THE EUROPEAN
PHYSICAL JOURNAL C

CrossMark

Letter

Comment on "Tetraquarks as diquark–antiquark bound systems"

M. R. Hadizadeh[a]

Department of Physics and Astronomy, Institute of Nuclear and Particle Physics, Ohio University, Athens, OH 45701, USA

Abstract We argue that the calculated masses of heavy tetraquarks obtained by solution of the spin-independent homogeneous Lippmann–Schwinger integral equation in a diquark–antiquark picture reported by Monemzadeh et al. (Phys Lett B 741:124, 2015), are incorrect. We have reexamined all of the published results and we believe that not only the reported tetraquark masses for states with zero total angular momentum are incorrect, but the reported masses for states with non-zero total angular momentum are quite misleading, because these states cannot be predicted by a spin-independent formalism.

In a recent letter by Monemzadeh et al. [1], the tetraquark bound state is studied as a two-body problem in a diquark–antiquark picture. The non-relativistic s-wave bound state of a diquark–antiquark system with reduced mass $\mu_{D\bar{D}}$ and the binding energy E_T, interacting with pair force $V(r)$ can be described in configuration space by the homogeneous Lippmann–Schwinger (LS) integral equation

$$\psi(r) = -\frac{2\,\mu_{D\bar{D}}}{4\pi} \int d^3\mathbf{r}' \, \frac{e^{-\kappa|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|} \, V(r') \, \psi(r'), \qquad (1)$$

where $\kappa = \sqrt{2\mu_{D\bar{D}}|E_T|}$. The corresponding equation in the published letter [1] (i.e. Eq. 2 and consequently Eqs. 4, 9 and 11) is missing a factor of $(2\pi)^{-1.5}$, which must be a typo, because our numerical analysis demonstrates that missing this factor leads to completely unreasonable tetraquark masses.[1]

In order to study tetraquark bound states, the spin-independent part of the diquark–antiquark potential given in Ref. [3] is used, with the strong coupling constant α_s of Ref. [4]

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{\ln\left(\frac{4\mu_{D\bar{D}}^2 + M_0^2}{\Lambda^2}\right)}, \qquad (2)$$

with $\beta_0 = 11 - \frac{2}{3}n_f$ (n_f is the number of flavor quarks), $M_0 = 2.24\sqrt{A} = 0.95\,\text{GeV}$ and $\Lambda = 0.413\,\text{GeV}$.

In Table 1 we have listed our numerical results for $1s$ heavy charm tetraquarks for states with total angular momentum equal to zero. As shown, our calculated masses are all larger than the predicted results of Ref. [3], which are obtained from a solution of the relativistic and spin-dependent LS integral equation in momentum space. Our results seem to be quite reasonable, because our numerical analysis indicates that the relativistic effects lead to a reduction in the calculated masses [5]. Clearly the reported results in Ref. [1] cannot be trusted, because, as shown in Table 1, not only the $J = 0$ states are not calculated correctly, they have reported the masses for non-zero J states, like $\frac{S\tilde{A}\pm A\tilde{S}}{\sqrt{2}}$ and $A\bar{A}$, which cannot be obtained in this formalism. It is a serious challenge and the authors should clarify how this spin-independent formalism can distinguish different spin states and consequently the solution of spin-independent LS integral equation can predict the masses of tetraquarks with non-zero total angular momentum.

Moreover, from numerical point of view the following issue should be considered. Since a regularized form of the diquark–antiquark potential is used to overcome the singularity of the confining potential at large distances, the calculated masses should be independent of the regularization cutoff r_c. In Ref. [1] the authors have chosen the non-zero root of the potential as a regularization cutoff and consequently the potential is fixed equal to zero for $r \geq r_c$. As we have shown in Table 2, clearly this cutoff is not high enough and

---

[a] e-mail: hadizadm@ohio.edu

[1] To verify the derivation of the LS integral equation (1) and test our numerical algorithm and code, we have solved it for the spin-independent Malfliet-Tjon Yukawa potential [2] and we have reproduced the same deuteron binding energy obtained from a solution of the LS equation in momentum space.

