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Finite anticanonical transformations in field–antifield formalism

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Abstract We study the role of arbitrary (finite) anticanonical transformations in the field–antifield formalism and the gauge-fixing procedure based on the use of these transformations. The properties of the generating functionals of the Green functions subjected to finite anticanonical transformations are considered.

1 Introduction

The field–antifield formalism [1,2], summarizing numerous attempts to find correct quantization rules for various types of gauge models [3–7], is a powerful covariant quantization method which can be applied to arbitrary gauge invariant systems. This method is based on the fundamental principle of BRST invariance [8,9] and has a rich new geometry [10]. One of the most important objects of the field–antifield formalism is an odd symplectic structure called antibracket and known to mathematicians as the Buttin bracket [11]. In terms of the antibracket the master equation and the Ward identity for generating functional of the vertex functions (effective action) are formulated. It is an important property that the antibracket is preserved under the anticanonical transformations which are dual to canonical transformations for a Poisson bracket. An important role and rich geometric possibilities of general anticanonical transformations in the field–antifield formalism have been realized in the procedure of gauge fixing [12] (see also [13]). The original procedure of gauge fixing [1,2] corresponds in fact to a special type of anticanonical transformation in an action being a proper solution to the quantum master equation. That type of transformations is capable to yield admissible gauge-fixing conditions in the form of equations of arbitrary Lagrangian surfaces (constraints in the antibracket involution) in the field–antifield phase space. Thereby, the necessary class of admissible gauges was involved actually. The latter made it possible to describe in [12] the structure and renormalization of general gauge theories in terms of anticanonical transformations. As the authors [12] assumed the use of regularizations in which $\delta(0) = 0$ in local field theories, they based themselves on the use of general anticanonical transformations in an action being a proper solution to the classical master equation. In turn, the gauge dependence and the structure of renormalization of the effective action have been analyzed by using infinitesimal anticanonical transformations only.

In the present article, we extend the use of anticanonical transformations in the field–antifield formalism from the infinitesimal level to the finite one, and we explore a gauge-fixing procedure for general gauge theories, based on arbitrary anticanonical transformations in an action being a proper solution to the quantum master equation with fixed boundary condition. Now it is worthy to notice the difference between the properties of the classical and quantum master equations under anticanonical transformations. The classical master equation is covariant under anticanonical transformations, as its left-hand side is the antibracket of the action with itself. In contrast to that, the form of the quantum master equation is not maintained under anticanonical transformations. One should accompany the anticanonical transformation by multiplying the exponential of $i/\hbar$ times the transformed action with the square root of the superjacobian of that anticanonical transformation. We will call such an operation an *anticanonical master transformation* and the corresponding action a *master-transformed action*. Thus, one can say that the form of the quantum master equation is maintained under the anticanonical master transformation.

We consider in all detail the relationship between the two descriptions (in terms of the generating functions and the generators) for arbitrary finite anticanonical transformations.

Finally, let us notice the study [14], among other recent developments, where a procedure was found to connect gen-

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erating functionals of the Green functions for a gauge system formulated in any two admissible gauges with the help of finite field-dependent BRST transformations.

2 Field–antifield formalism

The starting point of the field–antifield formalism [1] is a theory of fields $\{\mathcal{A}\}$ for which the initial classical action $S_0(\mathcal{A})$ is assumed to be invariant under the gauge transformations $\delta\mathcal{A} = R(\mathcal{A})\xi$. Here ξ are arbitrary functions of space-time coordinates, and $\{R(\mathcal{A})\}$ are generators of gauge transformations. The set of generators is complete but, in general, may be reducible and forms an open gauge algebra so that one works with general gauge theories. Here we do not discuss these points, referring to the original papers [1,2]. The structure of the gauge algebra determines the necessary content of the total configuration space of fields $\{\varphi^i\ (\varepsilon(\varphi^i) = \varepsilon_i)\}$ involving fields $\{\mathcal{A}\}$ of the initial classical system, ghost and antighost fields, auxiliary fields, and, in the case of reducible generators, pyramids of extra ghost and antighost fields as well as pyramids of extra auxiliary fields. To each field φ^i one introduces an antifield φ^*_i, whose statistics is opposite to that of the corresponding fields φ^i, $\varepsilon(\varphi^*_i) = \varepsilon_i + 1$. On the space of the fields φ^i and antifields φ^*_i one defines an odd symplectic structure (,) called the antibracket,

$$(F, G) \equiv F(\overleftarrow{\partial}_{\varphi^i}\overrightarrow{\partial}_{\varphi^*_i} - \overleftarrow{\partial}_{\varphi^*_i}\overrightarrow{\partial}_{\varphi^i})G, \tag{2.1}$$

and the nilpotent fermionic operator Δ,

$$\Delta = (-1)^{\varepsilon_i}\partial_{\varphi^i}\partial_{\varphi^*_i}, \quad \Delta^2 = 0, \quad \varepsilon(\Delta) = 1. \tag{2.2}$$

Here the notation

$$\partial_{\varphi^i} = \frac{\partial}{\partial\varphi^i}, \quad \partial_{\varphi^*_i} = \frac{\partial}{\partial\varphi^*_i} \tag{2.3}$$

is introduced. In terms of the antibracket and Δ-operator the quantum master equation is formulated as

$$\frac{1}{2}(\mathcal{S}, \mathcal{S}) = i\hbar\Delta\mathcal{S} \Leftrightarrow \Delta\ \exp\left\{\frac{i}{\hbar}\mathcal{S}\right\} = 0 \tag{2.4}$$

for a bosonic functional $\mathcal{S} = \mathcal{S}(\varphi, \varphi^*)$ satisfying the boundary condition

$$\mathcal{S}|_{\varphi^*=\hbar=0} = S_0(\mathcal{A}) \tag{2.5}$$

and being the basic object of the field–antifield quantization scheme [1,2]. Among the properties of the antibracket and Δ-operator we mention the Leibniz rule,

$$(F, GH) = (F, G)H + (F, H)G(-1)^{\varepsilon(G)\varepsilon(H)}, \tag{2.6}$$

the Jacobi identity,

$$((F, G), H)(-1)^{(\varepsilon(F)+1)(\varepsilon(H)+1)} + \mathsf{cycle}(F, G, H) \equiv 0, \tag{2.7}$$

the Δ-operator being a derivative to the antibracket,

$$\Delta(F, G) = (\Delta F, G) - (F, \Delta G)(-1)^{\varepsilon(F)}. \tag{2.8}$$

There exists a generating functional $Y = Y(\varphi, \Phi^*)$, $\varepsilon(Y) = 1$ of the anticanonical transformation,

$$\Phi^i = \partial_{\Phi^*_i}Y(\varphi, \Phi^*), \quad \varphi^*_i = Y(\varphi, \Phi^*)\overleftarrow{\partial}_{\varphi^i}. \tag{2.9}$$

The invariance property of the odd symplectic structure (2.1) on the phase space of (φ, φ^*) is dual to the invariance property of an even symplectic structure (a Poisson bracket) under a canonical transformation of canonical variables (p, q) (for further discussions of the relations between Poisson bracket and antibracket; see [15,16]).

The generating functional of the Green functions $Z(J)$ is defined in terms of the functional integral as [1,2]

$$\begin{aligned}\mathcal{Z}(J) &= \int \mathcal{D}\varphi\ \exp\left\{\frac{i}{\hbar}[S_e(\varphi) + J_i\varphi^i]\right\}\\ &= \exp\left\{\frac{i}{\hbar}W(J)\right\},\end{aligned} \tag{2.10}$$

where

$$S_e(\varphi) = \mathcal{S}(\varphi, \varphi^* = \partial_\varphi\psi(\varphi)), \tag{2.11}$$

$\psi(\varphi)$ is a fermionic gauge functional, $J_i\ (\varepsilon(J_i) = \varepsilon_i)$ are the usual external sources to the fields φ^i and $W(J)$ is the generating functional of the connected Green functions.

To discuss the quantum properties of general gauge theories, it is useful to consider, instead of the generating functional (2.10), the extended generating functionals $\mathcal{Z}(J, \varphi^*)$ and $W(J, \varphi^*)$ defined by the relations

$$\begin{aligned}\mathcal{Z}(J, \varphi^*) &= \int \mathcal{D}\varphi\ \exp\left\{\frac{i}{\hbar}[S(\varphi, \varphi^*) + J_i\varphi^i]\right\}\\ &= \exp\left\{\frac{i}{\hbar}W(J, \varphi^*)\right\}\end{aligned} \tag{2.12}$$

where

$$S(\varphi, \varphi^*) = \mathcal{S}(\varphi, \varphi^* + \partial_\varphi\psi(\varphi)). \tag{2.13}$$

Obviously, we have

$$\mathcal{Z}(J) = \mathcal{Z}(J, \varphi^*)\ |_{\varphi^*=0}, \quad W(J) = W(J, \varphi^*)\ |_{\varphi^*=0}. \tag{2.14}$$

The action $S = S(\varphi, \varphi^*)$ (2.13) satisfies the quantum master equation

$$\frac{1}{2}(S, S) = i\hbar\Delta S \Leftrightarrow \Delta\exp\left\{\frac{i}{\hbar}S\right\} = 0. \tag{2.15}$$

It follows from (2.15) that the Ward identities hold for the extended generating functionals $\mathcal{Z}(J, \varphi^*)$ and $W(J, \varphi^*)$

$$J_i\partial_{\varphi^*_i}\mathcal{Z}(J, \varphi^*) = 0, \quad J_i\partial_{\varphi^*_i}W(J, \varphi^*) = 0. \tag{2.16}$$

Indeed, we have

$$0 = \int \mathrm{d}\varphi \exp\left\{\frac{i}{\hbar}J\varphi\right\}\left(\Delta \exp\left\{\frac{i}{\hbar}S\right\}\right)$$
$$= \int \mathrm{d}\varphi(-1)^{\varepsilon_i}\partial_{\varphi^i}\left[\exp\left\{\frac{i}{\hbar}J\varphi\right\}\partial_{\varphi_i^*}\exp\left\{\frac{i}{\hbar}S\right\}\right]$$
$$-\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\int \mathrm{d}\varphi \exp\left\{\frac{i}{\hbar}(S+J\varphi)\right\} = -\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\mathcal{Z}(J,\varphi^*)$$
$$= -\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\exp\left\{\frac{i}{\hbar}W(\varphi^*,J)\right\} \Longrightarrow J_i\partial_{\varphi_i^*}W(\varphi^*,J)=0. \tag{2.17}$$

The generating functional of the vertex function (effective action) is defined via the Legendre transformation

$$\Gamma(\varphi,\varphi^*) = W(J,\varphi^*) - J\varphi,\ \varphi^i = \partial_{J_i}W(J,\varphi^*),\ \partial_{\varphi_i^*}W(J,\varphi^*) = \partial_{\varphi_i^*}\Gamma(\varphi,\varphi^*),\ \partial_{J_i} = \frac{\partial}{\partial J_i} \tag{2.18}$$

with the properties

$$J_i = -\Gamma(\varphi,\varphi^*)\overleftarrow{\partial}_{\varphi^i} = -(-1)^{\varepsilon_i}\Gamma_i,\quad \Gamma_i = \Gamma_i(\varphi,\varphi^*) = \partial_{\varphi^i}\Gamma(\varphi,\varphi^*). \tag{2.19}$$

It follows from (2.17) and (2.19) that the Ward identity for the effective action holds,

$$\Gamma\overleftarrow{\partial}_{\varphi^i}\partial_{\varphi_i^*}\Gamma = 0 \Longrightarrow \frac{1}{2}(\Gamma,\Gamma)=0, \tag{2.20}$$

which has the form of the classical master equation in the field–antifield formalism.

As pointed out for the first time in [12], the gauge-fixing procedure in the field–antifield formalism (2.13) can be described in terms of a special type of anticanonical transformation (2.9). Indeed, let us consider the anticanonical transformations of the variables (φ,φ^*) with the specific generating function

$$Y = Y(\varphi,\Phi^*) = \Phi_i^*\varphi^i - \psi(\varphi). \tag{2.21}$$

We have

$$\Phi^i = \varphi^i,\quad \varphi_i^* = \Phi_i^* - \partial_{\varphi^i}\psi(\varphi), \tag{2.22}$$

so that the transformed action $\tilde{S} = \tilde{S}(\varphi,\varphi^*)$

$$\tilde{S}(\varphi,\varphi^*) = S(\Phi,\Phi^*) = S(\varphi,\varphi^* + \partial_\varphi\psi(\varphi)) \tag{2.23}$$

coincides with (2.13). In particular, this fact made it possible to study effectively the gauge dependence and structure of renormalization of general gauge theories [12]. In what follows we explore a gauge-fixing procedure in the field–antifield formalism as an anticanonical transformation of general type with the only requirement for anticanonically generalized action: the supermatrix of the second field derivatives of this action must be non-degenerate. An essential difference in this point with the approach used in [12] is that we work with a general setting for an action (2.13) which satisfies the quantum master equation (not the classical master equation as in [12]).

