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Impact of family-non-universal Z' boson on pure annihilation $B_s \to \pi^+\pi^-$ and $B_d \to K^+K^-$ decays

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Abstract We study the $B_s \to \pi^+\pi^-$ and $B_d \to K^+K^-$ decays in the standard model and the family-non-universal Z' model. Since none of the quarks in the final states is the same as the initial quark, these decay modes can occur only via power-suppressed annihilation diagrams. Despite the consistency of the standard model prediction with the available data, room remains for a light Z' boson. Taking into account the Z' contribution, we find that theoretical results for the branching fractions can better accommodate the data. With the relevant data, we also derive a constraint on the parameter space for the Z'. Moreover, for $B_d \to K^+K^-$, both the direct and the mixing-induced CP asymmetry are sensitive to the couplings between Z' and fermions in the parameter spaces constrained by the data. The measurements at future experimental facilities, including the LHCb, Belle-II, and the proposed high energy e^+e^- collider, will provide us useful hints for direct searching for the light Z' boson.

1 Introduction

Since the discovery of the Higgs boson at the Large Hadron Collider (LHC) [1,2], the search for new physics (NP) degrees of freedom beyond the standard model (SM) has become one of the most important tasks in particle physics. In some NP models, when the initial group breaks down to $SU(2)_L \times U(1)_Y$ of the SM, an extra $U(1)'$ gauge symmetry will usually be produced, which results in an additional massive neutral gauge boson called Z'. Meanwhile, the resulting triangle anomalies could be canceled by the addition of new heavy chiral fermions. Quite a few models are of this type, such as grand unified theories based on the gauge group SO(10) model, see, for example, [3], the E_6 model [4–8], supersymmetric models [9–12], and some string inspired models [13–16] (for a review, see Refs. [17,18]). Although

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the $U(1)'$ charges are usually family-universal, it is not mandatory to be so, and the family-non-universal Z' has been realized in some models, such as in the aforementioned E_6 model [4–8].

On the experimental side, many efforts have been made to search for the Z' directly at the LEP, Tevatron, and LHC. With the assumption that the Z' couplings to the SM fermions are similar to those of the Z boson of the SM (the so-called sequential Z' model), the direct searches for the Z' can be performed in dilepton events. Now, the lower mass limit has been set as 2.86 TeV at the 95 % confidence level (CL) from collisions at 8 TeV with an integrated luminosity of 19.5 fb^{-1} by using e^+e^- and $\mu^+\mu^-$ [19] events, and this value becomes 1.90 TeV using the $\tau^+\tau^-$ events [20]. However, if the Z' boson does not couple to the charged leptons, the current constraints from the LHC are no longer valid. Theoretically, such leptophobic Z' bosons were first introduced several years ago in the context of the R_b–R_c puzzle [21], and as a possible explanation of anomalous high-E_T jet events at CDF [22]. Although these experimental effects ultimately disappeared, thereby removing the original motivation for such new physics, models with a leptophobic Z' still remain as viable candidates for physics beyond the SM, and it is therefore worthwhile to explore their phenomenology. In Refs. [23,24], it was shown that a leptophobic Z' can appear in E_6 models due to the mixing of the gauge boson kinetic terms. Very recently, the phenomenological studies in the possible colliders have been explored [25,26]. Complementary to the direct search, some features of the leptophobic Z' boson can also be constrained indirectly in the flavor physics. The family-non-universal Z' boson may induce tree-level flavor changing neutral currents (FCNCs) and thus they are severely bounded by experiment, most notably meson mixing [27–39]. Other effects of the FCNC induced by Z' in flavor physics have been studied extensively in past decades [40–61]. Motivated by the above arguments, in this work, we aim to perform a comprehensive analysis of the impact

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of a family-non-universal Z' boson on the pure annihilation decays $B_d \rightarrow K^+K^-$ and $B_s \rightarrow \pi^+\pi^-$. Since these modes are power suppressed in the heavy quark limit, their branching ratios are expected to be very small, and hence the sensitivity to NP can be enhanced.

Experimentally, the decay mode $B_s \rightarrow \pi^+\pi^-$ was first reported by the CDF collaboration,

$$\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = (0.60 \pm 0.17 \pm 0.04) \times 10^{-6} \text{ [62]}, \quad (1)$$

and it was soon confirmed by the LHCb collaboration with the 0.37 fb^{-1} data,

$$\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = (0.98^{+0.23}_{-0.19} \pm 0.07) \times 10^{-6} \text{ [63]}. \quad (2)$$

So, the averaged result is given as [64]

$$\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = (0.76 \pm 0.13) \times 10^{-6}. \quad (3)$$

The branching fraction of another pure annihilation decay mode $B_d \rightarrow K^+K^-$ has also been measured:

$$\mathcal{B}(B_d \rightarrow K^+K^-) = \begin{cases} (2.3 \pm 1.0 \pm 1.0) \times 10^{-7}, & \text{CDF [62];} \\ (1.2^{+0.8}_{-0.7} \pm 0.1) \times 10^{-7}, & \text{LHCb [63];} \end{cases} \quad (4)$$

and the averaged result is given as [64]

$$\mathcal{B}(B_d \rightarrow K^+K^-) = (1.3^{+0.6}_{-0.5}) \times 10^{-7}. \quad (5)$$

Note that the center value of CDF is larger by a factor of 2 than that of LHCb, though they are consistent with each other due to the large uncertainties.

