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Braneworld gravastars admitting conformal motion

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Abstract In this paper, we propose the braneworld gravastar configuration which is an alternative to the braneworld black hole. We study the Mazur and Mottola gravastar model within the context of the Randall–Sundrum type II braneworld scenario, based on the fact that our four dimensional spacetime is a three-brane, embedded in a five dimensional bulk. We present exact solutions of the modified field equations in each of the three regions making up the gravastar, namely, (I) the core, (II) the shell, and (III) the vacuum exterior. The junction conditions at each interface are fulfilled and we further explore interesting physical properties such as length, energy and entropy of the spherical distribution.

1 Introduction

Black holes are regions in spacetime where gravity is so intense that even light cannot escape to the exterior. These exotic objects have been the central focus of researchers over several decades. The extraordinary physics of black holes has its origin in the highly unexpected properties of the event horizon. The event horizon of a black hole is the boundary between its exterior and its interior; it acts like a one-way membrane providing a strong connection between gravitation, thermodynamics, and quantum theory. This connection was first introduced by Hawking and Bekenstein around 1974, when they looked at the nature of radiation emitted by black holes. It was soon realized that this prediction created an information loss problem (the black hole information paradox) which has become an important issue in quantum gravity and poses serious challenges to the foundations of theoretical physics. They showed that black holes surrounded by quantum fields will emit small amounts of radiation, causing them to shrink and eventually evaporating completely. These intrinsic problems associated with black hole horizons have led to a flurry of solutions attempting to remove these paradoxes. These semiclassical approaches generated by black hole horizons should be investigated because the full theory of quantum gravity is still unknown today.

Given the above, it has been suggested that alternative models of black holes, which do not involve horizons and could be stabilized under the exotic states of matter, should be studied. Among the various models proposed and studied thus far, the gravitational vacuum star (gravastar) has recently received widespread attention. The gravastar model was first proposed by Mazur and Mottola (MM) [1]. In this model it is suggested that a gravitationally collapsing star would force spacetime itself to undergo a phase transition that would prevent further collapse. The MM gravastar model consists of three regions: a de Sitter geometry in the interior filled with constant positive (dark) energy density ρ accompanied by an isotropic negative pressure $\rho = -p > 0$ which is connected via three intermediate layers to an outer vacuum Schwarzschild solution ($p = \rho = 0$). The intermediate relatively thin shell is composed of stiff matter ($p = \rho$). In order to achieve stability and compensate for discontinuities in the pressure profiles of the entire object, the MM gravastar requires two infinitesimally thin shells endowed with surface densities $\sigma_{\pm}$ and surface tensions $\vartheta_{\pm}$. As pointed out by Cattoen et al. [2] this 5-layer construction can be reduced to a 3-layer model by effectively removing the use of thin shells. The gravastar is now composed of three regions: (a) the de Sitter interior: $0 \le r \le r_1$, with equation of state $p = -\rho > 0$, (b) the shell: $r_1 < r < r_2$ with equation of state $p = +\rho$, and (c) the exterior: $r_2 < r$ with $p = \rho = 0$. Visser and Wiltshire [3] showed that the MM gravastar is dynamically stable against radial perturbations. Subsequent work by Carter [4], Chirenti and Rezzolla [5] and De Benedictis et al. [6] have shown that the MM gravastar is also stable against axial-perturbations. Investigations by Chan et al. [7] have shown that anisotropy of the interior fluid may affect the formation

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of a gravastar. They were able to show that the formation of a gravastar is possible only when the tangential pressure is greater than the radial pressure, at least in the neighborhood of the isotropic case. Numerous studies of gravastar models and their evolution can be found in [8–13], amongst others.

The Randall–Sundrum (RS) braneworld (BW) [14,15] model is based on the assumption that our four dimensional spacetime is a three-brane, embedded in a five dimensional bulk. Braneworld models have an appreciable impact in theoretical physics in so far as offering solutions to the mass hierarchy problem in particle physics. The Randall–Sundrum (RS) braneworld models play a significant role in cosmology, in particular, these models provide an explanation for the expansion rate of the Universe at high energies which differs from the prediction of standard general relativity [16].