3 Infinitesimal anticanonical transformations

As the first step in our study of anticanonical transformations in the field–antifield formalism, we consider the properties of the main objects subjected to infinitesimal anticanonical transformations. In the latter case, the generating functional Y reads

$$Y = Y(\varphi,\Phi^*) = \Phi_i^*\varphi^i + X(\varphi,\Phi^*),\quad \varepsilon(X)=1. \tag{3.1}$$

The functional X is considered as the infinitesimal one. Then the anticanonical transformations of the variables,

$$\Phi^i = \varphi^i + \partial_{\Phi_i^*}X(\varphi,\Phi^*),\quad \varphi_i^* = \Phi_i^* + \partial_{\varphi^i}X(\varphi,\Phi^*), \tag{3.2}$$

can be written down to the first order in X as

$$\begin{aligned}\Phi^i &= \varphi^i + \partial_{\varphi_i^*}X(\varphi,\varphi^*) + O(X^2),\\ \Phi_i^* &= \varphi_i^* - \partial_{\varphi^i}X(\varphi,\varphi^*) + O(X^2),\end{aligned} \tag{3.3}$$

or, in terms of the antibracket (2.1),

$$\begin{aligned}\Phi^i &= \varphi^i + (\varphi^i, X) + O(X^2),\\ \Phi_i^* &= \varphi_i^* + (\varphi_i^*, X) + O(X^2).\end{aligned} \tag{3.4}$$

The anticanonically transformed action $\tilde{S}$,

$$\tilde{S} = \tilde{S}(\varphi,\varphi^*) = S(\Phi,\Phi^*) = S + (S,X) + O(X^2) \tag{3.5}$$

does not satisfy the quantum master equation to the first approximation in X,

$$\frac{1}{2}(\tilde{S},\tilde{S}) - i\hbar\Delta\tilde{S} = i\hbar(S,\Delta X) + O(X^2) \neq 0. \tag{3.6}$$

Consider now the superdeterminant of the anticanonical transformation

$$\mathcal{J}(\varphi,\varphi^*) = \mathcal{J}(Z) = \mathrm{sDet}\,[\bar{Z}^A(Z)\overleftarrow{\partial}_B], \tag{3.7}$$

where

$$\bar{Z}^A = (\Phi^i,\Phi_i^*),\quad Z^A = (\varphi^i,\varphi_i^*),\quad \partial_A = \frac{\partial}{\partial Z^A}. \tag{3.8}$$

To the first-order approximation in X, $\mathcal{J}$ reads

$$\mathcal{J} = \exp\{2\Delta X\} + O(X^2) = \exp\left\{\frac{i}{\hbar}(-2i\hbar\Delta X)\right\} + O(X^2). \tag{3.9}$$

In contrast to the notation used in [17,18], now we refer to $S' = S'(\varphi,\varphi^*)$ constructed from $S = S(\varphi,\varphi^*)$ via the *anticanonical master transformation*,

$$S' = S'(\varphi,\varphi^*) = S(\Phi(\varphi,\varphi^*),\Phi^*(\varphi,\varphi^*)) \\ -i\hbar\frac{1}{2}\ln\mathcal{J}(\varphi,\varphi^*), \tag{3.10}$$

as the *master-transformed action*.

Note that, by itself, the anticanonical master transformation can be defined without reference on solutions of the quantum master equation. Namely, let us define a transformation of the form[1]

$$\exp\left\{\frac{i}{\hbar}G'\right\} = \exp\{-[F,\Delta]\}\exp\left\{\frac{i}{\hbar}G\right\}, \tag{3.11}$$

where G, F ($\varepsilon(G)=0,\ \varepsilon(F)=1$) are arbitrary functions of φ,φ^*, and $[\ ,\ ]$ stands for the supercommutator. Then we can prove (see Appendices C and D) the relation

$$G' = \exp\{\mathrm{ad}(F)\}G + i\hbar f(\mathrm{ad}(F))\Delta F, \\ f(\mathrm{ad}(F))\Delta F = -\frac{1}{2}\ln\mathcal{J},\ \ \mathrm{ad}(F)(\ldots) = (F,(\ldots)), \tag{3.12}$$

which repeats Eq. (3.10). In (3.12) the notation $f(x) = (\exp x - 1)x^{-1}$ is used.

The action S' (3.10) to the first order in X

$$S' = S + (S,X) - i\hbar\Delta X + O(X^2) \tag{3.13}$$

does satisfy the quantum master equation

$$\frac{1}{2}(S',S') - i\hbar\Delta S' = O(X^2). \tag{3.14}$$

Note that, due to the results of [17,18], the action (3.10) by itself satisfies the quantum master equation in the case of arbitrary anticanonical transformation, as well (see, also [13,19]).

Let us consider the generating functionals constructed with the help of the master-transformed action S' to the first order in X. We have

$$\mathcal{Z}' = \mathcal{Z}'(J,\varphi^*) = \int d\varphi\exp\left\{\frac{i}{\hbar}(S'+J\varphi)\right\} \\ = \exp\left\{\frac{i}{\hbar}W'(J,\varphi^*)\right\} \\ = \exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\left(1+\frac{i}{\hbar}\delta W(J,\varphi^*)\right), \tag{3.15}$$

$$\Gamma'(\varphi,\varphi^*) = W'(J,\varphi^*) - J\varphi = \Gamma(\varphi,\varphi^*) + \delta\Gamma(\varphi,\varphi^*), \\ \delta\Gamma(\varphi,\varphi^*) = \delta W(J(\varphi,\varphi^*),\varphi^*). \tag{3.16}$$

[1] In the present article, we only consider the case in which the generator F is a function; the case of an operator-valued F was studied in [20].

Therefore

$$\mathcal{Z}' - \mathcal{Z} = \delta\mathcal{Z} = \frac{i}{\hbar}\exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\delta W(J,\varphi^*) \\ = \frac{i}{\hbar}\exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\delta\Gamma(\varphi,\varphi^*) \\ = \frac{i}{\hbar}\int d\varphi[(S,X) - i\hbar\Delta X]\exp\left\{\frac{i}{\hbar}(S+J\varphi)\right\} \\ = \int d\varphi\exp\left\{\frac{i}{\hbar}J\varphi\right\}\Delta\left(X\exp\left\{\frac{i}{\hbar}S\right\}\right) \\ = -\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\left[\tilde{X}(J,\varphi^*)\exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\right] \\ = \exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\left[-\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\tilde{X}(J,\varphi^*)\right], \tag{3.17}$$

where

$$\tilde{X}(J,\varphi^*) = \exp\left\{-\frac{i}{\hbar}W(J,\varphi^*)\right\}\int d\varphi X\exp\left\{\frac{i}{\hbar}(S+J\varphi)\right\}. \tag{3.18}$$

When deriving (3.17), the Ward identity for $W(J,\varphi^*)$ (2.17), the quantum master equation for $S(\varphi,\varphi^*)$ (2.15), and the following identities:

$$i\hbar\exp\left\{\frac{i}{\hbar}S\right\}\Delta X = i\hbar\Delta\left(X\exp\left\{\frac{i}{\hbar}S\right\}\right) \\ +(S,X)\exp\left\{\frac{i}{\hbar}S\right\}, \tag{3.19}$$

$$\exp\left\{\frac{i}{\hbar}J\varphi\right\}\Delta\left(X\exp\left\{\frac{i}{\hbar}S\right\}\right) \\ = (-1)^{\varepsilon_i}\partial_{\varphi^i}\left[\exp\left\{\frac{i}{\hbar}J\varphi\right\}\partial_{\varphi_i^*}\left(Xe^{\frac{i}{\hbar}S}\right)\right] \\ -\frac{i}{\hbar}J_i\partial_{\varphi_i^*}\left(X\exp\left\{\frac{i}{\hbar}(S+J\varphi)\right\}\right), \tag{3.20}$$

are used. Rewriting (3.17) for a variation of the effective action $\Gamma = \Gamma(\varphi,\varphi^*)$, we obtain

$$\delta\Gamma(\varphi,\varphi^*) = -J_i\partial_{\varphi_i^*}\tilde{X}(J,\varphi^*) = (-1)^{\varepsilon_i}\Gamma_i\partial_{\varphi_i^*}\mathcal{X}(\varphi,\varphi^*) \\ -(-1)^{\varepsilon_i}\Gamma_i[\partial_{\varphi_i^*}J_j(\varphi,\varphi^*)]\,\partial_{J_j}\tilde{X}(J,\varphi^*)\Big|_{J=J(\varphi,\varphi^*)}, \tag{3.21}$$

where

$$\mathcal{X}(\varphi,\varphi^*) = \tilde{X}(J,\varphi^*)\Big|_{J=J(\varphi,\varphi^*)}. \tag{3.22}$$

One can rewrite Eq. (3.21) in terms of $\Gamma = \Gamma(\varphi,\varphi^*)$ as

$$\delta\Gamma(\varphi,\varphi^*) = \Gamma\overleftarrow{\partial}_{\varphi^i}\partial_{\varphi_i^*}\mathcal{X} - \Gamma\overleftarrow{\partial}_{\varphi_i^*}\partial_{\varphi^i}\mathcal{X} \\ = (\Gamma,\mathcal{X}) = -(\mathcal{X},\Gamma). \tag{3.23}$$

This result is proved in Appendix A. Equation (3.23) means that any infinitesimal anticanonical master transformation of the action S (3.5) with a generating functional X induces an infinitesimal anticanonical transformation in the effective action Γ (3.23) with a generating functional $\mathcal{X}$, provided the generating functional of the Green functions is constructed via the master-transformed action. An important goal of our present study is a generalization of this fact (for the first time found among the results of [12]) to the case of an arbitrary (finite) anticanonical transformation.

4 Finite anticanonical transformation

Consider an arbitrary (finite) anticanonical transformation described by a generating functional $Y = Y(\varphi, \Phi^*)$, $\varepsilon(Y) = 1$,[2]

$$\varphi^*_i = Y(\varphi, \Phi^*)\overleftarrow{\partial}_{\varphi^i}, \quad \Phi^A = \partial_{\Phi^*_i} Y(\varphi, \Phi^*). \tag{4.1}$$

Let Y have the form

$$Y(\varphi, \Phi^*) = \Phi^*_i \varphi^i + af(\varphi, \Phi^*), \quad \varepsilon(f(\varphi, \Phi^*)) = 1, \tag{4.2}$$

where a is a parameter. Then the solution of (4.1) up to second order in a can be written as

$$\begin{aligned}\Phi^i &= \varphi^i + af\overleftarrow{\partial}_{\varphi^*_i} - a^2(-1)^{(\varepsilon_i+1)(\varepsilon_j+1)} \\ &\quad f\overleftarrow{\partial}_{\varphi^*_j}\overleftarrow{\partial}_{\varphi^*_i}\overrightarrow{\partial}_{\varphi^j} f + O(a^3),\end{aligned} \tag{4.3}$$

$$\begin{aligned}\Phi^*_i &= \varphi^*_i - a\partial_{\varphi^i} f + a^2(-1)^{\varepsilon_i(\varepsilon_j+1)} \\ &\quad f\overleftarrow{\partial}_{\varphi^*_j}\overleftarrow{\partial}_{\varphi^i}\overrightarrow{\partial}_{\varphi^j} f + O(a^3),\end{aligned} \tag{4.4}$$

where $f \equiv f(\varphi, \varphi^*)$. Let us denote

$$Z^A = \{\varphi^i, \varphi^*_i\}, \quad \bar{Z}^A = \{\Phi^i, \Phi^*_i\}, \quad \varepsilon(\bar{Z}^A) = \varepsilon(Z^A) = \varepsilon_A, \tag{4.5}$$

and

$$\begin{aligned}F &= F(\varphi, \varphi^*; a) = -f(\varphi, \varphi^*) \\ &\quad + \frac{a}{2} f(\varphi, \varphi^*)\overleftarrow{\partial}_{\varphi^*_j}\overrightarrow{\partial}_{\varphi^j} f(\varphi, \varphi^*).\end{aligned} \tag{4.6}$$

