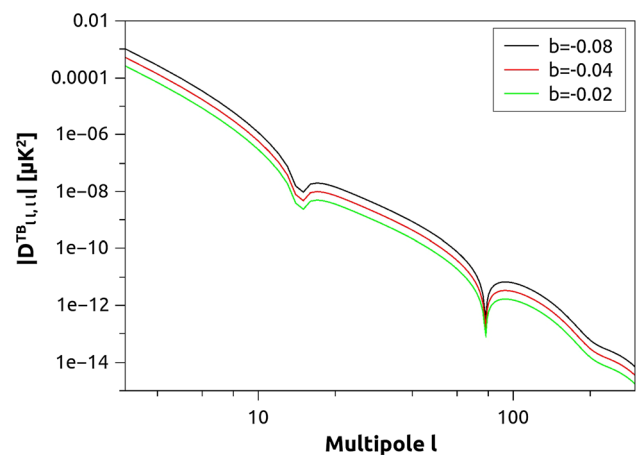


**Fig. 9** The anisotropic part of the  $BB$  correlation coefficients with  $m = 0$

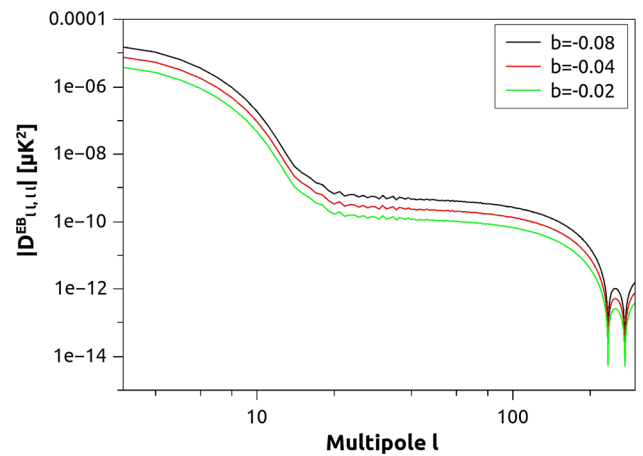


**Fig. 10** The anisotropic part of the  $BB$  correlation coefficients with  $m = l$

ous in Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12, which are generated by modifying the CAMB package [56]. At present, the observations of CMB, such as the Planck data



**Fig. 11** The anisotropic part of the  $TB$  correlation coefficients with  $m = l$



**Fig. 12** The anisotropic part of the  $EB$  correlation coefficients with  $m = l$

[57], do not give  $TT$ ,  $TE$ ,  $EE$ ,  $BB$  correlations for  $l' = l + 1$  and  $TB$ ,  $EB$  correlations for  $l' = l$ . Thus, to show the effect of the Finslerian modification for primordial tensor perturbations, we set the Finslerian parameter to be  $b = -0.02, -0.04, -0.08$ , which have the same order as the magnitude of the CMB dipole modulation [58]. The anisotropic part of the  $TT$ ,  $TE$ ,  $ET$ ,  $EE$  correlation coefficients for  $m = 0$  are shown in Figs. 1, 3, 5, 7, and 9, respectively. The anisotropic contributions to  $TT$ ,  $TE$ ,  $ET$ ,  $EE$  correlation coefficients for  $m = l$  are shown in Figs. 2, 4, 6, 8, and 10, respectively. The anisotropic contributions to  $TB$  and  $EB$  are shown in Figs. 11 and 12. Here, we have used the mean value of the cosmological parameters [55] to give the above figures. The tensor-to-scalar ratio  $r$  is set  $r = 0.05$ , which is compatible within the  $2\sigma$  level with the current observations [58,59]. The coefficients  $D_{l'l',mm'}^{XX'}$  in these figures are defined as  $D_{l'l',mm'}^{XX'} \equiv (2\pi)^{-1} \sqrt{l(l+1)l'(l'+1)} C_{XX',ll',mm'}$ . Here the pivot scale is set  $k_p = 0.01 \text{ Mpc}^{-1}$  for the tensor perturbations and  $0.05 \text{ Mpc}^{-1}$  for the scalar perturbations.

## 5 Conclusions and remarks

In this paper, we have investigated the gravitational wave in the flat Finsler spacetime. To get the plane-wave solution of gravitational waves, three constraints are involved. In the modified FRW spacetime (13) with tensor perturbations, we derived the perturbed gravitational field equation for tensor perturbations (19) by making use of the three constraints. From the solution of the perturbed gravitational field equation, we obtained the primordial power spectrum of tensor perturbations (21). The term  $3b\hat{k} \cdot \hat{n}_z$  in the primordial power spectrum (21) that violates the rotational symmetry and parity symmetry describes the statistical anisotropy of CMB temperature fluctuation, the E-mode and B-mode polarizations. We have used the primordial power spectrum (21) to derive the angular correlation coefficients  $C_{XX',ll',mm'}$ . The parity violation feature requires that the anisotropic effect appears in the  $TT$ ,  $TE$ ,  $EE$ ,  $BB$  correlations for  $l' = l + 1$  and  $TB$ ,  $EB$  correlations for  $l' = l$ . The numerical results for the anisotropic parts of the correlation coefficients show that they depend on  $m$ , and the  $TE$  and  $ET$  correlation coefficients are different.

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