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Shear-free condition and dynamical instability in $f(R, T)$ gravity

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Abstract The implications of the shear-free condition on the instability range of an anisotropic fluid in $f(R, T)$ are studied in this manuscript. A viable $f(R, T)$ model is chosen to arrive at stability criterion, where R is Ricci scalar and T is the trace of energy-momentum tensor. The evolution of a spherical star is explored by employing a perturbation scheme on the modified field equations and contracted Bianchi identities in $f(R, T)$. The effect of the imposed shear-free condition on the collapse equation and adiabatic index Γ is studied in the Newtonian and post-Newtonian regimes.

1 Introduction

In a recent work [1], we have studied the effect of an anisotropic fluid on the dynamical instability of a spherically symmetric collapsing star in $f(R, T)$ theory. Herein, we plan to explore the instability range of anisotropic spherically symmetric stars, considering shear-free condition. The role of the shear tensor in the evolution of gravitating objects and consequences of the shear-free condition have been studied extensively. Collins and Wainwright [2] studied the impact of shear on general relativistic cosmological and stellar models. Herrera et al. [3,4] worked out the homology and shear-free conditions for dissipative and radiative gravitational evolution.

The features of the gravitational evolution and its final outcome are of great importance in view of general relativity (GR) as well as in modified theories of gravity. Shear-free collapse accounting for heat flow is discussed in [5], where it is established that shear plays a critical role in the gravitational evolution and may lead to the formation of naked singularities [6]. It is mentioned in [6] that the occurrence of shearing effects near collapsing stars avoids the apparent horizon leading to the formation of a naked singularity. However, vanishing shear gives rise to the formation of an apparent horizon and so the evolving cloud ends in a black hole (BH). Thus, the relevance of the shear tensor in structure formation and its consequences on the dynamical instability range of a self-gravitating body is a well-motivated direction of study.

Stars shine by consuming their nuclear fuel; continuous fuel consumption causes imbalance between inwardly acting gravitational pull and outwardly drawn pressure, giving rise to collapse [7]. The outcome of gravitational evolution is dependent on the size as well as other physical aspects [8,9], such as isotropy, anisotropy, shear, radiation, dissipation etc. In comparison to the stars of mass around one solar masses, massive stars tend to lose nuclear fuel more rapidly and so they are more unstable. The pressure to density ratio, called the adiabatic index, denoted by Γ is utilized in the estimation of the stability/instability range of the stars. Chandrasekhar [10] explored the instability range of spherical stars in terms of Γ.

Herrera et al. [11–17] contributed substantially in addressing the instability problem in GR, accompanying various situations, i.e., isotropy, anisotropy, the shear-free condition, radiation, dissipation, the expansion-free condition, and shearing expansion-free fluids. In order to achieve a more precise and generic description of the universe, the dark energy components are incorporated by introducing modified theories of gravity. Modified theories are significant in the advancement toward accelerated expansion of the universe and to present corrections to GR on large scales. The modifications are introduced in the Einstein–Hilbert (EH) action by inducing a minimal or non-minimal coupling of matter and geometry [18–25].

The dynamical analysis of self-gravitating sources in modified theories of gravity has been discussed extensively in

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recent years. The null dust non-static exact solutions in $f(R)$ gravity are studied in [26], Cembranos et al. [27] studied the evolution of gravitating sources in the presence of a dust fluid. The instability range of spherically and axially symmetric anisotropic stars has been established in the context of $f(R)$ gravity [28–30], leading us to conclude that deviations from spherical symmetry complicate the subsequent evolution.

Harko et al. [31] presented the $f(R,T)$ theory of gravity as another alternative to GR and a generalization of $f(R)$ theory representing non-minimal matter to geometry coupling. The action in $f(R,T)$ gravity includes an arbitrary function of the Ricci scalar R and the trace of the energy-momentum tensor T to take into account the exotic matter. After the introduction of $f(R,T)$ gravity, its cosmological and thermodynamic implications were widely studied [32–40] including the energy conditions. Recently, we have studied the evolution of an anisotropic gravitating source with zero expansion [41]. Herein, we are interested in the exploration of the shear-free condition implications on a spherically symmetric gravitating source in $f(R,T)$ gravity.

The modified action in $f(R,T)$ gravity is as follows [31]:

$$\int dx^4\sqrt{-g}\left[\frac{f(R,T)}{16\pi G}+\mathcal{L}_{(m)}\right], \tag{1.1}$$

where $\mathcal{L}_{(m)}$ denotes the matter Lagrangian, and g represents the metric tensor. Various choices of $\mathcal{L}_{(m)}$ can be taken into account, each of which leads to a specific form of fluid. Many people worked out this problem in GR and modified theories of gravity, and the stability of general relativistic dissipative axially symmetric and spherically symmetric systems with a shear-free condition has been established in [42,43]. A dynamical analysis of the shear-free spherically symmetric sources in $f(R)$ gravity is presented in [44].