Then we have

$$\bar{Z}^A = \bar{Z}^A(Z; a) = \exp\{a\,\mathrm{ad}(F)\}Z^A + O(a^3), \tag{4.7}$$

where $\mathrm{ad}(F)$ means the left adjoint of the antibracket

$$\mathrm{ad}(F)(\ldots) = (F(Z; a), (\ldots)). \tag{4.8}$$

We call F in (4.6) a generator of the anticanonical transformation to the second order. It should be noticed that the generator of an anticanonical transformation does not coincide with the generating functional of this transformation already to the second order. A natural question arises: Does a generator exist for a given anticanonical transformation, actually? To answer this question, we begin with the claim that an operator $\exp\{\mathrm{ad}(F)\}$ generates an anticanonical transformation. Indeed, let Z^A be anticanonical variables so that the antibracket (2.1) can be presented in the form

$$\begin{aligned}&(H(Z), G(Z)) = H(Z)\overleftarrow{\partial}_A E^{AB}\overrightarrow{\partial}_B G(Z), \\ &(Z^A, Z^B) = E^{AB}, \quad \partial_A = \frac{\partial}{\partial Z^A},\end{aligned} \tag{4.9}$$

where E^{AB} is a constant supermatrix with the properties

$$E^{BA} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)}E^{AB}, \quad \varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1. \tag{4.10}$$

Then the transformation

$$Z^A \to \bar{Z}^A(Z) = \exp\{\mathrm{ad}F(Z)\}Z^A \tag{4.11}$$

is anticanonical,

$$(\bar{Z}^A(Z), \bar{Z}^B(Z)) = \bar{Z}^A(Z)\overleftarrow{\partial}_C E^{CD}\overrightarrow{\partial}_D \bar{Z}^B(Z) = E^{AB}. \tag{4.12}$$

To prove this fact we introduce a one-parameter family of transformations

$$\bar{Z}^A(Z, a) = \exp\{a\,\mathrm{ad}(F)\}Z^A, \quad \bar{Z}^A(Z, 0) = Z^A, \tag{4.13}$$

and the quantities $\bar{Z}^{AB}(Z, a)$,

$$\bar{Z}^{AB}(Z, a) = (\bar{Z}^A(Z, a), \bar{Z}^B(Z, a)), \quad \bar{Z}^{AB}(Z, 0) = E^{AB}. \tag{4.14}$$

It follows from the definitions (4.13) and (4.14) that the relations

$$\begin{aligned}&\frac{\mathrm{d}}{\mathrm{d}a}\bar{Z}^A(Z, a) = (F(Z), \bar{Z}^A(Z, a)), \\ &\frac{\mathrm{d}}{\mathrm{d}a}\bar{Z}^{AB}(Z, a) = ((F(Z), \bar{Z}^A(Z, a)), \bar{Z}^B(Z, a)) \\ &\quad + (\bar{Z}^A(Z, a), (F(Z), \bar{Z}^B(Z, a)))\end{aligned} \tag{4.15}$$

$$\begin{aligned}&= (F(Z), (\bar{Z}^A(Z, a), \bar{Z}^B(Z, a)) = (F(Z), \bar{Z}^{AB}(Z, a)) \\ &\quad = \mathrm{ad}(F(Z))\bar{Z}^{AB}(Z, a)\end{aligned} \tag{4.16}$$

hold, where the Jacobi identity (2.7) for antibrackets is used. A solution to Eq. (4.16) has the form

$$\begin{aligned}&\bar{Z}^{AB}(Z, a) = \exp\{a\,\mathrm{ad}(F(Z))\}\bar{Z}^{AB}(Z, 0) \\ &\quad = \exp\{a\,\mathrm{ad}(F(Z))\}E^{AB} = E^{AB},\end{aligned} \tag{4.17}$$

and the transformation (4.11) is really anticanonical. The inverse to this statement is valid as well: an arbitrary set of anticanonical variables $\bar{Z}^A(Z)$ can be presented in the form

$$\bar{Z}^A(Z) = \exp\{\mathrm{ad}(F(Z))\}Z^A \tag{4.18}$$

with some generator functional $F(Z)$, $\varepsilon(F(Z)) = 1$. In Appendix B, a proof of this fact is given.

Consider now a master-transformed action $S' = S'(\varphi, \varphi^*)$ (3.10). It was pointed out in [14] that there are presentations of S' in the following forms:

[2] Note that any anticanonical transformation can be described in terms of a generating functional.

$$\exp\left\{\frac{i}{\hbar}S'\right\} = \exp\{-[F,\Delta]\}\exp\left\{\frac{i}{\hbar}S\right\}, \tag{4.19}$$

or

$$S' = \exp\{\mathrm{ad}(F)\}S + i\hbar f(\mathrm{ad}(F))\Delta F, \tag{4.20}$$

where $S = S(\varphi,\varphi^*)$, and $F = F(\varphi,\varphi^*)$ is a generator functional of an anticanonical transformation, $f(x) = (\exp(x)-1)x^{-1}$. In accordance with (3.10), the first term in the right-hand side in (4.20) describes an anticanonical transformation of S with an odd generator functional F, while the second term is a half of a logarithm of the Jacobian (3.7) of that transformation, up to $(-i\hbar)$. In Appendix D, we give a proof of the latter statement.

Now we are in a position to study the properties of generating functionals of Green functions subjected to an arbitrary anticanonical transformation. We start with the generating functionals of Green and connected Green functions,

$$\begin{aligned}\mathcal{Z}' = \mathcal{Z}'(J,\varphi^*) &= \int \mathrm{d}\varphi \exp\left\{\frac{i}{\hbar}(S'(\varphi,\varphi^*) + J\varphi)\right\}\\ &= \exp\left\{\frac{i}{\hbar}W'(J,\varphi^*)\right\},\end{aligned} \tag{4.21}$$

where S' is defined in (4.19). The constructed generating functionals (4.21) obey the very important property of independence of F for physical quantities on-shell.[3] Indeed, for infinitesimal δF the variation of $\mathcal{Z}'$ (4.21),

$$\begin{aligned}\delta\mathcal{Z}' &= -\frac{i}{\hbar}\int \mathrm{d}\varphi[(S,\delta F) - i\hbar(\Delta\delta F)]\\ &\quad\times\exp\left\{\frac{i}{\hbar}(S(\varphi,\varphi^*) + J\varphi)\right\}\\ &= \frac{i}{\hbar}J_A\partial_{\varphi^*_A}\left[\delta\tilde{F}(J,\varphi^*)\exp\left\{\frac{i}{\hbar}W(J,\varphi^*)\right\}\right],\end{aligned} \tag{4.22}$$

is proportional to the external sources J. Due to the equivalence theorem [21], it means that the Green functions calculated with the help of the generating functionals $\mathcal{Z}(J,\varphi^*)$ and $\mathcal{Z}'(J,\varphi^*)$ give the same physical answers on-shell. In deriving (4.22), the result of the calculation (3.17) is used and the notation

$$\begin{aligned}&\delta\tilde{F}(J,\varphi^*) = Z^{-1}(J,\varphi^*)\\ &\int \mathrm{d}\varphi\ \delta F(\varphi,\varphi^*)\exp\left\{\frac{i}{\hbar}(S(\varphi,\varphi^*) + J\varphi)\right\}\end{aligned} \tag{4.23}$$

is introduced.

In the case of finite anticanonical transformations, we consider the following anticanonically generalized action S'':

$$\begin{aligned}&\exp\left\{\frac{i}{\hbar}S''(\varphi,\varphi^*)\right\}\\ &\quad= \exp\{-[F(\varphi,\varphi^*) + \delta F(\varphi,\varphi^*),\Delta]\}\exp\left\{\frac{i}{\hbar}S(\varphi,\varphi^*)\right\},\end{aligned} \tag{4.24}$$

where $\delta F = \delta F(\varphi,\varphi^*)$ is an infinitesimal functional. The following representation holds:

$$\begin{aligned}&\exp\{-[F(\varphi,\varphi^*) + \delta F(\varphi,\varphi^*),\Delta]\}\\ &\quad= \exp\{-[\delta\mathcal{F}(\varphi,\varphi^*),\Delta]\}\exp\{-[F(\varphi,\varphi^*),\Delta]\},\end{aligned} \tag{4.25}$$

where $\delta\mathcal{F}(\varphi,\varphi^*)$ is defined by the relation

$$\begin{aligned}&\exp\{-\mathrm{ad}(F(\varphi,\varphi^*)) - \mathrm{ad}(\delta F(\varphi,\varphi^*))\}\exp\{-\mathrm{ad}(F(\varphi,\varphi^*))\}\\ &\quad= \exp\{-\mathrm{ad}(\delta\mathcal{F}(\varphi,\varphi^*))\}.\end{aligned} \tag{4.26}$$

In Appendix C, a proof of Eqs. (4.25) and (4.26) is given. Due to (4.25), we can present the action S'' in the form

$$\begin{aligned}\exp\left\{\frac{i}{\hbar}S''(\varphi,\varphi^*)\right\} &= \exp\{-[\delta\mathcal{F}(\varphi,\varphi^*),\Delta]\}\\ &\quad\times\exp\left\{\frac{i}{\hbar}S'(\varphi,\varphi^*)\right\}.\end{aligned} \tag{4.27}$$

Although we need here the infinitesimal functional $\delta\mathcal{F}(\varphi,\varphi^*)$, the representation (4.27) by itself is valid for arbitrary functional $\delta\mathcal{F}$. In turn, the representation (4.27) allows us the use of the previous arguments concerning the case of infinitesimal anticanonical transformations and for the statement that the generating functionals $\mathcal{Z}''$ and $\mathcal{Z}'$ constructed with the help of the actions S'' and S', respectively, give the same physical results.

The next point of our study is connected with the behavior of generating functionals subjected to an arbitrary anticanonical transformation. Consider a one-parameter family of functionals $\mathcal{Z}'(J,\varphi^*;a)$,

$$\begin{aligned}\mathcal{Z}'(a) = \mathcal{Z}'(J,\varphi^*;a) &= \int \mathrm{d}\varphi\exp\left\{\frac{i}{\hbar}(S'(\varphi,\varphi^*;a) + J\varphi)\right\}\\ &= \exp\left\{\frac{i}{\hbar}W'(J,\varphi^*;a)\right\},\end{aligned} \tag{4.28}$$

$$\begin{aligned}&\exp\left\{\frac{i}{\hbar}S'(\varphi,\varphi^*;a)\right\}\\ &\quad= \exp\{-a[F(\varphi,\varphi^*),\Delta]\}\exp\left\{\frac{i}{\hbar}S(\varphi,\varphi^*)\right\},\end{aligned} \tag{4.29}$$

so that

$$\mathcal{Z}'(1) = \mathcal{Z}'. \tag{4.30}$$

[3] Note that in gauge theories the "on-shell" includes a definition of the physical state space.

Taking into account (3.17) and (4.29), we derive the relation

$$\begin{aligned}\partial_a\mathcal{Z}'(a) &= \frac{i}{\hbar}\mathcal{Z}'(a)\partial_a W'(J,\varphi^*;a) = \frac{i}{\hbar}\mathcal{Z}'(a)\partial_a\Gamma(\varphi,\varphi^*;a)\\ &= -\int \mathrm{d}\varphi \exp\left\{\frac{i}{\hbar}J\varphi\right\}[F(\varphi,\varphi^*),\Delta]\exp\left\{\frac{i}{\hbar}S'(\varphi,\varphi^*;a)\right\}\\ &= -\int \mathrm{d}\varphi \exp\left\{\frac{i}{\hbar}J\varphi\right\}\Delta\left(F(\varphi,\varphi^*)\exp\left\{\frac{i}{\hbar}S'(\varphi,\varphi^*;a)\right\}\right).\end{aligned} \tag{4.31}$$

By repeating similar calculations which lead us from (3.17) to (3.23) due to Eqs. (3.19), (3.20), and (A.1)–(A.7), we obtain

$$\partial_a\Gamma(\varphi,\varphi^*;a) = (\mathcal{F}(\varphi,\varphi^*;a),\Gamma(\varphi,\varphi^*;a)), \tag{4.32}$$

$$\begin{aligned}\mathcal{F}(\varphi,\varphi^*;a) &= \frac{1}{\mathcal{Z}'(J,\varphi^*;a)}\int \mathrm{d}\tilde{\varphi}F(\tilde{\varphi},\varphi^*)\\ &\times \exp\left\{\frac{i}{\hbar}(S'(\tilde{\varphi},\varphi^*;a)+J\tilde{\varphi})\right\}\Big|_{J=J(\varphi,\varphi^*;a)}.\end{aligned} \tag{4.33}$$

We will refer to (4.32) as the basic equation describing the dependence of the effective action on an anticanonical transformation in the field–antifield formalism. In Sect. 5, we present a solution to this equation.

