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Natural quasi-alignment with two Higgs doublets and RGE stability

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Abstract In the context of two Higgs doublet models, we study the conditions required in order to have stable quasi-alignment in flavour space. We show that stability under the renormalisation group equations imposes strong constraints on the flavour structure of the Yukawa couplings associated to each one of the Higgs doublets. In particular, we find a novel solution, where all Yukawa couplings are proportional to the so-called democratic matrix. This solution is rather unique, since it is the only stable solution which is a good starting point for reproducing the observed pattern of quark masses and mixing. We also show that this stable solution can be obtained by imposing on the Lagrangian a $Z_3 \times Z_3'$ flavour symmetry. Quark masses of the lighter quark generations are generated through the breaking of this discrete symmetry, and, at this stage, scalar-mediated flavour-changing neutral-currents arise, but they are naturally suppressed by the smallness of the light quark masses. In this way, we relate Higgs alignment to the hierarchy of the quark masses through a discrete family symmetry.

1 Introduction

One of the simplest extensions of the standard model (SM) consists of the addition of scalar doublets to the SM spectrum. Multi-Higgs extensions arise in a variety of frameworks, including supersymmetric extensions of the SM, as well as models with family symmetries. A two Higgs doublet model (2HDM) was first introduced by Lee [1], in order to achieve spontaneous breaking of the CP symmetry. If no extra symmetries are introduced, 2HDMs lead to too large tree-level scalar-mediated flavour-changing neutral-currents (FCNCs) [2,3]. In order to avoid these potentially dangerous currents, various schemes have been proposed:

(i) Glashow and Weinberg [4] have pointed out that one can avoid FCNCs at tree level by introducing a Z_2 symmetry under which the two Higgs doublets transform differently. The introduction of a Z_2 symmetry in 2HDMs prevents the generation of spontaneous CP breaking [5] unless the symmetry is softly broken [6].

(ii) Pich and Tuzon [7] have conjectured the existence of flavour alignment of the two Yukawa matrices, thus avoiding FCNCs at tree level. This is an interesting suggestion, but it has the drawback of being an ad-hoc assumption, not explained by any symmetry. Furthermore, it has been pointed out that in general this scheme is not stable under the renormalisation group [8]. There have been attempts to obtain alignment in various extensions of the SM [9–11].

(iii) Another possibility has been proposed some time ago [12] by Branco, Grimus and Lavoura (BGL) who have pointed out that there is a symmetry which, when imposed on the Lagrangian, constrains the Yukawa couplings in such a way that FCNCs do arise at tree level, but are entirely determined by the V_{CKM} matrix, with no other free parameters. In some of the BGL models, one has a strong natural suppression of the most dangerous FCNC, with, for example, the strangeness-changing neutral currents, proportional to $(V_{td}V_{ts}^*)^2$,

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which implies a very strong natural suppression of the contribution to the K^0–$\bar{K}^0$ transition. With this suppression, the neutral Higgs masses need not be too large. BGL models have been extended to the leptonic sector [13], their relation to minimal flavour violation models has been studied [14] and their phenomenological implications have been recently analysed [15–17].

This is not a complete list of possible schemes to avoid large FCNC in 2HDM, some other plausible solutions consist in postulating concrete Yukawa structures, such as the renowned Cheng and Sher [18] ansatz, or in simply decoupling the heavy Higgs states.

In this paper, we reexamine the question of the stability of flavour alignment under the renormalisation group. Assuming that the Yukawa couplings of the two Higgs doublets are aligned, i.e., proportional to each other, we study under what conditions the alignment is maintained by the renormalisation group. Apart from the conditions already found in Ref. [8], we find new solutions which can be of great physical interest. One of these solutions, corresponds to having all the Yukawa coupling matrices proportional to the so-called democratic matrix [19–21]. This solution is rather unique, since on the one hand it is stable under the renormalisation group equations (RGEs) and on the other hand it is the only stable solution which provides a good starting point for reproducing the observed pattern of quark masses and mixing. We then point out that this flavour democratic solution can be obtained as a result of a $Z_3 \times Z_3'$ flavour symmetry. In the framework that we propose, flavour alignment is exact in the limit where only the third family acquires mass. Once the two light generations acquire a mass, there are small deviations from alignment, which are suppressed by the strong hierarchy of quark masses. As a result, one obtains in this framework, a quasi-alignment of the Yukawa couplings, as a result of the $Z_3 \times Z_3'$ symmetry, together with the strong hierarchy of quark masses.

The paper is organised as follows. In the next section, we briefly describe the general flavour structure of the 2HDM, in order to settle our notation. In Sect. 3 we derive all the solutions for the Yukawa couplings, leading to alignment, stable under the renormalisation group. In Sect. 4 we show that the flavour democratic solution can be obtained as a result of a $Z_3 \times Z_3'$ flavour symmetry and propose an ansatz for the breaking of the $Z_3 \times Z_3'$ symmetry. In Sect. 5, we examine the suppression of scalar-mediated FCNCs in our framework. In Sect. 6, we perform a numerical analysis, showing how the pattern of quark masses and mixing can be obtained in the framework of our ansatz. Finally our conclusions are contained in Sect. 7. In the appendix we present a full study of the solutions of the alignment conditions.