Theoretically, within the QCD factorization (QCDF) approach [65–69], due to the existence of the endpoint singularity, only an estimation of the orders of magnitude can be given for these two decays by introducing new phenomenological parameters (ρ_A and ϕ_A) or an effective gluon propagator [70,71]. The predicted branching fractions are at the order of 10^{-8} [65–69,72–74]. The effects of SU(3) asymmetry breaking have also been discussed [75,76]. On the contrary, the perturbative QCD (PQCD) approach [77–79] retains the transverse momenta of all inner quarks and then smears the endpoint singularity, which makes the perturbative calculations reliable. On the basis of PQCD, the decays $B_s \rightarrow \pi^+\pi^-$ and $B_d \rightarrow K^+K^-$ have been explored in Refs. [80–82], respectively. In Ref. [83], the authors have revisited these two decays with new parameters (especially for the distribution amplitudes of light mesons), and the obtained results are

$$\mathcal{B}(B_s \rightarrow \pi^+\pi^-) = \left(5.10^{+1.96+0.25+1.05+0.29}_{-1.68-0.19-0.83-0.20}\right) \times 10^{-7}, \quad (6)$$

$$\mathcal{A}_{\text{CP}}(B_s \rightarrow \pi^+\pi^-) = \left(-2.3^{+0.0+0.3+0.1+0.1}_{-0.3-0.2-0.2-0.1}\right) \%, \quad (7)$$

$$\mathcal{B}(B_d \rightarrow K^+K^-) = \left(1.56^{+0.44+0.23+0.22+0.13}_{-0.42-0.22-0.19-0.09}\right) \times 10^{-7}, \quad (8)$$

$$\mathcal{A}_{\text{CP}}(B_d \rightarrow K^+K^-) = \left(18.9^{+0.2+1.4+0.1+0.8}_{-1.9-2.2-1.4-1.1}\right) \%, \quad (9)$$

where the first two errors come from the LCDAs of light mesons, and the third ones are from the uncertainties of the heavy mesons; the last uncertainties are from the uncertainties of the relevant CKM elements. It is apparent that with large uncertainties the above results are in agreement with the experimental data well. In spite of the agreement, by comparing the above predictions with the experimental results, one finds that the center value of the LHCb measurement of $B_s \rightarrow \pi^+\pi^-$ ($B_d \rightarrow K^+K^-$) is larger (smaller) than the theoretical prediction, which indicates that there may be some room left for the survival of a light Z' boson. In the following we will use the PQCD approach and investigate the impact of the family-non-universal leptophobic Z' model on the branching fractions and CP asymmetries of these two decays. Our results can be stringently tested at the LHCb experiment, Belle-II, and the future high energy e^+e^- collider.

This paper is organized as follows. In Sect. 2, after a brief introduction to the PQCD approach, we will present the numerical results of $B_s \rightarrow \pi^+\pi^-$ and $B_d \rightarrow K^+K^-$ in SM. In Sect. 3, we will discuss the effects of the Z' on the branching fractions and CP asymmetries of these two decay modes. Finally, conclusions will be drawn in Sect. 4.

2 SM calculation

In this section, we will start with the effective weak Hamiltonian for the $b \rightarrow D$ ($D = d, s$) transitions, which are given by [84]

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}}$$
$$\times \left\{ \sum_{q=u,c} V_{qb}V_{qD}^*(C_1 O_1^q + C_2 O_2^q) - V_{tb}V_{tD}^* \sum_{i=3}^{10} C_i O_i \right\} + \text{H.c.}, \quad (10)$$

where $V_{qb(D)}$ are the Cabibbo–Kobayashi–Maskawa (CKM) matrix elements. The explicit expressions of the local four-quark operators O_i ($i = 1, \ldots, 10$) and the corresponding Wilson coefficients C_i at different scales have been studied explicitly in the literature (for example, see Ref. [84]). For convenience, we list the numerical values of the Wilson coefficients at two typical scales, $\mu = 2.1, 1.0$ GeV, in Table 1. Note that $O_{1,2}^q$ are tree operators and the others O_{3-10} are penguin ones.

Based on the k_T factorization, the PQCD approach has been applied to calculate many non-leptonic B meson decays [77–80]. In this approach, the decay amplitude is conceptually written as

$$\mathcal{A} \sim \int dx_1 dx_2 dx_3 \int b_1 db_1 b_2 db_2 b_3 db_3 \text{Tr}[C(t)$$
$$\Phi_B(x_1, b_1)\Phi_2(x_2, b_2)\Phi_3(x_3, b_3)H(x_i, b_i, t)S_t(x_i)e^{-S(t)}], \quad (11)$$

Table 1 The Wilson coefficients C_i in the NDR scheme at the scale $\mu = 2.1, 1.0$ GeV

Scale μ	C_1	C_2	C_3	C_4	C_5	C_6	C_7/α_{em}	C_8/α_{em}	C_9/α_{em}	C_{10}/α_{em}
2.1 GeV	−0.283	1.135	0.021	−0.049	0.01	−0.060	−0.018	0.081	−1.266	0.321
1.0 GeV	−0.429	1.224	0.034	−0.072	0.01	−0.104	−0.023	0.134	−1.366	0.483

where x_i are the momentum fractions taken by light quarks in each mesons, and b_i are the conjugate variables of the transverse momenta of light quarks. "Tr" means the trace over both Dirac and color indices. In light of the factorization hypothesis, the Wilson coefficient $C(t)$ encapsulates the dynamics from m_W down to the scale t, where $t \sim O(\sqrt{M_B \Lambda_{QCD}})$ is the typical scale of the concerned annihilation type decays. The hard part H, involving the four-quark operators and the hard gluon, describes the hard dynamics characterized by the scale t, so it can be calculated perturbatively. The wave function Φ_M, standing for hadronization of the quark and anti-quark into the meson M, is independent of the specific processes and thus universal. The factor $S_t(x_i)$ arises from the resummation of the large double logarithms ($\ln^2 x_i$) on the longitudinal direction, while the Sudakov form factor $e^{-S(t)}$ is from the resummation of the double logarithm $\ln^2 k_T$. Fortunately, the endpoint could be smeared effectively with the help of these two functions, which makes our calculation reliable.