5 Solution to the basic equation

In what follows below, we will use a short notation for all quantities depending on the variables φ,φ^*,

$$\Gamma(\varphi,\varphi^*;a)\equiv\Gamma(a),\quad \Gamma(\varphi,\varphi^*)\equiv\Gamma,\quad \mathcal{F}(\varphi,\varphi^*;a)\equiv\mathcal{F}(a) \tag{5.1}$$

and so on. Then the basic equation (4.32) is written as

$$\partial_a\Gamma(a) = (\mathcal{F}(a),\Gamma(a)) = \mathrm{ad}(\mathcal{F}(a))\Gamma(a). \tag{5.2}$$

We will study solutions to (5.2) in the class of regular functionals in a, by using a power series expansion in this parameter. In the beginning, let us find a solution to this equation to the first order in a, presenting $\Gamma(a)$ and $\mathcal{F}(a)$ in the form

$$\Gamma_1(a)\equiv\Gamma(a) = \Gamma + a\Gamma_{1|1} + O(a^2), \tag{5.3}$$

$$\mathcal{F}_1(a)\equiv\mathcal{F}(a) = \frac{1}{a}\mathcal{F}_{1|1}(a) + O(a),\ \ \mathcal{F}_{1|1}(a) = a\mathcal{F}_{1|1}. \tag{5.4}$$

A straightforward calculation yields the following result:

$$\Gamma_{1|1} = (\mathcal{F}_{1|1},\Gamma) = \mathrm{ad}(\mathcal{F}_{1|1})\Gamma. \tag{5.5}$$

Introduce the notation $U_1(a) = \mathcal{F}_{1|1}(a) = a\mathcal{F}_{1|1}$ and the functional $\Gamma_2(a)$ by the rule

$$\Gamma_2(a) = \exp\{-\mathrm{ad}(U_1(a))\}\Gamma_1(a). \tag{5.6}$$

The dependence of $\Gamma_2(a)$ on a is described by the equation

$$\partial_a\Gamma_2(a) = (\mathcal{F}_2(a),\Gamma_2(a)) \tag{5.7}$$

where

$$\mathcal{F}_2(a) = [\exp\{-a\mathrm{ad}(\mathcal{F}_{1|1})\}\mathcal{F}_1(a) - \mathcal{F}_{1|1}]. \tag{5.8}$$

It follows from (5.6) that the functional $\Gamma_2(a)$ coincides with Γ up to the second order in a,

$$\Gamma_2(a) = \Gamma + O(a^2) = \Gamma + a^2\Gamma_{2|2} + O(a^3). \tag{5.9}$$

In turn, the functional $\mathcal{F}_2(a)$ vanishes to the first order in a,

$$\mathcal{F}_2(a) = O(a) = \frac{2}{a}\mathcal{F}_{2|2}(a) + O(a^2),\quad \mathcal{F}_{2|2}(a) = a^2\mathcal{F}_{2|2}. \tag{5.10}$$

To the second order in a, the solution to (5.7) reads

$$\Gamma_{2|2} = (\mathcal{F}_{2|2},\Gamma). \tag{5.11}$$

Then the functional $\tilde{\Gamma}_3(a)$ constructed by the rule

$$\tilde{\Gamma}_3(a) = \exp\{-\mathrm{ad}(\mathcal{F}_{2|2}(a))\}\Gamma_2(a) \tag{5.12}$$

coincides with Γ up to the third order in a

$$\tilde{\Gamma}_3(a) = \Gamma + O(a^3). \tag{5.13}$$

Introduce the functional $\Gamma_3(a)$,

$$\begin{aligned}\Gamma_3(a) &= \exp\{-\mathrm{ad}(U_2(a))\}\Gamma_1(a),\\ U_2(a) &= \mathcal{F}_{1|1}(a) + \mathcal{F}_{2|2}(a).\end{aligned} \tag{5.14}$$

Note that $\Gamma_3(a)$ coincides with $\tilde{\Gamma}_3(a)$ up to the third order in a,

$$\begin{aligned}\Gamma_3(a) &= \tilde{\Gamma}_3(a) + O(a^3) = \Gamma + O(a^3)\\ &= \Gamma + a^3\Gamma_{3|3} + O(a^4),\end{aligned} \tag{5.15}$$

so that we have

$$\begin{aligned}\Gamma_3(a) &= \exp\{-\mathrm{ad}(\mathcal{F}_{2|2}(a))\}\\ &\quad\times\exp\{-\mathrm{ad}(\mathcal{F}_{1|1}(a))\}\Gamma_1(a) + O(a^3)\\ &= \exp\{-\mathrm{ad}(\mathcal{F}_{2|2}(a))\}\Gamma_2(a) + O(a^3)\end{aligned} \tag{5.16}$$

due to the relation (B.4). It follows from (5.15) that

$$\partial_a\Gamma_3(a) = 3a^2\Gamma_{3|3} + O(a^3). \tag{5.17}$$

On the other hand, we have

$$\partial_a\Gamma_3(a) = (\mathcal{F}_3(a),\Gamma_3(a)) = \mathrm{ad}(\mathcal{F}_3(a))\Gamma_3(a), \tag{5.18}$$

where

$$\begin{aligned}\mathrm{ad}(\mathcal{F}_3(a)) = &-\exp\{-\mathrm{ad}(U_2(a))\}\partial_a\exp\{\mathrm{ad}(U_2(a))\}\\ &-\exp\{-\mathrm{ad}(U_2(a))\}\mathrm{ad}(\mathcal{F}_1(a))\exp\{\mathrm{ad}(U_2(a))\},\end{aligned} \tag{5.19}$$

the operators on the right-hand side of (5.19) have certainly the form of the ones of ad, see Eqs. (C.12), (C.15), and (C.7), (C.8). By using (C.4), (C.9), we derive from (5.19) and (5.18)

$$\begin{aligned}\mathrm{ad}(\mathcal{F}_3(a)) &= -\frac{2}{a}\mathrm{ad}(\mathcal{F}_{2|2}(a)) \\ &\quad + \exp\{-\mathrm{ad}(\mathcal{F}_{2|2}(a))\}\mathrm{ad}(\mathcal{F}_2(a)) \\ &\quad \times \exp\{\mathrm{ad}(\mathcal{F}_{2|2}(a))\} + O(a^2) \\ &= \frac{3}{a}\mathrm{ad}(\mathcal{F}_{3|3}(\varphi,\varphi^*;a)) + O(a^3), \\ &\quad \mathrm{ad}(\mathcal{F}_{3|3}(a)) = a^3\mathrm{ad}(\mathcal{F}_{3|3}), \end{aligned} \tag{5.20}$$

$$\Gamma_{3|3} = (\mathcal{F}_{3|3}, \Gamma). \tag{5.21}$$

Suppose that on the nth step of our procedure we have obtained the following relations:

$$\begin{aligned}\Gamma_n(a) &= \exp\{-\mathrm{ad}(U_{n-1}(a))\}\Gamma_1(a) = \Gamma + O(a^n) \\ &= \Gamma + a^n\Gamma_{n|n} + O(a^{n+1}),\ U_{n-1}(a) \\ &= \sum_{k=1}^{n-1}\mathcal{F}_{k|k}(a) \equiv \mathcal{F}_{[n-1|n-1]}(a),\end{aligned} \tag{5.22}$$

$$\partial_a\Gamma_n(a) = (\mathcal{F}_n(a), \Gamma_n(a)), \tag{5.23}$$

$$\mathcal{F}_n(a) = O(a^n) = \frac{n}{a}\mathcal{F}_{n|n}(a) + O(a^{n+1}), \quad \mathcal{F}_{n|n}(a) = a^n\mathcal{F}_{n|n}, \tag{5.24}$$

$$\Gamma_{n|n} = (\mathcal{F}_{n|n}, \Gamma). \tag{5.25}$$

We set

$$U_n(a) = \mathcal{F}_{[n|n]}(a). \tag{5.26}$$

Then we have

$$\begin{aligned}&\exp\{-\mathrm{ad}(\mathcal{F}_{n|n}(a)\}\Gamma_n(a) = \Gamma + O(a^{n+1}), \\ &\Gamma_{n+1}(a) = \exp\{-\mathrm{ad}(U_n(a))\}\Gamma_1(a) \\ &\quad = \exp\{-\mathrm{ad}(\mathcal{F}_{n|n}(a))\}\Gamma_n(a) + O(a^{n+1}) \end{aligned} \tag{5.27}$$

$$\quad = \Gamma + O(a^{n+1}) = \Gamma + a^{n+1}\Gamma_{n+1|n+1} + O(a^{n+2}). \tag{5.28}$$

In a similar manner, we derive the equation for $\Gamma_{n+1}(a)$

$$\partial_a\Gamma_{n+1}(a) = (\mathcal{F}_{n+1}(a), \Gamma_{n+1}(a)), \tag{5.29}$$

where

$$\begin{aligned}\mathrm{ad}(\mathcal{F}_{n+1}(a)) &= -\exp\{-\mathrm{ad}(U_n(a))\}\partial_a\exp\{\mathrm{ad}(U_n(a))\} \\ &\quad + \exp\{-\mathrm{ad}(U_n(a))\}\mathrm{ad}(\mathcal{F}_1(a))\exp\{\mathrm{ad}(U_n(a))\}.\end{aligned} \tag{5.30}$$

By the same reasons used at the previous stages, we conclude that

$$\begin{aligned}\mathrm{ad}(\mathcal{F}_{n+1}(a)) &= -\frac{n}{a}\mathrm{ad}(\mathcal{F}_{n|n}(a)) \\ &\quad + \exp\{-\mathrm{ad}(\mathcal{F}_{n|n}(a))\}\mathrm{ad}(\mathcal{F}_n(a)) \\ &\quad \times \exp\{\mathrm{ad}(\mathcal{F}_{n|n}(a))\} + O(a^n) \\ &= \frac{n+1}{a}\mathrm{ad}(\mathcal{F}_{n+1|n+1}(a)) + O(a^{n+1}),\end{aligned}$$

$$\mathcal{F}_{n+1|n+1}(a) = a^{n+1}\mathcal{F}_{n+1|n+1}, \tag{5.31}$$

$$\Gamma_{n+1|n+1} = (\mathcal{F}_{n+1|n+1}, \Gamma), \tag{5.32}$$

$$\begin{aligned}\Gamma(a) &= \exp\{\mathrm{ad}(U_n(a))\}\Gamma_{n+1}(a) \\ &= \exp\{\mathrm{ad}(U_n(a))\}\Gamma + O(a^{n+1}).\end{aligned} \tag{5.33}$$

Finally, by applying the induction method, we find that a solution to the basic equation (5.2) can be presented in the form

$$\Gamma(a) = \exp\{\mathrm{ad}(U(a))\}\Gamma, \tag{5.34}$$

which is nothing but an anticanonical transformation of Γ with a generator functional $U(a)$ defined by the functional $\mathcal{F}(a)$ in (5.2) as

$$U(a) = \sum_{k=1}^{\infty}\mathcal{F}_{k|k}(a). \tag{5.35}$$

In this proof, we have found a possibility to express the relation between $U(a)$ and $\mathcal{F}(a)$ in the form

$$\mathcal{F}(a) = -\exp\{\mathrm{ad}(U(a))\}\partial_a\exp\{-\mathrm{ad}(U(a))\}. \tag{5.36}$$

In turn, the relation (5.36) can be considered as a new representation of the functional (4.33). Let us notice that the functional $U(a)$ in (5.34) depends on the functional $F(a)$ only and does not depend on the choice of an initial data for $\Gamma(a)$. Equations (5.34)–(5.36) just represent the important relationship between the ordinary exponential and the path-ordered one.