Exact solutions describing braneworld stellar models are few and far in between. Germani and Maartens presented a static spherically star solution with uniform density [17]. An elegant overview of stars on the brane which include the matching conditions and projection of the Weyl stresses from the bulk was presented by Deruelle [18]. Some elegant work on star solutions in the braneworld scenario have been discussed in [19–22]. The gravitational collapse of bounded matter configurations on the brane was discussed by Govender and Dadhich [23]. Conformal symmetries are of some importance for better understanding of spacetime geometry because it helps to solve the geodesic equations of motion for the spacetime under consideration. Symmetry also helps in the search for a natural relationship between geometry and matter.

In relativity theory, the behavior of the metric is important when moving along curves on a manifold. A conformal Killing vector ξ is a vector field on a manifold so that if the metric is dragged along the curves created by ξ, its Lie derivative is directly proportional to itself, i.e.

$$\mathcal{L}_\xi g_{ik} = \psi g_{ik}, \tag{1}$$

for some scalar field ψ, known as the conformal factor. Here $\mathcal{L}$ represents the Lie derivative operator. The physical importance of this prerequisite is that when the metric is dragged along a specific congruence of curves it persists modulo some scale factor, ψ, which may differ from position to position on the manifold. One can note that ψ is not arbitrary but it depends on the conformal Killing vector ξ as $\psi(x^k) = \frac{1}{4}\xi^i_{;i}$ for a Riemannian space of dimension four.

The vector ξ characterizes the conformal symmetry, while the metric tensor g_{ik} is conformally mapped onto itself along ξ. The application of CKV provides deeper insights into the spacetime geometry. The conformally symmetric vacuum solutions of the gravitational field equations in the braneworld models have been found in [24]. Usmani et al. found the gravastar solution within the framework of Mazur–Mottola, admitting conformal motion in [25].

In this paper we present the solutions for gravastar in the context of Randall–Sundrum II type braneworld scenario. The paper is organized as follows: in Sects. 1 and 2 we briefly review the field equations in braneworld models by admitting conformal motion of Killing Vectors. In Sect. 4 we derive the stellar interior solution based on the fact that the matter in the core obeys a barotropic equation of state of the form $p = -\rho > 0$. The Weyl stresses, namely the scalar $\mathcal{U}$ and the anisotropy $\mathcal{P}$, in this region are completely determined. In Sect. 5 we present the complete gravitational and thermodynamical behavior of the shell of the star while the external region has been matched to the braneworld black hole solution in Sect. 6. In Sect. 7 we calculate the entropy of the fluid within the shell and present the junction conditions for the solutions under consideration in Sect. 8. Some final remarks are made in Sect. 9.

2 Field equations

Let us start by writing the modified Einstein field equations on the brane [26], which take the form

$$G_{\mu\nu} = k^2 T^{\text{eff}}_{\mu\nu}, \tag{2}$$

where $k^2 = 8\pi G$ and $T^{\text{eff}}_{\mu\nu}$ represent the effective energy-momentum tensor given by

$$T^{\text{eff}}_{\mu\nu} = T_{\mu\nu} + \frac{6}{\lambda} S_{\mu\nu} - \frac{1}{k^2}\mathcal{E}_{\mu\nu}. \tag{3}$$

We have chosen the bulk cosmological constant in such a way that the brane cosmological constant vanishes and λ is the brane tension which corresponds to the vacuum energy density on the brane. The high-energy and non-local corrections, respectively, are given by

$$S_{\mu\nu} = \frac{T T_{\mu\nu}}{12} - \frac{T_{\mu\alpha}T^\alpha_\nu}{4} + \frac{g_{\mu\nu}}{24}\left[3T_{\alpha\beta}T^{\alpha\beta} - T^2\right], \tag{4}$$

and

$$k^2\mathcal{E}_{\mu\nu} = -\frac{6}{\lambda}\left[\mathcal{U}\left(u_\mu u_\nu + \frac{1}{3}h_{\mu\nu}\right) + \mathcal{P}_{\mu\nu} + \mathcal{Q}_{(\mu u_\nu)}\right] \tag{5}$$

is a non-local source, arising from the 5-dimensional Weyl curvature with $\mathcal{U}$ representing the non-local energy density, $\mathcal{Q}_\mu$ is the non-local energy flux and non-local anisotropic pressure is $\mathcal{P}_{\mu\nu}$, respectively.