2 Yukawa couplings in the general two-Higgs-doublet-model (2HDM)

For completeness and in order to establish our notation we briefly review the flavour structure of the 2HDM, when no extra symmetries are introduced in the Lagrangian. The Yukawa couplings can be written:

$$\mathcal{L}_Y = -\overline{Q_L^0}\,\Gamma_1\Phi_1 d_R^0 - \overline{Q_L^0}\,\Gamma_2\Phi_2 d_R^0 - \overline{Q_L^0}\,\Omega_1\tilde{\Phi}_1 u_R^0$$
$$- \overline{Q_L^0}\,\Omega_2\tilde{\Phi}_2 u_R^0 - \overline{L_L^0}\,\Pi_1\Phi_1 l_R^0 - \overline{L_L^0}\,\Pi_2\Phi_2 l_R^0 + \text{h.c.}$$
(1)

where Φ_i denote the Higgs doublets and $\tilde{\Phi}_i \equiv i\,\tau_2\,\Phi_i^*$, and Γ_i, Ω_i and Π_i are matrices in flavour space. After spontaneous symmetry breaking, the following quark mass matrices are generated:

$$M_d = \frac{1}{\sqrt{2}}(v_1\Gamma_1 + v_2 e^{i\alpha}\Gamma_2),$$
$$M_u = \frac{1}{\sqrt{2}}(v_1\Omega_1 + v_2 e^{-i\alpha}\Omega_2)$$
(2)

where $v_i/\sqrt{2} \equiv |<0|\phi_i^0|0>|$ and α denotes the relative phase of the two vacuum expectation values (vevs) of the neutral components ϕ_i^0 of Φ_i. The neutral and the charged Higgs interactions with quarks are of the form

$$\mathcal{L}_Y(\text{quark, Higgs}) = -\overline{d_L^0}\,\frac{1}{v}\,[M_d H^0 + N_d^0 R + i\,N_d^0 I]\,d_R^0$$
$$-\overline{u_L^0}\,\frac{1}{v}\,[M_u H^0 + N_u^0 R + i\,N_u^0 I]\,u_R^0$$
(3)
$$+ \frac{\sqrt{2}H^+}{v}\left(\overline{u_L^0}\,N_d^0\,d_R^0 - \overline{u_R^0}\,N_u^{0\dagger}\,d_L^0\right) + \text{h.c.}$$

where $v \equiv \sqrt{v_1^2 + v_2^2} \approx 246$ GeV, and H^0 and R are orthogonal combinations of the fields ρ_j, arising when one expands [1] the neutral scalar fields around their vevs, $\phi_j^0 = \frac{e^{i\alpha_j}}{\sqrt{2}}(v_j + \rho_j + i\eta_j)$. Here we should choose H^0 in such a way that it has couplings to the quarks which are proportional to the mass matrices, as can be seen from Eq. (3). Similarly, I denotes the linear combination of η_j orthogonal to the neutral Goldstone boson. The matrices N_d^0 and N_u^0 are given by

$$N_d^0 = \frac{1}{\sqrt{2}}(v_2\Gamma_1 - v_1 e^{i\alpha}\Gamma_2),$$
$$N_u^0 = \frac{1}{\sqrt{2}}(v_2\Omega_1 - v_1 e^{-i\alpha}\Omega_2).$$
(4)

The quark mass matrices are diagonalised through

$$U_{dL}^\dagger M_d U_{dR} = D_d \equiv \text{diag}(m_d, m_s, m_b),$$
$$U_{uL}^\dagger M_u U_{uR} = D_u \equiv \text{diag}(m_u, m_c, m_t),$$
(5)

and the matrices N_d^0 and N_u^0 in the mass eigenstate basis transform into

$$U_{dL}^\dagger N_d^0 U_{dR} = N_d, \quad U_{uL}^\dagger N_u^0 U_{uR} = N_u.$$
(6)

There are similar expressions for the leptonic sector. We do not introduce neutrino masses since these are not relevant for our analysis.

3 Stability of the aligned 2HDM under RGE

The aligned two Higgs doublet model (A2HDM) is defined at tree level by the following relations involving the matrices introduced in Eq. (1):

$$\Gamma_2 = d \cdot \Gamma_1$$
$$\Omega_2 = u \cdot \Omega_1 \tag{7}$$
$$\Pi_2 = e \cdot \Pi_1$$

where d, u and e are constants. In this section we analyse the stability of the A2HDM under the RGEs. The one loop RGEs for the Yukawa couplings are [8,22]

$$\mathcal{D}\Gamma_k = a_\Gamma \Gamma_k$$
$$+ \sum_{l=1}^{2} \left[3\mathrm{Tr}\left(\Gamma_k \Gamma_l^\dagger + \Omega_k^\dagger \Omega_l \right) + \mathrm{Tr}\left(\Pi_k \Pi_l^\dagger + \Sigma_k^\dagger \Sigma_l \right) \right] \Gamma_l$$
$$+ \sum_{l=1}^{2} \left(-2\Omega_l \Omega_k^\dagger \Gamma_l + \Gamma_k \Gamma_l^\dagger \Gamma_l + \frac{1}{2}\Omega_l \Omega_l^\dagger \Gamma_k + \frac{1}{2}\Gamma_l \Gamma_l^\dagger \Gamma_k \right), \tag{8}$$