Let us state again that the dependence of the effective action on a finite anticanonical transformation with a generating functional $Y(\varphi, \Phi^*; a)$ is really described in terms of an anticanonical transformation with a generator functional $U(\varphi, \varphi^*; a)$. As an anticanonical transformation is a change of variables in Γ, in particular, it means that, on-shell, the effective action does not depend on gauges introducing with the help of anticanonical transformations.

6 Discussions

In the present article, we have explored a conception of a gauge-fixing procedure in the field–antifield formalism [1,2], based on the use of anticanonical transformations of general type. The approach includes an action (master-transformed action) constructed with the help of the anticanonical master transformation and being non-degenerate. The master-transformed action is a sum of two terms: one is an action subjected to an anticanonical transformation and the other is a term connected with a logarithm of a superdeterminant of this anticanonical transformation. This action satisfies the quantum master equation [13,19] (see also Appendix D). The generating functionals of the Green functions constructed via the master-transformed action obey the important property of the gauge independence of physical quantities on-shell, and they satisfy the Ward identity. We have found that any (finite) anticanonical master transformation of an action leads to the corresponding anticanonical transformation of effective action (generating functional of vertex functions) provided the generating functional of Green functions is constructed with the help of an anticanonical master action. We have proved the existence of a generator functional of an anticanonical transformation of the effective action. This result is essential when proving the independence of the effective action of anticanonical transformations on-shell and, on the other hand, it may supplement in a non-trivial manner the representation of anticanonical transformations in the form of a path-ordered exponential [13].

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Appendix A: Infinitesimal variation of effective action

Here we prove the possibility to present Eq. (3.21) in the form (3.23). To do this, we introduce the matrix of the second derivatives of Γ, Γ_{ij}, and its inverse, M^{ij},

$$\Gamma_{ij} \equiv \partial_{\varphi^i}\partial_{\varphi^j}\Gamma = (-1)^{\varepsilon_i\varepsilon_j}\Gamma_{ji},\\ \varepsilon(\Gamma_{ij}) = \varepsilon_i + \varepsilon_j, \tag{A.1}$$

$$M^{ij}\Gamma_{jk} = \delta^i_k, \quad \varepsilon(M^{ij}) = \varepsilon_i + \varepsilon_j,\\ M^{ji} = (-1)^{\varepsilon_i\varepsilon_j+\varepsilon_i+\varepsilon_j}M^{ij}. \tag{A.2}$$

From the Ward identity (2.20) written in the form

$$\Gamma_i\Gamma^{i^*} = 0, \quad \Gamma^{i^*} = \Gamma\overleftarrow{\partial}_{\varphi^*_i}, \quad \Gamma_i = \partial_{\varphi^i}\Gamma, \tag{A.3}$$

it follows that the relations

$$(-1)^{\varepsilon_j\varepsilon_k+\varepsilon_k}\Gamma_k\Gamma^{k^*}_j = (-1)^{\varepsilon_j}\Gamma^{k^*}\Gamma_{kj},\\ \Gamma^{k^*}_j = \partial_{\varphi^j}\partial_{\varphi^*_k}\Gamma \tag{A.4}$$

hold. By taking these relations into account, we have

$$(-1)^{\varepsilon_k}\Gamma_k[\partial_{\varphi^*_k}J_j(\varphi,\varphi^*)] = -(-1)^{\varepsilon_j+\varepsilon_k}\Gamma_k\partial_{\varphi^*_k}\partial_{\varphi^j}\Gamma\\ = -(-1)^{\varepsilon_j\varepsilon_k+\varepsilon_k}\Gamma_k\Gamma^{k^*}_j = -(-1)^{\varepsilon_j}\Gamma^{k^*}\Gamma_{kj} \tag{A.5}$$

and

$$\partial_{\varphi^k}\mathcal{X} = [\partial_{\varphi^k}J_j(\varphi,\varphi^*)]\partial_{J_j}\tilde{X} = -(-1)^{\varepsilon_j}\Gamma_{kj}\partial_{J_j}\tilde{X}\\ \Longrightarrow \partial_{J_j}\tilde{X}(J,\varphi^*)\Big|_{J=J(\varphi,\varphi^*)} = -(-1)^{\varepsilon_j}M^{jk}\partial_{\varphi^k}\mathcal{X}(\varphi,\varphi^*). \tag{A.6}$$

Therefore

$$(-1)^{\varepsilon_k}\Gamma_k[\partial_{\varphi^*_k}J_j(\varphi,\varphi^*)]\,\partial_{J_j}\tilde{X}(J,\varphi^*)\Big|_{J=J(\varphi,\varphi^*)}\\ = \Gamma^{k^*}\Gamma_{kj}M^{jk}\partial_{\varphi^k}\mathcal{X}(\varphi,\varphi^*) = \Gamma\overleftarrow{\partial}_{\varphi^*_k}\partial_{\varphi^k}\mathcal{X}. \tag{A.7}$$

Substituting (A.7) in (3.21), we have derived (3.23) for a variation of Γ.

Appendix B: Generator of anticanonical transformation

Here we give a proof that any anticanonical transformation can be described by the corresponding generator $\mathrm{ad}(F)$ in the sense of (4.18). Firstly, we note that if $\bar{Z}^A_l(Z)$, $l = 1, 2$, are anticanonical variables,

$$(\bar{Z}^A_1(Z), \bar{Z}^B_1(Z)) = (\bar{Z}^A_2(Z), \bar{Z}^B_2(Z)) = E^{AB}, \tag{B.1}$$

then the compositions of these variables, $\bar{Z}^A_{12}(Z) = \bar{Z}^A_1(\bar{Z}_2(Z))$ and $\bar{Z}^A_{21}(Z) = \bar{Z}^A_2(\bar{Z}_1(Z))$, are anticanonical as well. Indeed, we have

$$(\bar{Z}^A_{12}(Z), \bar{Z}^B_{12}(Z)) = \bar{Z}^A_{12}(Z)\overleftarrow{\partial}_C E^{CD}\overrightarrow{\partial}_D\bar{Z}^B_{12}(Z)\\ = \bar{Z}^A_1(\bar{Z}_2)\overleftarrow{D}_{2|M}[\bar{Z}^M_2(Z)\overleftarrow{\partial}_C E^{CD}\overrightarrow{\partial}_D\bar{Z}^N_2(Z)]\overrightarrow{D}_{2|N}\bar{Z}^B_1(\bar{Z}_2)\\ = \bar{Z}^Z_1(\bar{Z}_2)\overleftarrow{D}_{2|M}E^{MN}\overrightarrow{D}_{2|N}\bar{Z}^B_1(\bar{Z}_2) = E^{AB}, \quad D_{2|A} = \frac{\partial}{\partial\bar{Z}^A_2}. \tag{B.2}$$

In particular, the variables $\bar{Z}^A_{12}(Z) = \exp\{\mathrm{ad}F(Z)\}\bar{Z}^A_1(Z)$ are anticanonical if $\bar{Z}^A_1(A)$ are anticanonical variables. Indeed, we have

$$\bar{Z}^Z_{12}(Z) = \bar{Z}^Z_1(\bar{Z}_2(Z)), \quad \bar{Z}^A_2(Z) = \exp\{\mathrm{ad}F(Z)\}Z^A. \tag{B.3}$$

Secondly, the next remark is obvious

$$\exp\{\mathrm{ad}(F_{[n]}(Z;a))\}\exp\{\mathrm{ad}(F_{n+1}(Z;a))\}\\ = \exp\{\mathrm{ad}(F_{[n+1]}(Z;a))\} + O(a^{n+2}),\\ F_{[k]}(Z;a) = \sum_{l=1}^{k} F_l(Z;a), \quad F_l(Z;a) = a^l F_l(Z). \tag{B.4}$$

Now let $\bar{Z}^A(Z;a) \equiv \bar{Z}^A_1(Z;a) = Z^A + aZ^A_{1|1}(Z) + O(a^2)$ be anticanonical variables with a generating functional $Y(\varphi,\Phi^*;a) \equiv Y_1(\varphi,\Phi^*;a) = \Phi^*_i\varphi^i - af_{1|1}(\varphi,\Phi^*)+$

$O(a^2)$. Taking into account (4.2)–(4.4) and (4.6)–(4.8), we have

$$Z^A_{1|1}(Z) = (F_{1|1}(Z), Z^A),\ \ F_{1|1}(Z) = f_{1|1}(\varphi, \varphi^*) \implies \tag{B.5}$$

$$\bar{Z}^A_1(Z; a) = \exp\{\mathrm{ad}(F_{1|1}(Z; a))\}Z^A + O(a^2). \tag{B.6}$$

Then we introduce (anticanonical) variables $\bar{Z}^A_2(Z; a)$,

$$\bar{Z}^A_2(Z; a) = \exp\{-\mathrm{ad}(F_{1|1}(Z; a))\}\bar{Z}^A_1(Z; a) = Z^A + a^2 Z^A_{2|2}(Z) + O(a^3), \tag{B.7}$$

with the corresponding generating functional

$$Y_2(\varphi, \Phi^*; a) = \Phi^*_i \varphi^i - a^2 f_{2|2}(\varphi, \Phi^*) + O(a^3). \tag{B.8}$$

As a result, we have

$$Z^A_{2|2}(Z) = (F_{2|2}(Z), Z^A),\ \ F_{2|2}(Z) = f_{2|2}(\varphi, \varphi^*) \implies \tag{B.9}$$

$$\bar{Z}^A_2(Z; a) = \exp\{\mathrm{ad}(F_{2|2}(Z; a))\}Z^A + O(a^3) \implies \tag{B.10}$$

$$\bar{Z}^A_1(Z; a) = \exp\{\mathrm{ad}(F_{1|1}(Z; a))\}\bar{Z}^A_2(Z; a) = \exp\{\mathrm{ad}(F_{1|1}(Z; a))\}\exp\{\mathrm{ad}(F_{2|2}(Z; a))\}Z^A + O(a^3) = \exp\{\mathrm{ad}(F_{[2|2]}(Z; a))\}Z^A + O(a^3), \tag{B.11}$$

where the relation (B.4) is used.

Suppose that a representation of anticanonical variables $\bar{Z}^A_1(Z; a)$ does exist in the form

$$\bar{Z}^A_1(Z; a) = \exp\{\mathrm{ad}(F_{[n|n]}(Z; a))\}Z^A + O(a^{n+1}),\quad \mathrm{ad}(F_{[n|n]}(Z; a)) = \sum_{k=1}^{n} \mathrm{ad}(F_{k|k}(Z; a)). \tag{B.12}$$

Introduce the (anticanonical) variables $\bar{Z}^A_{n+1}(Z; a)$,

$$\bar{Z}^A_{n+1}(Z; a) = \exp\{-\mathrm{ad}(F_{[n|n]}(Z; a))\}\bar{Z}^A_1(Z; a) = Z^A + a^{n+1} Z^A_{n+1|n+1}(Z) + O(a^{n+2}). \tag{B.13}$$

The corresponding generating functional $Y_{n+1}(\varphi, \Phi^*; a)$ has the form

$$Y_{n+1}(\varphi, \Phi^*; a) = \Phi^*_i \varphi^i - a^{n+1} f_{n+1|n+1}(\varphi, \Phi^*) + O(a^{n+2}). \tag{B.14}$$

By the usual manipulations, we find

$$Z^A_{n+1|n+1}(Z) = (F_{n+1|n+1}(Z), Z^A),\ \ F_{n+1|n+1}(Z) = f_{n+1|n+1}(\varphi, \varphi^*), \tag{B.15}$$

$$\bar{Z}^A_{n+1}(Z; a) = \exp\{\mathrm{ad}(F_{n+1|n+1}(Z; a))\}Z^A + O(a^{n+2}), \tag{B.16}$$

$$\bar{Z}^A_1(Z; a) = \exp\{\mathrm{ad}(F_{[n|n]}(Z; a))\}\bar{Z}^A_{n+1}(Z; a) = \exp\{\mathrm{ad}(F_{[n|n]}(Z, a))\}\exp\{\mathrm{ad}(F_{n+1|n+1}(Z; a))\}Z^A + O(a^{n+2}) = \exp\{\mathrm{ad}(F_{[n+1|n+1]}(Z; a))\}Z^A + O(a^{n+2}). \tag{B.17}$$

Applying the induction method, we have proved that an arbitrary set of anticanonical variables $\bar{Z}^A(Z)$ can be really represented in the form (4.18).