The organization of this article is as follows: Sect. 2 comprises the modified dynamical equations in $f(R,T)$ gravity. Section 3 includes the model under consideration, the perturbation scheme, and the corresponding collapse equation along with the shear-free condition in the Newtonian and post-Newtonian eras. Section 4 contains concluding remarks followed by an appendix.

2 Dynamical equations in $f(R,T)$

In order to study the implications of the shear-free condition on the evolution of spherically symmetric anisotropic sources, modified field equations in $f(R,T)$ gravity are formulated by varying the action (1.1) with the metric g_{uv}. Here, we have taken $\mathcal{L}_{(m)}=\rho$ [36], for this choice of $\mathcal{L}_{(m)}$ the modified field equations in $f(R,T)$ gravity take the following form:

$$G_{uv}=\frac{1}{f_R}\left[(f_T+1)T^{(m)}_{uv}-\rho g_{uv}f_T+\frac{f-Rf_R}{2}g_{uv}\right.$$
$$\left.+(\nabla_u\nabla_v-g_{uv}\Box)f_R\right]. \tag{2.2}$$

Here $T^{(m)}_{uv}$ is the energy-momentum tensor for the usual matter taken to be locally anisotropic.

The three dimensional spherical boundary surface Σ is considered that constitutes two regions named ‘interior’ and ‘exterior’ spacetimes. The line element for the region inside the boundary Σ is

$$ds^2_-=A^2(t,r)dt^2-B^2(t,r)dr^2$$
$$-C^2(t,r)(d\theta^2+\sin^2\theta d\phi^2). \tag{2.3}$$

The line element for the region beyond Σ is [1]

$$ds^2_+=\left(1-\frac{2M}{r}\right)dv^2+2drdv-r^2(d\theta^2+\sin^2\theta d\phi^2), \tag{2.4}$$

where v is the retarded time and M denotes the total mass.

The expression for the anisotropic energy-momentum tensor $T^{(m)}_{uv}$ is given by

$$T^{(m)}_{uv}=(\rho+p_\perp)V_uV_v-p_\perp g_{uv}+(p_r-p_\perp)\chi_u\chi_v, \tag{2.5}$$

where ρ is the energy density, V_u describes the four-velocity of the fluid, χ_u is the radial four vector, and p_r and $p_\perp$ represent the radial and tangential pressure, respectively. These physical quantities are linked by

$$V^u=A^{-1}\delta^u_0,\quad V^uV_u=1,\quad \chi^u=B^{-1}\delta^u_1,\quad \chi^u\chi_u=-1. \tag{2.6}$$

The shear tensor denoted by σ_{uv} is defined as

$$\sigma_{uv}=V_{(u;v)}-a_{(u}V_{v)}-\frac{1}{3}\Theta(g_{uv}-V_uV_v), \tag{2.7}$$

where a_u is four acceleration and Θ is expansion scalar, given by

$$a_u=V_{(u;v)}V^v,\quad \Theta=V^u_{;u}. \tag{2.8}$$

The components of the shear tensor are found by variation of (2.7) and these are used to find the expression for the shear scalar in the following form:

$$\sigma=\frac{1}{A}\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right), \tag{2.9}$$

where a dot and a prime indicate time and radial derivatives, respectively. From the shear-free condition we arrive at a vanishing shear scalar, i.e., $\sigma=0$, implying $\frac{\dot{B}}{B}=\frac{\dot{C}}{C}$.

It is worth mentioning here that the expansion scalar and a scalar function described in terms of the Weyl tensor and the anisotropy of the pressure controls the departure from the shear-free condition. Such a function is related to the

Tolman mass and appears in a natural way in the orthogonal splitting of the Riemann tensor [45]. It is obvious that pressure anisotropy and density inhomogeneities have extensive implications on the stability of the shear-free condition, but it is not intuitively clear that their specific combination affects the stability [43]. Generically the shear-free condition remains unstable against the presence of pressure anisotropy. Alternatively, one can consider such a case that pressure anisotropy and density inhomogeneity are present in such a way that the scalar function appearing in an orthogonal splitting of the Riemann tensor vanishes, implying non-homogeneous anisotropic stable shear-free flow. Since we are dealing with a fluid evolving under the shear-free condition, we shall make use of this condition while evaluating the components of the field equations and also in the conservation equations.