Table 1 Masses of charm diquark–antiquark for 1s state with total angular momentum $J = 0$ calculated from non-relativistic Lippmann–Schwinger equation. S and A denote the scalar and axial vector diquarks

Diquark content	Mass (GeV)		
	Present	Ref. [1]	Ref. [3]
	$cq\bar{c}\bar{q}$		
$S\bar{S}$	3.885	3.70314	3.812
$A\bar{A}$	4.013	3.83908	3.852
	$cs\bar{c}\bar{s}$		
$S\bar{S}$	4.117	4.05390	4.051
$A\bar{A}$	4.250	4.09962	4.110

Table 2 Masses of charm diquark–antiquark for 1s state with total angular momentum $J = 0$ as a function of regularization cutoff r_c. The bold numbers are calculated for values of r_c which are non-zero roots of the diquark–antiquark potential and numbers in parentheses are the corresponding masses from Ref. [1]

r_c (GeV^{-1})	Mass (GeV)	
	$S\bar{S}$	$A\bar{A}$
	$cq\bar{c}\bar{q}$	
2.831	3.858	**3.987** (3.83908)
2.844	**3.859** (3.70314)	3.987
2.85	3.859	3.988
2.9	3.862	3.991
3.0	3.867	3.995
3.5	3.879	4.007
4.0	3.883	4.011
5.0	3.885	4.013
6.0	3.885	4.013
	$cs\bar{c}\bar{s}$	
2.808	4.092	**4.226** (4.09962)
2.821	**4.093** (4.05390)	4.227
2.85	4.094	4.228
2.9	4.097	4.231
3.0	4.101	4.235
3.5	4.112	4.245
4.0	4.116	4.248
5.0	4.117	4.250
6.0	4.117	4.250

in order to achieve the cutoff-independent results, converging with four significant digits, one needs to choose a cutoff at least equal to 5 GeV^{-1}. Choosing a regularization cutoff equal to a non-zero root of the diquark–antiquark potential

leads to smaller tetraquark masses, which are shown with bold numbers in Table 2; even with the same regularization cutoff the reported masses in Ref. [1] are different from our results.

We should mention that in our calculations we have used 100 mesh points of Gauss–Legendre quadrature for integration over the angle between **r** and **r**′ with a linear mapping, whereas the integration over r' is done by a hyperbolic-linear mapping with the sub-intervals $[0, \frac{r_c}{2}] + [\frac{r_c}{2}, r_c] + [r_c, 20]$ GeV^{-1} using 150 mesh points to achieve the converged masses with four significant digits.

Although the conclusions based on the incorrect results are misleading, but we do not understand of their conclusion: "our results are in good agreement with the results derived from complicated relativistic methods and can be a good replacement for them". How can a non-relativistic spin-independent method be an alternative to a relativistic spin-dependent method, even when the results of both methods are in good agreement?

In conclusion, our calculations demonstrate that all of the results published in the recent letter by Monemzadeh et al. [1], for the masses of tetraquarks with total angular momentum equal to zero, are incorrect, and the letter has serious numerical problems. The correct results should be as given in Table 1. Moreover, beside these incorrect results, we have difficulty to understand how the authors have obtained the masses of tetraquarks for non-zero total angular momentum in a spin-independent formalism!

References

1. M. Monemzadeh, N. Tazimi, P. Sadeghi, Phys. Lett. B **741**, 124 (2015)
2. R.A. Malfliet, J.A. Tjon, Nucl. Phys. A **127**, 161 (1969)
3. D. Ebert, R.N. Faustov, V.O. Galkin, Phys. Lett. B **634**, 214 (2006)
4. D. Ebert, R. Faustov, V. Galkin, Phys. Lett. B **696**, 241 (2011)
5. M.R. Hadizadeh, A. Khaledi-Nasab (in preparation)