

The horizon of the lightest black hole

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Abstract We study the properties of the poles of the resummed graviton propagator obtained by resumming bubble matter diagrams which correct the classical graviton propagator. These poles have been previously interpreted as black holes precursors. Here, we show using the horizon wavefunction formalism that these poles indeed have properties which make them compatible with being black hole precursors. In particular, when modeled with a Breit–Wigner distribution, they have a well-defined gravitational radius. The probability that the resonance is inside its own gravitational radius, and thus that it is a black hole, is about one half. Our results confirm the interpretation of these poles as black hole precursors.

1 Introduction

The aim of this paper is to investigate further the properties of the black holes precursors that have been identified in [1] using an effective theory approach for gravity and resummation techniques. In particular, we shall study whether these objects have an horizon and can thus truly be identified with black holes.

Obviously, quantum black holes are quantum gravitational objects, but while we are still far from having a theory of quantum gravity, effective field theory techniques can be reliably applied to general relativity coupled to matter at energy scales below the energy scale at which quantum gravitational effects become of the same magnitude as quantum effects generated by the other forces of nature [2–5]. The leading order terms of the effective field theory are given by

$$S = \int d^4x \sqrt{-g} \left[\frac{m_p^2}{16\pi} \mathcal{R}(g) + c_1 \mathcal{R}(g)^2 + c_2 \mathcal{R}_{\mu\nu}(g) \mathcal{R}^{\mu\nu}(g) + \mathcal{L}_{\text{SM}} + \mathcal{O}(\Lambda_c) \right], \quad (1.1)$$

where $\mathcal{R}(g)$ is the Ricci scalar, $\mathcal{R}^{\mu\nu}(g)$ is the Ricci tensor, the metric $g_{\mu\nu}$ describes the graviton when the action is linearized, and \mathcal{L}_{SM} stands for the Lagrangian of the standard model of particle physics. The action contains two energy scales, the Planck scale $m_p = 1.2209 \times 10^{19}$ GeV which is related to Newton's constant by $m_p = 1/\sqrt{G_N}$ and a scale Λ_c which is the energy scale at which we expect the effective field theory to break down. The constants c_1 and c_2 are dimensionless ones. We have suppressed the cosmological constant and a potential non-minimal coupling of the Higgs boson to the Ricci scalar which are not important for our considerations. It is important to realize that the two scales m_p and Λ_c need not to be identical. The Planck scale is the gravitational coupling constant which appears in the vertices of Feynman diagrams which involve gravitons. The other dimensionful parameter of the model, the cut-off of the effective field theory, Λ_c is related to the Planck scale, but as we shall see shortly, it has recently been shown to be dependent on the number of fields in the matter sector [1].

Working in linearized general relativity and in a Minkowski background, it is possible to resum loop diagrams involving matter fields which correct the graviton's propagator. This correction is calculated [6] in the large N limit, where $N = N_s + 3N_f + 12N_V$ (N_s , N_f , and N_V are, respectively, the number of real scalar fields, fermions and spin 1 fields in the model), while keeping $N G_N$ small. One uses dimensional analysis to regulate the integrals and absorb the divergent parts of the diagrams into the coefficients of \mathcal{R}^2 and $\mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu}$. Note that in the standard model $N_s = 4$, $N_f = 45$, and $N_V = 12$, so $N = 283$. In other words there are many more matter degrees of freedom than gravitational ones (we assume that there is only one massless graviton).

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Loops involving the graviton are thus suppressed by factors of $1/N$ compared to matter loops (at least as long as one considers energies below the Planck scale) and perturbation theory can be trusted.