The components of the modified Einstein tensor are

$$G_{00}=\frac{1}{f_R}\left[\rho+\frac{f-Rf_R}{2}+\frac{f_R''}{B^2}-\frac{3\dot{f}_R}{A^2}\frac{\dot{B}}{B}\right.\\ \left.-\frac{f_R'}{B^2}\left(\frac{B'}{B}-\frac{2C'}{C}\right)\right],\tag{2.10}$$

$$G_{01}=\frac{1}{f_R}\left[\dot{f_R}'-\frac{A'}{A}\dot{f}_R-\frac{\dot{B}}{B}f_R'\right],\tag{2.11}$$

$$G_{11}=\frac{1}{f_R}\left[p_r+(\rho+p_r)f_T-\frac{f-Rf_R}{2}\right.\\ \left.+\frac{\ddot{f}_R}{A^2}-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-\frac{2\dot{C}}{C}\right)-\frac{f_R'}{B^2}\left(\frac{A'}{A}+\frac{2C'}{C}\right)\right],\tag{2.12}$$

$$G_{22}=\frac{1}{f_R}\left[p_\perp+(\rho+p_\perp)f_T-\frac{f-Rf_R}{2}\right.\\ +\frac{\ddot{f}_R}{A^2}-\frac{f_R''}{B^2}-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-\frac{2\dot{B}}{B}\right)\\ \left.-\frac{f_R'}{B^2}\left(\frac{A'}{A}-\frac{B'}{B}+\frac{C'}{C}\right)\right].\tag{2.13}$$

The dynamical equations extracted from the conservation laws are vital in the study of stellar evolution. The conservation of the full field equations is considered to incorporate the non-vanishing divergence terms; the Bianchi identities are

$$G^{uv}_{;v}V_u=0,\quad G^{uv}_{;v}\chi_u=0,\tag{2.14}$$

and on simplification of (2.14), we have dynamical equations as follows:

$$\dot{\rho}-\rho\frac{\dot{f}_R}{f_R}+[1+f_T](3\rho+p_r+2p_\perp)\frac{\dot{B}}{B}+Z_1(r,t)=0,\tag{2.15}$$

$$(\rho+p_r)f_T'+(1+f_T)\left\{p_r'+\rho\frac{A'}{A}\right.\\ \left.+p_r\left(\frac{A'}{A}+2\frac{C'}{C}-\frac{f_R'}{f_R}\right)-2p_\perp\frac{C'}{C}\right\}\\ +f_T\left(\rho'-\frac{f_R'}{f_R}\right)+Z_2(r,t)=0,\tag{2.16}$$

where $Z_1(r,t)$ and $Z_2(r,t)$ are provided in the appendix as (5.1) and (5.2), respectively. Deviations from equilibrium in the conservation equations with the time transition leads to the stellar evolution, and a perturbation approach is devised to estimate the instability range.

3 Perturbation scheme and shear-free condition

We consider a particular $f(R,T)$ model of the form

$$f(R,T)=R+\alpha R^2+\lambda T,\tag{3.17}$$

where α and λ can be any positive constants. The perturbation approach is utilized to estimate the instability range of a spherical star with the shear-free condition. This scheme is utilized in the determination of more generic analytical constraints on the collapse equation, or rather to establish a dynamical analysis of special cases numerically. Also, the field equations are highly nonlinear differential equations; in such a scenario the application of a perturbation is beneficial to gaining insight.

It is assumed that initially all quantities are independent of time and with the passage of time the perturbed form depends on both time and radial coordinates. Taking $0<\varepsilon\ll 1$, the physical quantities and their perturbed form can be arranged as

$$A(t,r)=A_0(r)+\varepsilon D(t)a(r),\tag{3.18}$$

$$B(t,r)=B_0(r)+\varepsilon D(t)b(r),\tag{3.19}$$

$$C(t,r)=C_0(r)+\varepsilon D(t)\bar{c}(r),\tag{3.20}$$

$$\rho(t,r)=\rho_0(r)+\varepsilon\bar{\rho}(t,r),\tag{3.21}$$

$$p_r(t,r)=p_{r0}(r)+\varepsilon\bar{p}_r(t,r),\tag{3.22}$$

$$p_\perp(t,r)=p_{\perp0}(r)+\varepsilon\bar{p_\perp}(t,r),\tag{3.23}$$

$$m(t,r)=m_0(r)+\varepsilon\bar{m}(t,r),\tag{3.24}$$

$$R(t,r)=R_0(r)+\varepsilon D_1(t)e_1(r),\tag{3.25}$$

$$T(t,r)=T_0(r)+\varepsilon D_2(t)e_2(r),\tag{3.26}$$

$$f(R,T)=[R_0(r)+\alpha R_0^2(r)+\lambda T_0]+\varepsilon(D_1(t)e_1(r)[1\\ +2\alpha R_0(r)]+D_2(t)e_2(r)),\tag{3.27}$$

$$f_R=1+2\alpha R_0(r)+\varepsilon 2\alpha D_1(t)e_1(r),\tag{3.28}$$

$$f_T=\lambda.\tag{3.29}$$

Considering the Schwarzschild coordinate $C_0=r$ and implementing the perturbation scheme on the vanishing shear scalar implies