In particular, the wave functions $\Phi_{M,\alpha\beta}$ (α, β being Dirac indices) are decomposed in terms of the spin structures, $1_{\alpha\beta}$, $\gamma^\mu_{\alpha\beta}$, $(\gamma_5\sigma^{\mu\nu})_{\alpha\beta}$, $(\gamma^\mu\gamma_5)_{\alpha\beta}$ and $(\gamma_5)_{\alpha\beta}$. For the heavy pseudo-scalar meson B_q ($q = d, s$), the wave function $\Phi_{B,\alpha\beta}$ is given by

$$\Phi_{B,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}}\{(\not{p}_B\gamma_5)_{\alpha\beta} + m_B\gamma_{5\alpha\beta})\}\phi_B(x, b),$$
(12)

where $N_c = 3$ is the color degree of freedom, and p_B is the momentum of B meson. The scalar distribution amplitude ϕ_B is normalized by its own decay constant f_B,

$$\int_0^1 \phi_B(x, b = 0)dx = \frac{f_B}{2\sqrt{2N_c}}.$$
(13)

In this work, we employ the function

$$\phi_B(x, b) = N_B x^2(1 - x)^2 \exp\left[-\frac{1}{2}\left(\frac{xm_B}{\omega_B}\right)^2 - \frac{\omega_B^2 b^2}{2}\right],$$
(14)

where the shape parameter $\omega_{B_d} = 0.4$ GeV ($\omega_{B_s} = 0.45$ GeV) has been adopted in all previous analyses of exclusive $B_{d(s)}$ meson decays [77–81].

In contrast to the heavy meson, the wave functions of light meson ϕ_M are much complicated due to the non-negligible

chiral mass. Taking the K^+ meson as an example for illustration, we define its wave function as

$$\Phi_{K^+,\alpha\beta}(x, b) = \frac{i}{\sqrt{2N_c}}[(\gamma_5\not{p}_K)\phi_K^A(x, b) + m_{0K}\gamma_5\phi_K^P(x, b) + m_{0K}\gamma_5(\not{v}\not{n} - 1)\phi_K^T(x, b)]_{\alpha\beta},$$
(15)

where p_K is its momentum, and $m_{0K} = m_K^2/(m_u + m_s)$ is the aforementioned chiral mass. $\vec{v}$ and $\vec{n}$ are unit vectors, and $\vec{v}$ ($\vec{n}$) is (anti-)parallel to $\vec{p}_K$. As nonperturbative parameters, the light cone distribution amplitudes (LCDAs), $\phi_M^{A,P,T}$, should be fixed by experimental data in principle. Though there is no direct experimental measurement for the moment up to now, the non-leptonic charmless B_q decays already give much hints on them [77–80]. Since the PQCD approach had already given very good results for these decays, especially for the direct CP asymmetries in $B^0 \to \pi^+\pi^-$ and $B^0 \to K^+\pi^-$ decays, we shall adopt the well-constrained LCDAs of the mesons in these papers [77–81]:

$$\phi_\pi^A(x) = \frac{3f_\pi}{\sqrt{6}}x(1 - x)[1 + 0.44C_2^{3/2}(t)],$$

$$\phi_\pi^P(x) = \frac{f_\pi}{2\sqrt{6}}[1 + 0.43C_2^{1/2}(t)],$$

$$\phi_\pi^T(x) = -\frac{f_\pi}{2\sqrt{6}}[C_1^{1/2}(t) + 0.55C_3^{1/2}(t)],$$

$$\phi_K^A(x) = \frac{3f_K}{\sqrt{6}}x(1 - x)[1 + 0.17C_1^{3/2}(t) + 0.2C_2^{3/2}(t)],$$

$$\phi_K^P(x) = \frac{f_K}{2\sqrt{6}}[1 + 0.24C_2^{1/2}(t)],$$

$$\phi_K^T(x) = -\frac{f_K}{2\sqrt{6}}[C_1^{1/2}(t) + 0.35C_3^{1/2}(t)],$$
(16)

with the Gegenbauer polynomials defined as

$$C_1^{1/2}(t) = t, C_1^{3/2}(t) = 3t, C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1),$$

$$C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), C_3^{1/2}(t) = \frac{1}{2}t(5t^2 - 3),$$
(17)

and $t = 2x - 1$. It should be stressed that we have dropped the terms proportional to $C_4^{1/2,3/2}$, and only kept the first two terms, following the arguments of [81].

Now we turn to a calculation of the hard part H. According to the effective Hamiltonian, Eq. (10), we can draw four kinds of Feynman diagrams contributing to the $B_d \to K^+K^-$ and $B_s \to \pi^+\pi^-$ decays at the leading order, as shown in Fig. 1. The four diagrams are grouped into two types: (a) and (b) are

Fig. 1 The Feynman diagrams for annihilation contribution, with possible four-quark operator insertions
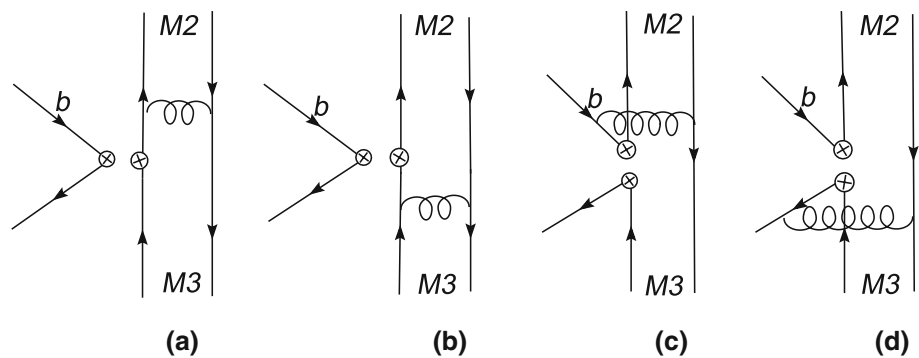
factorizable diagrams, and (c) and (d) are non-factorizable ones. Due to the current conservation, the contributions from the factorizable diagrams (a) and (b) will be canceled exactly by each other, so that the contributions from diagrams (a) and (b) are null. As far as diagrams (c) and (d) are concerned, by inserting the possible operators, we can obtain the amplitudes for the non-factorizable annihilation diagram M_{ann}^{LL} and M_{ann}^{SP}, where LL stands for the contribution from $(V-A)(V-A)$ operators, and SP for the contribution from $(S-P)(S+P)$ operators, which result from the Fierz transformation of the $(V-A)(V+A)$ operators. The expressions of M_{ann}^{LL} and M_{ann}^{SP} and the inner functions can be found in [81]. Finally, we obtain the total decay amplitudes for the decays of our concern as