$$\mathcal{D}\Omega_k = a_\Omega \Omega_k$$
$$+ \sum_{l=1}^{2} \left[3\mathrm{Tr}\left(\Omega_k \Omega_l^\dagger + \Gamma_k^\dagger \Gamma_l \right) + \mathrm{Tr}\left(\Sigma_k \Sigma_l^\dagger + \Pi_k^\dagger \Pi_l \right) \right] \Omega_l$$
$$+ \sum_{l=1}^{2} \left(-2\Gamma_l \Gamma_k^\dagger \Omega_l + \Omega_k \Omega_l^\dagger \Omega_l + \frac{1}{2}\Gamma_l \Gamma_l^\dagger \Omega_k + \frac{1}{2}\Omega_l \Omega_l^\dagger \Omega_k \right), \tag{9}$$

$$\mathcal{D}\Pi_k = a_\Pi \Pi_k$$
$$+ \sum_{l=1}^{2} \left[3\mathrm{Tr}\left(\Gamma_k \Gamma_l^\dagger + \Omega_k^\dagger \Omega_l \right) + \mathrm{Tr}\left(\Pi_k \Pi_l^\dagger + \Sigma_k^\dagger \Sigma_l \right) \right] \Pi_l$$
$$+ \sum_{l=1}^{2} \left(-2\Sigma_l \Sigma_k^\dagger \Pi_l + \Pi_k \Pi_l^\dagger \Pi_l + \frac{1}{2}\Sigma_l \Sigma_l^\dagger \Pi_k + \frac{1}{2}\Pi_l \Pi_l^\dagger \Pi_k \right), \tag{10}$$

where $\mathcal{D} \equiv 16\pi^2 \mu \, (\mathrm{d}/\mathrm{d}\mu)$ and μ is the renormalisation scale. The coefficients a_Γ, a_Ω and a_Π are given by

$$a_\Gamma = -8g_s^2 - \frac{9}{4}g^2 - \frac{5}{12}g'^2, \tag{11}$$

$$a_\Omega = -8g_s^2 - \frac{9}{4}g^2 - \frac{17}{12}g'^2, \tag{12}$$

$$a_\Pi = -\frac{9}{4}g^2 - \frac{15}{4}g'^2 \tag{13}$$

where g_s, g and g' are the gauge coupling constants of $\mathrm{SU}(3)_c$, $\mathrm{SU}(2)_L$ and $U(1)_Y$, respectively. The alignment relations given by Eq. (7) guarantee the absence of Higgs-

mediated FCNCs at tree level because both matrices M_d and N_d are proportional to Γ_1. Similarly both M_u and N_u are proportional to Ω_1 and M_l, N_l to Π_1. In general, these relations are broken at one loop level. From Eqs. (8)–(10) one can easily derive

$$\mathcal{D}(\Gamma_2) - d \cdot \mathcal{D}(\Gamma_1)$$
$$= (u^* - d)(1 + ud)\left\{ 3Tr\left(\Omega_1^\dagger \Omega_1\right) - 2\Omega_1 \Omega_1^\dagger \right\} \Gamma_1$$
$$+ (e - d)(1 + e^*d) Tr\left(\Pi_1^\dagger \Pi_1\right) \Gamma_1, \tag{14}$$

$$\mathcal{D}(\Omega_2) - u \cdot \mathcal{D}(\Omega_1)$$
$$= (d^* - u)(1 + ud)\left\{ 3Tr\left(\Gamma_1^\dagger \Gamma_1\right) - 2\Gamma_1 \Gamma_1^\dagger \right\} \Omega_1$$
$$+ (e^* - u)(1 + eu) Tr\left(\Pi_1^\dagger \Pi_1\right) \Omega_1, \tag{15}$$

$$\mathcal{D}(\Pi_2) - e \cdot \mathcal{D}(\Pi_1)$$
$$= 3(d - e)(1 + d^*e) Tr\left(\Gamma_1^\dagger \Gamma_1\right) \Pi_1$$
$$+ 3(u^* - e)(1 + eu) Tr\left(\Omega_1^\dagger \Omega_1\right) \Pi_1. \tag{16}$$

In order to enforce Eq. (7) at one loop level it is easy to realise that it is sufficient to impose

$$\mathcal{D}(\Gamma_2) - d \cdot \mathcal{D}(\Gamma_1) \propto \Gamma_1, \tag{17}$$

$$\mathcal{D}(\Omega_2) - u \cdot \mathcal{D}(\Omega_1) \propto \Omega_1, \tag{18}$$

$$\mathcal{D}(\Pi_2) - e \cdot \mathcal{D}(\Pi_1) \propto \Pi_1; \tag{19}$$

in fact the proportionality constants on the r.h.s. are the running[1] of d, u and e. Therefore, Eq. (16) does not impose any constraint: at one loop level the charged lepton sector remains aligned and there are no FCNCs in the leptonic sector. This result agrees with the findings of Refs. [23,24].

In Eqs. (14) and (15) the pieces that can break the alignment in the quark sector are the terms: $\Omega_1 \Omega_1^\dagger \Gamma_1$ and $\Gamma_1 \Gamma_1^\dagger \Omega_1$ respectively.[2] In order to have alignment at one loop level – fulfilling Eqs. (17) and (18) – there are two types of solutions:

1. $(u^* - d)(1 + ud) = 0$
2. $\Omega_1 \Omega_1^\dagger \Gamma_1 = \lambda_\Gamma \Gamma_1$ and $\Gamma_1 \Gamma_1^\dagger \Omega_1 = \lambda_\Omega \Omega_1$. With λ_Γ and λ_Ω complex numbers.