Appendix C: Some useful formulas

Consider a set of differential operators $\mathrm{ad}(A(Z))$, $\mathrm{ad}(B(Z)), \ldots, \varepsilon(A(Z)) = 1, \varepsilon(B(Z)) = 1, \ldots$ applied to any functional $M(Z)$ of anticanonical variables $Z = (\varphi, \varphi^*)$ as the left adjoint of the antibracket. If a multiplication operation is introduced as the commutator, then this set can be considered as a Lie superalgebra. Indeed, due to the symmetry properties and the Jacobi identity for the antibracket, we have

$$[\mathrm{ad}(A(Z)), \mathrm{ad}(B(Z)] = \mathrm{ad}(A(Z))\mathrm{ad}(B(Z)) - \mathrm{ad}(B(Z))\mathrm{ad}(A(Z)) = \mathrm{ad}(C_{A|B}(Z)), \tag{C.1}$$

$$C_{A|B}(Z) = (A(Z), B(Z)),\quad \varepsilon(C_{A|B}(Z)) = 1, \tag{C.2}$$

or, in more detail, by application to $M(Z)$,

$$(A(Z), (B(Z), M(Z))) - (B(Z), (A(Z), M(Z)))\ (A(Z), (B(Z), M(Z))) + (B(Z), (M(Z), A(Z))) = -(M(Z), (A(Z), B(z))) = ((A(Z), B(Z)), M(Z)) = \mathrm{ad}(C_{A|B}(Z))M(Z). \tag{C.3}$$

Note that the operators under consideration give a good example of odd first-order differential operations which are not nilpotent, $(\mathrm{ad}(A(Z)))^2 \neq 0$.

It is obvious that

$$\exp\{\mathrm{ad}(A_{n+1}(a))\}\exp\{\mathrm{ad}(A_{[n]}(a))\} = \exp\{\mathrm{ad}(A_{[n+1]}(a))\} + O(a^{n+2}), \tag{C.4}$$

$$A_{[n]}(a) = \sum_{k=1}^{n} A_k(a),\quad A_k(a) = a^k A_k \tag{C.5}$$

[see, also (B.4)].

Taking into account a series expansion

$$\exp\{\mathrm{ad}(A(Z))\}\mathrm{ad}(B(Z))\exp\{-\mathrm{ad}(A(Z))\} = \mathrm{ad}(B(Z)) + [\mathrm{ad}(A(Z)), \mathrm{ad}(B(Z))] + \frac{1}{2!}[\mathrm{ad}(A(Z)), [\mathrm{ad}(A(Z)), \mathrm{ad}(B(Z))]] + \ldots, \tag{C.6}$$

using relations similar to (C.1)–(C.3) and the Jacobi identity for the antibracket, we deduce the identity

$$\exp\{\mathrm{ad}(A(Z))\}\mathrm{ad}(B(Z))\exp\{-\mathrm{ad}(A(Z))\} = \mathrm{ad}(D_{A|B}(Z)),$$

$$D_{A|B}(Z) = B(Z) + (A(Z), B(Z)) \tag{C.7}$$

$$+ \frac{1}{2!}(A(Z), (A(Z), B(Z))) + \ldots = \exp\{\mathrm{ad}(A(Z))\}B(Z),\quad \varepsilon(D_{A|B}(Z)) = 1. \tag{C.8}$$

The useful identity

$$X = X(Z;a) = \exp\{\mathrm{ad}(A(Z;a))\}\partial_a \exp\{-\mathrm{ad}(A(Z;a))\}$$
$$= -\mathrm{ad}(D_A(Z;a)), \tag{C.9}$$
$$D_A(Z;a) = f(\mathrm{ad}(A(Z;a)))\partial_a A(Z;a), \tag{C.10}$$
$$f(x) = (\exp(x)-1)x^{-1},\ \varepsilon(A(Z;a))=1,\ \varepsilon(D_A(Z;a))=1, \tag{C.11}$$

holds, as well. Indeed, let us introduce the operator $X(t)$,

$$X(t) = X(Z;a;t) = \exp\{t\mathrm{ad}(A(Z;a))\}\partial_a$$
$$\times \exp\{-t\mathrm{ad}(A(Z;a))\},\ X(0)=0,\ X(1)=X. \tag{C.12}$$

Then we have

$$\partial_t X(t) = -\exp\{t\mathrm{ad}(A(Z;a))\}\mathrm{ad}(\partial_a A(Z;a))$$
$$\times \exp\{-t\mathrm{ad}(A(Z;a))\} = -\mathrm{ad}(C_{\partial_a A}(Z;a;t)), \tag{C.13}$$
$$C_{\partial_a A}(Z;a;t) = \exp\{t\mathrm{ad}(A(Z;a))\}\partial_a A(Z;a). \tag{C.14}$$

In deriving (C.13) and (C.14), the identities (C.7) and (C.8) are used. Using initial data for $X(t)$, it follows from (C.13) that

$$X(t) = -t\mathrm{ad}(D_{\partial_a A}(Z;a;t)),$$
$$D_{\partial_a A}(Z;a;t) = f(t\mathrm{ad}(A(Z;a)))\partial_a A(Z;a). \tag{C.15}$$

We will use the following convention and notation for applying the operators R and $\hat{R}$,

$$F(R)A(Z)(\ldots) = [F(R)A(Z)](\ldots),\ F(\hat{R})A(Z)(\ldots)$$
$$= F(R)[A(Z)(\ldots)], \tag{C.16}$$

where $F(R) = F(x)|_{x=R}$, $A(Z)$ is a function, and $(\ldots)$ means an arbitrary quantity.

Consider a first-order differential operator

$$N(Z)\partial \equiv N^A(Z)\partial_A,\quad \partial_A = \frac{\partial}{\partial Z^A},\quad \varepsilon(N^A(Z)) = \varepsilon(Z^A), \tag{C.17}$$

where $N^A(Z)$ are some functionals of Z. Let

$$\bar{Z}^A(Z) \equiv \exp\{N(Z)\partial\}Z^A, \tag{C.18}$$

then we have

$$\exp\{N(Z)\hat{\partial}\}Z^A \exp\{-N(Z)\hat{\partial}\}$$
$$= \sum_{k=0}\frac{1}{k!}[N(Z)\hat{\partial},[N(Z)\hat{\partial},\ldots[N(Z)\hat{\partial},Z^A]\ldots]]_{k\text{ times}}$$
$$= \sum_{k=0}\frac{1}{k!}[N(Z)\partial]^k Z^A = \exp\{N(Z)\partial\}Z^A = \bar{Z}^A(Z) \tag{C.19}$$

where the relation

$$[N(Z)\hat{\partial}, M(Z)] = N(Z)\partial M(Z) \tag{C.20}$$

is used. In general

$$\exp\{N(Z)\hat{\partial}\}g(Z)\exp\{-N(Z)\hat{\partial}\}$$
$$= \exp\{N(Z)\partial\}g(Z) = g(\bar{Z}). \tag{C.21}$$

Consider a more general differential operator than in (C.18),

$$L(a) = \exp\{aM(Z) + aN(Z)\hat{\partial}\} \tag{C.22}$$

where $M(Z)$ is a functional of Z and a is a parameter. We prove that there is a representation of this operator in the form

$$L(a) = H(Z,a)\exp\{aN(Z)\hat{\partial}\} \tag{C.23}$$

where $H(Z,a)$ is a functional. Indeed, it follows from (C.22) and (C.23) that

$$H(Z,a) = \exp\{aM(Z) + aN(Z)\hat{\partial}\}\exp\{-aN(Z)\hat{\partial}\}. \tag{C.24}$$

By differentiating $H(Z,a)$ with respect to a, one gets the relation

$$\frac{\mathrm{d}^n}{\mathrm{d}a^n}H(Z,a)$$
$$= \exp\{aM(Z) + aN(Z)\hat{\partial}\}h_n\exp\{-aN(Z)\hat{\partial}\}, \tag{C.25}$$

where

$$h_n = (M(Z) + N(Z)\hat{\partial})h_{n-1} - h_{n-1}N(Z)\hat{\partial},$$
$$h_0 = 1,\ h_1 = M(Z). \tag{C.26}$$

Suppose that h_k, $0 \le k \le n$ are some functionals, then

$$h_{n+1} = M(Z)h_n + N(Z)\hat{\partial}h_n - h_n N(Z)\hat{\partial}$$
$$= M(Z)h_n + N(Z)\partial h_n \tag{C.27}$$

is a functional, as well. The latter means that all a-derivatives of $H(Z,a)$ taken at $a=0$ are some functionals too and, as a consequence, $H(Z,a)$ is a functional.

Now we can derive a representation for $H(Z,a)$. We start with the equation

$$\frac{\mathrm{d}}{\mathrm{d}a}H(Z,a)$$
$$= \exp\{aM(Z) + aN(Z)\hat{\partial}\}M(Z)\exp\{-aN(Z)\hat{\partial}\}, \tag{C.28}$$

which can be rewritten as

$$\frac{\mathrm{d}}{\mathrm{d}a}H(Z,a)$$
$$= H(Z,a)(\exp\{aN(Z)\hat{\partial}\}M(Z)\exp\{-aN(Z)\hat{\partial}\})$$
$$= (\exp\{aN(Z)\partial\}M(Z))H(Z,a) \tag{C.29}$$

where the relation (C.21) is used. Integrating this equation leads to

$$H(Z)=H(Z,1)=\exp[f(x)M(Z)],\quad f(x)=\frac{\exp(x)-1}{x},\ x=N(Z)\partial. \tag{C.30}$$

Finally, we have

$$\exp\{M(Z)+N(Z)\hat{\partial}\}=\exp[f(x)M(Z)]\exp\{N(Z)\hat{\partial}\},\quad x=N(Z)\partial. \tag{C.31}$$

Appendix D: Master-transformed actions

Here we present a set of properties concerning master-transformed actions.

Firstly, we prove that an action S' constructed by the rule (4.19) from S, being a solution to the quantum master equation, satisfies the quantum master equation, as well. To do this, we consider a functional $X(Z)$ and the transformation $X(Z)\to X'(Z)=X(Z,1)$ of the form

$$\exp\left\{\frac{i}{\hbar}X(Z,a)\right\}=\exp\{-a[F(Z),\hat{\Delta}]\}\exp\left\{\frac{i}{\hbar}X(Z)\right\},\quad X(Z,0)=X(Z). \tag{D.1}$$

The transformation (D.1) has the property

$$\Delta\exp\left\{\frac{i}{\hbar}X(Z)\right\}=0\ \Longrightarrow\ \Delta\exp\left\{\frac{i}{\hbar}X(Z,a)\right\}=0. \tag{D.2}$$

Indeed, let us introduce a functional

$$Y(Z,a)=\Delta\exp\left\{\frac{i}{\hbar}X(Z,a)\right\},\quad Y(z,0)=\Delta\exp\left\{\frac{i}{\hbar}X(Z)\right\}. \tag{D.3}$$

Then we have

$$\frac{\mathrm{d}}{\mathrm{d}a}Y(Z,a)=-\hat{\Delta}\left([F(Z),\Delta]\right)\exp\left\{\frac{i}{\hbar}X(Z,a)\right\}=-\hat{\Delta}F(Z)Y(Z,a) \tag{D.4}$$

where the nilpotency of Δ operator is used. Integrating this equation gives

$$Y(Z,a)=\exp\{-a\hat{\Delta}F(Z)\}Y(Z,0)\ \Longrightarrow \tag{D.5}$$

$$\Delta\exp\left\{\frac{i}{\hbar}X(Z,a)\right\}=\exp\{-a\hat{\Delta}F(Z)\}\Delta\exp\left\{\frac{i}{\hbar}X(Z)\right\}. \tag{D.6}$$

Secondly, to prove the presentation of (4.20), we consider (D.1) in more detail. Note that

$$[F(z),\Delta]=(\Delta F(Z))-\mathrm{ad}(F(Z)), \tag{D.7}$$

and we have the following identification of (D.7) with the functions $M(Z)$ and the operator $N^A(Z)\partial_A$ from (C.23):

$$M(Z)=-\Delta F(Z),\ \ N^A(Z)\partial_A=\mathrm{ad}(F(Z)). \tag{D.8}$$

It follows from (C.31) that

$$X'=\exp\{\mathrm{ad}(F(Z))\}X+i\hbar f(\mathrm{ad}(F(Z)))\Delta F. \tag{D.9}$$

In the right-hand side in (D.9), the first term is an anticanonical transformation with finite fermionic generator F, while the second term is a half of a logarithm of the Jacobian of that transformation, up to $(-i\hbar)$. It is obvious that the inverse statement holds as well: the validity of the relation (D.9) implies Eq. (D.1).