For a static spherically symmetric matter distribution $\mathcal{Q}_\mu = 0$ and the non-local anisotropic pressure $\mathcal{P}_{\mu\nu}$ is given by

$$\mathcal{P}_{\mu\nu} = \mathcal{P}\left(r_\mu r_\nu - \frac{1}{3}h_{\mu\nu}\right), \tag{6}$$

where r_μ is the projected radial vector and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projected tensor with the 4-velocity u^μ and P is the pressure of the bulk. The above expression becomes

$$k^2\mathcal{E}_{\mu\nu} = -\frac{6}{\lambda}\left[\mathcal{U}u_\mu u_\nu + \mathcal{P}r_\mu r_\nu + h_{\mu\nu}\frac{(\mathcal{U}-\mathcal{P})}{3}\right], \tag{7}$$

where r_μ is a unit radial vector. We consider a perfect fluid energy-momentum tensor $T_{\mu\nu}$, having the explicit form

$$T_{\mu\nu} = \rho u_\mu u_\nu + p h_{\mu\nu}, \tag{8}$$

where u^μ is the 4-velocity. We consider the static spherically symmetric line element on the brane in the standard form

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right). \tag{9}$$

The gravitational field equations for the line element metric (9) must satisfy the effective 4-D Einstein equations (1) with the effective energy-momentum tensor [27–29]

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = k^2\rho^{\text{eff}}, \tag{10}$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = k^2\left(p^{\text{eff}} + \frac{4}{k^4\lambda}\mathcal{P}\right), \tag{11}$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}\nu'^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = k^2\left(p^{\text{eff}} - \frac{2}{k^4\lambda}\mathcal{P}\right), \tag{12}$$

$$p' + \frac{\nu'}{2}(\rho + p) = 0, \tag{13}$$

where primes denote differentiation with respect to r; and the effective energy density ρ^{eff}, the effective radial pressure $\tilde{p}_r$, and the effective transverse pressure $\tilde{p}_t$ are given by

$$\rho^{\text{eff}} = \tilde{\rho} = \rho\left(1 + \frac{\rho}{2\lambda}\right) + \frac{6}{k^4\lambda}\mathcal{U}, \tag{14}$$

$$p_r^{\text{eff}} = \tilde{p}_r = p + \frac{\rho}{2\lambda}(\rho + 2p) + \frac{2}{k^4\lambda}\mathcal{U} + \frac{4}{k^4\lambda}\mathcal{P}, \tag{15}$$

$$p_t^{\text{eff}} = \tilde{p}_t = p + \frac{\rho}{2\lambda}(\rho + 2p) + \frac{2}{k^4\lambda}\mathcal{U} - \frac{2}{k^4\lambda}\mathcal{P}. \tag{16}$$

Moreover, we observe from Eq. (5) that $\mathcal{E}_{\mu\nu} \to 0$ as $\lambda^{-1} \to 0$, i.e., using the limit in Eq. (3), we obtain $T_{\mu\nu}^{\text{eff}} = T_{\mu\nu}$, thereby recovering 4-dimensional general relativity. The extra dimensional effects produce anisotropy in the interior of the star distribution which can be written as

$$\tilde{p}_r - \tilde{p}_t = \frac{6}{k^4\lambda}\mathcal{P}. \tag{17}$$

Recently, Mazur and Mottola [30] made the interesting observation that the constant density interior Schwarzschild solution for a static, spherically symmetric collapsed star has a negative pressure when its radius is less than the Schwarzschild radius thereby describing a gravitational condensate star or gravastar. Furthermore, they showed that transverse stresses are induced within this region thereby abandoning the condition of pressure isotropy. In a more general approach Cattoen et al. [2] showed that stable gravastars must necessarily exhibit transverse pressures. It is interesting to note from (17) that our braneword construction of a gravastar naturally incorporates an effective transverse pressure arising from non-local Weyl stresses.