Solutions of type 1 include the usual 2HDM with natural flavour conservation, where the up and down quarks receive contributions from only one Higgs doublet. It is well known that this can be achieved through the introduction of a Z_2

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[1] The authors of Ref. [8] impose the condition that the r.h.s. of Eqs. (14)–(16) be equal to zero. This amounts to imposing alignment at one loop level and imposing additionally that there is no running of the parameters u, d and e.

[2] It can be readily seen that $\Omega_1 \Omega_1^\dagger \Gamma_1 \propto M_u M_u^\dagger M_d$ and $\Gamma_1 \Gamma_1^\dagger \Omega_1 \propto M_d M_d^\dagger M_u$. It is worth emphasizing that these structures are precisely the ones obtained in [23,24], which produce FCNCs at one loop level and break the alignment. Note that this radiatively induced misalignment is in general very small as also argued in [25,26].

symmetry. Here, we are not interested in this class of well known solutions. We are interested in the class of solutions of type 2, and in the appendix we study the complete set of matrices Ω_1 and Γ_1 that obey to the conditions required for this class of solutions. We show in the appendix that if one requires stability under the RGE and at the same time Yukawa structures which are, in leading order, in agreement with the observed pattern of quark masses and mixing, then one is lead to a unique solution, where the matrices Ω_1 and Γ_1 are of the form

$$\Omega_1 = c_1^d \Delta; \quad \Gamma_1 = c_1^u \Delta \tag{20}$$

with Δ the democratic mass matrix:

$$\Delta = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{21}$$

This solution corresponds to the limit where only the top and bottom quarks acquire mass, while the two first generations are massless. The up and down quarks are aligned in flavour space, so the V_{CKM} matrix equals the identity. The other stable solutions of type 2 correspond to non-realistic cases like for example having all up or down quarks massless or two up or two down quark masses degenerate or with a V_{CKM} very far from the identity matrix.

It is remarkable that the so-called democratic mass matrix is stable under RGE and that precisely this stability also enforces what could be called "quark alignment" in the sense that we also have a proportionality among Γ_i and Ω_i.

4 Natural quasi-alignment of Yukawa couplings

In this section we search for the minimal symmetry which when imposed on the Lagrangian, leads to the stable solution described in the previous section, corresponding to the democratic Yukawa couplings of Eq. (20). Before describing this symmetry, it is worth to analyse another type of alignment which is verified experimentally, the so-called up–down alignment in the quark sector.

4.1 The up–down alignment in the quark sector

In the quark sector, flavour mixing is small. This means that there is a weak basis (WB) where both M_u and M_d are close to the diagonal form. Experiment indicates that not only flavour mixing is small, but there is also up–down flavour alignment in the quark sector in the following sense. We can choose, without loss of generality, a WB where $M_u = \text{diag}(m_u, m_c, m_t)$. Of course, this is just a choice of ordering, with no physical meaning. Small mixing implies that in this WB M_d is almost diagonal. In principle, since the Yukawa couplings Y_u and Y_d are not constrained in the

SM, there is equal probability of M_d being close to $M_d = \text{diag}(m_d, m_s, m_b)$. corresponding to up–down alignment, or being close, for instance, to $M_d = \text{diag}(m_b, m_s, m_d)$ in which case there is up–down misalignment. It is clear that in the SM, assuming small mixing and hierarchical quark masses, the probability of obtaining up–down alignment is only 1/6. Given a set of arbitrary quark mass matrices M_u and M_d, one can derive necessary and sufficient conditions to obtain small mixing and up–down alignment, expressed in terms of WB invariants [27]. Since the experimentally verified up–down alignment is not automatic in the SM, one may wonder whether there is a symmetry which leads to up–down alignment. In the next subsection, we propose a symmetry which leads to up–down alignment in the quark sector and when extended to a 2HDM leads to a natural alignment of the two Higgs doublets in flavour space.

4.2 $Z_3 \times Z_3'$ symmetry and the two Higgs alignment

We introduce the $Z_3 \times Z_3'$ symmetry under which the quark left-handed doublets $Q_{L_i}^0$, the right-handed up quarks $u_{L_i}^0$ and the right-handed down quarks $d_{L_i}^0$ transform in the following way:

$$Q_{L_i}^0 \longrightarrow P_{ij}^\dagger Q_{L_j}^0,$$
$$u_{R_i}^0 \longrightarrow P_{ij} u_{R_j}^0,$$
$$d_{R_i}^0 \longrightarrow P_{ij} d_{R_j}^0, \tag{22}$$

where Z_3 corresponds to $P = \mathbb{1} + E_1$ and Z_3' to $P = \mathbb{1} + E_2$ with

$$E_1 = \frac{\omega - 1}{2} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix};$$
$$E_2 = \frac{\omega - 1}{3} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -1 \\ \frac{1}{2} & \frac{1}{2} & -1 \\ -1 & -1 & 2 \end{pmatrix} \tag{23}$$

and $\omega = e^{\frac{2\pi i}{3}}$. The Higgs doublets transform trivially under $Z_3 \times Z_3'$. The above symmetry leads to the following form for the Yukawa matrices: $\Gamma_j = c_j^d \Delta$, $\Omega_j = c_j^u \Delta$, corresponding to the stable solution of Eq. (20). This can easily be checked since $\Delta E_1 = \Delta E_2 = 0$. We thus conclude that the symmetry of Eqs. (22) and (23) leads to the alignment of the two Yukawa coupling matrices, with a democratic flavour structure. Note that this solution also guarantees an up–down alignment in the quark sector, as defined in the previous subsection.