This large N resummation leads to resummed graviton propagator given by [6]

$$i D^{\alpha\beta,\mu\nu}(q^2) = i(L^{\alpha\mu}L^{\beta\nu} + L^{\alpha\nu}L^{\beta\mu} - L^{\alpha\beta}L^{\mu\nu})\Delta(q^2), \tag{1.2}$$

with $L^{\mu\nu}(q) = \eta^{\mu\nu} - q^\mu q^\nu / q^2$ and

$$\Delta(q^2) = \frac{1}{2q^2 \left[1 - \frac{Nq^2}{120\pi m_p^2} \log\left(-\frac{q^2}{\mu^2}\right) \right]} \tag{1.3}$$

where μ is the renormalization scale. This resummation was first considered when studying the perturbative unitarity of the effective action (1.1) [6–12].

In [1], it has been proposed to interpret the massive poles of this propagator as Planck-size black hole precursors or quantum black holes. The position of the poles determines the mass and the width of the precursors: $p_0^2 = (M_{\text{BH}} + i \Gamma_{\text{BH}}/2)^2$. The poles of the resummed propagator (1.2) are given by

$$q_1^2 = 0$$

$$q_2^2 = (q_3^2)^* = \frac{\frac{120\pi}{N} m_p^2}{W\left(-\frac{120\pi}{N} \frac{m_p^2}{\mu^2}\right)} = (M_{\text{BH}} + i \Gamma_{\text{BH}}/2)^2, \tag{1.4}$$

where $W(x)$ is the Lambert W-function. The pole at $q^2 = 0$ corresponds to the usual massless graviton. The position of the pole and hence the energy scale at which non-perturbative effects are becoming important depends on the matter content of the model, i.e. on N . As mentioned above, in the standard model one has $N = 283$ and the complex pole at $q^2 = q_2^2$ corresponds to a particle with mass [1]

$$M_{\text{BH}} \simeq 7.2 \times 10^{18} \text{ GeV} \simeq \sqrt{\frac{120\pi}{N} \frac{m_p}{2}}, \tag{1.5}$$

and width

$$\Gamma_{\text{BH}} \simeq 6.0 \times 10^{18} \text{ GeV} \simeq \sqrt{\frac{120\pi}{N} \frac{m_p}{2}}. \tag{1.6}$$

As explained in [1], the mass and the width of the lightest of black holes depends on the parameter N . It is natural to interpret these poles as black hole precursors or non-local extended objects since the resummed propagator leads to non-local effects in gravity [13] and quantum field theory [14]. This interpretation is also compatible with generic arguments [15–18] based on quantum mechanics and general relativity which lead to the notion of a minimal length and thus some kind of non-locality. Obviously, these estimates

depend on the renormalization scale which is taken of the order of the Planck mass. One can use the spectral decomposition to write the propagator as

$$\Delta(q^2) = \frac{1}{q^2} + \frac{R_2}{q^2 - q_2^2} + \frac{R_3}{q^2 - q_3^2} + \int_{M_{\text{BH}}^2}^{\infty} ds \frac{\rho(s)}{s - q^2}, \tag{1.7}$$

where $R_{2/3}$ are the residues at the two non-trivial poles. The second complex pole at $q^2 = q_3^2$ would lead to acausal effects. Several mechanisms could eliminate this pole (see e.g. [13, 19–24], where the log-term is reinterpreted as a non-local interpolating function which leads to causal effects). However, we shall assume that this is the scale above which we cannot trust perturbation theory in the standard model.

The effective field theory does not provide reliable information about the spectral density function $\rho(s)$. However, we have some information as regards this function coming from black hole physics. We expect the classical regime to begin around 5–20 times the mass of the first black hole (see e.g. [25]). At that scale, we expect to have a continuum since semi-classical black holes are expected to have a continuous mass spectrum. Between M_{BH} and $(5 - 20) \times M_{\text{BH}}$, the situation is more difficult. In [26], it was argued that the mass spectrum of quantum black holes needs to be quantized, otherwise their virtual effects could lead to large effects in low energy experiments such as measurements of the anomalous magnetic moment of the muon. We will assume that $\rho(s)$ is discrete between M_{BH} and the continuous, semi-classical region. We assume that the resonances are sharply peaked and do not overlap much. We shall require that the spacing between the first quantum excitation which we identified as a pole of the resummed propagator and the next excitation is larger than the width of the black hole precursor. In that case, we should be able to trust the model up to a scale