3 Gravastar with conformal motion

We now demand that the interior spacetime admits conformal motion. This immediately places a restriction on the gravitational behavior of the gravastar. From Eq. (1) we can write

$$\mathcal{L}_\xi g_{ik} = \xi_{i;k} + \xi_{k;i} = \psi g_{ik}, \tag{18}$$

with $\xi_i = g_{ik}\xi^k$. From Eqs. (9) and (18), one can get the following expressions: $\xi^1\nu' = \psi, \xi^4 = C_1, \xi^1 = \frac{\psi r}{2}, \xi^1\lambda' + 2\xi^1_{,1} = \psi$.

Here, (1) and (4) represent the spatial and temporal coordinates r and t, respectively. The above set of equations imply

$$e^\nu = C_2^2 r^2, \tag{19}$$

$$e^\lambda = \left[\frac{C_3}{\psi}\right]^2, \tag{20}$$

$$\xi^i = C_1\delta_4^i + \left[\frac{\psi r}{2}\right]\delta_1^i, \tag{21}$$

where C_1, C_2, and C_3 are constants of integration. The field equations (Eqs. (10)–(12)) corresponding to the metric (9) take the following form:

$$\frac{1}{r^2}\left[1 - \frac{\psi^2}{C_3^2}\right] - \frac{2\psi\psi'}{rC_3^2} = k^2\rho^{\text{eff}}, \tag{22}$$

$$\frac{1}{r^2}\left[\frac{3\psi^2}{C_3^2} - 1\right] = k^2\left(p^{\text{eff}} + \frac{4}{k^4\lambda}\mathcal{P}\right), \tag{23}$$

$$\left[\frac{\psi^2}{C_3^2 r^2}\right] + \frac{2\psi\psi'}{rC_3^2} = k^2\left(p^{\text{eff}} - \frac{2}{k^4\lambda}\mathcal{P}\right). \tag{24}$$

The above system of equations represents a spherically symmetric matter distribution admitting conformal motion on the brane. From Eqs. (23) and (24) we can express the extra dimensional effects in terms of the conformal factor as

$$\frac{6}{k^2\lambda}\mathcal{P} = \frac{2\psi^2}{C_3^2 r^2} - \frac{1}{r^2} - \frac{2\psi\psi'}{C_3^2 r}, \tag{25}$$

In order to obtain the complete gravitational behavior of the model, the function ψ needs to be determined.

4 Interior region of the Braneworld gravastar

We now consider the inner portion of our gravastar model. When p = $-\rho$, Eq. (13) yields $\rho = const. = \rho_c$, therefore $p = p_c = -\rho_c$. Using (25) in (23) and together with the equation of state we obtain

$$\frac{2}{k^2\lambda}\mathcal{U} = -\frac{1}{3r^2} + \frac{5\psi^2}{3C_3^2 r^2} + \frac{4\psi\psi'}{3C_3^2 r} + k^2\rho_c\left(1+\frac{\rho_c}{2\lambda}\right). \tag{26}$$

Now using the values of $\mathcal{U}$ and $\mathcal{P}$ in Eq. (22) we obtain

$$\psi^2 + r\psi\psi' = \frac{C_3^2}{3}\left(1 - 2\mathcal{A}r^2\right), \tag{27}$$

where $\mathcal{A} = k^2\rho_c\left(1+\frac{\rho_c}{2\lambda}\right)$. Solving (27) we obtain

$$\psi^2 = \psi_0 + \frac{C_3^2}{3}\left(1 - \mathcal{A}r^2\right), \tag{28}$$

where ψ_0 is an integration constant. Now, the metric potential e^λ assumes the following form:

$$e^\lambda = C_3^2\left[\psi_0 + \frac{C_3^2}{3}\left(1-\mathcal{A}r^2\right)\right]^{-1}. \tag{29}$$