$$\frac{b}{B_0}=\frac{\bar{c}}{r}.\tag{3.30}$$

Using (3.18)–(3.29) and (3.30) in the dynamical equations i.e., (2.15) and (2.16), leads to the following expressions:

$$\dot{\bar{\rho}}+\left[\frac{2e\rho_0}{Y}+\lambda_1\frac{\bar{c}}{r}(2\rho_0+p_{r0}+4p_{\perp 0})+YZ_{1p}\right]\dot{D}=0, \tag{3.31}$$

$$\begin{aligned}&\lambda_1\left\{\bar{p_r}'+\bar{\rho}\frac{A_0'}{A_0}+\bar{p_r}\left(\frac{A_0'}{A_0}+\frac{2}{r}-\frac{2\alpha R_0'}{Y}\right)-\frac{2\bar{p_\perp}}{r}\right\}+\lambda\bar{\rho}'\\&+2\alpha\ddot{D}\left[\frac{1}{A_0^2}\left(e'+2e\frac{B_0'}{B_0}-\frac{\bar{c}}{r}R_0'\right)+B_0^2(Y)\left\{\frac{e}{B_0^2Y}\right\}'\right]\\&+D\left[\lambda_1[(\frac{a}{A_0})'(\rho_0+p_{r0})-2(p_{r0}+p_{\perp 0})(\frac{\bar{c}}{r})']\right.\\&-\frac{2\alpha}{Y}\left\{\lambda_1\left(p_{r0}'+\rho_0\frac{A_0'}{A_0}+p_{r0}\left(\frac{A_0'}{A_0}-\frac{2\alpha R_0'}{Y}+\frac{2}{r}\right)\right)\right\}\\&\left.+\lambda\left(e'+e[\rho_0'-\frac{2\alpha R_0'}{Y}]\right)+YZ_{2p}\right]=0,\end{aligned} \tag{3.32}$$

where Z_{1p} and Z_{2p} are given in the appendix. For the sake of simplicity we put Y in place of $1+2\alpha R_0$ and $\lambda_1=\lambda+1$, assuming that $D_1=D_2=D$ and $e_1=e_2=e$. The above mentioned perturbed dynamical equations and perturbed field equations shall be used to arrive at perturbed physical quantities such as $\bar{\rho}$, $\bar{p_r}$, and $\bar{p_\perp}$.

The expression for $\bar{\rho}$ can be found from (3.31), as follows:

$$\bar{\rho}=-\left[\frac{2e\rho_0}{Y}+\frac{\bar{c}}{r}(3\rho_0+p_{r0}+4p_{\perp 0})+YZ_{1p}\right]D. \tag{3.33}$$

The Harrison–Wheeler type equation of state relates $\bar{\rho}$ and $\bar{p_r}$; it is given by

$$\bar{p_r}=\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\bar{\rho}. \tag{3.34}$$

Putting $\bar{\rho}$ from (3.33) in (3.34), we find

$$\begin{aligned}\bar{p_r}=&-\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\left[\frac{2e\rho_0}{Y}+\lambda_1\frac{\bar{c}}{r}(3\rho_0\right.\\&\left.+p_{r0}+4p_{\perp 0})+YZ_{1p}\right]D.\end{aligned} \tag{3.35}$$

The perturbed form of the field equation (2.13) yields an expression for $\bar{p_\perp}$ that turns out to be

$$\begin{aligned}\bar{p_\perp}=&\left\{\frac{Y\bar{c}}{r}-2\alpha e\right\}\frac{\ddot{D}}{A_0^2}-\frac{\lambda\bar{\rho}}{\lambda_1}\\&+\left\{\left(p_{\perp 0}-\frac{\lambda}{\lambda_1}\rho_0\right)\frac{2\alpha e}{Y}+\frac{Z_3}{\lambda_1}\right\}D,\end{aligned} \tag{3.36}$$

Z_3 is the effective part of the field equation given in the appendix as (5.5).