$$\Lambda_c \simeq \sqrt{\frac{120\pi}{N}} m_p \simeq 1.4 \times 10^{19} \text{ GeV}, \tag{1.8}$$

which corresponds to twice the width of the black hole precursor. In other words, we model the mass spectrum between M_{BH} and the continuum and require that we can trust our model up to the scale Λ_c , which we take to be the cut-off for our model of quantum black holes.

2 Horizon wave-function

Our knowledge of black holes in general relativity suggests that these objects are states somewhat similar to hadrons in QCD, except that gravity democratically confines all sorts of particles above some critical scale, rather than just strongly interacting ones. This should be particularly true for quantum black holes [27, 28]. It is therefore very likely that, although their existence can be inferred within perturbation theory,

like we have recalled in the previous section, a full description of their quantum properties requires a non-perturbative approach, like the horizon wave-function (HWF) formalism (for the details, see Refs. [29–33]; for a similar picture of the black hole horizon, see Ref. [34]).

This approach assumes the validity of the Einstein equations in the non-perturbative regime, and it amounts to quantizing the Misner–Sharp mass for spherically symmetric sources, $m(r, t) = 4\pi \int_0^r \rho(\bar{r}, t) \bar{r}^2 d\bar{r}$, which in turn defines the gravitational radius of the system,

$$R_H = 2 \ell_p \frac{m}{m_p}. \tag{2.1}$$

The latter then identifies the location of a trapping surface if $R_H(r, t) = r$. If this relation holds in the vacuum outside the region where the source is located, R_H becomes the usual Schwarzschild radius, and the above argument gives a mathematical foundation to Thorne’s hoop conjecture [35], which roughly states that a black hole forms when the impact parameter b of two colliding small objects is shorter than $R_H = 2 \ell_p E/m_p$, where E is the total energy in the center of mass frame. This classical description becomes questionable for sources of the Planck size or lighter, since quantum effects may not be neglected. The Heisenberg principle of quantum mechanics introduces an uncertainty in the spatial localization of a particle of the order of the Compton–de Broglie length, $\lambda_m \simeq \ell_p m_p/m$. Since quantum physics is a more refined description of reality, we could argue that R_H only makes sense if $R_H \gtrsim \lambda_m$ or $m \gtrsim m_p$.

The HWF formalism starts from decomposing the particle’s state into energy eigenstates,

$$|\psi_S\rangle = \sum_E C(E) |\psi_E\rangle, \tag{2.2}$$

where the sum represents the spectral decomposition in Hamiltonian eigenmodes,

$$\hat{H} |\psi_E\rangle = E |\psi_E\rangle, \tag{2.3}$$

and H should be specified depending on the system at hand. The gravitational radius (2.1) is then quantized by expressing the energy $E = m$ in terms of the Schwarzschild radius r_H and define the corresponding wave-function¹

$$\psi_H(r_H) = \mathcal{N}_H C(r_H(E)), \tag{2.4}$$

whose normalization \mathcal{N}_H can be fixed by using the norm defined by the scalar product

$$\langle \psi_H | \phi_H \rangle = 4\pi \int_0^\infty \psi_H^*(r_H) \phi_H(r_H) r_H^2 dr_H. \tag{2.5}$$

Let us remark that this quantum description of the gravitational radius assumes that, in the static case, the only relevant

¹ Note we use the lower letter r_H to distinguish this quantum variable from the classical Schwarzschild radius R_H .

degrees of freedom associated with the gravitational structure of space-time (which classically give rise to trapping surfaces) are those turned on by the degrees of freedom of the matter source. This implies that we can just consider “on-shell” states, for which Eq. (2.1) holds as an operator equation, and neglect gravitational fluctuations, which could be studied by employing standard background field method techniques.