Now we show that the equality holds of

$$\exp\{-[F_2(Z)+F_1(Z),\Delta]\}=\exp\{-[\mathcal{F}_2(Z),\Delta]\}\exp\{-[F_1(Z),\Delta]\}, \tag{D.10}$$

where $\mathcal{F}_2(Z)$ is determined by the relation

$$\exp\{[\mathrm{ad}(F_2(Z))+\mathrm{ad}(F_1(Z))]\}\exp\{-\mathrm{ad}(F_1(Z))\}=\exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}. \tag{D.11}$$

The existence of Eqs. (D.10) and (D.11) means that transformations generated by $\exp\{-[F(Z),\Delta]\}$ and $\exp\{\mathrm{ad}(F(Z))\}$ obey a group property.

Consider anticanonical transformations generated by the fermionic functions $F_1(Z)$, $F_1(Z)+F_2(Z)$, and $\mathcal{F}_2(Z)$

$$\bar{Z}_1^A(Z)=\exp\{\mathrm{ad}(F_1(Z))\}Z^A,\ \ \bar{Z}_2^A(Z)=\exp\{[\mathrm{ad}(F_2(Z))+\mathrm{ad}(F_1(Z))]\}Z^A, \tag{D.12}$$

$$\bar{\mathcal{Z}}_2^A(Z)=\exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}Z^A. \tag{D.13}$$

Then, due to (D.11), we have

$$\bar{Z}_2^A(Z)=\exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}\exp\{\mathrm{ad}(F_1(Z))\}Z^A=\bar{Z}_1^A(\bar{\mathcal{Z}}_2(Z)). \tag{D.14}$$

For a given action $S(Z)$, the relations

$$S_1(Z)=\exp\{\mathrm{ad}(F_1(Z))\}S(Z)=S(\bar{Z}_1(Z)), \tag{D.15}$$

$$S_2(Z)=\exp\{[\mathrm{ad}(F_2(Z))+\mathrm{ad}(F_1(Z))]\}S(Z)=S(\bar{Z}_2(Z))=S(\bar{Z}_1(\bar{\mathcal{Z}}_2(Z))=\exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}S_1(Z) \tag{D.16}$$

hold. Using the chain rule and multiplication rule for superdeterminants, one obtains for the logarithm of the superdeterminant of the anticanonical transformation (D.14)

$$\begin{aligned}
&\ln\,\mathrm{sDet}\,[\bar{Z}_2^A(Z)\overleftarrow{\partial}_B] \\
&\quad= \ln\,\mathrm{sDet}\,[\,\bar{Z}_1^A(\bar{\mathcal{Z}}_2)\overleftarrow{\partial}_{\bar{\mathcal{Z}}_2^C}\Big|_{\bar{\mathcal{Z}}_2^C\to\bar{\mathcal{Z}}_2^C(Z)}(\bar{\mathcal{Z}}_2^C(Z)\overleftarrow{\partial}_B] \\
&\quad= \ln\,\mathrm{sDet}\,[(\bar{Z}_1^A(\bar{\mathcal{Z}}_2)\overleftarrow{\partial}_{\bar{\mathcal{Z}}_2^B})(Z)] \\
&\qquad+ \ln\,\mathrm{sDet}\,[\bar{\mathcal{Z}}_2^A(Z)\overleftarrow{\partial}_B] \\
&\quad= \exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}\ln\,\mathrm{sDet}\,[(\bar{Z}_1^A(Z)\overleftarrow{\partial}_B)] \\
&\qquad+ \ln\,\mathrm{sDet}\,[\bar{\mathcal{Z}}_2^A(Z)\overleftarrow{\partial}_B].
\end{aligned} \tag{D.17}$$

Consider the action S_2' constructed from an action S with the help of anticanonical master transformation with the generator functional F_1+F_2 (D.12). We obtain

$$S_2'(Z) = S_2(Z) - \frac{i\hbar}{2}\ln\,\mathrm{sDet}\,[\bar{Z}_2^A(Z)\overleftarrow{\partial}_B] \tag{D.18}$$

where $S_2(Z)$ is defined by the first equality in (D.16), and $\bar{Z}_2^A$ is given by the second equality in (D.12). It follows from (D.17) and (D.18) that

$$\begin{aligned}
S_2'(Z) &= \exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}\left(S_1(Z) - \frac{i\hbar}{2}\ln\,\mathrm{sDet}\,[\bar{Z}_1^A(Z)\overleftarrow{\partial}_B]\right) \\
&\quad - \frac{i\hbar}{2}\ln\,\mathrm{sDet}\,[\bar{\mathcal{Z}}_2^A(Z)\overleftarrow{\partial}_B] \\
&= \exp\{\mathrm{ad}(\mathcal{F}_2(Z))\}S_1'(Z) - \frac{i\hbar}{2}\ln\,\mathrm{sDet}\,[\bar{\mathcal{Z}}_2^A(Z)\overleftarrow{\partial}_B],
\end{aligned} \tag{D.19}$$

where S_1' is the master-transformed action S under the anticanonical transformation of variables Z with the generator functional $F_1(Z)$, and, as a result, S_2' is presented as a master-transformed action S' corresponding to the anticanonical master transformation of Z with the generator functional $\mathcal{F}_2$, i.e., in the form of successive anticanonical master transformations. From (D.19) we deduce the relations

$$\begin{aligned}
&\exp\{-[F_2(z)+F_1(Z),\Delta]\}\exp\left\{\frac{i}{\hbar}S(Z)\right\} \\
&\quad= \exp\{-[\mathcal{F}_2(Z),\Delta]\}\exp\left\{\frac{i}{\hbar}S_1'(Z)\right\} \\
&\quad= \exp\{-[\mathcal{F}_2(Z),\Delta]\}\exp\{-[F_1(Z),\Delta]\}\exp\left\{\frac{i}{\hbar}S(Z)\right\}
\end{aligned} \tag{D.20}$$

being valid for arbitrary functional $S(Z)$. The latter proves the relation (D.10).

Finally, we give a proof of the relation

$$\begin{aligned}
&\frac{1}{2}\ln\,\mathrm{sDet}\,[\bar{Z}^A\overleftarrow{\partial}_B] \\
&\quad= -f(\mathrm{ad}(F))\Delta F,\quad \bar{Z}^A = \exp\{\mathrm{ad}(F)\}Z^A,
\end{aligned} \tag{D.21}$$

used in the representation of the master-transformed actions (3.10) and (4.20). To do this, we introduce a one-parameter family of anticanonical transformations

$$\bar{Z}^A(a) = \exp\{a\,\mathrm{ad}(F)\}Z^A, \tag{D.22}$$

and the corresponding set of logarithms of superdeterminants

$$D(a) = \ln\,\mathrm{sDet}\,[\bar{Z}^A(a)\overleftarrow{\partial}_B]. \tag{D.23}$$

Consider anticanonical transformations with an infinitesimal variation of the parameter a,

$$\bar{Z}_2^A = \bar{Z}^A(a+\delta a) = \exp\{(a+\delta a)\mathrm{ad}(F)\}Z^A, \tag{D.24}$$

and functionals

$$D(a+\delta a) = \ln\,\mathrm{sDet}\,[\bar{Z}_2^A\overleftarrow{\partial}_B] = \ln\,\mathrm{sDet}\,[\bar{Z}^A(a+\delta a)\overleftarrow{\partial}_B]. \tag{D.25}$$

Taking into account the Eqs. (D.10), (D.11), (D.12), and (D.13), we have the following identification:

$$F_1 = aF,\quad F_2 = \delta a F,\quad \mathcal{F}_2 = \delta a F \tag{D.26}$$

and the representations up to the second order in δa

$$\begin{aligned}
&\exp\{\mathrm{ad}(\mathcal{F})_2\} = 1 + \delta a\,\mathrm{ad}(F) + O((\delta a)^2), \\
&\quad \bar{\mathcal{Z}}_2^A = Z^A + \delta a F\overleftarrow{\partial}_C E^{CA} + O((\delta a)^2),
\end{aligned} \tag{D.27}$$

$$\begin{aligned}
&\ln\,\mathrm{sDet}\,[\bar{\mathcal{Z}}_2^A\overleftarrow{\partial}_B] = \delta a\,\mathrm{sTr}\,[F\overleftarrow{\partial}_C E^{CA}\overleftarrow{\partial}_B] + O((\delta a)^2) \\
&\quad = -2\delta a\Delta F + O((\delta a)^2).
\end{aligned} \tag{D.28}$$

From (D.27), (D.28), and (D.17) follows the differential equation for $D(a)$,

$$\partial_a D(a) = \mathrm{ad}(F)D(a) - 2\Delta F,\quad D(0) = 0. \tag{D.29}$$

Let us seek a solution to this equation in the form

$$D(a) = \exp\{a\,\mathrm{ad}(F)\}D_1(a),\quad D_1(0) = 0. \tag{D.30}$$

Then we obtain

$$\partial_a D_1(a) = -2\exp\{-a\,\mathrm{ad}(F)\}\Delta F \tag{D.31}$$

and

$$\begin{aligned}
&D_1(a) = -2a\exp\{-a\,\mathrm{ad}(F)\}f(a\,\mathrm{ad}(F))\Delta F + C, \\
&\quad C = D_1(0) = 0.
\end{aligned} \tag{D.32}$$

Finally, we find

$$\begin{aligned}
&D(a) = -2af(a\,\mathrm{ad}(F))\Delta F,\quad \ln\,\mathrm{sDet}\,[\bar{Z}^A\overleftarrow{\partial}_B] \\
&\quad = D(1) = -2f(\mathrm{ad}(F))\Delta F.
\end{aligned} \tag{D.33}$$

Appendix E: Factorization of the Jacobian of anticanonical transformation

For the sake of completeness of our study of anticanonical transformations, let us present here a simple proof of the factorization property of the grand Jacobian of an anticanonical transformation within the field–antifield formalism [1,2]. The result is known at least since Ref. [18] of Batalin and Vilkovisky, although the proof was omitted therein.

We will proceed with the use of antisymplectic Darboux coordinates Z^A in the form of an explicit splitting into fields ϕ^i and antifields ϕ^*_i,

$$Z^A = \{\phi^i, \phi^*_i\}, \quad \varepsilon(Z^A) = \varepsilon_A, \quad \varepsilon(\phi^*_i) = \varepsilon(\phi^i) + 1, \tag{E.1}$$

so that

$$(Z^A, Z^B) = E^{AB}, \quad \varepsilon(E^{AB}) = \varepsilon_A + \varepsilon_B + 1, \tag{E.2}$$

where E^{AB} is a constant invertible antisymplectic metric with the following block structure:

$$E^{AB} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix} \tag{E.3}$$

and the antisymmetry property

$$E^{AB} = -(-1)^{(\varepsilon_A+1)(\varepsilon_B+1)} E^{BA}. \tag{E.4}$$

Let $F = F(Z)$ be a fermion generator of an anticanonical transformation,

$$Z^A \;\to\; \bar{Z}^A(t) = \exp\{t\,\mathrm{ad}(F)\} Z^A,$$
$$\bar{Z}^A(t=0) = Z^A, \quad \bar{Z}^A = \{\Phi^i, \Phi^*_i\}. \tag{E.5}$$

$\bar{Z}^A$ satisfies the Lie equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\bar{Z}^A = (\bar{F}, \bar{Z}^A)_{\bar{Z}}, \tag{E.6}$$

where $\bar{F} = F(\bar{Z}) = F(Z)$.