Substitution of $\bar{\rho}$, $\bar{p_r}$, and $\bar{p_\perp}$ from (3.33), (3.35), and (3.36) into (3.32) leads to a collapse equation,

$$\begin{aligned}&\ddot{D}\left[\frac{2\alpha}{A_0^2Y}\left\{e'+2e\frac{B_0'}{B_0}-\frac{\bar{c}}{r}R_0'\right\}-2\alpha B_0^2\left\{\frac{e}{B_0^2Y}\right\}'\right.\\&\left.+\frac{1}{A_0^2}\{\frac{Y\bar{c}}{r}-2\alpha e\}\right]+D\left[\frac{1}{Y}\left\{\lambda_1\left((\rho_0+p_{r0})\left(\frac{a}{A_0}\right)'\right.\right.\right.\\&\left.\left.-2(\rho_0+p_{\perp 0})\left(\frac{\bar{c}}{r}\right)'\right)\right\}-\frac{2\alpha}{Y}\left\{\lambda\left(e'-\rho_0'-\frac{2\alpha R_0'}{Y}\right)\right.\\&\left.+\lambda_1\left(e'p_{r0}+e[p_{r0}'+\rho_0\frac{A_0'}{A_0}+p_{r0}(\frac{A_0'}{A_0}+\frac{2}{r}-\frac{2\alpha R_0'}{Y})]\right)\right\}\\&-\left(\lambda+\lambda_1\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\right)\left\{\rho_0\frac{2e}{Y}+\lambda_1\frac{\bar{c}}{r}(3\rho_0+p_{r0}\right.\\&\left.+4p_{\perp 0})+YZ_{1p}\right\}_{,1}+\left\{\frac{A_0'}{A_0}+\frac{2}{r}\frac{\lambda}{\lambda_1}\right.\\&+\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\left(\frac{A_0'}{A_0}+\frac{2}{r}-\frac{2\alpha R_0'}{Y}\right)+\lambda_1\left(\Gamma\frac{p_{r0}}{\rho_0+p_{r0}}\right)'\Bigg\}\\&\times\left\{\frac{2e\rho_0}{Y}+\lambda_1\frac{\bar{c}}{r}(3\rho_0+p_{r0}+4p_{\perp 0})+(Y)Z_{1p}\right\}\\&\left.\left.+\frac{2}{r}\frac{1}{\lambda_1}Z_3\right\}+Z_{2p}\right]=0.\end{aligned} \tag{3.37}$$

Matching conditions at the boundary surface together with the perturbed form of (2.13) can be written in the simplified form as follows:

$$\ddot{D}(t)-Z_4(r)D(t)=0, \tag{3.38}$$

provided that

$$\begin{aligned}Z_4=&\frac{rA_0^2}{Y\bar{c}-2\alpha er}\left[\frac{2\alpha e}{Y}p_{\perp 0}+\lambda\frac{\bar{c}}{r}(3\rho_0+p_{r0}\right.\\&\left.+4p_{\perp 0})+YZ_{1p}+\frac{Z_3}{\lambda_1}\right].\end{aligned} \tag{3.39}$$

The valid solution of (3.38) turns out to be

$$D(t)=-e^{\sqrt{Z_4}t}. \tag{3.40}$$

The terms of Z_4 must be constrained in such a way that all terms maintain positivity. The impact of the shear-free condition on the dynamical instability of N and pN regimes is covered in the following subsections.

3.1 Newtonian regime

In order to establish the instability range in the Newtonian era, we set $\rho_0\gg p_{r0}$, $\rho_0\gg p_{\perp 0}$, and $A_0=1$, $B_0=1$. Insertion of these assumptions and (3.40) into (3.37) leads to the instability condition, relating the usual matter and dark source contribution,

$$\Gamma < \frac{Z_4X_3 + X_4 + \lambda\rho_0(X_2 + YZ_{1p(N)})_{,1} + X_1X_2 - \frac{2}{r\lambda_1}Z_{3(N)} + YZ_{2p(N)}}{\lambda_1 p_{r0}X_2' + \left\{p_{r0}\left(\frac{2\alpha R_0'}{Y} - \frac{2}{r}\right)\right\}X_2}, \tag{3.41}$$

where

$$X_1 = (\lambda\rho_0' + \frac{2\lambda}{r\lambda_1}), \quad X_2 = \frac{2e}{Y} + 3\lambda_1 b,$$
$$X_3 = -2\alpha^2 bR_0' + Yb,$$
$$Z_4 = \lambda_1\left[\rho_0 a' + 2(p_{r0} + p_{\perp 0})b'\right] + \frac{2\alpha}{Y}\left[\lambda\left(\frac{2\alpha R_0'}{Y} - \rho_0' + e'\right) + \lambda_1\left\{p_{r0} + e[p_{r0}' + p_{r0}\left(\frac{2}{r} - \frac{2\alpha R_0'}{Y}\right)]\right\}\right].$$

The quantities $Z_{1p(N)}$ and $Z_{2p(N)}$ are terms of Z_{1p} and Z_{2p} belonging to the Newtonian era. The gravitating source remains stable in the Newtonian approximation until the inequality for Γ is satisfied, for which the following constraints must be met:

$$2\alpha R_0' < Y, \quad \frac{2\alpha R_0'}{Y} > \rho_0' - e'.$$

The case when $\alpha \to 0$ and $\lambda \to 0$ leads to GR corrections and results for $f(R)$ can be retrieved by setting $\lambda \to 0$.