In order to give mass to the first two quark generations, the $Z_3 \times Z_3'$ symmetry has to be broken. This breaking will also lead to Higgs-mediated FCNCs, but these couplings will be suppressed by the smallness of the quark masses. In order to illustrate how a realistic pattern of quark masses and mixing

can be obtained, we shall assume that the breaking of the $Z_3 \times Z_3'$ symmetry occurs in two steps. In the first step the symmetry $Z_3 \times Z_3'$ is broken into just one of the Z_3 and the second generation acquires mass and finally in the last step the masses of the quarks u and d are generated. In the first step the symmetry $Z_3 \times Z_3'$ is broken to Z_3 generated by $P = \mathbb{1} + E_1$. One can check that

$$\Gamma_j = c_j^d \left(\Delta + \varepsilon_d \, A\right); \quad \Omega_j = c_j^u \left(\Delta + \varepsilon_u \, A\right);$$

$$A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{24}$$

are invariant under this Z_3 symmetry. Note that $A \, E_1 = 0$. At this stage the second generation acquires mass. Finally, the lightest quarks, u and d acquire mass through a small perturbation, proportional to $\hat{\delta}_{d,u}$, which breaks this Z_3 symmetry. We assume that

$$\Gamma_2 = c_2^d \left(\Delta + \varepsilon_d \, A + \hat{\delta}_d \, B_d\right), \tag{25}$$

while

$$\Gamma_1 = c_1^d \left(\Delta + \varepsilon_d \, A\right); \tag{26}$$

with equivalent expressions for the up sector. Here

$$B_u = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}; \quad B_d = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & \eta \end{pmatrix} \tag{27}$$

where η is some complex number with modulus of order one. The symmetry is broken, and neither B_u nor B_d are invariant under the $Z_3 \times Z_3'$ symmetry.

5 Suppression of scalar-mediated FCNC

In order to study the suppression of a scalar-mediated FCNC, it is useful to start by analysing the parameter space in our framework.

5.1 The parameter space

From Eqs. (2), (25) and (26) it follows that, in leading order,

$$m_b = \frac{3}{\sqrt{2}} |c_1^d v_1 + c_2^d v_2 e^{i\alpha}|; \quad m_t = \frac{3}{\sqrt{2}} |c_1^u v_1 + c_2^u v_2 e^{-i\alpha}|. \tag{28}$$

Writing $v \equiv \sqrt{v_1^2 + v_2^2} = v_1 \sqrt{1 + t^2}$, with

$$t = \frac{v_2}{v_1} \tag{29}$$

we obtain in leading order the following relations:

$$\frac{|c_1^d + c_2^d \, t e^{i\alpha}|}{\sqrt{1 + t^2}} = \frac{\sqrt{2}}{3} \frac{m_b}{v}; \quad \frac{|c_1^u + c_2^u \, t e^{-i\alpha}|}{\sqrt{1 + t^2}} = \frac{\sqrt{2}}{3} \frac{m_t}{v}, \tag{30}$$

which impose restrictions on the allowed parameter space. A priori, we do not assume any conspiracy between parameters and take $t = O(1)$. It is then clear from Eq. (30) that the c_i^u are generically of order one, while c_i^d are smaller and may assume values of order $O(\frac{m_b}{m_t})$. This is an important ingredient which, as we shall see, will play a rôle in the evaluation of the strengths of the FCNCs and the allowed parameter space for the Higgs masses.

Next we give the structure of the flavour-changing neutral Yukawa couplings. To this aim, it is useful to express the quark mass matrices in Eq. (2) in terms of the perturbations given in Eqs. (25) and (26):

$$\begin{aligned} M_d &= \tfrac{v_1}{\sqrt{2}} \left(c_1^d + c_2^d t e^{i\alpha}\right) \left[\Delta + \varepsilon_d \, A + \delta_d B_d\right] \\ M_u &= \tfrac{v_1}{\sqrt{2}} \left(c_1^u + c_2^u t e^{-i\alpha}\right) \left[\Delta + \varepsilon_u \, A + \delta_u B_u\right] \end{aligned}; \quad \begin{aligned} \delta_d &\equiv \tfrac{c_2^d t e^{i\alpha}}{c_1^d + c_2^d t e^{i\alpha}} \, \widehat{\delta}_d, \\ \delta_u &\equiv \tfrac{c_2^u t e^{-i\alpha}}{c_1^u + c_2^u t e^{-i\alpha}} \, \widehat{\delta}_u. \end{aligned} \tag{31}$$

Then we derive the expressions for the matrices which couple to the Higgs scalars in Eqs. (3) and (4). In the basis where the up and down quark matrices are diagonal, the matrices N_d and N_u of Eq. (6) become

$$N_d = t \, D_d - \frac{v_1}{\sqrt{2}} \left(1 + t^2\right) e^{i\alpha} \, U_{d_L}^\dagger \, \Gamma_2 \, U_{d_R},$$

$$N_u = t \, D_u - \frac{v_1}{\sqrt{2}} \left(1 + t^2\right) e^{-i\alpha} \, U_{u_L}^\dagger \, \Omega_2 \, U_{u_R} \tag{32}$$

where we have used Eqs. (4) and (6) with Eq. (2). Finally, from Eq. (32) combined with Eqs. (25) and (31) we find