The radiation energy density and pressure of the bulk are obtained; thus, respectively,

$$\frac{2}{k^2\lambda}\mathcal{U} = \frac{1}{3r^2}\left(\frac{2}{3} + \frac{5\psi_0}{C_3^2 r}\right), \tag{30}$$

$$\frac{6}{k^4\lambda}\mathcal{P} = \frac{1}{3r^2}\left(\frac{6\psi_0}{C_3^2} - 1\right), \tag{31}$$

For a real conformal factor, it is clear from Eq. (28) that $r < \sqrt{\left[\frac{3\psi_0}{\mathcal{A}C_3^2} + \mathcal{A}\right]}$. This gives a clue of the upper limit of the interior region. Since the conformal factor ψ may vary from place to place on the manifold, this implies that it places a restriction on the size of the interior region.

For $\psi_0 = 0$, one can find that both $\mathcal{U}$ and $\mathcal{P}$ are inversely proportional to r^2 but with opposite signs. Note that for $\psi_0 > \frac{C_3^2}{6}$ both $\mathcal{U}$ and $\mathcal{P}$ are positive, however, for $\psi_0 < \frac{C_3^2}{6}$, $\mathcal{U}$ takes a positive value but $\mathcal{P}$ has a negative value. For $\psi_0 = \frac{C_3^2}{6}$, the radiation pressure of the bulk vanishes and only the radiation energy density $\mathcal{U}$ of the bulk survives.

The active gravitational mass, $M(r)$, can be obtained from Eq. (22), which may be expressed in the form

$$M(r) = 4\pi\int_0^r \rho^{\mathrm{eff}}\tilde{r}^2 d\tilde{r} = \frac{1}{2G}r\left[1 - \left(\frac{\psi_0}{C_3^2} + \frac{1}{3}\left(1-\mathcal{A}r^2\right)\right)\right]. \tag{32}$$

The radiation energy density and pressure of the bulk fail to be regular at the origin, however, the effective gravitational mass is always positive and vanishes as $r \to 0$. This implies that the effective gravitational mass is singularity free.

5 Shell of the gravastar

For the shell of the gravastar we consider an ultra-relativistic fluid of soft quanta obeying the EOS $p = \rho$, which represents a stiff fluid. This EOS is referred to as the Zel'dovich Universe; it has been studied by various authors in the cosmological context [31–33] and in astrophysical settings [34–36].

Now, using the EOS $p = \rho$, we obtain from Eq. (13)

$$p = \rho = \frac{m}{r^2}, \tag{33}$$

where m is an integration constant. Using the same expression given in (25), we are in a position to determine $\mathcal{U}$, from Eq. (22), which is given by

$$\frac{2}{k^2\lambda}\mathcal{U} = -\frac{1}{3r^2} + \frac{5\psi^2}{3C_3^2 r^2} + \frac{4\psi\psi'}{3C_3^2 r} - \frac{mk^2}{r^2} - \frac{3k^2m^2}{2\lambda r^4}. \tag{34}$$

Using the values of $\mathcal{U}$ and $\mathcal{P}$ in Eq. (22) we obtain

$$\psi^2 + r\psi\psi' = \frac{C_3^2}{3}\left[(1+mk^2) + \frac{2k^2m^2}{\lambda r^2}\right], \tag{35}$$

which is easily solved to yield

$$\psi^2 = \psi_1 + \frac{C_3^2}{3}\left[(1+mk^2) + \frac{4k^2m^2}{\lambda}\frac{\ln r}{r^2}\right]. \tag{36}$$

[ψ_1 is an integration constant]