3.2 Post-Newtonian regime

We assume $A_0 = 1 - \frac{m_0}{r}$ and $B_0 = 1 + \frac{m_0}{r}$ to evaluate the stability condition in the pN regime. On substitution of these assumptions in (3.37), we have the following inequality for Γ to be fulfilled for the stability range:

$$\Gamma < \frac{Z_4X_5 + X_6 + \lambda\rho_0(X_7 + YZ_{1p(PN)})_{,1} + X_8X_7 - \frac{2}{r\lambda_1}Z_{3(PN)} + YZ_{2p(PN)}}{\lambda_1 p_{r0}X_7' + \left\{p_{r0}\left(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2}{r}\right)\right\}X_7}, \tag{3.42}$$

where

$$X_5 = \frac{2\alpha r^2}{(r-m_0)^2}\left\{e' - \frac{r}{r+m_0}\left(bR_0' + 2e\frac{m_0}{r}\right)\right\} + Y\left[\frac{r^2}{(r-m_0)^2}\left\{2\alpha e - Y\frac{\bar{c}}{r}\right\} - \frac{2\alpha(r+m_0)^2}{r^2}\left\{\frac{er^2}{Y(r+m_0)^2}\right\}'\right],$$
$$X_6 = \lambda_1\left\{\rho_0\left(\frac{ar}{r-m_0}\right)' - 2(p_{r0} + p_{\perp 0})b'\right\} - \frac{2\alpha}{Y}\left[(\lambda_1 p_{r0} + \lambda)e' + e\left\{\lambda_1(p_{r0}' + \frac{\rho_0 m_0}{r(r-m_0)}) + p_{r0}\left(\frac{2}{r} - \frac{2\alpha R_0'}{Y}\right)\right\} - \lambda\left(\rho_0' - \frac{2\alpha R_0'}{Y}\right)\right],$$
$$X_7 = \frac{2e}{Y} + \lambda_1 b\left(2 + \frac{r}{r+m_0}\right),$$
$$X_8 = \left(\frac{m_0}{r(r-m_0)} + \frac{2\alpha R_0'}{Y} + \frac{2\lambda}{\lambda_1 r} + \lambda\rho_0'\right).$$

$Z_{1p(PN)}$ and $Z_{2p(PN)}$ are terms of Z_{1p} and Z_{2p} that lie in the post-Newtonian era. The above inequality (3.42) holds for positive definite terms and describes the stability range of the subsequent evolution. The positivity of each term appearing in (3.42) leads to the following restrictions:

$$\frac{r}{r+m_0}\left(bR_0' + \frac{2em_0}{r}\right) < e', \quad 2\alpha e - Yb > \frac{(r^2-m_0^2)^2}{r^4}\left\{\frac{er^2}{Y(r+m_0)^2}\right\}' \times \left(\frac{ar}{r-m_0}\right)' > 2(p_{r0} + p_{\perp 0})b', \quad \rho_0' < \frac{2\alpha R_0'}{Y}.$$

4 Concluding remarks

In this manuscript, we carried out a study of the implications of the shear-free condition on the stability of spherically symmetric anisotropic stars in $f(R,T)$. Our exploration regarding the viability of the $f(R,T)$ model reveals that the selection of $f(R,T)$ model for dynamical analysis is constrained to the form $f(R,T) = f(R) + \lambda T$, where λ is an arbitrary positive constant. The restriction on the form of $f(R,T)$ originates from the complexities of nonlinear terms of the trace in an analytical formulation of the field equations. The model under consideration is of the form $f(R,T) = R + \alpha R^2 + \lambda T$, representing a viable substitute to dark source and the exotic matter, both satisfying the viability criterion (positivity of radial derivatives up to second order).

In $f(R,T)$ gravity, the non-minimal matter–geometry coupling includes the terms of the trace T in the action (1.1) that is beneficent in the description of quantum effects or so-called exotic matter. The components of the modified field equations together with the implementation of the shear-free condition are developed in Sect. 2. Further conservation laws are considered in order to arrive at the dynamical equations by means of the Bianchi identities. These equations are utilized to estimate the variations in the gravitating system with the passage of time.

The complexities of more generic analytical field equations are dealt with by using a linear perturbation of the physical quantities. The perturbation scheme induces a significant ease in the description of the dynamical system, or rather to present a stability analysis by means of numerical simulations. The analytic approach we have employed here is more general and substantially important in explorations regarding structure formation. The perturbed shear-free condition together with the dynamical and field equations leads to the evolution equation, relating Γ with the usual and dark source terms. It is found that the induction of the trace of the energy-momentum tensor in the action (1.1) contributes a positive addition to Γ, which slows down the subsequent evolution considerably.