$$
\begin{aligned}
&\mathcal{A}(\overline{B}_s \to \pi^+\pi^-) \\
&= \frac{G_F}{\sqrt{2}} \Bigg\{ V_{ub}V_{us}^* M_{\text{ann}}^{LL}[C_2] - V_{tb}V_{ts}^* \bigg(M_{\text{ann}}^{LL}[C_4 + C_{10}] \\
&\quad + M_{\text{ann}}^{SP}[C_6 + C_8] + M_{\text{ann}}^{LL}\left[C_4 - \frac{1}{2}C_{10}\right]_{\pi^- \leftrightarrow \pi^+} \\
&\quad + M_{\text{ann}}^{SP}\left[C_6 - \frac{1}{2}C_8\right]_{\pi^- \leftrightarrow \pi^+} \bigg) \Bigg\};
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&\mathcal{A}(\overline{B}_d \to K^+K^-) \\
&= \frac{G_F}{\sqrt{2}} \Bigg\{ V_{ub}V_{ud}^* M_{\text{ann}}^{LL}[C_2] - V_{tb}V_{td}^* \bigg(M_{\text{ann}}^{LL}[C_4 + C_{10}] \\
&\quad + M_{\text{ann}}^{SP}[C_6 + C_8] + M_{\text{ann}}^{LL}[C_4 - \frac{1}{2}C_{10}]_{K^- \leftrightarrow K^+} \\
&\quad + M_{\text{ann}}^{SP}[C_6 - \frac{1}{2}C_8]_{K^- \leftrightarrow K^+} \bigg) \Bigg\}.
\end{aligned}
\tag{19}
$$

In Eq. (18), when π^+ and π^- are exchanging, the M_{ann}^{LL} results are obtained because of the SU(2) symmetry. Furthermore, if we ignore the small x_1 (the momentum fraction of s quark in the $\overline{B}_s$ meson) in the denominators, M_{ann}^{LL} is the same as M_{ann}^{SP}, too.[1] However, for $\overline{B}_d \to K^+K^-$, $M_{\text{ann}}^{LL(SP)}$ do not have the same formulas as $M_{\text{ann}}^{LL(SP)}|_{K^- \leftrightarrow K^+}$ due to the difference between the mass of the up (down) quark and

that of the strange quark, and such a difference might affect the direct CP asymmetry.

In fact, in our calculations there are many uncertainties, the most important one of which is from the distribution amplitude of initial heavy meson, because it cannot be calculated directly from QCD up till now. In the following work, we shall vary the shape parameter $\omega_{B_d} = 0.40 \pm 0.05$ and $\omega_{B_s} = 0.50 \pm 0.05$. Furthermore, the contributions from the next-leading order (NLO) have not been treated. In the current work, to estimate the uncertainties of NLO, we simply vary t from $0.8t$ to $1.2t$, where t is the largest scale in each diagram and its expressions have been given in Ref. [81]. Combining all above uncertainties, we obtain the CP-averaged branching fractions of two decay modes

$$
\mathcal{B}(B_s \to \pi^+\pi^-) = (5.5^{+1.1}_{-0.9}) \times 10^{-7}, \tag{20}
$$
$$
\mathcal{B}(B_d \to K^+K^-) = (1.9^{+0.3}_{-0.3}) \times 10^{-7}. \tag{21}
$$

Since the uncertainties from the π, K meson distribution amplitudes are very small, we will not discuss them. Note that by employing the DAs of light mesons based on the QCD sum rules from Refs. [85–93], Xiao et al. have explored these two decays [83]. They found that not only the DAs of B meson but also those of the light mesons will lead to large uncertainties for the branching fractions, as shown in Eqs. (6)–(9). In principle, the DAs of light mesons in B meson decays should be fitted or tested from the data. However, if we calculate the form factors of $B \to K, \pi$ within the DAs, the form factors obtained ($q^2 = 0$) are much smaller than the values abstracted from the data. On the contrary, the DAs we used (Eq. (16)) are fitted not only from the B meson semi-leptonic decays [94,95], but also from the non-leptonic B decays, such as $B \to K\pi, \pi\pi$ [77–79] and $B_s \to K\pi, \pi\pi$ [81]. In addition, the DAs are based on the Gegenbauer expansion, while the PQCD approach is on the basis of heavy quark expansions, so it is implausible to adopt the values of [91–93] directly.

In discussing the B meson decays, we usually define the direct CP asymmetry as

$$
\mathcal{A}_{CP}^{\text{dir}} \equiv \frac{|A(\overline{B}_q \to f)|^2 - |A(B_q \to \overline{f})|^2}{|A(\overline{B}_q \to f)|^2 + |A(B_q \to \overline{f})|^2}. \tag{22}
$$

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[1] In Ref. [83], there are typos in Eqs. (27) and (28).