$$\begin{aligned} N_d &= \frac{c_1^d t - c_2^d e^{i\alpha}}{c_1^d + c_2^d t e^{i\alpha}} \, D_d - \frac{v}{\sqrt{2}} \frac{c_1^d \sqrt{1+t^2}}{t} \delta_d \, U_{d_L}^\dagger \, B_d \, U_{d_R} \\ N_u &= \frac{c_1^u t - c_2^u e^{-i\alpha}}{c_1^u + c_2^u t e^{-i\alpha}} \, D_u - \frac{v}{\sqrt{2}} \frac{c_1^u \sqrt{1+t^2}}{t} \delta_u \, U_{u_L}^\dagger \, B_u \, U_{u_R} \end{aligned} \tag{33}$$

where $D_d \equiv \mathrm{diag}(m_d, m_s, m_b)$ and $D_u \equiv diag(m_u, m_c, m_t)$.

The crucial point is that in our scheme these matrices have an extra suppression factor, proportional to $\delta_{d,u}$. Using the expressions given in Eqs. (31) and (27), and computing the trace, the second invariant and the determinant for the squared quark mass matrices $H_{u,d} \equiv \left(M M^\dagger\right)_{u,d}$, one can find that in leading order

$$\begin{aligned} \delta_d &= \sqrt{3} \sqrt{\frac{m_d}{m_s}} \frac{m_s}{m_b} = O\left(\lambda^3\right) \\ \delta_u &= \sqrt{3} \sqrt{\frac{m_u}{m_c}} \frac{m_c}{m_t} = O\left(\lambda^5\right) \end{aligned} \tag{34}$$

where $\lambda \equiv 0.2$ is of the order of the Cabibbo angle.

From Eq. (31) it follows that in leading order $U_{d_L} = U_{u_L} = U_{d_R} = U_{u_R} = F$, where

$$F == \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ -1/\sqrt{2} & 1/\sqrt{6} & 1/\sqrt{3} \\ 0 & -2/\sqrt{6} & 1/\sqrt{3} \end{pmatrix} \quad (35)$$

is the matrix that diagonalises the exact democratic limit Δ. Thus, taking into account Eq. (27), the matrix contributions from $U_{d_L}^\dagger B_d U_{d_R}$ and $U_{u_L}^\dagger B_u U_{u_R}$ are both of order one. One can thus conclude that:

– for the down sector, with the assumptions made after Eq. (30), we have a total suppression factor of $O(\frac{m_b}{m_t}) \cdot O(\lambda^3)$
– for the up sector, we have a suppression factor of $O(\lambda^5)$ or smaller depending on the value that we choose to assume for c_1^u, but which, as explained, it is reasonable to take of order one.

6 Numerical analysis

The matrices of Eq. (31) may be explicitly written

$$M_u = c_u \begin{pmatrix} 1 & 1 & 1+\varepsilon+\delta \\ 1 & 1 & 1+\varepsilon \\ 1+\varepsilon+\delta & 1+\varepsilon & 1+\varepsilon+\delta \end{pmatrix}_u,$$

$$M_d = c_d \begin{pmatrix} 1 & 1 & 1+\varepsilon+\delta \\ 1 & 1 & 1+\varepsilon \\ 1+\varepsilon+\delta & 1+\varepsilon & 1+\varepsilon+\eta\delta \end{pmatrix}_d, \quad (36)$$

where we have introduced $c_d \equiv \frac{v_1}{\sqrt{2}}(c_1^d + c_2^d t e^{i\alpha})$ and $c_u \equiv \frac{v_1}{\sqrt{2}}(c_1^u + c_2^u t e^{-i\alpha})$. Although these two coefficients are in general complex, and since the physically meaningful matrices are those defined as $H = M M^\dagger$, both coefficients may be taken as real for our numerical exercise. If one then parametrises the remaining variables as

$$\varepsilon = \varepsilon_m \exp(i\,\varepsilon_f), \quad \delta = \delta_m \exp(i\,\delta_f),$$
$$\eta = \eta_m \exp(i\,\eta_f), \quad (37)$$

one is left with 12 real parameters that compose the quark mass matrices in our scheme.

In order to check if this parameter space could accommodate the flavour sector, a numerical survey was made where we looked for one combination that could fit the observed values of the quark masses given at the scale of the Z boson mass [28], the moduli of the entries of the CKM matrix [29], the strength of CP violation I_{CP} and $\sin 2\beta$ and γ [29], with β and γ being two of the angles of the unitarity triangle. A simple run of all 12 parameters produced a "reference point":

	Up sector	Down sector
c	56.73	0.89
ε_m	1.6×10^{-2}	0.11
ε_f	-5.6×10^{-3}	0.41
δ_m	8.1×10^{-4}	2.2×10^{-2}
δ_f	$\pi + 0.32$	2.26
η_m	–	4.99
η_f	–	$\pi + 0.62$

which yields the output values:

$$D_d = \text{diag}(0.00204, 0.05824, 2.85356) \text{ GeV},$$
$$D_u = \text{diag}(0.00114, 0.61736, 171.684) \text{ GeV},$$
$$|V_{CKM}| = \begin{pmatrix} 0.9745 & 0.2244 & 0.0036 \\ 0.2243 & 0.9737 & 0.0415 \\ 0.0087 & 0.0407 & 0.9991 \end{pmatrix}, \quad (38)$$
$$|I_{CP}| = 3.0 \times 10^{-5},$$
$$\sin 2\beta = 0.69,$$
$$\gamma = 69.3°.$$