The outcome of the gravitational evolution is size dependent, and we have as well other physical aspects such as isotropy, anisotropy, shear, radiation, dissipation, etc. The instability range for N and pN approximations is considered, which imposes some restrictions on the physical variables. It is observed that the terms appearing in Γ are less constrained for both the regimes (N and pN) in comparison to the anisotropic sources [1]. Thus, the shear-free condition benefits in more stable anisotropic configurations. Corrections to GR and $f(R)$ establishments can be made by setting $\alpha \to 0$, $\lambda \to 0$, and $\lambda \to 0$, respectively. The local isotropy of the model can be settled by assuming $p_r = p_\perp = p$. The extension of this work for a shearing expansion of the free evolution of anisotropic spherical and cylindrical sources is in process.

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Appendix

We have

$$\begin{aligned}
Z_1(r,t) &= f_R A^2\left[\left\{\frac{1}{f_R A^2}\left(\frac{f-Rf_R}{2}-\frac{3\dot{f}_R}{A^2}\frac{\dot{B}}{B}-\frac{f'_R}{B^2}\left(\frac{B'}{B}-\frac{2C'}{C}\right)+\frac{f''_R}{B^2}\right)\right\}_{,0}\right.\\
&\quad\left.+\left\{\frac{1}{f_R A^2 B^2}\left(\dot{f}'_R-\frac{A'}{A}\dot{f}_R-\frac{\dot{B}}{B}f'_R\right)\right\}_{,1}\right]-\frac{\dot{f}_R}{A^2}\left\{\left(3\frac{\dot{B}}{B}\right)^2+\frac{9\dot{A}}{A}\frac{\dot{B}}{B}\right\}\\
&\quad+\frac{3\ddot{f}_R}{A^2}\frac{\dot{B}}{B}+\frac{\dot{A}}{A}(f-Rf_R)-\frac{2f'_R}{B^2}\left\{\frac{\dot{A}}{A}\left(\frac{B'}{B}-\frac{C'}{C}\right)+\frac{\dot{B}}{B}\left(\frac{3A'}{A}-\frac{2C'}{C}+\frac{B'}{B}\right)\right\}\\
&\quad+\frac{f''_R}{B^2}\left(\frac{2\dot{A}}{A}+\frac{\dot{B}}{B}\right)+\frac{1}{B^2}\left(\dot{f}'_R-\frac{A'}{A}\dot{f}_R\right)\left(\frac{3A'}{A}+\frac{B'}{B}+\frac{2C'}{C}\right),
\end{aligned}\tag{5.1}$$

$$\begin{aligned}
Z_2(r,t) &= f_R B^2\left[\left\{\frac{1}{f_R A^2 B^2}\left(\dot{f_R}'-\frac{A'}{A}\dot{f}_R-\frac{\dot{B}}{B}f'_R\right)\right\}_{,0}+\left\{\frac{1}{f_R B^2}\left(\frac{Rf_R-f}{2}\right.\right.\right.\\
&\quad\left.\left.\left.-\frac{\dot{f}_R}{A^2}\left(\frac{\dot{A}}{A}-\frac{2\dot{C}}{C}\right)-\frac{f'_R}{B^2}\left(\frac{A'}{A}+\frac{2C'}{C}\right)+\frac{\ddot{f}_R}{A^2}\right)\right\}_{,1}\right]+(Rf_R-f)\frac{B'}{B}\\
&\quad-\frac{1}{A^2}\frac{\dot{f}_R}{A^2}\left\{\frac{A'}{A}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}\right)+\frac{B'}{B}\left(\frac{\dot{A}}{A}-\frac{2\dot{C}}{C}\right)\right\}+\left(\frac{\dot{A}}{A}+\frac{5\dot{B}}{B}\right)\left(\dot{f_R}'-\frac{A'}{A}\dot{f}_R-\frac{\dot{B}}{B}f'_R\right)\\
&\quad-\frac{f'_R}{B^2}\left\{\frac{A'}{A}\left(\frac{A'}{A}+\frac{3B'}{B}\right)+\frac{2C'}{C}\left(\frac{3B'}{B}+\frac{C'}{C}\right)\right\}+\frac{\ddot{f}_R}{A^2}\left(\frac{A'}{A}+\frac{2B'}{B}\right)+\frac{f''_R}{B^2}\left(\frac{A'}{A}+\frac{2C'}{C}\right);
\end{aligned}\tag{5.2}$$