This allows us to write

$$\mathcal{U} = \frac{k^2\lambda\psi_1}{6C_3^2 r}\left(\frac{5}{r} - 4\right) + \frac{k^2\lambda}{9r^2}\left(1 - 2k^2m\right) + \frac{k^4m^2}{9r^4}\left(2\ln r - \frac{11}{4}\right) \tag{37}$$

and

$$\mathcal{P}=\frac{k^2\lambda\psi_1}{3C_3^2 r}\left(1+\frac{1}{r}\right)-\frac{k^2\lambda}{18r^2}+\frac{mk^4}{9r^2}\left(\lambda-\frac{2m}{r^2}+8m\frac{\ln r}{r^4}\right). \quad (38)$$

Here we assume the interfaces at $r=R$ and $r=R+\epsilon$ which describe the matching surface of two regions i.e., interior and exterior region, are very thin. This means that ϵ is very small i.e., $0<\epsilon\ll 1$. The proper thickness between two interfaces is obtained:

$$l=\int_R^{R+\epsilon}\sqrt{e^{\lambda}}dr=\int_R^{R+\epsilon}\frac{1}{\sqrt{f(r)}}dr. \quad (39)$$

Let us introduce H as the primitive of $\frac{1}{\sqrt{f(r)}}$ i.e., $\frac{dH}{dr}=\frac{1}{\sqrt{f(r)}}$, which provides the proper thickness:

$$l=[H]_R^{R+\epsilon}. \quad (40)$$

Now, using the Taylor series expansion $H(R+\epsilon)$ up to a first order approximation about R, we obtain $H(R+\epsilon)\simeq$ H(R)+ϵ H'(R), and we can write $l\approx\epsilon\frac{dH}{dr}\mid_R$. Therefore, using Eq. (39), the proper thickness is finally given by

$$l\approx\epsilon\left(\frac{C_3^2}{\psi_1}+\frac{3}{(1+mk^2)+\frac{4k^2m^2}{\lambda}\frac{\ln R}{R^2}}\right)^{1/2}. \quad (41)$$

Now, the energy $\mathcal{E}$, within the shell is given by

$$\mathcal{E}=4\pi\int_R^{R+\epsilon}\rho^{eff}\,r^2dr=\left[4\pi m\left(r-\frac{m}{2\lambda r}\right)+\frac{4\pi\psi_1}{k^2C_3^2}(5r-2r^2)\right.$$
$$\left.+\frac{8\pi r}{3k^2}\left(1-2k^2m\right)+\frac{8\pi m^2}{9\lambda}\left(-\frac{\ln r}{r}+\frac{7}{4r}\right)\right]_R^{R+\epsilon}. \quad (42)$$

It is to be noted that the energy within the shell depends on the thickness of the shell as well as brane tension.

6 Exterior region of the gravastar

In general the exterior spacetime on the brane is nonempty, ie., ρ^{eff} and p^{eff} are nonzero. This is due to the presence of the Weyl stresses brought about by bulk graviton effects. In order to close the system of equations in the exterior region one has to prescribe $\mathcal{P}^+$ and $\mathcal{U}^+$, which are not unique. The simplest choice, $\mathcal{P}^+=\mathcal{U}^+=0$, ensures that the exterior is the vacuum Schwarzschild solution. For non-vanishing Weyl stresses in the exterior ($p=\rho=0$), the spacetime is described by the metric

$$ds^2=-f(r)dt^2+f^{-1}(r)dr^2+r^2\left(d\theta^2+\sin^2\theta d\phi^2\right), \quad (43)$$

where $f(r)=1-\frac{2M}{r}+\frac{q}{r^2}$.

This exterior spacetime corresponds to a tidal charged black hole characterized by two parameters: its mass M and dimension-less tidal charge, q. Here, the tidal charge parameter q emerges from the prognosis on the brane of free gravitational field effects in the bulk. This tidal charge q may assume positive and negative values. For a positive tidal charge the metric (43) corresponds to a Reissner–Nordström black hole solution. The presence of the tidal charge increases the gravitational field of this black hole.