Moreover, because the final states $\pi^+\pi^-$, K^+K^- have definite CP-parity, one can measure the time-dependent decay width of the $B_q \to f$ decay [96]:

$$\Gamma(B(t) \to f) \propto \cosh\left(\frac{\Delta\Gamma t}{2}\right)$$
$$+ \mathcal{H}_f \sinh\left(\frac{\Delta\Gamma t}{2}\right) - \mathcal{A}_{CP}^{\text{dir}}\cos(\Delta m t) - \mathcal{S}_f \sin(\Delta m t),$$
(23)

where $\Delta m = m_H - m_L > 0$ is the mass difference, and $\Delta\Gamma = \Gamma_H - \Gamma_L$ is the difference of the decay widths for the heavier and lighter B_q^0 mass eigenstates. Correspondingly, the time-dependent decay width $\Gamma(\overline{B}_q^0(t) \to f)$ is obtained from the above expression by flipping the signs of the $\cos(\Delta m t)$ and $\sin(\Delta m t)$ terms. $\mathcal{S}_f$ and $\mathcal{H}_f$, which can be extracted from the time-dependent decay width, are defined as

$$\mathcal{S}_f \equiv \frac{2Im[\lambda]}{1+|\lambda|^2}, \quad \mathcal{H}_f \equiv \frac{2Re[\lambda]}{1+|\lambda|^2},$$
(24)

with

$$\lambda = \eta_f e^{2i\epsilon}\frac{A(\overline{B}_q \to f)}{A(B_q \to \bar{f})},$$
(25)

where η_f is $+1(-1)$ for a CP-even (CP-odd) final state f and $\epsilon = \arg[-V_{cb}V_{tq}V_{cq}^*V_{tb}^*]$. In SM, the predicted results are listed as

$$\begin{cases} \mathcal{A}_{CP}^{\text{dir}}(B_s \to \pi^+\pi^-) = (-1.5 \pm 0.2)\%, \\ \mathcal{S}_f(B_s \to \pi^+\pi^-) = 0.11 \pm 0.01, \\ \mathcal{H}_f(B_s \to \pi^+\pi^-) = 0.99; \end{cases}$$
(26)

$$\begin{cases} \mathcal{A}_{CP}^{\text{dir}}(B_d \to K^+K^-) = (37^{+5}_{-7})\%, \\ \mathcal{S}_f(B_d \to K^+K^-) = -0.81 \pm 0.05, \\ \mathcal{H}_f(B_d \to K^+K^-) = -0.45 \pm 0.05. \end{cases}$$
(27)

For $B_s \to \pi^+\pi^-$, both branching fraction and CP asymmetry parameters agree with previous studies [80,81,83], and small differences are from the uncertainties of the CKM matrix elements. For $B_d \to K^+K^-$, our branching fraction is consistent with the prediction of [83], but the direct CP asymmetry is much smaller than their result because they might have omitted the effect of the SU(3) asymmetry in the LCDAs of K meson.[2] Compared with the experimental results (especially from LHC), although our branching fractions are consistent with the data after considering the uncertainties of both the theoretical and the experimental sides, the center value of $B_d \to K^+K^-$ ($B_s \to \pi^+\pi^-$) is a bit larger (smaller) than the data, which means there is a little room for us to search for possible effects of NP. Unfortunately, the CP

asymmetries of these two decays have not been measured in the current experiments.

3 The contribution of the Z' boson

Now we shall study the possible contributions of the extra gauge boson Z' in these two decay modes. Ignoring the interference between the Z and Z' bosons, we write the Lagrangian with Z' on the gauge interaction basis as

$$\mathcal{L}^{Z'} = -g_2 Z'^\mu \sum_{i,j} \overline{\psi}_i^I \gamma_\mu[(\epsilon_{\psi_L})_{ij}P_L + (\epsilon_{\psi_R})_{ij}P_R]\psi_j^I,$$
(28)

where the field ψ_i stands for the ith family fermion, g_2 for the coupling constant, ϵ_{ψ_L} (ϵ_{ψ_R}) for the left-handed (right-handed) chiral coupling, and $P_{L,R} = (1 \mp \gamma_5)/2$. After rotating to the physical basis, the mass eigenstates will be obtained by $\psi_{L,R} = V_{\psi_{L,R}}\psi_{L,R}^I$, and the usual CKM matrix is given by $V_{CKM} = V_{u_L}V_{d_L}^\dagger$. Similarly, we can get the coupling matrices in the physical basis of up- (down-)type quarks,

$$B_u^X \equiv V_{u_X}\epsilon_{u_X}V_{u_X}^\dagger, \quad B_d^X \equiv V_{d_X}\epsilon_{d_X}V_{d_X}^\dagger \quad (X = L, R).$$
(29)

If $\epsilon_{u(d)L(R)}$ is not proportional to the identity matrix, the nonzero off-diagonal elements in the $B_{u,d}^{L,R}$ will appear, which induces the FCNC interactions at the tree level. In the current work, we assume that the up-type coupling matrix $\epsilon_{u_{L(R)}}$ is proportional to the unit matrix, and the right-handed couplings are flavor-diagonal for simplicity. Thereby, the effective Hamiltonian of $b \to s\bar{q}q$ ($q = u, d$) transition mediated by the Z' is given by

$$\mathcal{H}_{\text{eff}}^{Z'} = \frac{2G_F}{\sqrt{2}}\left(\frac{g_2 m_Z}{g_1 m_{Z'}}\right)^2 B_{bs}^L(\bar{s}b)_{V-A}\sum_q (B_{qq}^L(\bar{q}q)_{V-A}$$
$$+ B_{qq}^R(\bar{q}q)_{V+A}) + \text{h.c.},$$
(30)

where $g_1 = e/(\sin\theta_W\cos\theta_W)$ and $m_{Z'}$ is the mass of Z' boson. The diagonal elements of the effective coupling matrices $B_{qq}^{L,R}$ are required to be real because of the hermiticity of the effective Hamiltonian. However, for the off-diagonal one of B_{bs}^L, it might be a complex number and a new weak phase ϕ_{bs} is introduced, which might play important roles in explaining the large CP asymmetries in $B \to K\pi$ [72–74]. Since the above operators of the forms $(\bar{s}b)_{V-A}(\bar{q}q)_{V\mp A}$ already exist in SM, we shall represent the Z' effect by modifying the Wilson coefficients of the corresponding operators. As a result, we reorganize Eq. (30) as

$$\mathcal{H}_{\text{eff}}^{Z'} = -\frac{G_F}{\sqrt{2}}V_{tb}V_{ts}^*\sum_q (\Delta C_3^s O_3^{(q)}$$
$$+ \Delta C_5^s O_5^{(q)} + \Delta C_7^s O_7^{(q)} + \Delta C_9^s O_9^{(q)}) + \text{h.c.}$$
(31)