The normalized wave-function ψ_H yields the probability that the gravitational radius has size $r = r_H$, but this radius is “fuzzy”, like the energy. Moreover, having defined the ψ_H associated with a given ψ_S , we can also define the conditional probability density that the particle lies inside its own gravitational radius as

$$\mathcal{P}_<(r < r_H) = P_S(r < r_H) \mathcal{P}_H(r_H), \tag{2.6}$$

where

$$P_S(r < r_H) = \int_0^{r_H} \mathcal{P}_S(r) dr = 4\pi \int_0^{r_H} |\psi_S(r)|^2 r^2 dr \tag{2.7}$$

is the usual probability that the system lies within the size $r = r_H$, and

$$\mathcal{P}_H(r_H) = 4\pi r_H^2 |\psi_H(r_H)|^2 \tag{2.8}$$

is the probability density that the gravitational radius has size $r = r_H$. One can also view $\mathcal{P}_<(r < r_H)$ as the probability density that the sphere $r = r_H$ is a trapping surface, so that the probability that the system is a black hole (of any horizon size), will be obtained by integrating (2.6) over all possible values of r_H , namely

$$P_{BH} = \int_0^\infty \mathcal{P}_<(r < r_H) dr_H. \tag{2.9}$$

Note that the Planck mass m_p and length ℓ_p play a crucial role in the above construction, since they explicitly appear in the definition of the gravitational radius (2.1). In the following, we shall assume their standard values. This is consistent with our effective theory approach since we do not consider corrections to the coefficient of the Ricci scalar in the effective action.

2.1 Gravitational radius and uncertainty

We can now derive the HWF for the non-trivial pole corresponding to a well-defined one-particle state (1.4). For simplicity, we model the lightest black hole using a Breit–Wigner distribution

$$\psi_S^*(E) \psi_S(E) \equiv \rho(E) = \frac{\mathcal{N}}{(E^2 - M_{BH}^2)^2 + M_{BH}^2 \Gamma_{BH}^2}, \tag{2.10}$$

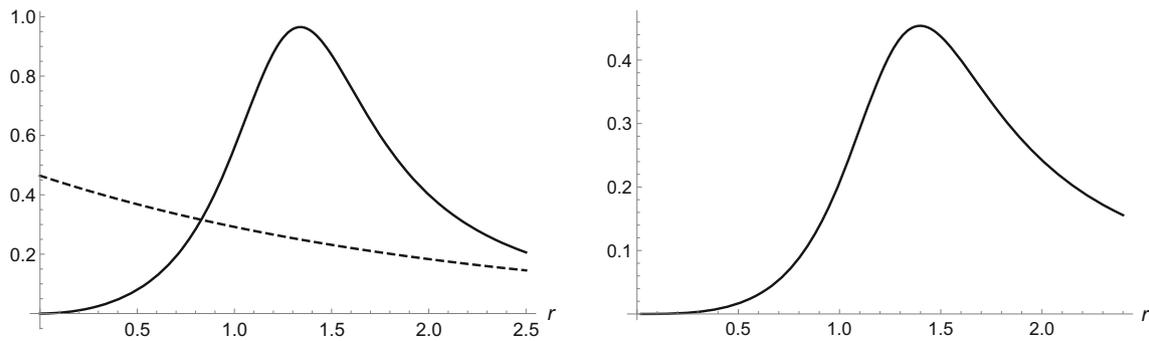


Fig. 1 *Left panel* probability density \mathcal{P}_H in Eq. (2.12) for the horizon size (solid line) compared with the probability density \mathcal{P}_S of the resonance size (dashed line). *Right panel* probability density $\mathcal{P}_<$ in