7 Entropy within the shell

We shall try to calculate the entropy of the fluid within the shell by adapting the concept of Mazur and Mottola [1], which is given by

$$\mathcal{S}=4\pi\int_R^{R+\epsilon}sr^2\sqrt{e^{\lambda}}\,dr=4\pi\epsilon sr^2\sqrt{e^{\lambda}}. \quad (44)$$

Here, s(r) is the entropy density for the local temperature T(r), which may be written as

$$s(r)=\frac{\alpha^2k_B^2T(r)}{4\pi\hbar^2G}=\alpha\left(\frac{k_B}{\hbar}\right)\sqrt{\frac{p}{2\pi G}}, \quad (45)$$

where α^2 is a dimensionless constant.
Thus the entropy of the fluid within the shell is [applying the same procedure to obtain Eq. (39)] given by

$$\mathcal{S}=\sqrt{\frac{m}{2\pi G}}\left(\frac{4\pi\epsilon\alpha k_BR}{\hbar}\right)\left(\frac{C_3^2}{\psi_1}+\frac{3}{(1+mk^2)+\frac{4k^2m^2}{\lambda}\frac{\ln R}{R^2}}\right)^{1/2}. \quad (46)$$

8 Junction interface and surface stresses

Here we match the interior gravastar geometry, given in Eq. (9), with an exterior braneworld black hole solution at a junction interface Σ. The junction surface Σ is a timelike hypersurface defined by the parametric equation $f\left(x^{\mu}(\xi^i)\right)=0$, where $\xi^i=(\tau,\theta,\phi)$ are the intrinsic coordinates on the hypersurface with proper time τ.

The extrinsic curvature (second fundamental form) associated with a hypersurface Σ is given by

$$K_{ij}^{\pm}=-\eta_{\nu}\left(\frac{\partial^2x^{\nu}}{\partial\xi^i\partial\xi^j}+\Gamma_{\alpha\beta}^{\nu\pm}\frac{\partial x^{\alpha}}{\partial\xi^i}\frac{\partial x^{\beta}}{\partial\xi^j}\right), \quad (47)$$

where η_ν represents the unit normal ($\eta^\nu \eta_\nu = 1$) at the junction defined by

$$n_\nu = \pm \left| g^{\alpha\beta} \frac{\partial f}{\partial x^\alpha} \frac{\partial f}{\partial x^\beta} \right|^{-1/2} \frac{\partial f}{\partial x^\nu}. \tag{48}$$

Since K_{ij} is discontinuous across the Σ, the discontinuity of the metric is usually described by $k_{ij} = K^+_{ij} - K^-_{ij}$.

Now using the Lanczos equation [37–39], we can obtain the intrinsic stress-energy tensor as $S^i_j = \text{diag}\,(-\sigma, \mathfrak{p}, \mathfrak{p})$, where σ is the line energy density and $\mathfrak{p}$ is the line tension, defined by

$$\sigma = -\frac{1}{4\pi a}\left[\sqrt{e^{-\lambda}}\right]^+_- = -\frac{1}{4\pi a}\left[\sqrt{1 - \frac{2M}{a} + \frac{q}{a^2}} - \sqrt{\frac{\psi_0}{C_3^2} + \frac{1}{3}\left(1 - \mathcal{A}a^2\right)}\right]. \tag{49}$$

$$\mathfrak{p} = \frac{1}{8\pi a}\left[\left(1 + \frac{a\nu'}{a}\right)\sqrt{e^{-\lambda}}\right]^+_- = \frac{1}{4\pi a}\left[\frac{1 - M/a}{2\sqrt{1 - \frac{2M}{a} + \frac{q}{a^2}}} - \sqrt{\frac{\psi_0}{C_3^2} + \frac{1}{3}\left(1 - \mathcal{A}a^2\right)}\right]. \tag{50}$$

The surface mass of the thin shell is defined by

$$m_s = 4\pi a^2 \sigma = -a\left[\sqrt{1 - \frac{2M}{a} + \frac{q}{a^2}} - \sqrt{\frac{\psi_0}{C_3^2} + \frac{1}{3}\left(1 - \mathcal{A}a^2\right)}\right]. \tag{51}$$