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2 If we keep the SU(3) symmetry, the direct CP asymmetry of $B_d \to K^+K^-$ is $\mathcal{A}_{CP}^{\text{dir}}(B_d \to K^+K^-) = 19.5\%$, which is almost the same as the result of [83].

Correspondingly, the contributions of the extra Z' boson to the SM Wilson coefficients at the m_W scale is given

$$\Delta C_{3(5)}^s = -\frac{2}{3V_{tb}V_{ts}^*}\left(\frac{g_2 m_Z}{g_1 m_{Z'}}\right)^2 B_{bs}^L(B_{uu}^{L(R)} + 2B_{dd}^{L(R)}),$$
(32)

$$\Delta C_{9(7)}^s = -\frac{4}{3V_{tb}V_{ts}^*}\left(\frac{g_2 m_Z}{g_1 m_{Z'}}\right)^2 B_{bs}^L(B_{uu}^{L(R)} - B_{dd}^{L(R)}).$$
(33)

One can see that Z' contributes to the electro-weak penguins $\Delta C_{9(7)}$ as well as the QCD penguins $\Delta C_{3(5)}$. In order to show that the new physics is primarily manifest in the electro-weak penguins, we simply assume $B_{uu}^{L(R)} \simeq -2B_{dd}^{L(R)}$, and this relation has been used widely [27–61]. Therefore, the Z' contributions to the Wilson coefficients are

$$\Delta C_{3(5)}^s = 0, \quad \Delta C_{9(7)}^s = 4\frac{|V_{tb}V_{ts}^*|}{V_{tb}V_{ts}^*}\zeta_s^{LL(R)}e^{i\phi_{bs}},$$
(34)

where

$$\zeta_s^{LX} = \left(\frac{g_2 m_Z}{g_1 m_{Z'}}\right)^2\left|\frac{B_{bs}^L B_{dd}^X}{V_{tb}V_{ts}^*}\right| \quad (X=L,R), \phi_{bs}=\text{Arg}[B_{bs}^L].$$
(35)

Note that the other SM Wilson coefficients at a scale lower than m_b will also receive contributions from the Z' boson through renormalization group evolution. Since in this model there is no new particle below m_W, the renormalization group evolution of the modified Wilson coefficients is exactly the same as the one in SM (for a review, see [84]).

Similarly, we also obtain the hamiltonian $b \to d\bar{q}q$, and the corresponding Wilson coefficients and inner functions are given as

$$\Delta C_{3(5)}^d = 0, \quad \Delta C_{9(7)}^d = 4\frac{|V_{tb}V_{td}^*|}{V_{tb}V_{td}^*}\zeta_d^{LL(R)}e^{i\phi_{bd}},$$
(36)

$$\zeta_d^{LX} = \left(\frac{g_2 m_Z}{g_1 m_{Z'}}\right)^2\left|\frac{B_{bd}^L B_{ss}^X}{V_{tb}V_{td}^*}\right| \quad (X=L,R), \phi_{bd}=\text{Arg}[B_{bd}^L].$$
(37)

Now, we are in a position to discuss the possible parameter spaces of $\zeta_{s,d}^{LL,LR}$ and $\phi_{bs,bd}$. In particular, we assume $g_2/g_1 \sim 1$, because we expect that the hypercharges of the $U(1)_Y$ gauge group and of the extra $U(1)'$ have the same origin from some grand unified models. Furthermore, we also hope $m_Z/m_{Z'}$ is at the order of $\mathcal{O}(10^{-1})$, so that the neutral Z' boson could be detected at the LHC experiment directly. Note that the mass of a leptophobic $m_{Z'}$ boson has not been constrained till now, as mentioned in Sect. 1. In addition, we need to determine the other parameters $|B_{bs}^L|$, $|B_{bd}^L|$, $|B_{qq}^X|$, and new weak phases $\phi_{bd,bs}$ with the accurate data from B factories and LHCb experiment. For example, $B_{bs,bd}^L$ and

$\phi_{bs,bd}$ could be extracted from B_q^0–$\bar{B}_q^0$ ($q = d, s$) mixing. In order to explain the mass differences between B_q^0 and $\bar{B}_q^0$ with the new Z' boson, $|B_{bs(d)}^L| \sim |V_{tb}V_{ts(d)}^*|$ is required. Then, with the experimental data of $B_{d,s}$ the non-leptonic charmless decays, $B_{qq}^{L,R} \sim 1$ could be extracted. For the new introduced phases ϕ_{bs} and ϕ_{bd}, they have not been constrained totally, although many efforts have been made [57–61]; we therefore set them as free parameters. How to constrain these parameters globally is beyond the scope of the current work and can be found in many references [40–61]. So as to probe the new physics effect for a maximum range, we assume $\zeta \sim \zeta_{d,s}^{LL} \sim \zeta_{d,s}^{LR} \in [0.001, 0.02]$, i.e., the range of $m_{Z'}$ is about [636, 2800] GeV, and $\phi_{bd,bs} \in [-180°, 180°]$.