Let us consider the (grand) Jacobian, $J(t)$, of the anticanonical transformation (E.5),

$$J(t) = \mathrm{sDet}\,[\bar{Z}^A(t)\overleftarrow{\partial}_B], \tag{E.7}$$

together with its logarithm

$$\ln J(t) = \mathrm{sTr}\,\ln[\bar{Z}^A(t)\overleftarrow{\partial}_B]. \tag{E.8}$$

By using (E.6) and the relations

$$(\bar{Z}^A\overleftarrow{\partial}_C)(Z^C\overleftarrow{\partial}_{\bar{B}}) = \delta^A_B, \quad (Z^A\overleftarrow{\partial}_{\bar{C}})(\bar{Z}^C\overleftarrow{\partial}_B) = \delta^A_B, \tag{E.9}$$

which are valid for any invertible transformation $Z^A \to \bar{Z}^A$, together with the formula for a δ-variation,

$$\delta\,\mathrm{sTr}\,\ln M = \mathrm{sTr}\,M^{-1}\delta M, \quad (M^{-1})^A_C M^C_B = \delta^A_B, \tag{E.10}$$

we derive the equation for $\ln J$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\ln J &= \mathrm{sTr}\left[(Z^A\overleftarrow{\partial}_{\bar{C}})\frac{\mathrm{d}}{\mathrm{d}t}(\bar{Z}^C\overleftarrow{\partial}_B)\right] \\ &= (-1)^{\varepsilon_A}(Z^A\overleftarrow{\partial}_{\bar{C}})(\dot{\bar{Z}}^C\overleftarrow{\partial}_A) \\ &= (-1)^{\varepsilon_A}(Z^A\overleftarrow{\partial}_{\bar{C}})((\bar{F}, \bar{Z}^C)\overleftarrow{\partial}_A) \\ &= -(-1)^{\varepsilon_C}(\overrightarrow{\partial}_{\bar{C}}Z^A)\overrightarrow{\partial}_A(\bar{Z}^C, \bar{F}) \\ &= -(-1)^{\varepsilon_C}\overrightarrow{\partial}_{\bar{C}}(\bar{Z}^C, \bar{F}) = -2\bar{\Delta}\bar{F},\end{aligned} \tag{E.11}$$

where the operators Δ, $\bar{\Delta}$ are defined[4] by

$$\begin{aligned}\Delta = \Delta_Z &= \frac{1}{2}(-1)^{\varepsilon_A}\partial_A(Z^A, \ldots) \\ &= \frac{1}{2}(-1)^{\varepsilon_A}\partial_A E^{AB}\partial_B, \\ \bar{\Delta} = \Delta_{\bar{Z}} &= \exp\{\mathrm{ad}(tF)\}\Delta\exp\{\mathrm{ad}(-tF)\}.\end{aligned} \tag{E.12}$$

Here ∂_A and $\partial_{\bar{A}}$ denote the partial Z^A- and $\bar{Z}^A$-derivative, respectively.

Now, let J_ϕ be the Jacobian in the sector of fields,

$$J_\phi(t) = \mathrm{sDet}\,[\Phi^i(t, \phi, \phi^*)\overleftarrow{\partial}_j], \tag{E.13}$$

together with its logarithm,

$$\ln J_\phi(t) = \mathrm{sTr}\,\ln[\Phi^i(t, \phi, \phi^*)\overleftarrow{\partial}_j], \tag{E.14}$$

where ∂_i denotes the partial ϕ^i-derivative. In what follows below, the symbols $\partial_{\bar{k}}$ and $\partial^{*\bar{k}}$, with barred indices, will be used to denote the partial Φ^k- and Φ^*_k-derivatives, respectively. To get the t-derivative of $\ln J_\phi$, one needs the inverse to the matrix $\Phi^i\overleftarrow{\partial}_j$.

Let us consider an anticanonical transformation in the sector of fields,

$$\phi^i \;\to\; \Phi^i = \Phi^i(t, \phi, \phi^*). \tag{E.15}$$

Let us resolve that equation for initial fields ϕ^i, with t and ϕ^*_i kept fixed,

$$\phi^i = \phi^i(t, \Phi, \phi^*), \tag{E.16}$$

so that

$$\phi^i(t, \Phi(t, \phi, \phi^*), \phi^*) \equiv \phi^i. \tag{E.17}$$

It follows from (E.17) that the relation

$$(\phi^i(t, \Phi, \phi^*)\overleftarrow{\partial}_{\bar{k}})(\Phi^k(t, \phi, \phi^*)\overleftarrow{\partial}_j) = \delta^i_j \tag{E.18}$$

holds, because the initial fields ϕ^i are inverse functions to $\Phi^i(t, \phi, \phi^*)$ at the fixed values of t and ϕ^*_i. From now on, the variables Φ^i, Φ^*_i are considered as functions of t, ϕ^i, ϕ^*_i, while the fields ϕ^i are functions of t, Φ^i, ϕ^*_i, so that the short notation will be used naturally,

$$\phi^i(t, \Phi, \phi^*) = \phi^i, \quad \Phi^i(t, \phi, \phi^*) = \Phi^i,$$
$$\Phi^*_i(t, \phi, \phi^*) = \Phi^*_i. \tag{E.19}$$

Now, we derive the following equation for $\ln J_\phi$:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\ln J_\phi = &-\bar{\Delta}\bar{F} - \frac{1}{2}(-1)^{\varepsilon_k}\bar{F}\overleftarrow{\partial}^{*\bar{k}}\overleftarrow{\partial}^{*\bar{m}} \\ &\times[(\Phi^*_m\overleftarrow{\partial}_i)(\phi^i\overleftarrow{\partial}_{\bar{k}}) - (k \leftrightarrow m)(-1)^{\varepsilon_k\varepsilon_m}].\end{aligned} \tag{E.20}$$

[4] Notice that in (E.11) we mean just the second equality (E.12) so as to define the transformed operator $\bar{\Delta}$. That definition is maintained by the two following motivations: it respects both the nilpotency property and the multiplicative composition $\overline{\Delta G} = \bar{\Delta}\bar{G}$, $\bar{G} = G(\bar{Z})$, with arbitrary function $G = G(Z)$.

In turn, let us consider the quantity

$$T_{jk} = (\Phi^*_j \overleftarrow{\partial}_i)(\phi^i \overleftarrow{\partial}_{\bar{k}}) - (\Phi^*_k \overleftarrow{\partial}_i)(\phi^i \overleftarrow{\partial}_{\bar{j}})(-1)^{\varepsilon_j \varepsilon_k}. \tag{E.21}$$

Then, by multiplying (E.21) subsequently from the right by the two Jacobi matrices accompanied with a special sign factor, we have

$$T_{jk}(\Phi^k \overleftarrow{\partial}_l)(\Phi^j \overleftarrow{\partial}_m)(-1)^{\varepsilon_j \varepsilon_l} = (\overrightarrow{\partial}_l \Phi^*_j)(\Phi^j \overleftarrow{\partial}_m) - (m \leftrightarrow l)(-1)^{\varepsilon_m \varepsilon_l}. \tag{E.22}$$

The latter can be rewritten in the form

$$T_{jk}(\Phi^k \overleftarrow{\partial}_l)(\Phi^j \overleftarrow{\partial}_m)(-1)^{\varepsilon_j \varepsilon_l} = (\overrightarrow{\partial}_l \Phi^*_j)(\Phi^j \overleftarrow{\partial}_m) - (\overrightarrow{\partial}_l \Phi^j)(\Phi^*_j \overleftarrow{\partial}_m). \tag{E.23}$$

Now, let us introduce the quantity

$$L_{AB} = (\overrightarrow{\partial}_A \bar{Z}^C) E_{CD} (\bar{Z}^D \overleftarrow{\partial}_B), \tag{E.24}$$

where E_{AB} is the inverse to E^{AB}, with the following block structure:

$$E_{AB} = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad \varepsilon(E_{AB}) = \varepsilon_A + \varepsilon_B + 1 \tag{E.25}$$

and the antisymmetry property

$$E_{AB} = -(-1)^{\varepsilon_A \varepsilon_B} E_{BA}. \tag{E.26}$$

Notice that the field–field components of (E.24),

$$L_{ij} = (\overrightarrow{\partial}_i \Phi^*_k)(\Phi^k \overleftarrow{\partial}_j) - (\overrightarrow{\partial}_j \Phi^k)(\Phi^*_k \overleftarrow{\partial}_i), \tag{E.27}$$

do coincide with (E.23). By taking the relation

$$(Z^A, Z^B)_{\bar{Z}} = (Z^A \overleftarrow{\partial}_{\bar{C}}) E^{CD} (\overrightarrow{\partial}_{\bar{D}} Z^B) = E^{AB} \tag{E.28}$$

into account, we have

$$\begin{aligned} E^{AC} L_{CB} &= (Z^A, Z^C)_{\bar{Z}} L_{CB} \\ &= (Z^A \overleftarrow{\partial}_{\bar{C}}) E^{CD} (\overrightarrow{\partial}_{\bar{D}} Z^E)(\overrightarrow{\partial}_E \bar{Z}^F) E_{FG} (\bar{Z}^G \overleftarrow{\partial}_B) \\ &= (Z^A \overleftarrow{\partial}_{\bar{C}}) E^{CD} \delta^F_{\bar{D}} E_{FG} (\bar{Z}^G \overleftarrow{\partial}_B) \\ &= (Z^A \overleftarrow{\partial}_{\bar{C}})(\bar{Z}^C \overleftarrow{\partial}_B) = \delta^A_B. \end{aligned} \tag{E.29}$$

The latter implies[5]

$$L_{AB} = E_{AB}, \quad L_{ij} = 0. \tag{E.30}$$

Thus, we obtain the equation for the Jacobian J_ϕ in the sector of fields,

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln J_\phi = -\bar{\Delta} \bar{F}. \tag{E.31}$$

In the same way, we derive the equation

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln J_{\phi^*} = -\bar{\Delta} \bar{F} \tag{E.32}$$

for the Jacobian $J_{\phi*}$ in the sector of antifields,

$$J_{\phi^*}(t) = \mathrm{sDet}[\Phi^*_i(t, \phi, \phi^*) \overleftarrow{\partial}^{*j}], \quad \ln J_{\phi^*}(t) = \mathrm{sTr} \ln[\Phi^*_i(t, \phi, \phi^*) \overleftarrow{\partial}^{*j}]. \tag{E.33}$$

It follows from (E.11), the initial data (E.5), (E.31), and (E.32) that

$$J_\phi = J_{\phi^*} = J^{1/2}, \tag{E.34}$$

and, finally, we have the factorization property,

$$J = J_\phi J_{\phi^*}. \tag{E.35}$$

It seems to be rather useful to mention here the main properties of the grand Jacobian J of anticanonical transformations, within the field–antifield formalism. Let $Z^A \to \bar{Z}^A$ be an anticanonical transformation with a fermion generator F.

Consider Eq. (E.11) as rewritten in the form

$$\frac{\mathrm{d}}{\mathrm{d}t} \ln J^{1/2} = -\bar{\Delta} \bar{F}, \tag{E.36}$$

where the $\bar{\Delta}$-operator is defined in (E.12). A formal solution to (E.36) has the form

$$\ln J^{1/2} = -[(\exp\{\mathrm{ad}(tF)\} - 1)/\mathrm{ad}(F)] \Delta F. \tag{E.37}$$

It follows immediately from (E.12) that

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} \bar{\Delta} &= \exp\{\mathrm{ad}(tF)\}[\mathrm{ad}(F), \Delta] \exp\{\mathrm{ad}(-tF)\} = \\ &= \mathrm{ad}(-\exp\{\mathrm{ad}(tF)\} \Delta F) = \frac{\mathrm{d}}{\mathrm{d}t} \mathrm{ad}(\ln J^{1/2}), \end{aligned} \tag{E.38}$$

which implies[6]

$$\bar{\Delta} = \Delta + \mathrm{ad}(\ln J^{1/2}). \tag{E.39}$$

That is just the transformation property of the Δ operator under anticanonical transformation. Further, it follows from (E.36) that

$$\bar{\Delta} \frac{\mathrm{d}}{\mathrm{d}t} (\ln J^{1/2}) = 0. \tag{E.40}$$

By substituting (E.39), we get

$$\frac{\mathrm{d}}{\mathrm{d}t} \left[\frac{1}{2} (\ln J^{1/2}, \ln J^{1/2}) + \Delta \ln J^{1/2} \right] = 0, \tag{E.41}$$

which implies

$$\Delta \exp\{\ln J^{1/2}\} = \Delta(J^{1/2}) = 0. \tag{E.42}$$

That is just the antisymplectic counterpart to the Hamiltonian Liouville theorem [13,18].

[5] The same result follows via t-differentiation of (E.21) and the use of the Lie equation (E.6).

[6] The same result follows from (E.12) and the use of the anticanonical invariance of E^{AB}.

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