Here M can be interpreted as the total mass of the braneworld black hole solution. It can be written in the following form:

$$M = \frac{1}{2a}\left[a^2 + q - a^2\zeta^2 + 2am_s\zeta - m_s^2\right], \tag{52}$$

where $\zeta^2(a) = \frac{\psi_0}{C_3^2} + \frac{1}{3}\left(1 - \mathcal{A}a^2\right)$. As an interesting observation we note that the line tension is negative, which implies that there is a line pressure as opposed to a line tension. In our configuration, the junction interference i.e. the region (b) contains two different types of fluid: one is the ultra-relativistic fluid obeying $p = \rho$, and the other is a matter component which arises from the discontinuity of the second fundamental form. This provides an extra surface stress energy and surface tension at the junction interface.

9 Final remarks

In our work we have focused our attention on the problem of modeling gravastars within the context of Randall–Sundrum type II braneworld scenario, based on the fact that our four dimensional spacetime is a three-brane, embedded in a five dimensional bulk. The star model has three distinct regions with different equations of state: (a) the interior solution: $0 \le r \le r_1$, with equation of state $p = -\rho > 0$, (b) the shell: $r_1 < r < r_2$ with equation of state $p = +\rho$, and (c) the exterior: $r_2 < r$ with $p = \rho = 0$ and the solution is found under the assumption of conformal motion.

In our model, we have shown that the upper limit of the interior region can be derived for real conformal factor (i.e., $\psi > 0$). As interesting observation, the dimensionless integration constant, ψ_0, plays an important role in determining the nature of the bulk Weyl scalar $\mathcal{U}$ and non-local anisotropic pressure $\mathcal{P}$. $\psi_0 = C_3^2/3$ represents a barrier, where the radiation pressure of the bulk vanishes but the effect of the bulk Weyl scalar is nonzero and takes positive values at greater radii. In the core region both $\mathcal{U}$ and $\mathcal{P}$ fail to be regular at the origin, however, the effective gravitational mass is always positive and vanishes at $r = 0$, which provides a singularity free solution for our model. Next we have discussed the explicit form of the shell of the star by considering an ultra-relativistic fluid of soft quanta obeying the EOS $p = \rho$, and we found the proper thickness of the shell. We also explicitly show that the energy within the shell depends on the thickness of the shell as well as brane tension. We have also shown that the junction interference contains two different types of matter, namely an ultra-relativistic fluid and matter components appear due to the discontinuity of the affine connections at the region b. The latter is a thin shell of matter content with negative energy density. This newly developed stress-energy tensor supports the consideration of the Casimir effect between compact objects at arbitrary separations [40]. It is argued that these two fluids do not interact and characterize the shell of the gravastar. It is shown that the junction interference between interior and exterior regions contains a thin shell of matter with negative energy density. It seems that this negative energy is very similar to the Casimir effect reported in Ref. [40], in which the authors have obtained the electromagnetic Casimir interaction between compact objects at arbitrary separations. We conclude with a vital point: this paper is not intended to confirm an exact alternative to the braneworld black hole, but this is the first attempt of a project to propose the braneworld gravastar configuration which is an alternative to the braneworld black hole. We constructed a model of a braneworld gravastar by requiring that the interior spacetime admits conformal motion. We have integrated the resulting equations and obtained a class of exact solutions that describe gravastars on the brane which are particular alternatives to the braneworld black hole. Now, it is an obvious query whether this braneworld construction changes in any way the implications of a gravastar solution to the black hole paradoxes already proposed without any braneworld. It needs further research. Hopefully, it will be a subject for future study. However, we must point out that the braneworld construction of a gravastar naturally incorporates anisotropic pressure via the brane embedding. This may not be the case in standard general relativity, that is to say, the anisotropy required for the stability of gravastars needs to be assimilated into the energy-momentum tensor a priori.

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