In Fig. 2, we explore the possible effects of the Z' boson on the decay mode $B_s \to \pi^+\pi^-$. In the left panel, we present the variation of the CP-averaged branching fraction as a function of the new weak phase ϕ_{bs} with different $\zeta = 0.001$ (dotdashed), 0.01 (dotted), 0.02 (dashed). The experimental region (filled by horizontal lines) and the SM predictions (filled by vertical lines) are also shown for comparison. From this figure, one can see that the SM is consistent with the data within 1σ. Including the Z' contribution, it is apparent that the parameter space $|\phi_{bs}| < 80°$ will be excluded. For $|\phi_{bs}| > 80°$, if $\zeta < 0.01$, the contribution of the Z' boson will be buried by the uncertainties of the SM. One also sees that when $\zeta = 0.02$ the branching ratio will be enhanced to 7.6×10^{-7}, which is larger than the SM prediction. Note that the averaged experimental have large errors, and the small band will help us to determine the magnitude of ζ. It is emphasized that the LHCb had obtained a bit larger result, which indicates the existence of a light Z'. In the right panel, we plot the relation between the branching ratio and the direct CP asymmetry $\mathcal{A}_{\text{CP}}^{\text{dir}}$ with an extra Z' boson. The region edged by a blue curve is the possible region with parameter $\zeta < 0.02$ and $\phi_{bs} \in [-180°, 180°]$. With the experimental data, the lower half of the region can be excluded. In the permitted region, the range of the direct CP asymmetry is $[-3.2\%, 0.1\%]$, which is much larger than the estimation of the SM (the gray region). The future measurement of these values in the LHCb (LHC-II) experiment and the high energy e^+e^- collider will help us to probe the effects of Z'. Note that when discussing the effects of the Z' boson, we will not include the uncertainties induced by wave functions and scale t, because the major objective of this work is to search for the possibility of a new physics signal, rather than to produce acute numerical results.

Similarly, the effects of the extra Z' boson in $B_d \to K^+K^-$ are also presented in Fig. 3. In the left panel, it is clear that the position of the SM prediction is on the top of the experimental data, although some parts of them overlap with each other. Furthermore, the heavy Z' contributions ($\zeta < 0.1$) are not apparent due to the uncertain-
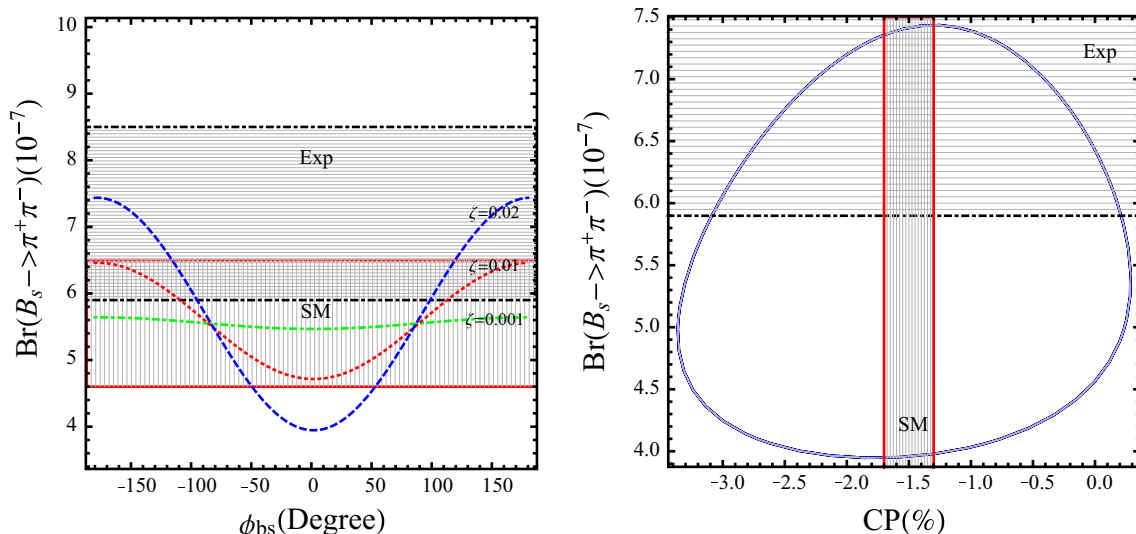
Fig. 2 The contribution of Z' to the decay mode $B_s \rightarrow \pi^+\pi^-$. The *left panel* represents the branching fraction as functions of ϕ_{bs}, the *dot-dashed* (*green*), *dotted* (*red*), and *dashed* (*blue*) *lines* represent results from $\zeta = 0.001$, 0.01, 0.02, respectively. The region edged by *dot-dashed lines* (*black*) is for the experimental data, while the 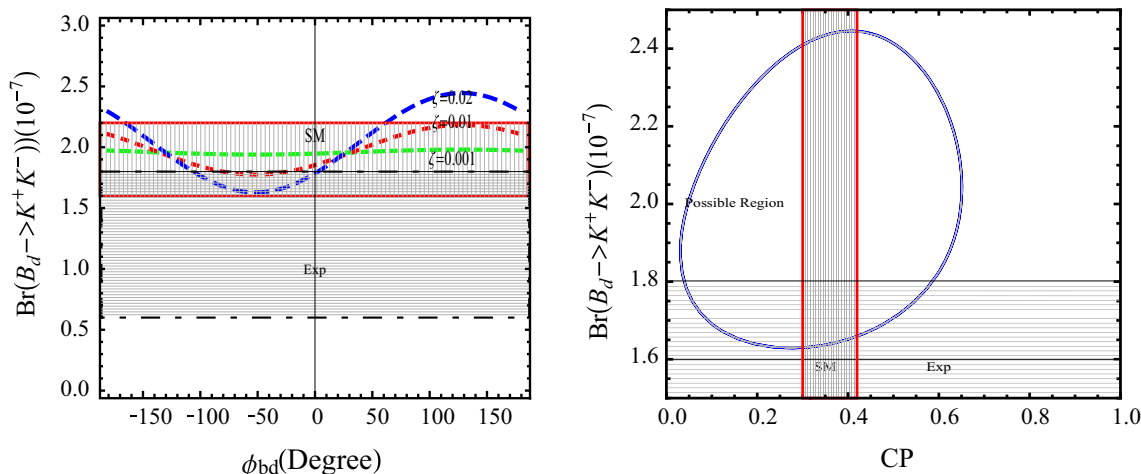one edged by the *solid lines* (*red*) is for the prediction of the SM. The *right panel* stands for the relation between the branching fraction and the direct CP asymmetry, the region edged by *dot-dashed lines* (*black*) is the experimental data, while the one edged by the *solid lines* (*red*) is the prediction of the SM