Eq. (2.6) the resonance is located within its gravitational radius of size r . All lengths are in units of ℓ_p

where \mathcal{N} is a normalization factor, and $E < E_c$, with E_c a cut-off corresponding to the beginning of the continuum spectrum in Eq. (1.7). In the following we shall assume for simplicity, and in agreement with (1.7), that there is no other discrete resonance in the spectrum below the cut-off for our model, so that

$$E_c \simeq 2 M_{\text{BH}} \simeq \Lambda_c. \tag{2.11}$$

The corresponding HWF is then obtained by assuming the unnormalized HWF $|\tilde{\psi}_H|^2 \simeq \rho$, and the corresponding probability density (2.8) then reads

$$\begin{aligned} \mathcal{P}_H dr_H &= \frac{m_p^3 M_{\text{BH}}}{4 \ell_p^3 F_0(\gamma_{\text{BH}}, \Lambda)} \left[\frac{r_H^2 dr_H}{\left(\frac{m_p^2 r_H^2}{4 \ell_p^2} - M_{\text{BH}}^2\right)^2 + M_{\text{BH}}^2 \Gamma_{\text{BH}}^2} \right] \\ &= \frac{(x+1)^{\frac{1}{2}}}{x^2 + \gamma_{\text{BH}}^2} \frac{dx}{F_0(\gamma_{\text{BH}}, \Lambda)}. \end{aligned} \tag{2.12}$$

where $F_0(\gamma_{\text{BH}}, \Lambda) \simeq F_0(0.83, 3)$ is a number of order one (see Appendix A). We can now compute expectation values of powers of the gravitational radius,

$$\langle \hat{r}_H^n \rangle \simeq \left(2 \ell_p \frac{M_{\text{BH}}}{m_p} \right)^n \frac{F_n(0.83, 3)}{F_0(0.83, 3)}, \tag{2.13}$$

from which, in particular, one finds

$$\langle \hat{r}_H \rangle \simeq 1.4 \ell_p, \tag{2.14}$$

and

$$\langle \hat{r}_H^2 \rangle \simeq 2.2 \ell_p^2, \tag{2.15}$$

so that the relative uncertainty in the gravitational radius is given by

$$\sqrt{\frac{\langle \hat{r}_H^2 \rangle - \langle \hat{r}_H \rangle^2}{\langle \hat{r}_H \rangle^2}} \simeq 0.3, \tag{2.16}$$

which means that the gravitational radius is well defined for such a quantum object.

2.2 Black hole probability

We cannot yet claim the resonance is a black hole. For that, we need to show that the quantum state of this resonance is located mostly inside the gravitational radius.

First of all, we obtain the resonance wave-function in position space by projecting ψ_S in Eq. (2.10) on the spherical Bessel function

$$j_0(E, r) = \frac{\sin(E r)}{E r}, \tag{2.17}$$

that is,

$$\begin{aligned} \psi_S(r) &= \int_0^\infty \psi_S(E) j_0(E, r) dE \\ &\simeq \int_0^\infty \frac{E^2 dE}{E^2 - M_{\text{BH}}^2 + i M_{\text{BH}} \Gamma_{\text{BH}}} \frac{\sin(E r)}{E r} \\ &\simeq \frac{1}{r} \exp \left[-i \frac{M_{\text{BH}}}{m_p} \sqrt{1 - i \frac{\Gamma_{\text{BH}}}{M_{\text{BH}}} \frac{r}{\ell_p}} \right], \end{aligned} \tag{2.18}$$

where we omitted a normalization factor for simplicity. We can then compute the probability density in Eq. (2.7) for the resonance size, the probability (2.6) that the resonance is inside its own gravitational radius, and the probability (2.9) that it is a black hole

$$P_{\text{BH}}(M_{\text{BH}}, \Gamma_{\text{BH}}) \simeq 0.48. \tag{2.19}$$

See also Fig. 1 for plots of the above quantities. This result is interesting as it is compatible with the interpretation of the poles in the resummed graviton propagators as black holes precursors. If the probability had been much smaller than one, the interpretation as black hole would have been challenged. If it had been close to one, we would expect the black hole to be semi-classical but this would be inconsistent with our expectation and model for the mass spectrum described

above. A probability around one half is precisely what one would expect from a black hole precursor.