It should be noted that the 12 parameters fix not only V_{CKM} and the quark mass spectrum, but also the strengths of all the FCNC couplings. In order to evaluate the numerical stability of this reference point, we performed a numerical check, varying the input parameters randomly around the values that produced the reference point above; the new results were then combined in the scatter plots shown in Fig. 1 where the reference point is highlighted. In order to obtain an estimate of the lower bound for the flavour-violating Higgs masses, we consider the contribution to K^0–$\overline{K}^0$ mixing. Apart from the SM box diagram one now has a new physics contribution arising from the scalar-mediated FCNC tree-level diagrams thus making the total transition amplitude equal to $M_{12} = M_{12}^{SM} + M_{12}^{NP}$, Ref. [16] with

$$M_{12}^{NP} = \sum_{H=R,I} \frac{f_M^2 m_M}{96 v^2 m_H^2} \left\{ \left[1 + \left(\frac{m_M}{m_{q1}+m_{q2}} \right)^2 \right] \right.$$
$$\left. \times C_1(H) - \left[1 + 11 \left(\frac{m_M}{m_{q1}+m_{q2}} \right)^2 \right] C_2(H) \right\}, \quad (39)$$

where

$$C_1(R) = \left[(N_{q2q1})^* + N_{q1q2} \right]^2,$$
$$C_2(R) = \left[(N_{q2q1})^* - N_{q1q2} \right]^2, \quad (40)$$

and

$$C_1(I) = -\left[(N_{q2q1})^* - N_{q1q2} \right]^2,$$
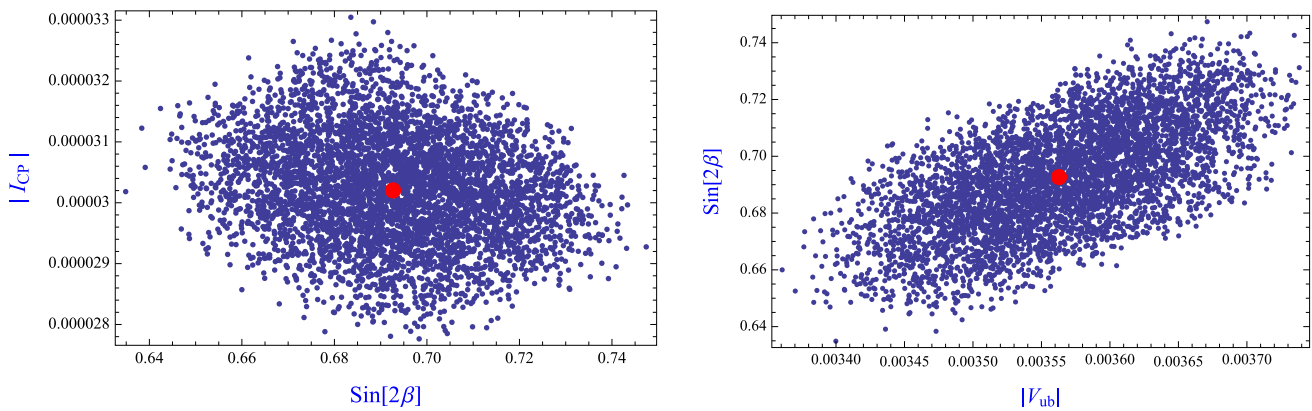$$C_2(I) = -\left[(N_{q2q1})^* + N_{q1q2} \right]^2. \quad (41)$$

Fig. 1 We present *scatter plots* showing $|I_{CP}|$ versus $\sin 2\beta$ and $\sin 2\beta$ versus $|V_{ub}|$ obtained by varying randomly the input parameters around the reference point
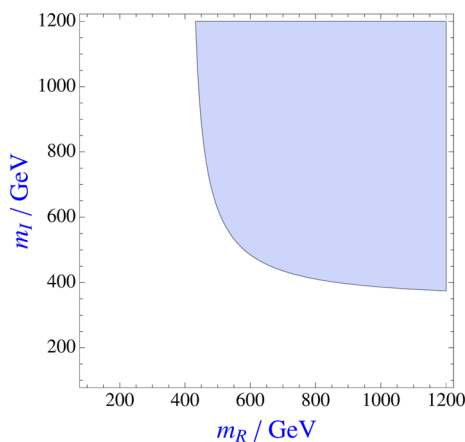

Fig. 2 *Plot* showing the allowed region for m_I and m_R, taking into account the constraint on Δm_K

The indices q_1 and q_2 refer to the valence quarks of the meson M, and N is N_u or N_d, depending on the meson system considered.

In this framework it is a good approximation to use the matrix F for both U_{d_L} and U_{d_R}. Using the values we obtained for δ_d and taking, as already discussed, $t \simeq 1$ and $c_1^d \simeq \frac{\sqrt{2}}{3}\frac{m_b}{v}$, the new physics contribution to M_{12}^K becomes solely dependent on f_K, m_K, m_R and m_I. In K^0–$\overline{K}^0$, both M_{12}^K and Γ_{12}^K are relevant for the mass difference Δm_K. It is reasonable to impose the constraint that M_{12}^{NP} in the neutral kaon system does not exceed the experimental value of Δm_K. Adopting as input values the PDG experimental determinations of f_K, m_K and Δm_K [30], one is left with combinations of m_R and m_I where our model respects the inequality $M_{12}^{NP(K)} < \Delta m_K$. The region plot that we have obtained is presented in Fig. 2. It is clear that in this framework the masses of the flavour-violating neutral Higgs can be below the TeV scale, so that they could be discovered at the next run of the LHC.