$$
\begin{aligned}
Z_{1p} &= 2\alpha A_0^2\left[\frac{1}{A_0^2B_0^2Y}\left\{e'-e\frac{A_0'}{A_0}-\frac{b}{B_0}R_0'\right\}\right]_{,1}+\frac{1}{Y}[e-[\lambda T_0\\
&\quad -\alpha R_0^2]\left(\frac{a}{A_0}+\frac{e}{Y}\right)+\frac{2\alpha}{B_0^2}\left\{\left(\frac{B_0'}{B_0}-\frac{2}{r}\right)\left(e'-2R_0'\left(\frac{a}{A_0}+\frac{b}{B_0}\right)\right.\right.\\
&\quad \left.+\frac{2\alpha e}{Y}R_0'\right)+R_0''\left(\frac{2a}{A_0}+\frac{b}{B_0}\right)-2R_0'\left(\frac{b}{B_0}\left(\frac{2A_0'}{A_0}+\frac{B_0'}{B_0}+\frac{1}{r}\right)\right.\\
&\quad \left.\left.\left.+\frac{\bar{c}}{r}\left(\frac{A_0'}{A_0}-\frac{3}{r}\right)\right)+\left(e'-e\frac{A_0'}{A_0}\right)\left(\frac{3A_0'}{A_0}+\frac{B_0'}{B_0}+\frac{2}{r}\right)\right\}\right] \qquad (5.3)\\
Z_{2p} &= B_0^2Y\left[\frac{1}{B_0^2Y}\left\{e-\frac{2\alpha}{B_0^2}\left\{\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left(e-\left[\frac{2\alpha e}{Y}+\frac{4b}{B_0}\right]R_0'\right)+R_0'[\left(\frac{a}{A_0}\right)'\right.\right.\right.\\
&\quad \left.\left.\left.+\left(\frac{\bar{c}}{r}\right)']\right\}-[\lambda T_0-\alpha R_0^2]\left(\frac{b}{B_0}+\frac{e}{Y}\right)\right\}\right]_{,1}+bB_0Y\left[\frac{1}{B_0^2Y}\left\{\lambda T_0-\alpha R_0^2\right.\right.\\
&\quad \left.\left.-\frac{4\alpha}{B_0^2}\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)R_0'\right\}\right]_{,1}+\frac{2\alpha}{B_0^2}\left[R_0''\left\{\left(\frac{a}{A_0}\right)'-2\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)\left(\frac{b}{B_0}+\frac{e}{Y}\right)\right.\right.\\
&\quad \left.+\left(\frac{\bar{c}}{r}\right)'\right\}-R_0'\left\{\frac{A_0'}{A_0}\left[2\left(\frac{a}{A_0}\right)'+3\left(\frac{b}{B_0}\right)'\right]+3\frac{B_0'}{B_0}\left[\left(\frac{a}{A_0}\right)'+2\left(\frac{\bar{c}}{r}\right)'\right]\right.\\
&\quad \left.+\frac{2}{r}\left[\left(3\frac{b}{B_0}\right)'+2\left(\frac{\bar{c}}{r}\right)'\right]\right\}+\left(\frac{2b}{B_0}R_0'-e\right)\left\{3\frac{B_0'}{B_0}\left(\frac{A_0'}{A_0}+\frac{2}{r}\right)+\left(\frac{A_0'}{A_0}\right)^2\right.\\
&\quad \left.\left.+\frac{2}{r^2}\right\}\right]+e\frac{B_0'}{B_0}-[\lambda T_0-\alpha R_0^2]\left(\frac{b}{B_0}+\frac{2e}{Y}\frac{B_0'}{B_0}\right); \qquad (5.4)\\
Z_3 &= \frac{Y}{B_0^2}\left[\frac{a''}{A_0}+\frac{\bar{c}''}{r}-\frac{A_0''}{A_0}\left(\frac{a}{A_0}+\frac{2b}{B_0}\right)+\frac{A_0'}{A_0}\left\{\frac{2b}{B_0}\left(\frac{B_0'}{B_0}-\frac{1}{r}\right)+\left(\frac{\bar{c}}{r}\right)'\right.\right.\\
&\quad \left.\left.-\left(\frac{b}{B_0}\right)'\right\}+\frac{B_0'}{B_0}\left\{\frac{2bB_0'}{rB_0}-\left(\frac{a}{A_0}\right)'-\left(\frac{\bar{c}}{r}\right)'\right\}+\frac{1}{r}\left\{\left(\frac{a}{A_0}\right)'-\left(\frac{b}{B_0}\right)'\right\}\right]\\
&\quad -\frac{2\alpha e}{Y}\left\{\frac{\lambda T_0-\alpha R_0^2}{2}-\frac{2\alpha}{B_0^2}\left(R_0'\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{1}{r}\right)-R_0''\right)\right\}-\frac{2\alpha}{B_0^2}\left\{e''\right.\\
&\quad \left.+\frac{2b}{B_0}R_0''+\left(\frac{A_0'}{A_0}-\frac{B_0'}{B_0}+\frac{1}{r}\right)\left(\frac{2b}{B_0}R_0'-e'\right)\right\}. \qquad (5.5)
\end{aligned}
$$

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