2.3 Decay time

The decay time τ of a common resonance can be estimated from the uncertainty relation

$$\tau \Delta E \simeq \ell_p m_p, \tag{2.20}$$

and would be extremely short for our lightest black hole, namely

$$\tau_{\text{BH}} \simeq \frac{\ell_p m_p}{\Gamma_{\text{BH}}} \simeq \ell_p = \tau_p. \tag{2.21}$$

However, if the probability P_{BH} is significantly close to one, the resonance should decay more slowly. Given the non-local nature of P_{BH} , a precise estimate would require a numerical analysis [29], but we can obtain a rough estimate by simply considering that the (initial) decay probability is reduced by $(1 - P_{\text{BH}})$, so that

$$\tau_{\text{BH}} \simeq \frac{\ell_p m_p}{\Gamma_{\text{BH}} (1 - P_{\text{BH}})} \simeq 2 \tau_p. \tag{2.22}$$

Again this result confirms the interpretation of the poles as black hole precursors, since their lifetime is close to the Planck time.

We emphasize that all estimates in this section were obtained by assuming that the proper mass and length scales in the definition of the gravitational radius (2.1) have their traditional values (i.e. $m_p \sim 10^{19}$ GeV). This is consistent, since the coefficient of the Ricci scalar in the effective action is not affected by the quantum corrections we have considered. Here, we have not considered the running of the Planck mass, since there are not many particles in the standard model, this would be a small effects [36–38]. There are, however, well-known models which can affect significantly the value of the coefficient of the Ricci scalar. For example, models with a large extra-dimensional volume [39–41]. Note that these models would not only affect the value of the coefficient of the Ricci scalar, but they also affect the effective theory itself and thus the resummed propagator calculation as well. This effect would have to be carefully studied in these models.

3 Conclusions

In this paper, we have studied the properties of the poles of the resummed graviton propagator obtained by resumming bubble matter diagrams which correct the classical graviton propagator. These poles had been interpreted as black holes precursors previously. Here, we have shown using the HWF formalism that these poles indeed have properties which make

them compatible with being black hole precursors. In particular, when modeled with a Breit–Wigner distribution, they have a well-defined gravitational radius. The probability that the resonance is inside its own gravitational radius, and thus that it is a black hole is roughly 50 %. The mass, width, and gravitational radius as well as the existence of an horizon depends on the matter content of the theory. Here we have assumed that the particle content is that of the standard model of particle physics. Our results confirm the previously proposed interpretation of these poles as black hole precursors.

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A Useful integrals

In order to compute integral of functions such as the one in Eq. (2.12), it is useful to define the dimensionless variables

$$x + 1 = \frac{m_p^2 r_H^2}{4 \ell_p^2 M_{\text{BH}}^2}$$

$$\gamma_{\text{BH}} = \frac{\Gamma_{\text{BH}}}{M_{\text{BH}}} \simeq 0.83, \tag{A.1}$$

and

$$\Lambda = \frac{m_p^2 R_c^2}{4 \ell_p^2 M_{\text{BH}}^2} - 1 = \left(\frac{E_c}{M_{\text{BH}}} \right)^2 - 1 \simeq 3. \tag{A.2}$$

We can then write

$$F_n(\gamma_{\text{BH}}, \Lambda) = \int_{-1}^{\Lambda} \frac{(x + 1)^{\frac{n+1}{2}}}{x^2 + \gamma_{\text{BH}}^2} dx, \tag{A.3}$$

and obtain, in particular,

$$F_0(6/7, 3) \simeq 2.8 \tag{A.4}$$

$$F_1(6/7, 3) \simeq 3.5 \tag{A.5}$$

$$F_2(6/7, 3) \simeq 4.6. \tag{A.6}$$

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