Fig. 3 The contribution of Z' to the decay mode $B_d \rightarrow K^+K^-$. The legends are the same as in Fig. 2

ties of the SM. Moreover, for the ϕ_{bd}, the ranges $[0, 180°]$ and $[-180, -110°]$ will be excluded. We also plot the region (edged by a curve) related to the direct CP asymmetry and the branching fraction, as shown in the right panel. Note that the SM prediction is 30–42 %, but with a light Z' the estimated range is to be 5–60 % after considering the constraint from the experimental data. It is concluded that for $B_d \rightarrow K^+K^-$ the future measurement of the direct CP asymmetry will help us to search for the possible effect of a light Z', though its contribution to the branching fraction is polluted by the SM uncertainties.

Finally, we shall discuss the Z' effect on the CP symmetry parameters $\mathcal{S}_f$ and $\mathcal{H}_f$. For $B_s \rightarrow \pi^+\pi^-$, as the weak phase of $V_{tb}V_{ts}^*$ is very small, both $\mathcal{S}_f$ and $\mathcal{H}_f$ are not sensitive to the NP. On the contrary, for $B_d \rightarrow K^+K^-$, $\mathcal{S}_f$ and $\mathcal{H}_f$ are sensitive to the extra leptophobic Z' boson. In Fig. 4, we plot the relations of $\mathcal{S}_f$ (right panel) and $\mathcal{H}_f$ (left panel) with varying ϕ_{bd} from -180 to $180°$, when $\zeta = 0.01$ and $\zeta = 0.02$. The estimates of SM (edged by the lines) are also presented. From the figures, one can see that in the permitted range of ϕ_{bd}, with a light Z' boson ($\zeta = 0.02$), $\mathcal{S}_f$ could reach -0.55, which is larger than the prediction of the SM. For $\mathcal{H}_f$, its values could reach to -0.75 when
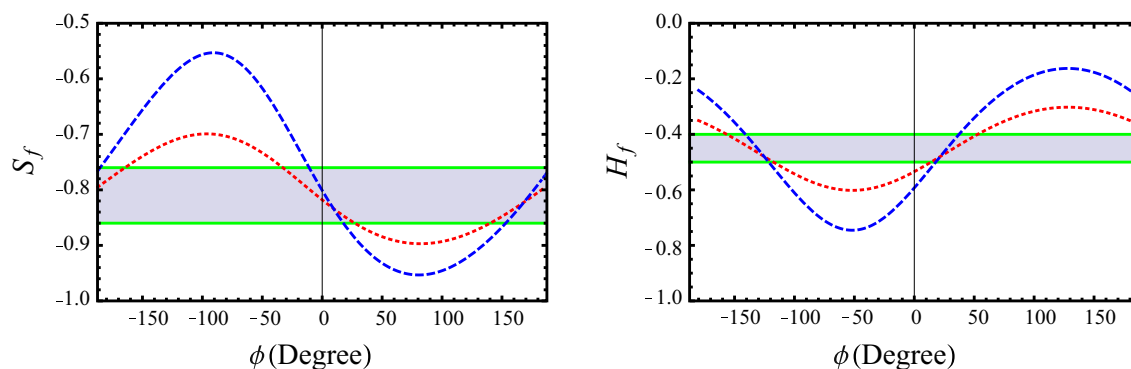
Fig. 4 The CP symmetry parameters $\mathcal{S}_f$ (*left panel*) and $\mathcal{H}_f$ (*right panel*) as a function of the weak phase ϕ_{bd}, the *dotted* (*red*) and *dashed* (*blue*) *lines* represent results from the $\zeta = 0.01, 0.02$, and the regions edged by *solid line* (*green*) are the predictions of the SM

$\phi_{bd} = -50°$. The future measurement of them will help us to further constrain the parameters, which might be helpful for direct searching for a light Z' boson.

4 Summary

In this work, we have studied the pure annihilation decays $B_d \rightarrow K^+ K^-$ and $B_s \rightarrow \pi^+ \pi^-$ in the SM and the family-non-universal leptophobic Z' model. Although the SM predictions in the PQCD approach are in agreement with the experimental data, the branching fraction of $B_s \rightarrow \pi^+ \pi^-$ ($B_d \rightarrow K^+ K^-$) is a little bigger (smaller) than the LHCb result, which may indicate the survival space for a light Z' boson. Inspired by this fact, we have constrained the $U(1)$ phase as $\phi_{bd} \in [-110, 0°]$, and ϕ_{bs} is $|\phi_{bs}| > 80°$. Within the allowed space range, the direct CP asymmetry for $B_s \rightarrow \pi^+ \pi^-$ is predicted as $[-3.2, 0.1 \%]$, while it is 5–60 % for $B_d \rightarrow K^+ K^-$. Furthermore, we have also calculated the mixing-induced CP asymmetries and found that the parameters $\mathcal{S}_f$ and $\mathcal{H}_f$ of $B_d \rightarrow K^+ K^-$ are very sensitive to the effect of Z'. With a light Z', the maximum (minimal) value of $\mathcal{S}_f (\mathcal{H}_f)$ can reach -0.55 (-0.75). The future measurements of these observables may provide us with some hints for direct searching for a light leptophobic Z' boson.

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