7 Conclusions

We have studied in detail, in the framework of 2HDM, the question of stability of alignment, under the renormalisation group. It was shown that there are new stable solutions, apart from those found in Ref. [8]. Stability under the RGE puts very strict restrictions on the flavour structure of the Higgs Yukawa couplings. If one imposes the stability conditions and at the same time requires that the flavour structure is in agreement with the observed pattern of quark masses and mixing, then one is lead to a unique solution, where all Higgs flavour matrices are proportional to the so-called democratic matrix. We have also shown that these flavour structures leading to stable alignment can be obtained by imposing on the Lagrangian a $Z_3 \times Z_3'$ symmetry. In the limit where this symmetry is exact, only the third generation of quarks acquires a mass. Non-vanishing masses for the two first generations are obtained through the breaking of the discrete symmetry which in turn generates scalar-mediated FCNCs which are suppressed by the smallness of the light quark masses.

It should be pointed out that the proposed framework does not automatically imply a natural suppression of CP violating effects such as edms [31,32]. In this respect this framework has the same features as the standard aligned model of Ref. [7].

The scenario presented in this paper provides a natural framework for the alignment hypothesis and establishes a possible intriguing link between stability of alignment in 2HDM and the observed pattern of quark masses and mixing.

Appendix: Solutions to the alignment conditions

The solutions to the alignment conditions

$$\Omega_1 \Omega_1^\dagger \Gamma_1 = \lambda_\Gamma \Gamma_1; \quad \Gamma_1 \Gamma_1^\dagger \Omega_1 = \lambda_\Omega \Omega_1 \qquad (42)$$

can be obtained by the following steps. First we define the Hermitian matrices

$$H_\Gamma = \Gamma_1 \Gamma_1^\dagger; \quad H_\Omega = \Omega_1 \Omega_1^\dagger. \qquad (43)$$

It is easy to show that λ_Γ and λ_Ω are real. This can be achieved by multiplying the first equation by its Hermitian conjugate and inserting the second equation (and vice versa) to get

$$\lambda_\Omega H_\Omega^2 = |\lambda_\Gamma|^2 H_\Gamma, \qquad (44)$$

$$\lambda_\Gamma H_\Gamma^2 = |\lambda_\Omega|^2 H_\Omega; \qquad (45)$$

it follows from these equations that both λ_Ω and λ_Γ should be real since one has two identities among Hermitian matrices. Now multiplying each of the Eq. (42) on the right by $\Gamma_1^\dagger$ and $\Omega_1^\dagger$, respectively, we get

$$H_\Omega H_\Gamma = \lambda_\Gamma H_\Gamma; \quad H_\Gamma H_\Omega = \lambda_\Omega H_\Omega, \qquad (46)$$

and taking Hermitian conjugates

$$H_\Gamma H_\Omega = \lambda_\Gamma H_\Gamma; \quad H_\Omega H_\Gamma = \lambda_\Omega H_\Omega; \qquad (47)$$

therefore,

$$\lambda_\Gamma H_\Gamma = \lambda_\Omega H_\Omega, \qquad (48)$$

and we conclude that

$$[H_\Gamma, H_\Omega] = 0, \qquad (49)$$

implying that $V_{\text{CKM}} = I$ up to permutations of rows or columns. Denoting the usual bi-unitary diagonalisation procedure by

$$\Gamma_1 = V_L^\Gamma D_\Gamma V_R^{\Gamma\dagger}; \quad \Omega_1 = V_L^\Omega D_\Omega V_R^{\Omega\dagger}, \qquad (50)$$

from Eq. (49) we conclude that we can always choose the unitary matrices V_L^Γ and V_L^Ω equal to each other,

$$V_L^\Gamma = V_L^\Omega, \qquad (51)$$

and the alignment conditions can easily be reduced to conditions among the diagonal matrices D_Γ and D_Ω. From Eq. (46), it then follows that

$$D_\Omega^2 D_\Gamma = \lambda_\Gamma D_\Gamma; \quad D_\Gamma^2 D_\Omega = \lambda_\Omega D_\Omega. \qquad (52)$$

It can be checked that there are only two types of solutions. Those with λ_Γ and λ_Ω different from zero (solutions 1–3) and the remaining ones (solutions 4, 5).

1. $D_\Gamma = a P_3$ and $D_\Omega = \alpha P_3$ and changes of P_3 by P_2 or P_1.
2. $D_\Gamma = a (I - P_1)$ and $D_\Omega = \alpha (I - P_1)$ and changes of P_1 by P_2 or P_3.
3. $D_\Gamma = aI$ and $D_\Omega = \alpha I$.
4. $D_\Gamma = 0$ and D_Ω arbitrary and vice versa.
5. $D_\Gamma = a P_i$ and $D_\Omega = \alpha (I - P_i)$

where P_i stand for the projection operators

$$(P_i)_{jk} = \delta_{ij}\delta_{ik}. \qquad (53)$$

Solutions 2–4 cannot be good approximations to the actual quark spectra due to the implied degeneracy. Solution 5 gives rise to V_{CKM} matrix very different from the identity matrix. Only solution 1 provides, in leading approximation the correct pattern of quark masses and mixing. In a suitable weak basis, this solution can be written as a democratic matrix Δ.

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